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INTERNATIONAL MONETARY FUND

Research Department

International Policy Coordination in a
World with Model Uncertainty

Prepared by Atish R. Ghosh* and Paul R. Masson

Authorized for Distribution by Michael P. Dooley

December 4, 1987

Abstract

This paper considers the design and desirability of rules for the international coordination of macroeconomic policies in a world characterized by model uncertainty. It is argued that the presence of model uncertainty, far from rendering cooperation unattractive, may actually provide a compelling incentive to coordinate policies, provided policymakers recognize that they cannot know the true model. In order to quantify the benefits from coordination, confidence regions are specified for the parameters in MINIMOD; it is assumed that policymakers in the United States and the rest of the world use these ranges as their subjective probability priors. It is shown that the expected value of gains from coordination of policies may be large, despite substantial uncertainty about the parameters.

JEL Classification Numbers: 0260, 4200

* Harvard University and Oxford University. This paper was written while Mr. Ghosh was working as a summer intern.

The authors are grateful to Guillermo Calvo, Max Corden, Bob Flood, and Swati Ghosh for helpful comments.

<u>Contents</u>	<u>Page</u>
I. Introduction	1
II. The Theory of Model Uncertainty and Policy Coordination	3
III. The Structure of the Econometric Model	5
IV. Optimal Strategies Under Model Uncertainty	11
V. Simulation Results	16
VI. Conclusion	22
Text Tables	
1. Parameter Values of U.S. Model for Different Simulations	8
2. Degree of Reduced-Form Model Uncertainty of Policy Instruments on Selected Variables	19
3. Gains from Coordination Under Model Uncertainty	20
Appendix	24
References	27

I. Introduction

There is now a substantial literature that establishes the conditions under which international coordination of economic policies can be expected to lead to better outcomes, and a few empirical studies that attempt to quantify those gains. 1/ The consensus of these empirical studies is that there exist clear gains to policy coordination, but that their magnitude is not enormous (Oudiz and Sachs [1984], McKibbin and Sachs [1987]). Most empirical studies to date have not, however, attempted to take into account the fact that the true model describing the economy is not known. 2/

Furthermore, policymakers have long doubted the value of international macroeconomic coordination in an environment characterized by model uncertainty. Typifying this view, Martin Feldstein, then Chairman of the Council of Economic Advisers, wrote:

"Economists armed with econometric models of the major countries of the world can, under certain circumstances identify co-ordinated policies that, quite apart from the balance of payments constraints, are better than outcomes of unco-ordinated choices. But in practice, the overwhelming uncertainty about the quantitative behaviour of individual economies and their interaction, the great difficulty of articulating policy rules in a changing environment all make any such international fine tuning unworkable."

The Economist, June 11, 1983

Consistent with this view, Frankel [1987] shows that if policymakers choose their policies on the basis of one set of empirical macromodels, each one having an equal chance of being correct, and policymakers simply ignore the reality of model uncertainty, then coordinated policies may be inferior to the uncoordinated, or Nash, equilibrium. 3/

There are at least two grounds for criticism of such an approach. First, it attributes irrationality to the policymakers, since they must certainly recognize the reality of model uncertainty. Second, although such an argument appears to be an indictment of international policy coordination, it is rather an argument against the pursuit of any activist policy--coordinated or uncoordinated--when its effects are not known. If, in contrast, policymakers in each country are assumed to set policies with due regard to the model uncertainty, the presence of model uncertainty cannot make coordination welfare deteriorating. On the contrary, model uncertainty may provide an additional incentive to coordinate macroeconomic policies. 4/ Policy need not be more activist in the coordinated outcome

1/ For surveys of the theoretical and empirical literature see Cooper [1985], Horne and Masson [1987], and Fischer [1987].

2/ Canzoneri and Minford [1986] consider uncertainty about other countries' actions, but do not treat model uncertainty.

3/ See also Frankel and Rockett [1986], Holtham and Hughes Hallett [1987].

4/ See Ghosh [1986a, 1986b], Ghosh [1987], Ghosh and Ghosh [1987].

than the Nash, and the relative activism of the two policies will be shown below to be important in determining the effect of uncertainty on the expected gains from coordination.

In this paper, the focus is on the case in which governments are assumed to take explicit account of any model uncertainty when forming optimal policies. A theoretical section shows that whether model uncertainty increases or decreases the attractiveness of coordination depends upon the source of the uncertainty in the reduced form of the model. Uncertainty about the effects of the foreign instruments on the home country (i.e., the transmission effects) invariably raises the gains from coordination relative to the Nash equilibrium. In contrast, uncertainty regarding the effects of the country's own instruments upon itself has ambiguous implications for the benefits from coordination.

The effects of model uncertainty on the ex ante expected gains from policy coordination are then assessed in a two region econometric model of the world economy. Uncertainty is introduced at the level of the structural model rather than the reduced form. In particular, uncertainty concerning parameters such as the interest elasticity of money demand, price elasticities in import and export equations, the slope of the Phillips Curve, the degree of substitution between government and private consumption, and the strength of accelerator effects on investment is considered. This approach allows tracing the effect of uncertainty about econometric estimates of key structural parameters on the reduced-form multipliers, and ultimately on the welfare implications of coordinated versus uncoordinated macroeconomic policies.

Assessment of gains to coordination involves a comparison of dynamic programming solutions that maximize expected utility (either independently or jointly) under uncertainty about the effects of the instruments. The welfare criterion used is the expected (ex ante) value of an objective function where the expectation is calculated over the prior probability weights assigned to each model. The analysis, therefore, may be viewed as an extension of the seminal work of Brainard [1967] to a two-player, strategic setting. It should also be stressed that since we focus upon the expected gains from coordination, we do not have to specify, arbitrarily, the "true" model of the world economy.

The plan of the paper is as follows. Section II reviews some of the theoretical arguments about the effects of model uncertainty on the gains from policy coordination. Section III describes the econometric model, MINIMOD, used for the simulation work and specifies confidence regions for the key structural parameters. Section IV explains the procedure used to calculate the gains from coordination under model uncertainty. Section V presents estimates of those gains, while Section VI offers some brief concluding remarks.

II. The Theory of Model Uncertainty and Policy Coordination

To begin our discussion, it is useful to consider the effects of model uncertainty on the gains from policy coordination in the simplest possible theoretical structure, and one which has been widely used in the policy coordination literature; 1/ the discussion here follows Ghosh and Ghosh [1987]. Let policymakers have the quadratic objective function defined over the price level:

$$V = - \max (1/2)\{p^2\}$$

subject to the linear reduced-form equation:

$$p = \theta_1 M + \theta_2 M^* + \epsilon \quad \epsilon > 0$$

where p is the price level, ϵ is an inflation shock, M is the home country's monetary policy, M^* is the foreign country's monetary policy. It is assumed that $\theta_1 > 0$, $\theta_1 + \theta_2 > 0$. The coefficient θ_1 represents the effects of the home country's monetary policy on its own price level, while θ_2 gives the effects of the foreign instrument on the home country's price level; these coefficients are termed domestic and transmission multipliers respectively. A similar objective function and model applies to the symmetric foreign country:

$$p^* = \theta_1 M^* + \theta_2 M + \epsilon \quad \epsilon > 0$$

Note that θ_1 and θ_2 again represent domestic and transmission multipliers respectively. Under a cooperative regime a single planner maximizes a weighted average of the two countries' welfare functions:

$$W = \omega V + (1-\omega)V^*$$

In a world with symmetric countries it is natural to choose the relative welfare weight of $\omega = 0.5$ so that any gains from coordination are shared equally. In the absence of coordination, each country maximizes its respective objective function, V or V^* , choosing its policies under the assumption that the other country's policy settings are given. Since countries are mirror images, we may concentrate on the home country with the understanding that the effects on the foreign country are identical.

In the absence of model uncertainty, the unique symmetric Nash equilibrium is characterized by the following policy settings:

$$M = M^* = -\epsilon/(\theta_1 + \theta_2)$$

Similarly, under cooperation:

$$M = M^* = -\epsilon/(\theta_1 + \theta_2)$$

The Nash and cooperative equilibria coincide so that there are no gains from coordination in this case. Here each country has one instrument and

1/ See Oudiz and Sachs [1984], Cooper [1985].

one target so that the two countries can achieve their optimum optimum without coordination (see Oudiz and Sachs [1984]).

Now suppose that the multipliers θ_1 and θ_2 are uncertain, with means μ_{θ_1} , μ_{θ_2} and variances $\sigma_{\theta_1}^2$, $\sigma_{\theta_2}^2$ respectively. For expositional purposes it is convenient to separate the effects of uncertainty about domestic multipliers from the effects of uncertainty about transmission multipliers. 1/ The objective function is now:

$$V = - \max (1/2) E (p^2)$$

where the expectation is taken over the uncertain parameters. The symmetric Nash strategies are now given by:

$$M = M^* = -\epsilon / (\mu_{\theta_1} + \mu_{\theta_2} + \sigma_{\theta_1}^2 / \mu_{\theta_1}) \quad (2.1)$$

while the cooperative equilibrium is given by:

$$M = M^* = -\epsilon / (\mu_{\theta_1} + \mu_{\theta_2} + (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) / (\mu_{\theta_1} + \mu_{\theta_2})) \quad (2.2)$$

therefore, unless

$$\mu_{\theta_1} \sigma_{\theta_2}^2 = \mu_{\theta_2} \sigma_{\theta_1}^2 \quad (2.3)$$

the cooperative and Nash equilibria differ and there would be gains from coordination. 2/ When there is no uncertainty, i.e. $\sigma_{\theta_1} = \sigma_{\theta_2} = 0$, then

(2.3) necessarily holds, but in general this will not be the case.

The reason that model uncertainty introduces gains from coordination has been discussed in some detail by Ghosh and Ghosh [1987]. As Brainard [1967] shows, under instrument uncertainty, the "efficiency" of policy instruments decreases and the optimal program calls for the modified use of policy. The use of policy instruments brings the mean values of the target variables closer to their bliss points; however, it also increases their variances around their means. The planner therefore faces a mean-variance trade-off in the use of his instruments. 3/ In the Nash equilibrium each government treats the variance from the foreign instrument as exogenous

1/ When parameter uncertainty is introduced at the level of the structural (as opposed to the reduced-form) model, however, the presence of domestic multiplier uncertainty will generally be associated with uncertainty about the transmission effects so that this separation is not possible.

2/ When θ_1 and θ_2 are correlated the expressions for the optimal strategies become more complicated, but the condition for efficiency of the Nash equilibrium remains the same. Note that the Nash strategy can be over or underinterventionist relative to the cooperative policy.

3/ Ghosh and Ghosh [1987] use a modified mean-variance portfolio analysis diagram to illustrate the inefficiency of the Nash strategy.

additive uncertainty, i.e., as independent of its own instrument. In equilibrium, however, the foreign instrument is a function of the home instrument (at the symmetric equilibrium $dM^*/dM = 1$) so that the Nash player incorrectly estimates the efficiency of his instrument and chooses an inappropriate degree of intervention.

Starting from an equilibrium in which there are no gains from coordination, therefore, the introduction of uncertainty about either the domestic or the transmission multipliers unambiguously raises the incentive to coordinate. More generally, the effects of introducing model uncertainty into a model in which there already exist gains from coordination depend upon whether there is uncertainty about the domestic or the transmission multipliers.

Consider a model in which each country has two targets, the price level and the level of output, but only one instrument, the money supply. It can be shown that increasing the degree of domestic multiplier uncertainty has ambiguous effects on the gains from policy coordination (Ghosh and Ghosh [1987]). For example, in the limiting case of an infinite increase in the home multiplier uncertainty, that is, where $\sigma_{\theta_1}^2 \rightarrow \infty$, the optimal policy, both under Nash and under cooperative behaviour, is to refrain from any intervention whatsoever:

$$M = M^* = 0$$

The Nash and cooperative strategies therefore converge at this point, eliminating all gains from coordination. For intermediate values the incentive to coordinate may be either enhanced or diminished when domestic multiplier uncertainty is increased. An increase in parameter uncertainty will lead to a less active use of the policy instrument; if the Nash equilibrium is too interventionist, then an increase in domestic multiplier uncertainty will reduce the use of policy and bring the Nash closer to the cooperative solution, thereby reducing gains from coordination. In contrast, the divergence between the Nash and cooperative cases is always an increasing function of the degree of transmission uncertainty and so, correspondingly, is the gain from coordination. The difference in impact of domestic and transmission multiplier uncertainty will figure prominently in the discussion of quantitative estimates of gains to coordination presented below.

III. The Structure of the Econometric Model

The model we have used to estimate the gains of policy coordination is a small, two-region macroeconomic model called MINIMOD (see Haas and Masson [1986]). This model contains blocks for the United States and an aggregate of the rest of the world countries, with about 30 equations per region. The regions are linked through uncovered interest parity and bilateral trade flows.

Within each economy, output is demand determined; the gap between potential output--based on a Cobb-Douglas production function--and actual

output affects the rate of change of prices, through an expectations-augmented Phillips curve. Domestic demand, or absorption, is divided into private consumption, which depends on disposable income, wealth and the real interest rate; investment, which is modeled as the lagged adjustment of capital to its optimal level, with the optimal level depending on GDP and the user cost of capital; and government spending, which is an exogenous policy instrument. A parameter determines the degree to which government spending and consumption are substitutes, and hence the extent that increased government spending directly crowds out private consumption.

The monetary policy variable in each region is taken to be the monetary base, and a multiplier relationship links it to the M1 stock, whose demand depends on the level of GDP, the GDP deflator, and the short-term rate of interest. Short-term and long-term rates of interest are linked through an arbitrage relationship that makes expected holding-period returns equal on the two instruments. Similarly, open interest parity enforces equality between expected returns on domestic and foreign short-term bonds, taking due account of expected changes in the exchange rate. The rate of change of goods prices, expected long-term bond rates, and expected exchange rates are all assumed consistent with the predictions of the model; that is, expectations are formed rationally.

Each region is assumed to produce a single composite good, which is an imperfect substitute for the other region's good. Each region's imports are assumed to depend on the relative price of the two goods--that is, the ratio of the GDP deflators, multiplied by the nominal exchange rate--and on the level of absorption. The current account balance is the sum of net exports, modelled as above, and net investment income, which is simply the product of the U.S. short-term interest rate and the level of net foreign claims. The model includes stock/flow interactions that permit achievement of a steady state equilibrium in the long run. Private sector net wealth is defined to be the sum of high-powered money, the stock of government debt--multiplied by a factor that measures the degree of Ricardian non-Equivalence--the stock of net foreign claims, and the physical capital stock.

The stability of the model requires that positive feedbacks of wealth on consumption be large, and also that government bonds not be allowed to grow without limit--i.e., the government's intertemporal budget constraint must be satisfied. The model therefore includes a simulation rule that modifies tax rates in order to stabilize government debt as a ratio to GNP, and a sufficiently high value of the marginal propensity to consume out of wealth (see Masson [1987] for details). These features are sufficient to ensure that net foreign liabilities settle down as a ratio to GNP in both economies, assuming that in addition the exogenous combined rate of technical progress and labor force growth is equal in the two regions.

Since MINIMOD is non-linear, it was first linearized around a point on a steady-state growth path. A linear model is necessary in order to make the policy optimisation tractable; the essential conclusions about the effects of model uncertainty on the gains from coordination are not,

however, specific to such a model. ^{1/} The linearized model exhibits saddlepoint stability, and has as many unstable eigenvalues as jumping variables--namely five, corresponding to the exchange rate, two long-term bond rates, and two inflation rates. For these nonpredetermined variables, instead of initial conditions, terminal conditions are imposed such that they do not grow without bound. As a result, a unique stable rational expectations path exists. The linearized version of the model is a function of the structural parameters of the original, non-linear, model, so we can study the effects of uncertainty concerning parameters that can be given an intuitive, economic interpretation.

An oft-stated argument for policy coordination is that, since under cooperation governments could choose the same policies they were pursuing at the Nash equilibrium, by revealed preference, coordination must be welfare improving. However, in MINIMOD, expectations are formed rationally and governments are assumed unable to pre-commit their actions so that private sector reaction functions faced by the cooperative planner will, in general, differ from that faced by the governments acting independently. As Rogoff [1985] demonstrates, the standard argument that coordination must be beneficial thus breaks down in the presence of a forward-looking private sector. In particular, monetary expansion may be more tempting when pursued jointly, since exchange rate depreciation would not result. Therefore, if governments are unable to pre-commit themselves not to pursue expansionary policies, the private sector will build in a (rational) forecast of looser monetary policy into their wage demands and the inflation-output trade-off will be less favorable in the coordinated regime than in the Nash. Whether this effect is sufficiently strong to render the gains from coordination negative depends upon the specific parameters of the model. In the simulations reported below, the gains from coordination in fact always turn out to be positive, so that the possibility raised by Rogoff does not apply to MINIMOD, at least for the policies and shocks being considered.

The key structural parameters which govern both the domestic effects of policy changes and their transmission to the foreign economy are listed in Table 1, together with their values in MINIMOD and notional ranges which are intended to capture 95 percent confidence regions. In fact, the parameters of MINIMOD were not in most cases directly estimated, but rather derived from other empirical work, primarily the Federal Reserve's Multicountry Model (MCM). Uncertainty in the parameters cannot therefore be directly associated with estimated standard errors. In any case, parameter uncertainty in empirical models is likely to be considerably greater than estimated standard errors, because specification searches have usually been used to achieve a good fit and to accord with priors concerning the size of coefficients. A better measure of model uncertainty is the range of estimates resulting from different estimation strategies and different maintained hypotheses. Parameter changes will also result from different sample periods, and if parameters do change over time then

^{1/} See Ghosh and Ghosh [1987] for a discussion of the effects of model uncertainty on the gains from coordination in a model with general functional forms.

Table 1. Parameter Values of U.S. Model for Different Simulations

Parameters Changed		High	Low	Base
1. None: Baseline				
2. Elasticity of money demand with respect to current and lagged interest rate	current	-0.4842	-0.1948	-0.3390
	lag 1	-0.1859	-0.0748	-0.1303
3. Degree of direct crowding out of private consumption by government expenditure		0.3	-0.3	0.0
4. Slope of the Phillips curve: effects of current and lagged capacity utilization rate on GNP deflator.	current	0.0405	0.0045	0.0225
	lag 1	0.0866	0.0096	0.0481
	lag 2	-0.0580	-0.0064	-0.0322
5. Elasticity of import demand with respect to activity		2.9273	1.6273	2.2773
6. Elasticity of import demand with respect to real exchange rate		-0.8226	-0.3226	-0.5726
7. Elasticity of export demand with respect to foreign activity		1.3148	0.2148	0.7648
8. Elasticity of export demand with respect to real exchange rate		1.3339	0.8739	1.1039
9. Effect of GDP on investment activity		0.0427	0.0134	0.0294

this variation, which may be important, also may not be correctly captured by estimated standard errors in a model with constant coefficients.

Another issue is whether parameters vary endogenously as a result of a change in the policy regime, a possibility raised by Lucas [1976]. This is obviously relevant here, as policy coordination would constitute a clear change in regime relative to the historical period. Parameter ranges should be wide enough to allow for this possibility.

The first key parameter that is considered in Table 1 is the interest elasticity of money demand in the United States. The demand for narrow money--the M1 aggregate--has been affected in recent years by two major structural changes: financial deregulation and technological innovations that have changed the costs of carrying out financial transactions. Simpson [1984, pp. 261-62], described some of these changes and concluded:

"...rapid financial change continues to affect the behavior of M1 and thus the setting of M1 growth objectives. Considerable uncertainty surrounds the contours of the relationship that now exists among M1, income, interest rates, and other economic developments and that will exist once the transition phase has drawn to a close...The preceding discussion strongly suggests that the interest elasticity of M1 demand will be much lower once the deregulation process has ended and that the demand to hold M1 balances will continue to be 'noisy,' especially over short periods..."

The equation favored by Simpson includes an own-rate of interest (in particular, the rate on NOW accounts, which are included in M1) as well as the market rate. He predicts that the elasticity with respect to the overall level of the interest rates will fall over time--relative to the "standard" elasticity of 0.10--as the proportion of M1 that is subject to an unregulated own-rate grows. He cites in opposition, however, a model developed at the Federal Reserve Bank of San Francisco, that explains strong M1 growth in the 1982-83 period not by deregulation but rather by a high interest elasticity--equal to 0.15--that is asserted not to have changed in recent years. Thus we quantify uncertainty concerning money demand elasticities by postulating a range of 0.05 to 0.15 for the interest elasticity: this range includes both a substantial decline from the standard elasticity and the opposing view of the San Francisco Fed. This range, plus or minus half the point estimate, is applied to the MINIMOD coefficient. It should be noted that since Simpson's article, the Fed has in fact abandoned M1 targets, citing instability in the money demand, income and interest rate relationship.

Another parameter that is of direct relevance to the effects of policy is the degree to which government spending is a direct substitute for private consumption expenditure. If government programs can just as easily be provided directly by the private sector then there need be no real effects from increases in government spending (providing also that the financing of that spending has no real effects, that is, Ricardian equivalence holds). In contrast, if government spending is of quite a

different nature, then it should not affect the utility of private consumption. There is a parameter in MINIMOD that measures this degree of substitutability; it is provisionally set to zero so that there is no direct offset through private consumption of government spending.

Empirical estimates of the degree of offset are quite mixed. It should be emphasized that government and private spending can be either substitutes or complements. If complements, then increases in government spending tend to increase private consumption; for instance, improved roads or parks may stimulate expenditures on cars and induce people to travel. Several authors have discovered complementarity (Buiter and Tobin [1979], Feldstein [1982] and Kormendi [1983]); in contrast, Hernandez-Cata [1982] and Ahmed [1986] find substitutability, i.e., a direct offset on private spending. While the data with which the models were estimated and also the precise specification differ considerably, the studies cited above suggest a range of -0.3 to 0.3, with a midpoint at the MINIMOD estimate of zero.

Clearly a crucial feature of any model is the degree of wage/price stickiness, or alternatively the slope of the Phillips curve. In the limiting case of perfect flexibility, monetary policy has no real effects. In MINIMOD, the degree of flexibility is captured by coefficients on the rate of capacity utilization in the equation for the rate of change of the GDP deflator. Uncertainty about these parameters is directly related to uncertainty concerning the "sacrifice ratio" between the cumulative output loss relative to potential needed to achieve a given reduction in the rate of inflation. The higher the coefficients on capacity utilization, the lower the sacrifice ratio. There is a considerable range of estimates of the "sacrifice ratio"; according to Sachs [1985], traditional estimates ranged from 6 to 18, with Arthur Okun's best guess being 10. Sachs himself calculates the sacrifice ratio to be 3 in the 1982-84 disinflationary period. A reasonable range therefore might be 2 to 18, or plus or minus 0.8 times the point estimate. Such a range is applied to the MINIMOD coefficients in Table 1.

Trade equations have been the object of numerous empirical studies, and most have used the standard specification, or a variant of it, embodied in MINIMOD--imports by each region in volume terms depends on real domestic expenditure and on relative prices, that is, on the real exchange rate. Despite the consensus concerning this specification, there is still controversy about the relevant elasticities, and considerable debate about their implications for future U.S. current account deficits. ^{1/} A comparison of 7 detailed models of the U.S. current account revealed that for both imports and exports, activity and price elasticities varied widely. For non-oil imports, income elasticities of demand ranged from 1.2 to 2.5, while price elasticities ranged from 0.7 to 1.2 (see Brookings

^{1/} A recent conference at the Brookings Institution compared the recent tracking and future projections of several models of the U.S. current account; it is summarized in Bryant and Holtham [1987]. See also Krugman and Baldwin [1987]. For a general survey of empirical work on trade equations, see Goldstein and Khan [1985].

[1987], Table V-1). Similarly, for non-agricultural goods exports, foreign income elasticities range from 1.0 to 2.1, and relative price elasticities from 0.5 to 1.0. These elasticities apply to different data than those in MINIMOD: trade flows include only a subset of merchandise trade, while MINIMOD's trade flows include all merchandise trade as well as non-factor services. Nevertheless, they give a good measure of the uncertainty relating to the parameters in trade equations. On the basis of the above results, the range of absorption elasticities is taken to be MINIMOD's estimate plus or minus 65 percent of the mean for imports, and 55 percent for exports. Relative price elasticities are taken to include values 25 percent on either side of MINIMOD's coefficient for imports, and 23 percent for exports.

A final important linkage for both monetary and fiscal policies is the accelerator effect of income variations on investment. MINIMOD's equation for the change in the capital stock is based on a conventional lagged adjustment to the optimal level, the latter depending on GDP and the user cost of capital. The accelerator effect thus depends on the speed of adjustment to the optimal capital stock. Several alternative specifications give a similar positive effect of changes in GDP on investment. Clark [1979] estimates equations with different theoretical underpinnings; his estimate for the contemporaneous effects of output range from 0.03 to 0.08, when coefficients for producers' durable equipment and non-residential structures are weighted together using 1986 relative shares. Applying a similar range to the point estimate in MINIMOD gives the range presented in Table 1 for that parameter.

The ranges of parameter estimates are very large--larger in general than those implied by estimated standard errors from any single model. Though the ranges do not allow for a change in sign of the structural parameter (with the exception of the direct effect of government expenditure on consumption), reduced-form multipliers do in fact change signs. The range of resulting policy effects would in all likelihood include the various model results considered by Frankel [1987].

IV. Optimal Strategies Under Model Uncertainty

Calculation of the gains from policy coordination involves comparing the expected utility achievable under the dynamic time consistent Nash equilibrium (Basar and Olsder [1982]) with the expected utility enjoyed by each country when a single planner maximizes a weighted sum of the two countries' welfares. In the simulations reported below, equal weights were assigned to each country, $\frac{1}{2}$ and the gains from coordination were assessed by reference to this global welfare function.

In the baseline simulations--where there is no model uncertainty--we

$\frac{1}{2}$ For convenience, the rest of the world region in MINIMOD is referred to as a country.

use the standard dynamic programming solution for the optimal control of an economy with forward-looking variables (see Oudiz and Sachs [1985])). Once we introduce model uncertainty, however, the optimal control solution must be modified to take into account the risks associated with the use of policy. We therefore extend the algorithms of Chow [1975] and Kendrick [1981] to deal with forward looking-variables and strategic behavior.

The logic of the model is as follows. In period t , policy-makers observe the inherited state vector x_t which is predetermined; the timing convention of the linearized MINIMOD is such that all non-jumping endogenous variables are determined at the beginning of the period and are thus included in x_t . Policy-makers then set their instruments with a view to influencing x_{t+1} and any jumping variables in period t , e_t . At the time that policies must be decided, the true model describing the world economy is unknown and policymakers must use their prior beliefs over the possible models. In period $t+1$, x_{t+1} is observed so that the model which applied to period t may be inferred. However, since the realisation of the true model is assumed independent in each period, policymakers will use the same priors in the next period; updating of priors is not required.

The home government's welfare function is assumed given by:

$$V = \text{Max} \sum_{t=0}^{\infty} \beta^t E(y_t' \Omega y_t) \quad (4.1)$$

where y_t is a n vector of endogenous variables, measured as deviation from the bliss point; Ω is an $(n \times n)$ matrix of utility weights; and $E(\cdot)$ is the expectation operator. All of the model's endogenous variables are contained within y_t , and the non-zero terms of Ω assign weights to those variables targeted by the home planner. The foreign government maximizes a similar objective function.

The world economy's dynamics are given by:

$$\begin{bmatrix} x_{t+1} \\ e_{t+1} \end{bmatrix} = \begin{bmatrix} A^i & B^i \\ D^i & F^i \end{bmatrix} \begin{bmatrix} x_t \\ e_t \end{bmatrix} + \begin{bmatrix} C^i \\ G^i \end{bmatrix} U_t \quad i=1, \dots, k \quad (4.2)$$

where i indexes the k possible models describing the world economy; $(A^i, B^i, C^i, D^i, F^i, G^i)$ are constant coefficient matrices for model i 1/; x_t is a p vector of pre-determined variables, e_t is an m vector of forward-looking variables, e_{t+1} is the expectation as of period t of the

1/ These matrices have dimension $\{(pxp), (pxq), (pxr), (qxp), (qxq), (qxr)\}$ respectively.

jumping variables in period $t+1$, and U_t is a r vector of home and foreign instruments.

In each time period, there are four vectors of potentially different probability priors assigned to each of the k possible models. The home and foreign governments use the subjective priors Π_t and Π_t^* to derive their optimal policies. The private sector assigns the priors $\hat{\Pi}_t$ to the models. Finally, there is an objective probability, $\tilde{\Pi}_t$ that any given model is the correct description of the world economy. The priors are exogenously given and are not updated either through active or passive learning.

The specification of the objective probability vector, $\tilde{\Pi}_t$, depends upon the treatment of model uncertainty. If it is assumed that there is a unique correct model, say model j , that describes the world economy over the relevant horizon, but that policymakers have yet to discover it, then $\tilde{\Pi}_t$ is degenerate i.e., it is a vector of zeros except for the j^{th} element, which is unity. An alternative treatment of model uncertainty, and one which is implemented in what follows, assumes that the world economy is too complex to be captured by a single model. Rather, the correct model changes from period to period and may be viewed as a truly stochastic realization from the set of k possible models, where each outcome has the associated objective probability $\tilde{\pi}_t^i$.

The objective priors are used to calculate the expected welfare for each country:

$$V = \text{Max} \sum_{t=0}^{\infty} \beta^t E\{y_t' \Omega y_t\} = \text{Max} \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^k \tilde{\pi}_t^i \{y_t^i' \Omega y_t^i\} \quad (4.3)$$

where the notation y_t^i reads "the value of y_t implied by model i ", so that V represents the true mathematical expectation of the country's welfare. Rational expectations by the private and public sectors in each country obtains when their subjective priors coincide with the objective probabilities:

$$\hat{\pi}_t^i = \pi_t^i = \pi_t^{i*} = \tilde{\pi}_t^i$$

In the simulation analysis below, it is assumed that these conditions of rational expectations obtain. In contrast, Frankel [1987] explicitly assumes that policymakers do not exhibit such rational expectations.

Although we are interested in the infinite horizon game, it is convenient to start with a finite horizon problem and let the number of periods go to infinity 1/. Let the horizon be T periods, and consider the final period. It is assumed that by period T the world economy has attained a steady state and that all forward-looking variables have

1/ This algorithm is an extension to that used by Oudiz and Sachs [1985] in their deterministic model; wherever possible, a similar notation has been adopted. The appendix provides a fuller derivation.

stabilized. The world economy has inherited a state vector x_T and the objective function for the home country is:

$$v_T(x_T) = \text{Max} -(1/2) E\{y_T' \Omega y_T\} = \text{Max} -(1/2) \sum_{i=1}^k \pi_T^i (y_T^i' \Omega y_T^i) \quad (4.4)$$

A corresponding expression gives the foreign country's objective function, v_T^* . Since the forward-looking variables have stabilized by period T:

$$e_{T+1}^i = e_T^i \quad i=1, \dots, k \quad (4.5)$$

where again, e^i reads "the value of the forward-looking variables implied by model i." Substitution into the respective models yields:

$$e_T^i = J_T^i x_T + K_T^i U_T \quad i=1, \dots, k \quad (4.6)$$

where J_T^i and K_T^i are matrices specific to model i. In period T, x_T does not have an i superscript, since it represents the (observed) inherited state vector. The vector of endogenous variables, y_T^i is given by:

$$y_T^i = \begin{bmatrix} x_T \\ e_T^i \end{bmatrix} = M_T^i x_T + N_T^i U_T \quad i=1, \dots, k \quad (4.7)$$

where again, M_T^i and N_T^i are matrices specific to model i. Substituting (4.7) into (4.4) yields the objective functions.

When the countries are in a cooperative agreement, a single global planner chooses the entire vector of instruments, U, to maximize a weighted sum of the utility functions:

$$w_T(x_T) = \omega v_T(x_T) + (1-\omega) v_T^*(x_T)$$

It is noteworthy that the global planner uses the each country's probability priors to evaluate their respective expected utilities. If $\Pi \neq \Pi^*$ then at least one of the governments does not have rational expectations. However, as long as the global planner uses each country's own priors in evaluating v and v^* , disagreements over the true model do not preclude the possibility of coordination. Indeed, such disagreements over the prior vector can, by themselves, give rise to gains from coordination in models where there are no other conflicts between the countries. 1/ Given the linear-quadratic structure of the model, the solution to the planner's optimization problem may be written:

$$U_T^c = \Gamma_T^c x_T$$

where the c superscript denotes the cooperative solution.

When countries are acting non-cooperatively, each chooses the subset of U under its control - u and u^* - to maximize $v_T(\cdot)$ and $v_T^*(\cdot)$ respectively, taking the actions of the other government as given. The

1/ See Frankel [1987] and Ghosh [1987] for a discussion.

resulting equilibrium may be written:

$$U_T^N = \Gamma_T^N x_T$$

Hence, from (4.6):

$$e_T^i = (J_T^i + K_T^i \Gamma_T^i) x_T = \Theta_T^i x_T \quad (4.8)$$

where Γ denotes Γ^C or Γ^N as appropriate. Rational expectations of the forward-looking variables are:

$$\tilde{E}_T\{e_T\} = \sum_{i=1}^k \tilde{\pi}_T^i \Theta_T^i x_T = \tilde{\Theta}_T x_T \quad (4.9)$$

Finally, the period T value functions for the home country is:

$$v_T^C(x_T) = -(1/2) \sum_{i=1}^k \tilde{\pi}_T^i x_T' (M_T^i + N_T^i \Gamma^C)' \Omega (M_T^i + N_T^i \Gamma^C) x_T = x_T' S_T^C x_T \quad (4.10)$$

Similarly, substituting for Γ^N yields the value under non-cooperative behavior:

$$v_T^N(x_T) = x_T' S_T^N x_T$$

Correspondingly, for the foreign country:

$$v_T^{C*}(x_T) = x_T' S_T^{C*} x_T \quad \text{and} \quad v_T^{N*}(x_T) = x_T' S_T^{N*} x_T$$

The functions Γ_T^C , Γ_T^N , $\tilde{\Theta}_T$, S_T^C , S_T^{C*} , S_T^N , S_T^{N*} give the optimal policies, the evolution of the forward-looking variables, and the value functions, given that the state vector is x_T .

Now consider period T-1, to which the state vector x_{T-1} has been bequeathed. At T-1 the home government faces the optimization problem:

$$v_{T-1}(x_{T-1}) = \text{Max} \quad -(1/2) \sum_{i=1}^k \pi_{T-1}^i (y_{T-1}^i' \Omega y_{T-1}^i) + \beta \sum_{i=1}^k \pi_{T-1}^i v_T(x_T^i) \quad (4.11)$$

where $v(\cdot)$ represents the value function under cooperative or non-cooperative behavior, as appropriate. Note that x_T^i now has an i superscript: in period T-1 it is not known which x_T will be bequeathed since the model pertinent to period T-1 is not known. Hence, in period T-1, $E_{T-1}\{v_T(\cdot)\}$ is found by taking the expectation over all possible x_T^i . Expanding (4.11) makes this clearer:

$$v_{T-1}(x_{T-1}) = \text{Max} \quad -(1/2) \sum_{i=1}^k \pi_{T-1}^i (y_{T-1}^i' \Omega y_{T-1}^i)$$

$$+ \beta \sum_{i=1}^k \pi_{T-1}^i \sum_{i=1}^k \pi_T^i x_T' (M_T^i + N_T^i \Gamma^C)' \Omega (M_T^i + N_T^i \Gamma^C) x_T \quad (4.12)$$

The inner summation--over π_T --yields the expected utility in period T given that the inherited state vector is x_T . The outer summation--over π_{T-1} --yields the expectation of $v_T(x_T^i)$ over the possible x_T^i bequeathed from period T-1.

The optimization now yields $S_{T-1}(\cdot)$ and so forth. As described in the appendix, this procedure is repeated until stationary functions $\{\theta(x), S(x), S^*(x)\}$ are obtained. When these functions become stationary the truncation error from solving a (large) but finite period problem rather than the true infinite horizon becomes negligible. Starting from an initial state vector x_0 , the present value of the gains from coordination accruing to the world are then given by:

$$\begin{aligned} \Psi(x_0) &= \omega[v^C(x_0) - v^N(x_0)] + (1-\omega)[v^{C*}(x_0) - v^{N*}(x_0)] \\ &= \omega[x_0'(S^C - S^N)x_0] + (1-\omega)[x_0'(S^{C*} - S^{N*})x_0] \end{aligned}$$

V. Simulation Results

Model uncertainty about the structural parameters for the U.S. block in MINIMOD was introduced to examine its effects on the gains from coordination. Table 1 indicates which parameters were assumed uncertain in each simulation. The simulations also require specification of the utility weights and probability priors. The choice of the utility function parameters is likely not to be critical here because the primary focus is on the effects of model uncertainty relative to the baseline (no model uncertainty) results. It is assumed that policymakers in each country target GDP, the rate of change of prices, and the current account balance; The square of each of these variables, expressed as a percentage of its value at the point of linearization, enters the objective function; there are no cross-product terms. 1/ The relative weights on the targets are taken to be 1, 10, 1.5, respectively. In addition, we include penalties for the use of monetary and fiscal policy, measured as a percentage deviation from the levels of the policy variables in the baseline, and assign a relative welfare weight of 0.5 to each quadratic term. The discount factor, β , for each government was chosen to be $\beta = 0.95$ and the weight assigned each country in the global planner's objective function was $\omega = 0.5$. Throughout the simulation exercises we assume that all policymakers, together with the private sector, exhibit rational expectations, and that the priors are not updated:

1/ See Oudiz and Sachs [1984] for a similar objective function.

$$\pi_t^i = \pi_t^{*i} = \hat{\pi}_t^i = \tilde{\pi}_t^i = \bar{\pi}^i \quad \forall i, t$$

In each of the simulations there are two competing models with probability weights $\bar{\pi}^i = 0.5$, $i=1,2$.

As discussed in Section II, the implications of model uncertainty for the gains from coordination depend, inter alia, upon whether it is the domestic multipliers which are uncertain or the transmission effects from abroad which are unknown. It is useful, therefore, to gain some understanding of how the structural parameter uncertainty translates into multiplier uncertainty in the reduced form of the model. Since the forward-looking variables are functions of the entire future sequence of policy settings, however, it is not possible--in a rational expectations setting--to define a measure of multiplier uncertainty which is independent of whether governments are acting cooperatively or non-cooperatively. As a partial solution to this problem, we solve for a rational expectations path along which future monetary and fiscal policies are at their baseline values, and concentrate on uncertainty concerning the impact effects of policy. Using a recursive form of the Blanchard-Kahn [1980] solution method we obtain the dynamic systems:

$$z_t = \begin{bmatrix} x_{t+1} \\ e_t \end{bmatrix} = \begin{bmatrix} \Lambda_x^i \\ \Lambda_e^i \end{bmatrix} x_t + \begin{bmatrix} \Phi_x^i \\ \Phi_e^i \end{bmatrix} U_t \quad i=1, \dots, k \quad (5.1)$$

where Λ^i and Φ^i are semi-reduced form coefficients, derived under the assumption that $U_{t+j} = 0$, $\forall j > 0$. The mean value of the effect of an instrument u_t^M on a target variable z_{t+1}^j is:

$$\mu_M^j = \sum_{i=1}^k \pi^i \Phi_{jM}^i$$

where Φ_{jM}^i is the (j,M) element of Φ^i and the average has been taken over the k possible models (here k equals 2). Similarly, the variance of each model's multipliers is given by:

$$(\sigma_M^j)^2 = \sum_{i=1}^k \pi^i (\Phi_{jM}^i - \mu_M^j)^2$$

For each target z_t^j , we then define a measure of the parameter uncertainty 1/ regarding the use of instrument u^M as:

$$\zeta_M^j \equiv \mu_M^2 / (\mu_M^2 + \sigma_M^2)$$

so that $\zeta_M^j \in (0,1)$ is a decreasing function of the severity of model

1/ This measure is a simple transformation of the coefficient of variation, which Brainard [1967] uses as a measure of policy effectiveness.

uncertainty. Note that ζ is a dimensionless statistic, satisfying $\zeta = 1$ when there is no uncertainty and $\zeta \rightarrow 0$ as the quality of the instrument worsens.

Table 2 gives the ζ values for each combination of the instruments and targets in each set of simulations. In the baseline simulation, of course, ζ is always unity since, by construction, there is no model uncertainty. More generally, the prevalence of model uncertainty implies that ζ be less than unity, at least for the effects of some of the instruments on some of the targets. A number of points emerge from this Table. First, there is usually greatest disagreement among the models concerning the effects of policies--domestic or foreign--on the current account. The greater the welfare weight placed on the current account, the more important will be the implications of this form of model uncertainty for the gains from coordination. Second, any uncertainty about parameters of the home country, such as the degree of fiscal crowding-out, or the slope of the Phillips curve, is often magnified in the transmission process. For example, in simulation 3, where the degree of fiscal crowding out in the U.S. is the only uncertain parameter, uncertainty about the effects of U.S. fiscal policy on ROW GDP and on ROW inflation is greater than uncertainty surrounding the effects on U.S. GDP or inflation.

In the symmetrical two-country model without expectations that was discussed in Section II, uncertainty about transmission effects--the effect of home policy on the foreign country and vice versa--always increased the gains from coordination, while uncertainty about the domestic effects had ambiguous implications for the incentive to coordinate. (Though in the limiting cases where $\zeta = 0$ for domestic instruments the gains from coordination are necessarily eliminated.) In Table 2, uncertainty about the transmission multipliers has been indicated by an asterisk. In simulations 3 and 4 the degree of transmission uncertainty is almost always greater than the corresponding uncertainty on the domestic targets. For these simulations, therefore, the analysis of Section II suggests that the presence of model uncertainty should serve to raise the gains from coordination. For the remaining simulations, however, since domestic multiplier uncertainty is large, there is no clear presumption about the effects of model uncertainty.

The gains from coordination were calculated for a standard shock of a 1 percent decline in GDP, relative to baseline, together with a 1 percent rise in the consumer price level, again relative to baseline, to both the U.S. and ROW sectors.^{1/} The shock is purely transitory, occurring in the initial period, and returning to zero immediately. However, because of the dynamics of the model, effects on endogenous variables may persist for a

^{1/} In general, the joint distribution describing the error terms in the model's equations will be related to the distribution describing the parameters. With an estimated model, one could choose a drawing from these distributions that was consistent with the historical data. This was not done here, however.

Table 2. Degree of Reduced-Form Model Uncertainty of
Policy Instruments on Selected Variables

(Value of Unity Signifies no Model Uncertainty Policy Instrument)

Simulation	Endogenous Variables	UG	UM	RG	RM
1. Baseline	RGDP	1.000*	1.000*	1.000	1.000
	UGDP	1.000	1.000	1.000*	1.000*
	UCURRBAL	1.000	1.000	1.000	1.000
	RPI	1.000*	1.000*	1.000	1.000
	UPI	1.000	1.000	1.000*	1.000*
2. Interest Elasticity of U.S. money de- mand uncertain.	RGDP	0.999*	0.859*	0.999	0.999
	UGDP	0.999	0.855	0.997*	0.995*
	UCURRBAL	0.983	0.854	0.998	0.999
	RPI	0.975*	0.850*	0.986	0.999
	UPI	0.984	0.850	0.997*	0.999*
3. Direct crowding out effect of U.S. go- vernment spending uncertain	RGDP	0.913*	1.000*	1.000	1.000
	UGDP	0.920	1.000	1.000*	1.000*
	UCURRBAL	0.938	1.000	1.000	1.000
	RPI	0.869*	1.000*	1.000	1.000
	UPI	0.879	1.000	1.000*	1.000*
4. Slope of U.S. Philips curve uncertain	RGDP	0.179*	0.431*	0.998	0.162
	UGDP	0.966	0.898	0.912*	0.555*
	UCURRBAL	0.613	0.265	0.740	0.366
	RPI	0.471*	0.419*	0.467	0.414
	UPI	0.493	0.410	0.379*	0.412*
5. Activity elasticity of U.S. imports uncertain	RGDP	0.915*	0.999*	0.999	0.996
	UGDP	0.996	0.998	0.999*	0.999*
	UCURRBAL	0.964	0.968	0.994	0.978
	RPI	0.827*	0.949*	0.917	0.967
	UPI	0.730	0.967	0.983*	0.970*
6. Real exchange rate elasticity of U.S. imports uncertain	RGDP	0.997*	0.995*	0.999	0.981
	UGDP	0.999	0.999	0.999*	0.992*
	UCURRBAL	0.964	0.994	0.996	0.942
	RPI	0.940*	0.994*	0.773	0.929
	UPI	0.908	0.990	0.927*	0.939*
7. Activity elasticity of ROW imports uncertain	RGDP	0.999*	0.994*	0.999	0.999
	UGDP	0.999	0.999	0.682*	0.951*
	UCURRBAL	0.998	0.995	0.490	0.991
	RPI	0.995*	0.984*	0.417	0.994
	UPI	0.997	0.987	0.999*	0.995*
8. Real exchange rate elasticity of ROW imports uncertain	RGDP	0.998*	0.996*	0.999	0.996
	UGDP	0.999	0.999	0.999*	0.981*
	UCURRBAL	0.991	0.998	0.992	0.983
	RPI	0.958*	0.993*	0.880	0.971
	UPI	0.972	0.994	0.968*	0.973*
9. Effect of GDP on U.S. investment uncertain	RGDP	0.996*	0.999*	0.999	0.996
	UGDP	0.915	0.998	0.999*	0.999*
	UCURRBAL	0.964	0.968	0.994	0.978
	RPI	0.827*	0.967*	0.917	0.967
	UPI	0.730	0.949	0.983*	0.970*

UG - U.S. fiscal policy
UM - U.S. monetary policy
RG - ROW fiscal policy
RM - ROW monetary policy
UGDP - U.S. GDP
RGDP - ROW GDP
UCURRBAL - U.S. current account balance
UPI - U.S. consumer inflation
RPI - ROW consumer inflation
* denotes transmission effect

Table 3. Gains from Coordination Under Model Uncertainty

Simulation	Gain under Uncertainty	Gain under Certainty Equivalent	Ratio
	$\Psi(\pi^1, \pi^2)$	$\bar{\Psi}(\pi^1, \pi^2)$	$\Psi(\pi^1, \pi^2) / \bar{\Psi}(\pi^1, \pi^2)$
1. Baseline	19.5	19.5	1.00
2. Interest Elasticity of U.S. money de- mand uncertain	17.0	9.9	1.42
3. Direct crowding out effect of U.S. go- vernment spending uncertain	29.2	12.2	2.38
4. Slope of U.S. Philips curve uncertain	25.1	10.9	2.29
5. Activity elasticity of U.S. imports uncertain	23.9	19.2	1.24
6. Real exchange rate elasticity of U.S. imports uncertain	3.4	12.5	0.27
7. Activity elasticity of ROW imports uncertain	23.9	16.8	1.41
8. Real exchange rate elasticity of ROW imports uncertain	19.4	17.1	1.13
9. Effect of GDP on U.S. investment uncertain	9.6	17.8	0.53

considerable number of periods. To focus upon the effects of model uncertainty per se the gains from coordination under each set of parameter values must be normalised by the corresponding "certainty equivalent" model. Otherwise, the effects of the model uncertainty cannot be distinguished from the fact that different models, when known to be realized with unit probability, imply different gains from coordination. For example, let there be two possible models, which obtain with probability π^1 and π^2 respectively, and let the gains from coordination be denoted by $\Psi(\pi^1, \pi^2)$. The certainty equivalent gains from coordination, $\bar{\Psi}(\pi^1, \pi^2)$, are given by:

$$\bar{\Psi}(\pi^1, \pi^2) = \pi^1 \Psi(1, 0) + \pi^2 \Psi(0, 1)$$

The effects of model uncertainty are beneficial to the case for coordination if and only if:

$$\Psi(\pi^1, \pi^2) > [\pi^1 \Psi(1, 0) + \pi^2 \Psi(0, 1)]$$

Table 3 reports the gains from coordination, under model uncertainty, $\Psi(\pi^1, \pi^2)$, and for the certainty equivalent model $\bar{\Psi}(\pi^1, \pi^2)$, under each of the parameter values listed in Table 1. The gains reported are the total benefit accruing to the world economy over the infinite horizon and are expressed in terms of the one quarter U.S. GDP equivalent. Thus a gain of 19.5 is equivalent to the utility associated with a 19.5 percent increase in U.S. GDP sustained over one quarter. These are the estimated gains in the baseline simulation, with the original MINIMOD parameters and no model uncertainty. This estimate of the gains from coordination is roughly commensurate with previous empirical estimates (e.g., Oudiz and Sachs [1984] who found gains of about 1 percent, but sustained over three years), though of course they depend upon the specific utility function and shocks under consideration.

From Table 3 it is apparent that the presence of model uncertainty generally raises the gains from policy coordination. Only for simulations 6 and 9, in which there is uncertainty about the real exchange rate elasticity of import demand by the United States and about the effect of U.S. GDP on U.S. investment demand, respectively, does model uncertainty serve to reduce the gains from coordination. Inspection of Table 2 reveals that in these simulations domestic multiplier uncertainty generally exceeds the corresponding transmission uncertainty. Specifically, the effects of ROW government expenditure on ROW inflation exhibits considerable parameter uncertainty in simulation 6, while in simulation 9 all the transmission effects are less uncertain than are the corresponding domestic multipliers. In the other simulations, however, the effect of model uncertainty is to raise the gains from coordination. Indeed, in simulations 3 and 4, where there is generally much greater transmission uncertainty relative to domestic uncertainty, the gains from coordination are more than doubled; while for the other simulations the effects of model uncertainty are far from negligible.

VI. Conclusions

This paper has examined the desirability of international policy coordination in an environment characterized by model uncertainty. Rather than use the reduced forms of different models, we chose a single two region macro-econometric model, and introduced parameter uncertainty at the structural level of the model. The primary advantage of such an approach is that it allows tracing the effect of uncertainty about structural parameters on reduced-form multipliers and, in turn, on the gains from coordination.

The major finding in this paper has been that model uncertainty, far from precluding policy coordination, may in fact provide a strong incentive for countries to coordinate their macroeconomic policies. By contrast, in a series of articles, 1/ Frankel reaches exactly the opposite conclusion.

The essential difference between Frankel's approach and ours is that Frankel explicitly assumes that policymakers do not exhibit rational expectations. Policymakers are assumed to ignore the presence of model uncertainty in choosing their optimal plans even though they disagree on which is the correct model of the world economy. Within our framework, therefore, the probability priors used by his policymakers, Π and Π^* , do not coincide with the objective probabilities $\bar{\Pi}$. Since the expected value of each country's welfare is evaluated at the true probability weights $\bar{\Pi}$, policymakers are effectively maximizing the wrong objective function. If in the Nash equilibrium policymakers are undertaking incorrect policies, and if under cooperation they become more efficient at making mistakes, it is scarcely surprising that coordination may be welfare reducing.

However, the general argument that ignorance of the correct model is likely to negate gains from coordination is problematic. First it attributes an extraordinary degree of irrationality to policymakers to suggest that they simply ignore model uncertainty. Second, this argument has very little to do with policy coordination: it is simply an indictment of activist policy when the effects are unknown. If there is some doubt about the effects of policies, it is clearly desirable for the authorities to be cautious in setting those policies.

Although the gains from coordination in the baseline simulation without model uncertainty were rather modest, if policymakers explicitly recognize model uncertainty when framing their policies the gains from coordination relative to the Nash equilibrium increase considerably. Policy setting in the latter does not properly take into account the feedback effect on overall uncertainty that results from induced changes in foreign policies. In terms of GDP-utility equivalents, model uncertainty can increase the gains from coordination by as much as a factor of two. Furthermore, in line with theoretical arguments, the implications of

1/ Frankel [1987], Frankel and Rockett [1986].

uncertainty concerning MINIMOD parameters for the incentive to coordinate depend crucially on the source of that uncertainty. When transmission effects are unknown, the greater the uncertainty, the greater the gains from coordination. When the effects of policies on the domestic economy are uncertain, however, model uncertainty can either increase or decrease the gains from coordination, depending on a number of other factors.

This paper is partly methodological--exploring the design of coordination regimes in dynamic rational expectations models in the presence of model uncertainty--and there remain a number of possible extensions to our work. First, we intend to undertake more extensive simulation analyses to see how structural parameter uncertainty is embodied into the reduced-form multipliers of such models. This is of considerable interest since the disagreements between multi-country models may be reducible to econometric estimates of certain key structural parameters. Second, although we have employed a dynamic model, we have not included any learning mechanism, whether active or passive. It seems reasonable, however, that policymakers may start with some vector of priors which are slowly updated until they converge to the rational (objective) probabilities over the models. If policymakers learn about the domestic effects of their policies faster than they do about transmission effects, an increase in the gains from coordination may occur over time. Conversely, if policymakers attempt active learning--i.e. perturbing the economy in order to learn about it--then the Nash equilibrium may become even more inefficient since each government will introduce excessive noise into the world economy. 1/

Finally, it is worth exploring whether coordination decreases the overall level of uncertainty, by making clearer what the reactions of foreign governments are likely to be. In such a framework, the relevant uncoordinated equilibrium may not be the Nash, but rather one where there is the anticipation of some reaction by the foreign government, but of an unknown nature. 2/ In any case, this paper has demonstrated that the implications of model uncertainty for the incentive to coordinate must be explored more fully before policymakers dismiss the potential benefits of macroeconomic cooperation in an uncertain world.

1/ Ghosh [1986b] explores this possibility in a theoretical model.

2/ This possibility was suggested by Guillermo Calvo. See also Schwartz [1987].

This appendix derives the Nash and cooperative strategies under model uncertainty using a dynamic programming solution technique.

Assume, in period t , that:

$$v_{t+1}(x_{t+1}) = x'_{t+1} S_{t+1} x_{t+1} \quad (A1)$$

$$v^*_{t+1}(x_{t+1}) = x'_{t+1} S^*_{t+1} x_{t+1} \quad (A2)$$

$$\text{and } E(e_{t+1}) = \theta_{t+1} x_{t+1} \quad (A3)$$

where $v(\cdot)$ is the home country's value function, $v^*(\cdot)$ the foreign country's value function, x_t is the p dimensional state vector in period t , e_t is the q dimensional vector of jumping variables, S and S^* are matrices. The k models are given by:

$$x^i_{t+1} = A^i x_t + B^i e_t + C^i U_t \quad (A4)$$

$$e^i_{t+1} = D^i x_t + F^i e_t + G^i U_t \quad (A5)$$

where U_t is an r dimensional vector of controls. Substituting (A3) into (A4) and (A5):

$$\begin{aligned} e^i_t &= (\theta_{t+1} B^i - F^i)^{-1} \{ (D^i - \theta_{t+1} A^i) x_t + (G^i - \theta_{t+1} C^i) U_t \} \\ &= J^i_{t+1} x_t + K^i_{t+1} U_t \end{aligned}$$

and $x^i_{t+1} = (A^i + B^i J^i_{t+1}) x_t + (B^i K^i_{t+1} + C^i) U_t$

$$\begin{aligned} &= H^i_{t+1} x_t + L^i_{t+1} U_t \\ y^i_t &= \begin{bmatrix} x_t \\ e^i_t \end{bmatrix} = \begin{bmatrix} I_p \\ J^i_{t+1} \end{bmatrix} x_t + \begin{bmatrix} 0 \\ K^i_{t+1} \end{bmatrix} U_t \\ &= M^i_{t+1} x_t + N^i_{t+1} U_t \end{aligned}$$

The home country's value function is given by:

$$\begin{aligned} v_t(x_t) &= \sum_{i=1}^k \pi^i \{ (M^i_{t+1} x_t + N^i_{t+1} U_t)' \Omega (M^i_{t+1} x_t + N^i_{t+1} U_t) \\ &\quad + \beta (H^i_{t+1} x_t + L^i_{t+1} U_t)' S_{t+1} (H^i_{t+1} x_t + L^i_{t+1} U_t) \} \end{aligned}$$

Where Ω are the objective function weights and S_{t+1} summarizes the future discounted utility stream. The foreign country's expected value function is given by a similar expression. Under the Nash equilibrium, each country chooses a subset of U_t , u_t and u^*_t . Let lower case letters denote conformable partitions of M , N , H , and L . Solving for the Nash equilibrium yields:

$$U_t^N = - \left[\begin{array}{cc} \sum_{i=1}^k \pi^i (n_{t+1}^i, \Omega n_{t+1}^i + \beta l_{t+1}^i, S_{t+1} l_{t+1}^i) & \sum_{i=1}^k \pi^i (n_{t+1}^i, \Omega n_{t+1}^{i*} + \beta l_{t+1}^i, S_{t+1} l_{t+1}^{i*}) \\ \sum_{i=1}^k \pi^i (n_{t+1}^{i*}, \Omega^* n_{t+1}^{i*} + \beta l_{t+1}^{i*}, S_{t+1}^* l_{t+1}^{i*}) & \sum_{i=1}^k \pi^i (n_{t+1}^{i*}, \Omega^* n_{t+1}^i + \beta l_{t+1}^{i*}, S_{t+1}^* l_{t+1}^i) \end{array} \right]^{-1} \left[\begin{array}{c} \sum_{i=1}^k \pi^i (n_{t+1}^i, \Omega M_{t+1}^i + \beta l_{t+1}^i, S_{t+1} H_{t+1}^i) \\ \sum_{i=1}^k \pi^i (n_{t+1}^{i*}, \Omega^* M_{t+1}^i + \beta l_{t+1}^{i*}, S_{t+1}^* H_{t+1}^i) \end{array} \right] x_t$$

$$= \Gamma^N x_t \quad (A6)$$

Under coordination, the corresponding first order condition is:

$$U_t^C = [\sum_{i=1}^k \pi^i (N_{t+1}^i, \Omega^G N_{t+1}^i + \beta L_{t+1}^i, S_{t+1}^G L_{t+1}^i)]^{-1} [\sum_{i=1}^k \pi^i (N_{t+1}^i, \Omega^G M_{t+1}^i + \beta L_{t+1}^i, S_{t+1}^G H_{t+1}^i)] x_t$$

$$= \Gamma^C x_t \quad (A7)$$

where Ω^G and S^G are matrices of the global planner's value function:

$$\Omega^G = \omega \Omega + (1-\omega) \Omega^*$$

$$S^G = \omega S + (1-\omega) S^*$$

The home country's value function therefore becomes:

$$v_t(x_t) = \sum_{i=1}^k \pi^i \{ x_t' (M_{t+1}^i + N_{t+1}^i \Gamma)' \Omega (M_{t+1}^i + N_{t+1}^i \Gamma) x_t + \beta x_t' (H_{t+1}^i + L_{t+1}^i \Gamma)' S_{t+1} (H_{t+1}^i + L_{t+1}^i \Gamma) x_t \}$$

$$= x_t' S_t x_t \quad (A8)$$

where Γ denotes Γ^C or Γ^N as appropriate. Similarly, the foreign country's value function may be written:

$$v_t^*(x_t) = x_t' S_t^* x_t$$

The expected value of the forward-looking variables is:

$$E_{t-1}\{e_t\} = \sum_{i=1}^k \pi^i (J_{t+1}^i x_t + K_{t+1}^i \Gamma x_t) \equiv \theta_t x_t \quad (A9)$$

Replacing S_{t+1} by S_t , S_{t+1}^* by S_t^* , and θ_{t+1} by θ_t , we obtain recursive rules to compute the equilibria. The recursion is started by choosing $S_T = S_T^* = 0$ and by requiring that $e_{T+1} = e_T$.

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