IMF WORKING PAPER

WP/87/45

INTERNATIONAL MONETARY FUND

Research Department

Dual Exchange Rates in the Presence of Incomplete Market Separation: Long-Run Effectiveness and Implications for Monetary Policy

Prepared by Daniel Gros*

Authorized for Distribution by Donald J. Mathieson

July 2, 1987

Abstract

The existing literature on dual exchange rate regimes assumes that the separation between the two foreign exchange markets is perfect. In this paper, by contrast, a divergence between the two exchange rates induces a flow of arbitrage activity, the magnitude of which depends on both the costs of evading exchange controls and the size of the exchange rate differential. These arbitrage flows lead to a gradual convergence of the two exchange rates. In the long run, therefore, a dual exchange rate regime with a fixed commercial rate imposes the same constraints as a fixed unified exchange rate.

JEL Classification Number:
4314

*This paper was written while the author was a member of the Research Department. He is currently at the Universite Catholique de Louvain and the Centre for European Policy Studies in Belgium. He wishes to thank Charles Adams for helpful discussions and Peter Isard for extensive comments.

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Summary

Most of the literature on dual exchange rate regimes assumes that the separation between the two foreign exchange markets is perfect. By contrast, this paper assumes that a divergence between the two exchange rates induces a flow of arbitrage activity by agents who are able to evade controls by incurring costs. These agents buy foreign exchange at the lower exchange rate, usually the fixed commercial rate, and sell it at the higher rate, usually the freely floating financial rate. The magnitude of the arbitrage flows depends on the costs of evading controls and on the size of the exchange rate differential. The arbitrage flows cause the two exchange rates to converge in the long run unless the authorities are prepared on a continuous basis to offset the effects of the arbitrage. A dual exchange rate regime imposes, therefore, the same constraints on monetary policy in the long run as does a fixed unified exchange rate.

In the short run, however, monetary policy (or disturbances in the financial markets) can have strong effects on the exchange rate differential. An unanticipated increase in domestic credit of the central bank (or an unanticipated increase in international interest rates) would lead to a jump in the financial rate and thus the exchange rate differential. Through the effects of arbitrage, the exchange rate differential then diminishes gradually toward zero. The initial jump is an increasing function of the degree of separation of the two foreign exchange markets.

The costs sustained by private agents in their efforts to evade the controls are a loss from a social welfare point of view. It might, therefore, be argued that stricter controls would be beneficial, ceteris paribus, insofar as these would reduce the scope for evasion. For the case of quadratic arbitrage costs, however, the paper shows that stricter controls would instead lead to more waste and welfare losses through arbitrage because they would tend to slow down the reduction in the exchange rate differential, increasing the incentive for arbitrage activity.

I. Introduction

Dual exchange rate regimes have been discussed extensively in the economic literature, where it has been emphasized that the separation of the goods and financial markets that can be achieved by having different exchange rates for commercial and financial transactions may be useful in insulating the goods markets of the home economy from the effects of disturbances in the financial markets. 1/ For the most part, however, the existing models of the effects of dual exchange rates have not challenged the assumption that the authorities succeed in separating the two markets, or equivalently, that private arbitrage activity has no important consequences. 2/ In contrast, this paper develops a model of the scale of private arbitrage activity and focuses on the important consequences of such activity for the viability and effectiveness of dual exchange rate regimes.

Taking private arbitrage activity into account leads to the conclusion that dual exchange rates (as well as capital controls) could only succeed temporarily in dampening the effects (on the domestic goods market) of shocks to financial or other markets. To offset such effects permanently, the authorities would have to induce a steadily increasing differential between the two exchange rates. But this would also lead to a steadily increasing incentive for private operators to circumvent the regulations that separate the two markets by buying foreign exchange at the lower rate (usually the controlled or commercial rate) and selling foreign exchange at the higher rate (usually the free or financial rate). Such arbitrage activity would expand as the differential between the two exchange rates widened and would thus limit the size of the differential the authorities would be able to support.

By the same token, these considerations suggest that any differential between the two exchange rates would tend to disappear over time unless the authorities were prepared to offset continuously the effects of private arbitrage activity by adjusting one of the exchange rates or by using domestic credit policy. 3/ Data from Belgium, which has had a dual exchange rate since 1946, confirms that the differential between the commercial and financial exchange rates has usually been very small (Chart 1). However, substantial differentials have emerged during periods of tension in the financial markets (especially when a readjust-

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1/ See, for example, Dornbusch (1976) and (1985). The insulating effects of dual exchange rate regimes are also discussed in Flood and Marion (1982).

2/ The only exception seems to be Bhandari and Decaluwe (1984). Many other authors, however, do acknowledge that the difference between the two exchange rates should not become sufficiently large to create problems in enforcing the separation between the two markets.

3/ A series of shocks would tend to raise the differential each time a shock hit the system, but after each shock the differential should, on average, decline.

ment of the parities inside the EMS is expected). A similar pattern can be observed in the case of the Mexican two-tear exchange market (Chart 2). The discount of the peso on the free exchange market (relative to the controlled exchange rate) started at nearly 100 percent at the end of 1982 and then declined continuously until June 1985.

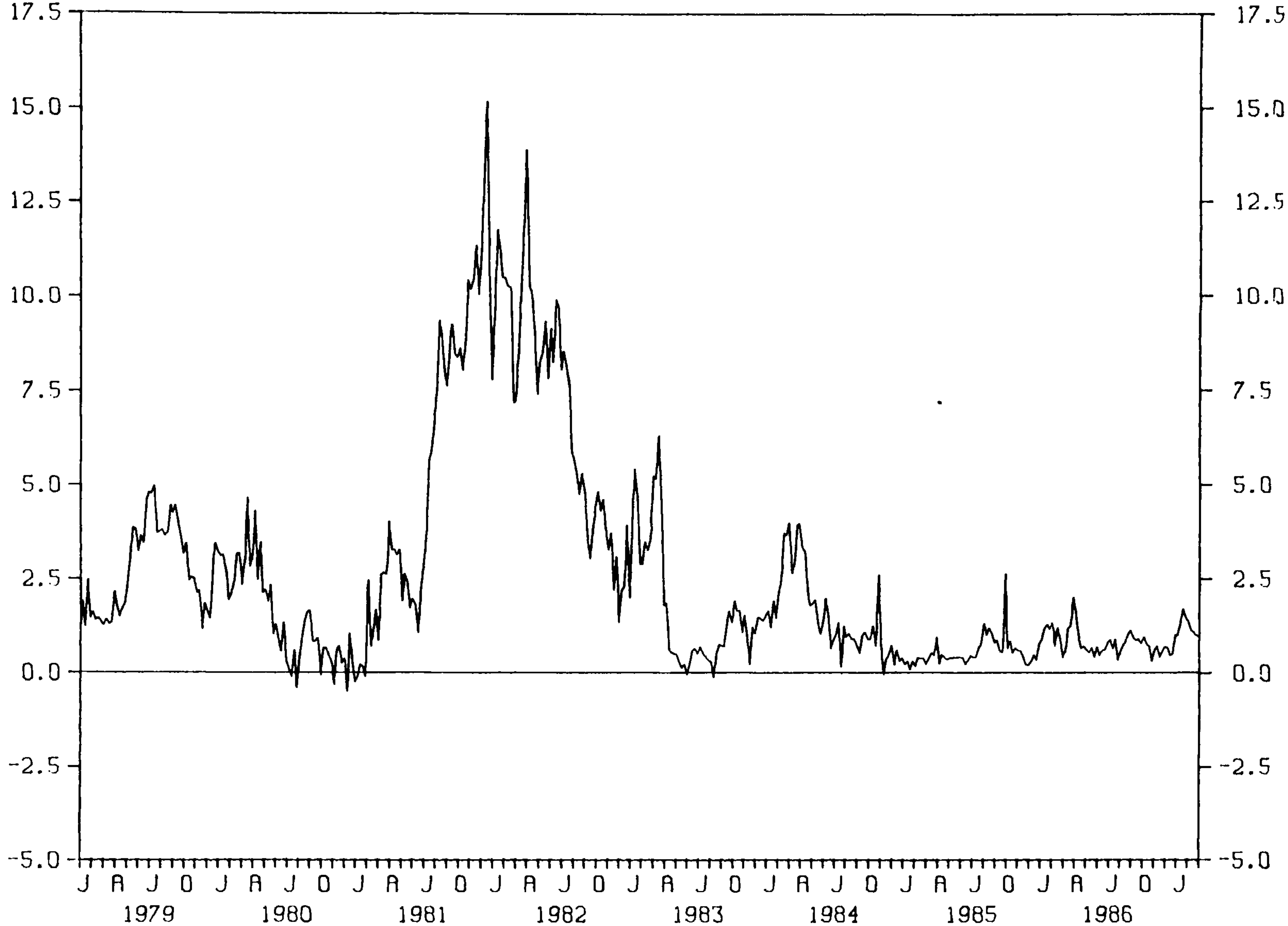
By combining the simple model of arbitrage activity with an equally simple macroeconomic model, the paper also analyses the consequences of various shocks such as changes in the international interest rate or devaluations of the commercial exchange rate. The analysis suggests that an unanticipated devaluation of the commercial rate would lead to a jump in the financial rate which could either over- or undershoot the amount of the devaluation of the commercial rate and might thus increase or decrease the differential. The analysis also suggests that the anticipation of a future devaluation of the commercial rate would lead to an immediate depreciation of the financial rate and would thus give rise to a differential between the two rates; this differential would be eliminated, or at least diminished, only when the devaluation of the commercial rate actually took place. This result seems intuitively plausible because the financial rate is a forward looking variable.

Another more surprising result is that an increase in the degree of separation of the two markets, as represented by the costs for potential arbitrageurs, would increase the effects of disturbances in the financial markets (e.g., changes in the international interest rate) on the financial exchange rate and thus, given the commercial exchange rate, on the exchange rate differential. If the aim of a dual exchange rate regime is to insulate the goods markets from disturbances in the financial markets, this result implies that the price for this insulation of the goods markets might be a higher degree of variability of some shadow prices in the financial markets and an increased dead-weight loss because of the increase in private arbitrage and evasion activity.

The long-run ineffectiveness of the controls that have to support the dual exchange rate regime is also illustrated by the discussion of the effects of monetary policy. It is shown that with a fixed commercial exchange rate no independent monetary policy is possible in the long run. Any increase in domestic credit would be offset in the long run by an equal loss of reserves. In terms of the so-called offset coefficient this implies that the impact offset coefficient is different from (minus) one, but the long run offset coefficient is always equal to one.

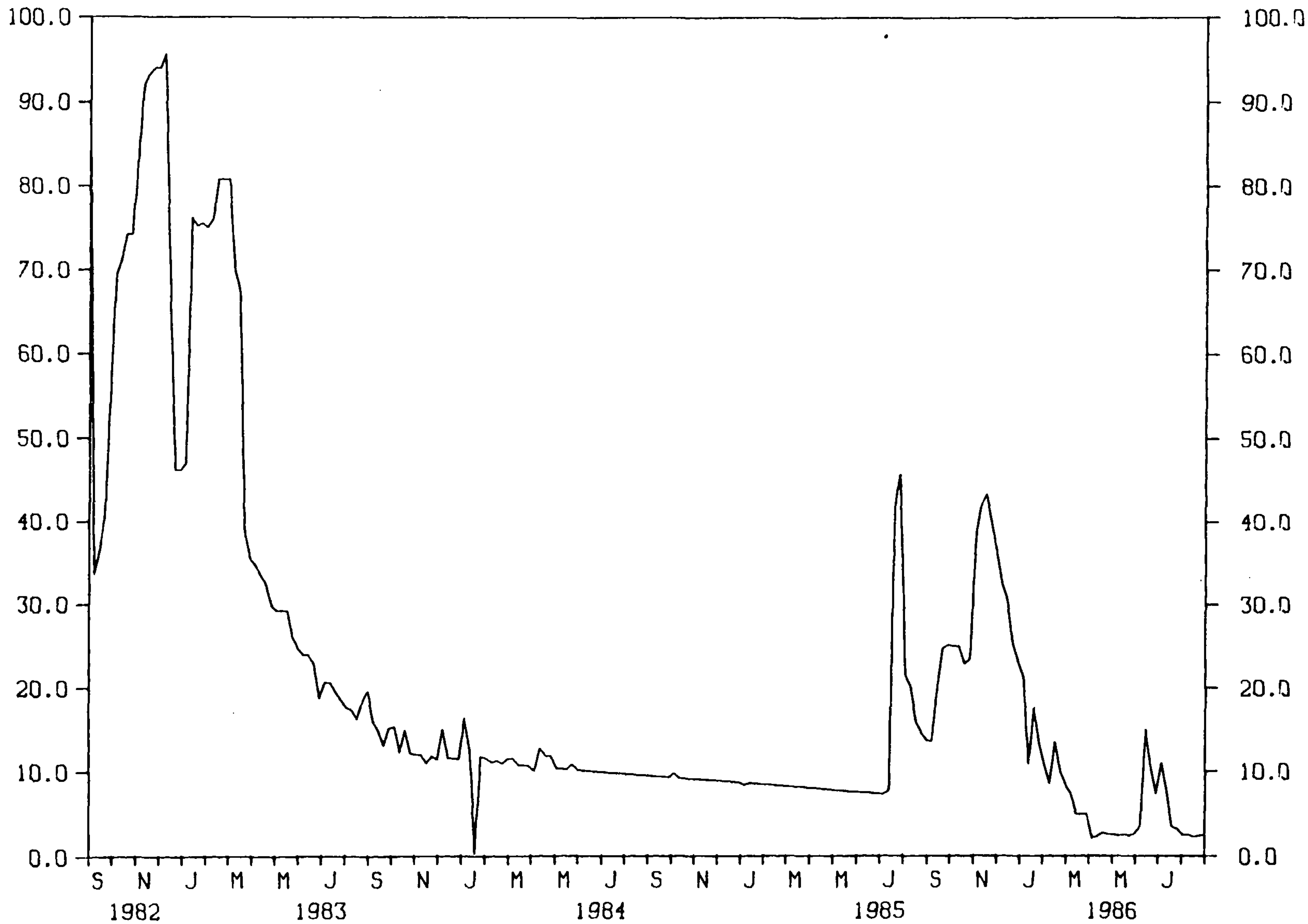
By considering the relevance of private arbitrage activity, the paper also highlights the importance of two points that are often overlooked in policy discussions about dual exchange rate regimes. The two points can be developed by first considering the proposition that in the absence of private arbitrage activity, a dual exchange rate regime is operative, in the sense that it induces a different capital and current account than a unified floating or fixed exchange rate, only if the expected rates of change of the two exchange rates differ. This propo-

Chart 1. The Belgian Two-Tier Exchange Rate Market:
The Percent Discount on the Financial Franc 1/



1/ The discount is computed as the natural logarithm of the ratio of the financial exchange rate to the commercial exchange rate.

Chart 2. The Mexican Two-Tier Exchange Market:
The Percent Discount on the Free Peso 1/



1/ The discount is computed as the natural logarithm of the ratio of the U.S. dollar price of one peso on the free exchange market to the U.S. dollar price of one peso on the controlled market.

sition can be shown to hold in a variety of models and has been emphasized recently by Adams and Greenwood (1985) and Frenkel and Razin (1986). The intuition behind the proposition is that the savings and investment decisions that determine the capital account depend on the expected intertemporal terms of trade, which in turn are a function of the rates of change of the two exchange rates rather than their levels. The proposition must be modified, however, in the presence of private arbitrage activity, as in the model developed in this paper; in that case a dual exchange rate regime can also affect the equilibrium current and capital accounts when the differential between the two rates is constant because of the arbitrage activity induced by the differential.

A first corollary of this modified proposition, given the tendency of the differential between the two exchange rates to disappear over time, is that a dual exchange rate regime cannot be effective in constraining the (cumulative) capital or current account in the long run. Any effect that is obtained when the differential between the financial and commercial rate is widening would be offset by an opposite effect when the differential is falling back to zero.

A second corollary of the modified proposition, contrary to the arguments of Adams and Greenwood (1985) and Frenkel and Razin (1986), is that in the presence of incomplete market separation, there is not an exact equivalence between a regime of capital controls that attempts to insulate domestic interest rates from international interest rates and a dual exchange rate regime. The reason is that private arbitrage activity reacts differently to the two regimes. Under capital controls, the incentive for private arbitrage activity exists only if the capital controls are effective in creating a differential between international and domestic interest rates. Under dual exchange rates, however, the incentive for private arbitrage activity exists even if the differential between the two exchange rates is constant and there is no differential between international and domestic interest rates.

A more technical result of the paper is that taking into account private arbitrage activity serves to determine the <u>level</u> of the financial exchange rate even in the context of models in which it would otherwise be indeterminate, such as the basic Dornbusch (1976) model. In most other models of dual exchange rates, the level of the financial exchange rate is determined via wealth effect type considerations. However, when private arbitrage activity is introduced as a function of the difference between the levels of the financial and commercial rates, the level of the financial exchange rate is determined even in the basic Dornbusch model.

The remainder of this paper is organized as follows. Section II sets up the model that links arbitrage flows to the exchange rate differential and Section III then incorporates this model of arbitrage flows into a simple general equilibrium model. Section IV describes the effects of anticipated (future) and unanticipated devaluations of the commercial rate. It shows that the differential between the financial and the commercial exchange rates could either increase or decrease in response to an unanticipated devaluation, but that an anticipated future devaluation would always have an unambiguous effect on the differential by leading to an immediate devaluation of the financial exchange rate. Section V describes the effects of anticipated and unanticipated changes in monetary policy. Given the fixed commercial exchange rate, monetary policy consists of changes in the domestic credit of the central bank; accordingly this section shows that an increase in domestic credit, whether anticipated or not, would lead to an immediate depreciation of the financial rate. The magnitude of this depreciation is an increasing function of the degree of separation of the two exchange markets. Section VI addresses the social welfare loss associated with activity to evade the controls associated with a dual exchange rate regime. Section VII contains some concluding remarks.

II. A Model of Arbitrage Activity in Dual Exchange Rate Regimes

Under a dual exchange rate regime, the exchange rate applicable to current account transactions (the commercial rate) may differ from the exchange rate applicable to capital account transactions (the financial rate). 1/ The commercial rate is usually fixed by the authorities, whereas the financial rate is sometimes set in a free market and sometimes set by the authorities. Since dual exchange rate regimes have mostly been used by countries trying to limit capital outflows, the financial rate, whether determined by a free market or by the authorities, has usually priced foreign exchange at a premium compared with the commercial rate.

Dual exchange rate regimes and capital controls can be regarded as analogous since under a dual exchange rate the authorities can either set (a path for) the financial rate in order to achieve a given capital account target or impose controls on capital movements and let the financial rate be determined in the market. Often the implicit target for the capital account (excluding any change in official reserves) is zero; in this case the authorities maintain the commercial rate at a certain level, but the financial rate is determined in a free market without government intervention in that market. This is the case in Belgium, where the authorities maintain the commercial franc within the limits imposed by the EMS, whereas the financial Belgian franc floats freely.

The existence of a dual exchange rate offers economic agents arbitrage opportunities in the sense that they would like to buy foreign

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1/ It is assumed here that interest payments are also converted at the financial exchange rate, thus the commercial exchange rate does not apply to all current account transactions.

exchange at the lower rate and sell foreign exchange at the higher rate. Since the financial rate usually prices foreign currency at a premium compared with the commercial rate, it is henceforth assumed that the arbitrage opportunity consists of buying foreign exchange at the commercial and selling it at the financial rate. Because of this arbitrage opportunity, all dual exchange rates have to be complemented by a set of rules that define current account (i.e., commercial) and capital account (i.e., financial) foreign exchange transactions.

One way to circumvent the rules designed to keep the two markets separate would be for an exporter to underinvoice its foreign clients and invest the unrecorded payments in foreign financial assets. The proceeds from the sale of these assets could then be repatriated at the financial rate. An importer would correspondingly let himself be over-invoiced and would use the foreign exchange, bought at the commercial rate, to acquire foreign assets, which could likewise be repatriated at a profit. It is assumed here that importers and exporters incur costs each time they over- or underinvoice because each transaction (export or import) is subject to the control of the enforcement agencies. 1/ (There might also be costs in repatriating the profits at the financial exchange rate.) These costs might take the form of penalties that are assessed by the enforcement agencies and of side payments that have to be made to foreign suppliers and clients in order to induce them to collaborate in the over- or underinvoicing. The potential arbitrageur maximizes the difference between the arbitrage profits and his costs, given by g(y);

$$(1)\quad \text{Max } y_t \left(\frac{E_t}{\bar{C}} - 1\right) - g(y_t)$$

where y_t represents the amount of over- or underinvoicing that the arbitrageur undertakes, measured as the amount of domestic currency that the arbitrageur effectively uses to buy foreign exchange at the fixed commercial rate, $\bar{C}$ in order to earn a profit by selling at the financial rate, E_t. (Both these exchange rates are in terms of domestic currency per unit of foreign currency.) It is apparent that (1) is not an intertemporal problem because the arbitrage opportunity, which arises if $(E_t/\bar{C})$ exceeds one, 2/ and the costs occur at the same time. The arbitrageur, therefore, maximizes profits when the original cost of

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1/ This is in contrast to the model of capital controls developed elsewhere. With capital controls, arbitrageurs incur cost only once, when they shift funds to the international market. Keeping the funds on the international market, where they earn higher interest rates, does not involve any additional costs. However, with a dual exchange rate regime, arbitrageurs have to make the entire round trip each time to earn the arbitrage profit.

2/ As mentioned above, only situations in which the financial rate is at a discount compared to the commercial rate are considered here.

increasing the amounts over- or underinvoiced is equal to the proportional discount of the financial exchange rate:

$$(2)\quad [\frac{E_t}{\bar{C}} - 1] = g'(y_t)$$

An interior equilibrium exists only if $g''(y_t) > 0$: that is, if $g(.)$ is convex or has increasing marginal costs. The assumption of increasing marginal costs can be defended in two ways. First, in practice dual exchange rate regimes have not collapsed immediately through large-scale evasion activity, contrary to what might be expected if evasion activity showed decreasing marginal costs. Second, it seems plausible that in reality importers and exporters would find it increasingly difficult to justify their official prices or terms of payment as they increased the scale of their over- or underinvoicing.

These considerations suggest that as long as the financial rate is at a discount, funds will flow to take advantage of the arbitrage opportunity, but the magnitude of the arbitrage flows will depend on the exact form of the "cheating" function $g(y_t)$. A small positive exchange rate differential might be compatible with no arbitrage flows if the marginal cost of even small amounts of arbitrage is positive; formally, the threshold at which arbitrage flows will stop is given by:

$$(3)\quad (\frac{E_t}{\bar{C}} - 1) = g'(0)$$

In the remainder of the paper, it is convenient to use a specific form of $g(.)$ that yields a linear relationship between the discount and the scale of arbitrage flows. Such a linear relationship results if the arbitrage cost function is quadratic, i.e., $g(y_t) = (\phi/2)y_t^2$, in this case the arbitrage flows are given by:

$$(4)\quad y_t = (\frac{E_t}{\bar{C}} - 1)/\phi$$

This specific functional form implies that the arbitrage flows stop only if the discount disappears. This seems to be an appropriate assumption for a country like Belgium which lies in the heart of Europe, is well integrated in international markets (exports are equal to 75 percent of GNP), and thus has a considerable scope for arbitrage flows through over- or underinvoicing. The assumption also seems consistent with the fact that in the case of Belgium, during periods of calm in the foreign exchange markets (presumably associated with little or no arbitrage flows), the discount of the financial rate has always been close to zero.

III. The General Equilibrium

This section discusses the general equilibrium consequences of the model of arbitrage activity developed here in the context of a streamlined macroeconomic model. The model is kept as simple as possible to illustrate the main channel through which the arbitrage flows affect conditions in the financial markets.

The model consists of three equations. The first equation describes the trade balance and thus the intervention activity of the government, which purchases any excess supply of foreign exchange resulting from commercial transactions, or provides for any excess demand, in order to keep the (nominal) commercial rate constant. The first important point about the model is that the arbitrage flows induced by the dual exchange rate regime influence the intervention activity of the government because the overinvoicing of imports (underinvoicing of exports) leads to an increased demand for foreign exchange on the commercial market. The "true" trade balance is a function of the (commercial) real exchange rate, denoted by $(\bar{c} - p - \bar{k})$, where $\bar{k}$ denotes the long run equilibrium real exchange rate. This implies that the flow of intervention activity (in units of domestic currency), denoted by $D(F_t)$, needed to keep the commercial exchange rate at c is given by:

$$(5) \quad D(F_t) = \beta(\bar{c} - p_t - \bar{k}) - (e_t - \bar{c})/\phi$$

In this expression, e_t and $\bar{c}$ represent the natural logarithms of the financial and commercial exchange rates and p_t represents the logarithm of the price level. The second expression on the right hand side of equation (5) is equivalent to equation (4) for small values of the discount.

The financial market for foreign exchange is not controlled; this implies that risk neutral arbitrageurs ensure that uncovered interest rate parity holds. The domestic interest rate, i, is therefore equal to the foreign interest rate, denoted by i^*, plus the expected rate of depreciation, $D^e(e_t)$. Combining this with a simple money demand function yields the second equation of the model:

$$(6) \quad i_t = D^e(e_t) + i^* = -\lambda[\ln(F_t + \overline{DC}) - p_t]$$

where F represents the foreign assets of the central bank, $\overline{DC}$ represents domestic credit, $F + \overline{DC} = M$ represents the money stock, and λ is one over the semi-elasticity of money demand. $D^e(e_t)$ represents the expected rate of change of financial exchange rate; in this perfect foresight model no distinction is made between actual and expected values. The superscript e is, therefore, suppressed for the rest of the paper. The model is closed with a conventional price adjustment rule:

(7) $D(p_t) = \alpha(\bar{c}-p_t-\bar{k})$

The simple model consisting of equations (5) through (7) has three dynamic variables: the price level, p_t; the foreign assets of the central bank, F_t; and the financial exchange rate, e_t. These variables are functions of the commercial exchange rate, $\bar{c}$ and the stock of domestic credit, $\overline{DC}$ which represent the policy variables. It is apparent from equations (5) through (7) that in the long run the dual exchange rate regime is equivalent to a fixed unified exchange rate regime if the effects of private arbitrage activity are taken into account. Indeed, at the steady state the fixed commercial exchange rate has to be equal to the financial exchange rate; in addition, the commercial exchange rate determines the price level, the money supply, and hence, given domestic credit, the foreign exchange reserves of the central bank. 1/

It is also apparent from equations (5) and (6) that without the arbitrage flows the level of the financial exchange rate is not determined. In the absence of arbitrage flows (i.e., if ϕ is equal to infinity), the model would determine only the rate of change of the financial exchange rate. This result has already been noted in Dornbusch (1976); in general it depends on the absence of wealth effects, which are used in Dornbusch (1976) and (1985) and Frenkel and Razin (1986) to determine the level of the financial exchange rate.

The viability of the dual exchange rate regime depends on the objectives of the government. If the government is just concerned with the level of the differential between the financial and commercial rates it can maintain any constant value of the differential if it is prepared to offset the impact of the arbitrage activity by continuously reducing domestic credit, $\overline{DC}$, at the rate $(e_t-\bar{c})/\phi$. However, the purpose of a dual exchange rate regime is often to evade the consequences of an inflationary policy by keeping domestic interest rates at an artificially low level. This purpose would not be achieved by a policy that keeps the premium constant and thus keeps $D(e_t) = 0$.

Equation (6) also illustrates the proposition mentioned in the introduction that the dual exchange rate regime is only operative in insulating domestic capital markets if the financial rate is expected to depreciate at a different rate than the commercial rate. The first equality sign in equation (6) implies immediately that if $D(e_t) = 0$ the domestic interest rate is equal to the international interest rate. This result is a direct consequence of the assumption that only the rate of change of the financial exchange (as opposed to its level) matters for financial markets.

1/ Formally this is implied by the steady state conditions $p = \bar{c}$, $e = \bar{c}$, and $F = \exp - (i^* + \lambda\bar{c}) - \overline{DC}$.

The effects of monetary policy on e_t, $D(e_t)$, and hence on the domestic interest rate, are discussed in Sections IV and V below. The dynamic behavior of the system (5) through (7) which is discussed in these sections can be analyzed by computing the roots of the system:

$$(8) \quad \begin{bmatrix} \mu & 1/\phi & \beta \\ \lambda/M & \mu & -\lambda \\ 0 & 0 & \mu+\alpha \end{bmatrix} \begin{bmatrix} F_t \\ e_t \\ P_t \end{bmatrix} = 0$$

The system has three roots, given by:

$$(9) \quad \mu_1 = -\alpha \text{ and } \mu_{2,3} = \pm [\lambda/\phi M]^{(1/2)}$$

The first, stable root, $\mu_1 = -\alpha$, determines the path of prices which then act as an exogenous forcing variable in the remaining dynamic system of F_t and e_t because the complete system (5) through (7) is recursive. The dynamic sub-system consisting of F and e has two roots, (μ_2, μ_3), of opposite sign; thus, the sub-system exhibits the usual saddlepath instability. The absolute value of these roots is determined only by the parameters that characterize the financial markets, ϕ and λ. The complete solution of the model that uses also the initial condition is presented in the appendix.

IV. The Effects of Devaluations of the Commercial Exchange Rate

This section uses the model of the previous section to analyse the behavior of the financial exchange rate in response to unanticipated and anticipated future changes in the commercial exchange, $\bar{c}$.

The reaction of the financial exchange rate to changes in the commercial exchange rate is of particular interest in the case of Belgium because the Belgian commercial franc is fixed within the EMS (neglecting the $\pm$ 2.25 percent bands) vis-a-vis the Deutsche mark and other European currencies. Since the beginning of the EMS until September 1986 there had been ten realignments, six of which involved changes in the central parity of the Belgian commercial franc vis-a-vis the Deutsche mark. 1/ Chart 1 shows that around the realignment dates, the financial franc is at a discount. The discount of the financial franc usually rises in periods of tension in the foreign exchange markets, that is, before each EMS realignment, even before those realignments at which the parity of the Belgian (commercial) franc was not adjusted. During 1981 and 1982, the discount sometimes reached 15 percent and oscillated in general between 7 and 10 percent. This pattern in the financial exchange rate of the Belgian franc can be explained in terms of the model presented here, as will be shown in the remainder of this section which discusses the reaction of the financial exchange rate to unanticipated and future anticipated changes (devaluations) of the commercial rate. 2/

An unanticipated change in $\bar{c}$ of $\Delta\bar{c} = \bar{c}' - \bar{c}$ would lead to a jump in the financial exchange rate, Δe_t, equal to: 3/

$$(10) \quad \Delta e_t = \phi \left[\frac{\beta - \alpha M}{1 + (\alpha/\mu)} + 1\right] \Delta\bar{c}$$

Whether or not the financial exchange rate would jump by more, less, or the same amount as the commercial exchange rate depends on the relative magnitudes of B and αM (assuming that the economy was initially at its steady state equilibrium). The financial rate would jump by the same amount as $\bar{c}$ to a new steady state, and the money market would remain in equilibrium (after the change in $\bar{c}$) with the financial rate constant (i.e., with $D(e_t) = 0$) only if the devaluation of the commercial rate had no effect on real balances. This could happen only if the effects of the rising price level on the demand for money were exactly offset by the increased inflows through the trade account. (Note that αM describes the effect of the rising level of the logarithm of prices on the real demand for money (not expressed as a logarithm) while β describes the effect of the devaluation on trade flows.) In this

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1/ The dates and the rates of devaluation of the Belgian franc against the Deutschemark (as measured by changes in central bilateral DM/BF parity) were: September 24, 1979 (-2 percent); October 5, 1981 (-5.5 percent); February 22, 1982 (-8.5 percent); June 14, 1982 (-4.25 percent); March 21, 1983 (-4 percent); and April 7, 1986 (2 percent). There were no changes in the DM/BF central parity during the realignments on November 30, 1979, March 23, 1981, July 22, 1985 and August 4, 1986.

2/ In the case of Belgium, a certain class of exporters has the option of using either the financial or the commercial exchange rate. This means that the financial rate can never be lower than commercial rate because the arbitrage costs apply only to outflows. This is at variance with the model developed above, which has implicitly assumed that arbitrage costs apply symmetrically to outflows and inflows. The consequences of the assymmetry in the Belgian system will be taken into account whenever they become relevant in the remainder of the paper.

3/ See the Appendix, equations (A.3) through (A.9), for derivations.

special case, the potential for arbitrage activity has no impact because no difference emerges between the financial and the commercial rates. By contrast, if the trade account reacts strongly to a devaluation of the commercial rate (i.e., if β is relatively high), as one would expect in Belgium where the proportion of tradables in GNP is very high, the financial exchange rate would overshoot its long-run equilibrium. In this case arbitrage activity would emerge and would influence both the initial jump of the financial rate and its entire future path. 1/ Since the (absolute value of the) root is a decreasing function of the parameter ϕ that describes the severity of the controls the authorities use to separate the two exchange rate markets, it is not apparent from equation (10) whether an increase in the severity of the controls would tend to reduce or increase the over- or undershooting of the financial exchange rate. However, in the Appendix (see equations (A.12) and (A.13)) it is proved that an increase in the severity of the controls would thus tend to increase the observed differential between the financial and commercial exchange rates after unanticipated devaluations.

It has often been argued that if the financial exchange rate is at a discount, the authorities should devalue the commercial rate to eliminate the differential and thus the incentive for arbitrage activity. However, the above results imply that sometimes an unexpected devaluatfon might increase the differential between the financial and commercial rates.

In equations (7) and (5) the equilibrium real exchange rate, $\bar{k}$, has the same effect (but with the opposite sign) on inflation and the trade balance as the commercial exchange rate, $\bar{c}$. This implies that a change in the equilibrium real exchange rate of $\Delta\bar{k} > 0$ (which implies a long-run real depreciation of the domestic currency) would have similar effects on the financial exchange rate as a devaluation of the commercial exchange rate ($\Delta\bar{c} > 0$). In the appendix it is shown that this is indeed the case; the initial jump in the financial exchange rate, Δe_t, in response to an unanticipated depreciation of the equilibrium real exchange rate is equal to:

$$(11) \quad \Delta e_t = -\phi \left[\frac{\beta-\alpha M}{1+(\alpha/\mu)}\right] \Delta\bar{k}$$

Comparing equations (11) and (10) sbows that the same parameters that determine whether the financial exchange rate over- or undershoots in response to an unanticipated devaluation also determine whether the financial exchange rate appreciates or depreciates in response to an

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1/ For Belgium, in the case of $\alpha M > \beta$, an undershooting would not be possible because the financial rate cannot be below the commercial rate. Instead there would be instantaneous inflows of money which would force the financial rate to be equal to the commercial rate.

unanticipated depreciation of the equilibrium real exchange rate. This implies that if $\beta > \alpha M$ an unanticipated depreciation of the equilibrium real exchange rate would lead to an overshooting of the financial exchange rate. (That is an immediate discrete depreciation followed by a continuous appreciation that leads the financial exchange rate back to its original level.) The similarity between equations (10) and (11) also implies that an increase in the severity of the controls would tend to increase the magnitude of the over- or undershooting.

In contrast to the effects of a current unanticipated devaluation, it is clear that a future anticipated devaluation of the commercial rate has to raise the discount on the financial rate at the time the market begins to expect the future devaluation of the commercial rate. The amount by which the financial exchange rate has to jump at the time the news about the future devaluation reaches the market is determined by the condition that there should be no jump in the financial exchange rate at the time the commercial exchange rate is actually devalued (provided that the amount and the timing of the devaluation was correctly anticipated by the market). However, the differential that arises as the financial rate depreciates tends to lead to a loss of reserves because of the arbitrage flows. This loss of reserves implies (see equation (6)) that the expected rate of devaluation has to rise and the financial exchange rate will therefore continue to depreciate at an increasing rate until the devaluation of the commercial rate takes place. At this time, the arbitrage flows should stop or may even be reversed, depending on how long the devaluation was anticipated and on tha value of $(\beta-\alpha M)$. In the case of Belgium, the differential cannot be negative and the initial jump of the financial rate is therefore determined also by the condition that the financial exchange rate is expected to be at least equal to the new (devalued) commercial rate. However, if β is much larger than αM so that an unanticipated devaluation would result in overshooting, the finanical rate might remain at a discount even compared to the new commercial rate after the devaluation. In this case the differential would only be reduced by the impact of the devaluation of the commercial rate and the differential would disappear only gradually over time.

V. The Effects of Monetary Policy on the Exchange Rate Differential

The model can also be used to calculate the effects of an expansionary monetary policy, represented here by an unanticipated increase in domestic credit of the central bank. In the context of this model, the steady state money supply is determined by the fixed commmercial exchange rate. The increase in domestic credit must therefore be offset, in the long run, by an equivalent reserve loss. Since the behavior of prices and thus the trade account is also determined by the fixed commercial exchange rate, the reserve loss can be caused only by the arbitrage activity, which in turn has to be induced by a differential between the financial and commercial exchange rates. Formally, this can be shown by computing the initial jump in the financial exchange rate, Δe_t, as a function of the change in domestic credit, $\Delta\overline{DC}$: 1/

$$(12)\quad \Delta e_t = [\phi\lambda/M]^{1/2}\Delta\overline{DC}$$

This expression implies that the initial jump of the financial exchange rate is an increasing function of the parameter ϕ that characterizes the degree of separation of the two exchange rate markets. A higher degree of separation (in the sense of a high cost for potential arbitrageurs and a high value of ϕ) implies a larger initial jump in the financial exchange rate because the required outflow of reserves can take place only if there is a large differential that causes the arbitrage flows in spite of the high costs associated with them. After the initial discrete depreciation of the financial rate, the rate of depreciation of the financial rate becomes negative (i.e., the financial rate appreciates). The financial rate continues to appreciate at a decreasing rate until the differential disappears.

An equivalent result can be easily obtained for the effects of a change in the international interest rate, as is immediately apparent from the correspondance between changes in i* and change in $\overline{DC}$ in equation (6). This implies that the exchange rate differential depends not only on domestic policies, but also on developments on the world financial markets.

The effectiveness of monetary policy has often been discussed in terms of the so-called offset coefficient, which measures by how much any given change in domestic credit is offset by reserve flows. In this framework, the offset coefficient is a function of time as can be shown by computing the change in reserves induced by a given change in domestic credit (see the Appendix for details of computations):

$$(13)\quad F_t - F_o = \Delta\overline{DC}(e^{-\mu t}-1)$$

Given the continuous time formulation, the impact coefficient (that is, for t=0) is equal to zero. However, over time the (absolute value of the) offset coefficient rises and goes to (minus) one in the long run, (that is, as t goes to infinity).

The use of dual exchange rate regimes has often been advocated on the grounds that a dual exchange rate isolates goods markets from the effects of disturbances in financial markets. In this model, the goods market (characterized by the level of the real exchange rate) is indeed isolated from the effects of disturbances in the financial markets, but since the effects of disturbances in financial markets cannot be

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1/ See Appendix equation (A.11) for a derivation of this result.

eliminated they show up in the shadow price associated with the differential between the financial and the commercial exchange rates. This shadow price has to move by more, for a given disturbance, if the cost of moving from one market to the other is high, because only in this way can this shadow price induce the flows of funds that ultimately neutralize the effects of the original disturbance. Since the dead-weight loss associated with the arbitrage flows is an increasing function of the differential between the two exchange rates, a better separation of the two markets might not be welfare improving. In general, this consideration thus contributes another argument against the use of dual exchange rate regimes.

The effects of an anticipated future increase in domestic credit can be derived from the requirement that there should be no discrete jump in the financial exchange rate at the time the increase in domestic credit actually occurs. This requirement implies that the financial exchange rate has to jump at the time the news of the future increase in domestic credit policy is received. The exchange rate differential that is then created leads immediately to capital outflows, so that the money supply starts to fall and the rate of depreciation of the financial exchange rate becomes positive. The financial rate will then continue to depreciate at an increasing rate until the anticipated increase in domestic credit actually occurs. At that point in time, the rate of depreciation of the financial rate becomes negative (i.e., the financial rate appreciates), and it continues to be negative until the exchange rate differential disappears.

Since the domestic interest rate is equal to the international interest rate, i*, plus the expected rate of depreciation of the financial rate, the results concerning the effects of monetary policy have important consequences for the viability and usefulness of dual exchange rate regimes. If the aim of the dual exchange rate regime is to keep domestic interest rates below international interest rates, the results of this section suggest that an increase in the money supply (i.e., an increase in domestic credit) could achieve this objective only if it comes as a surprise; but even then the effects would be only temporary. If a one-time expansion of domestic credit was anticipated, it would have the undesired effect of raising domestic interest rates initially. A continuously expansionary domestic credit policy that was correctly anticipated would have no effects on domestic interest rates in the long run either. Such a policy would lead to a constant premium of the financial exchange rate that was just large enough to induce arbitrage outflows that neutralized the effect of the domestic credit expansion. Such a situation would be viable, but it would also imply that the only effect of the dual exchange rate system was to offer arbitrageurs the opportunity to make profits.

By the same token, if in response to an increase in international interest rates the authorities tried to neutralize the effects of arbitrage flows on domestic interest rates by increasing the rate of domestic credit expansion, the differential between the two exchange rates would widen, thus increasing the arbitrage flows. Any attempt by the authorities to conduct an independent money supply policy would then lead to an unstable spiral of depreciations of the financial rate and increasing arbitrage flows. A dual exchange regime (with a fixed commercial exchange rate) imposes, therefore, essentially the same constraints on the conduct of monetary policy as a unified fixed exchange rate.

VI. Enforcement and the Welfare Loss Through Evasion Activity

The central theme of this paper is that the effectiveness of dual exchange rate regimes is limited because in reality the separation between the two foreign exchange markets is never perfect. In particular, a differential between the two exchange rates induces a flow of arbitrage activity by agents who are able to avade the controls, and the arbitrage flows in turn act to close the differential between the two exchange rates.

This section analyzes the loss of social welfare that results from evasion of the exchange controls associated with a dual exchange rate regime. The evasion activity is motivated by the pursuit of profits that exceed the costs incurred in evading the controls. Viewed in isolation from the costs of evasion activity, the profits received by arbitrageurs on their exchange transactions are simply the losses incurred by the authorities operating the dual exchange market; financial wealth is redistributed through the exchange market transactions. However, in adition to those redistribution effects, which may or may not be regarded as a social welfare loss, the real resources that are absorbed in evasion activity are wasted from the social point of view.

Without attempting to assess the overall net social gains or losses associated with a dual exchange rate regime, it may be asked whether a tightening of exchange controls would reduce the amount of real resources that were wasted in evasion activity. By using stricter regulations and imposing heavier fines, the authorities could make it more costly for private agents to transform "commercial" foreign exchange into "financial" foreign exchange. In the context of the present model this would amount to an increase in the parameter ϕ.

A stricter enforcement mechanism, ceteris paribus, would decrease the arbitrage flow between the two foreign exchange markets, but it would also make each unit actually transferred more costly for private agents. It is, therefore, not clear, a priori, whether a stricter enforcement mechanism would reduce the total waste of resources, which is equal to the total costs sustained by the arbitrageurs. In the context of the present model, the net result of these two effects can be calculated by using the solution for the exchange rate differential that arises in response to certain shocks, as presented in the Appendix. For example, an unanticipated shock to domestic credit of $\Delta\overline{DC}$ would lead to the following path for the exchange rate differential: 1/

(14) $$e_t - \bar{c} = \left(\frac{\phi\lambda}{M}\right)^{1/2} \Delta\overline{DC}\ e^{-\mu t}$$

The exchange rate differential and the flows between the two foreign exchange markets are linked by $y_t = (e_t - \bar{c})/\phi$. Using this relationship in equation (14) leads to:

(15) $$y_t = \left(\frac{\lambda}{\phi M}\right)^{1/2} \Delta\overline{DC}\ e^{-\mu t}$$

The total costs sustained (wasted from a social welfare point of view) by private agents are given by $g(y_t) = (\phi/2)\ (y_t)^2$. Using this relationship in equation (15) leads to:

(16) $$g(y_t) = \frac{\phi}{2} \frac{\lambda}{\phi M} (\Delta\overline{DC})^2\ e^{-2\mu t}$$

It is apparent from equation (15) that for this specific shock (and under a quadratic cost function) the two effects mentioned above exactly cancel each other. However, a change in ϕ would also have a third effect, since it would influence the value of the root, μ, thereby affecting the speed of adjustment of the system. The impact of this effect can be seen by computing the present value of all present and future arbitrage costs caused by the shock to domestic credit. Assuming a social discount rate of ρ this present value, denoted by PVAC, is equal to:

(17) $$PVAC = \int_0^\infty g(y_t) e^{-\rho t} dt = \frac{\lambda}{2M} (\Delta\overline{DC})^2 \int_0^\infty e^{-(2\mu+\rho)t} dt$$

The solution of this integral is:

(18) $$PVAC = \frac{\lambda}{2M} (\Delta\overline{DC})^2 \frac{1}{2\mu+\rho}$$

Since $\mu = (\lambda/\phi M)^{1/2}$, condition (18) implies that the present value of arbitrage costs increases with ϕ. A tighter enforcement mechanism causes higher welfare losses because it delays the approach towards

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1/ See equation (A.11) in the Appendix.

equilibrium, keeping the exchange rate differential higher, and thus offering more incentives for private agents to arbitrage between the two foreign exchange markets. Equation (18) also shows that the welfare loss is proportional to the square of the shock to domestic credit. This implies that even with the protection of a dual exchange rate regime, a steady monetary policy is very important.

Since it has been demonstrated above that changes in domestic credit and changes in the international interest rate are equivalent, condition (18), also implies that an increase in the variability of the international interest rate would lead to higher welfare costs. Dual exchange rates have often been advocated on the grounds that they protect the real sector of the economy from the effects of shocks to financial markets. According to this argument, a higher variability of international interest rates would, therefore, increase the usefulness of dual exchange rates. However, the results of this section suggest that a high degree of instability in international financial markets could also lead to higher welfare losses through the attempt to earn arbitrage profits. It is, therefore, not clear whether a high degree of instability in international financial markets makes dual exchange rates more desirable.

VII. Concluding Remarks

This paper has analyzed the consequences of incomplete market separation in dual exchange rate regimes by developing an explicit model of the arbitrage flows that occur when the two exchange rates differ. While it has often been argued informally that the differential between the two exchange rates should not be allowed to become too large, it appears that the consequences of arbitrage flows in response to the exchange rate differential have not previously been analyzed formally within a macroeconomic framework.

The main result of the paper is to show that these arbitrage flows would eliminate the exchange rate differential over time. This is in contrast to the existing literature in which there is no force that reduces the exchange rate differential so that the exchange rate differential could settle down to any value in the long run. However, the data on the Mexican dual exchange rate market shows that the discount of the financial exchange rate tends to decline over time, in the absence of new shocks. The experience of the Belgian dual exchange rate regime, which has been in place for almost 40 years, also shows that the discount of the financial rate tends to stay close to zero except in times of turbulence when devaluations of the commercial rate are expected. The data from these two countries thus support the framework proposed here.

An important corollary of the tendency of the two exchange rates to converge is that any attempt by the authorities to permanently neutralize the effects of these arbitrage flows would lead to an unstable situation. Dual exchange regimes should therefore not be used to attempt to permanently offset the effects of permanent shocks.

By using a simple macroeconomic model the paper also analyses the effects of various disturbances on the exchange rate differential. An unanticipated devaluation of the commercial exchange rate (or a fall in the long run equilibrium real exchange rate) would lead to an overshooting of (and thus to a discount on) the financial exchange rate if the trade balance is sufficiently elastic with respect to changes in the commercial exchange rate. An unanticipated increase in domestic credit would always lead to a discount on the financial exchange rate; moreover, this discount is an increasing function of the degree of severity of the regulations that are used to keep the two exchange markets separate.

Since arbitrage flows link the two exchange rates the financial exchange rate, which is forward-looking, reacts to the anticipation of future disturbances. A future devaluation of the commercial rate or a future increase in domestic credit would therefore lead to an immediate discount on the financial rate. This is also confirmed by the data for Belgium which show that the discount on the financial rate tends to increase sharply before each devaluation of the commercial Belgian Franc inside the EMS.

Derivations

The system of equations (5) through (7) is recursive; the path of prices is determined by equation (7) alone and acts like an exogenous, forcing variable in the reduced system (5) through (6). The particular solution for the two variable F and e, therefore, contains not only a constant but also a term in exp $(-\alpha t)$. The sum of the particular and the homogenous solutions for F_t and e_t can therefore be written as:

$$\text{(A.1)} \quad F_t = Ae^{-\mu t} + F_{p,t} = Ae^{-\mu t} + Be^{-\alpha t} + \bar{F}$$

$$\text{(A.2)} \quad e_t = Ce^{-\mu t} + e_{p,t} = Ce^{-\mu t} + De^{-\alpha t} + \bar{c}$$

Where A, B, C, and D are constants to be determined by the initial conditions. Equation (A.2) already uses the result that the stationary state implies that $\bar{e} = \bar{c}$.

I. The Reaction of the Financial Exchange Rate to a Devaluation of the Commercial Rate

To calculate the jump in the financial exchange rate that results from an unanticipated devaluation of the commercial rate by $\Delta c>0$ from $\bar{c}$ to $\bar{c}'$, it is convenient to assume that initially the economy was at a stationary state with $p = \bar{c} - \bar{k}$. The particular solution (the last two terms in (A.1) and (A.2)) has to hold at each point in time. Using this fact and the solution for the price level $p_t = -\Delta\bar{c}e^{-\alpha t} + \bar{c}' - \bar{k}$ in equations (5) and (6) yields:

$$\text{(A.3)} \quad -\beta\Delta\bar{c} = \alpha B - D/\phi$$

$$\text{(A.4)} \quad -\lambda\Delta\bar{c} = -\alpha D + \lambda B/M$$

Solving (A.3) for B and substituting into (A.4) yields:

$$\text{(A.5)} \quad -\lambda\Delta\bar{c} = -\alpha D + \lambda[-\beta\Delta\bar{c} + (D/\phi)]/M\alpha$$

or

$$\text{(A.6)} \quad D = \frac{\beta - M\alpha}{(1 - \alpha^2 \frac{M\phi}{\lambda})}\Delta\bar{c} = \phi\frac{\beta - M\alpha}{1 - (\alpha/\mu)^2}\Delta\bar{c}$$

where the second equality sign comes from the result that $\mu = \pm(\lambda/M\phi)^{1/2}$. At the time of the jump in the commercial exchange rate, the money supply is given because it is a slowly adjusting variable. Since the system is assumed to start from a stationary state, this implies that $\dot{e}_o = 0$. Using this initial condition in the complete solution for the exchange rate (A.2) yields:

$$(A.7) \qquad 0 = -\mu C - \alpha D$$

Combining this with (A.6) implies that the exchange rate at time zero is given by:

$$(A.8) \qquad e_o = [1 - \frac{\alpha}{\mu}] \phi \frac{\beta - M\alpha}{1 - (\alpha/\mu)^2} \Delta\bar{c} + \bar{c}'$$

or, since the financial rate was equal to $\bar{c}$ before the devaluation:

$$(A.9) \qquad \Delta e = e_o - \bar{c} = \phi[\frac{\beta - M\alpha}{1 + (\alpha/\mu)} + 1] \Delta\bar{c}$$

II. The Effects of a Change in the Equilibrium Real Exchange Rate

The effects of a change in the equilibrium real exchange rate from $\bar{k}$ to $\bar{k}'$ by $\Delta\bar{k} < 0$ can be calculated using equations (A.3) through (A.7), where Δc has been substituted by $-\Delta k$. The only difference is in equations (A.8) and (A.9); the new steady rate for the financial rate is still given by $\bar{c}$, which imples that the initial value of the financial exchange rate is given by:

$$(A.8') \qquad e_o = [1- \frac{\alpha}{\mu}] \frac{\beta-M\alpha}{1-(\alpha/\mu)^2} (-\Delta\bar{k}) + \bar{c}$$

The initial jump in the financial exchange rate is therefore given by:

$$(A.9') \qquad \Delta e = e_o - \bar{c} = [\frac{\beta - M\alpha}{1 + (\alpha/\mu)}](-\Delta\bar{k})$$

III. The Effects of Domestic Credit Policy

The effect of an unanticipated jump in domestic credit of $\Delta\overline{DC}$ (from $\overline{DC}$ to $\overline{DC}'$) is more straightforward to calculate. In this case $p=p=c$ and the particular solutions consist only of a constant term. The initial condition in this case is that $\Delta e = -i - \lambda[\ln(F-\overline{DC}') - \bar{p}]$. Using equations (A.1) and (A.2), this implies

$$\text{(A.10)} \qquad \frac{\Delta\overline{DC}\ \lambda}{\bar{M}} = \mu C$$

The initial jump of e_t is equal to C; this implies that:

$$\text{(A.11)} \qquad e_o - \bar{c} = \Delta\overline{DC}\ \frac{\lambda}{M\mu}$$

Using $\mu = (\lambda/M\phi)^2$ yields equation (12) in the text.

IV. The Effects of Making Evasion Activity More Costly on the Initial Jump of the Financial Rate

The effect of a change in the parameter ϕ on the absolute value of the initial jump in the financial exchange rate that occurs in response to an unanticipated devaluation of the commercial exchange rate can be calculated from equation (10):

$$\text{(A.12)} \qquad \frac{\partial|\Delta e|}{\partial\phi} = |\beta-\alpha M|\left[\frac{1+\alpha\frac{\phi M}{\lambda} - \phi\alpha\frac{1}{2}\left(\frac{\phi M}{\lambda}\right)^{-\frac{1}{2}}\frac{M}{\lambda}}{\left[1+\alpha\left(\frac{\phi M}{\lambda}\right)^{1/2}\right]^2}\right]$$

where the term (α/μ) in the denominator of (10) has been substituted using the result $\mu = (\lambda/\phi M)^{1/2}$. The expression above can be simplified to:

$$\text{(A.13)} \qquad \frac{\partial|\Delta e|}{\partial\phi} = |\beta-\alpha M|\left[1+\alpha\left(\frac{\phi M}{\lambda}\right)^{1/2}\right]^{-2}\left[1+(1/2)\alpha\left(\frac{\phi M}{\lambda}\right)^{1/2}\right]$$

which is always positive.

References

Adams, Charles, and Jeremy Greenwood, "Dual Exchange Rate Systems and Capital Controls: An Investigation," Journal of International Economics, Vol. 18 (February 1985), pp. 43-63.

Bhandari, Jagdeep D., and Bernard Decaluwe, "Stochastic Implications of Incomplete Separation Between Commercial and Financial Exchange Markets," unpublished manuscript (1984).

Dornbusch, Rudiger, "Special Exchange Rates for Capital Account Transactions," Working Paper No. 1659, National Bureau of Economic Research (July 1985).

_____, "The Theory of Flexible Exchange Rate Regimes and Macroeconomic Policy," Scandinavian Journal of Economics, No. 2 (May 1976), pp. 255-75.

Flood, Robert P., and Nancy Peregrim Marion, "The Transmission of Disturbances Under Alternative Exchange-Rate Regimes With Optimal Indexing," The Quarterly Journal of Economics (February 1982), pp. 43-66.

Frenkel, Jacob A., and Assad Razin, "The Limited Viability of Dual Exchange-Rate Regimes," Working Paper No. 1902, National Bureau of Economic Research (April 1986).