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Investment, Openness and Country Risk

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Abstract

The purpose of this study is to draw attention to the linkages between country risk and the openness of an economy, and to demonstrate that in the long run the openness of an economy is endogenously determined by the interaction between endowments and policies. The presence of country risk poses a problem for the smooth operation of international credit markets: the ex-ante first best policy is for countries to pre-commit themselves to no-default policies. Such a commitment, however, may not be credible because it may not be the optimal ex-post policy. This suggests a special role for policies leading towards investment in openness - as a way to increase the credibility of a no-default commitment. The paper studies the optimal implementation of these policies.

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I. Introduction and Summary

The evolution of external indebtedness during the 1970's and the 1980's has demonstrated the unique dimension of international banking in the presence of country risk. The lack of simple enforcement mechanisms for debt repayment tends to degenerate the international credit market into an equilibrium where the volume of international credit is limited by the effective penalties associated with defaults. These penalties are the results of potential embargoes, being associated with restrictions on the flow of both temporal and intertemporal trade (i.e., trade in goods and financial assets, respectively).

We can gain further insight into this issue by noting that the borrower's 'first best' policy (i.e., the policy that will maximize the expected welfare of the borrowing nation) is to pre-commit itself to no-default. This is because a default is associated with a net waste of resources (resulting from the embargo) that is not captured by any party and thus results in welfare losses. The problem with a no-default commitment is that it is not a credible one, because it is time inconsistent. The time consistent policy regarding the default decision is based on a periodic cost-benefit assessment. Indeed, the existing literature has focused on analyzing the properties of the time consistent equilibrium. Typically, the default decision is arrived by comparing the saving resulting from the default to the default penalty, which is taken as exogenously given. 1/

The purpose of the present paper is to focus on the role of investment policies in the presence of country risk. The importance of this issue stems from the observation that a trade embargo has the consequence of eliminating the gains from trade. Thus, a default penalty is tied directly to the openness of the economy. In the long run the openness of an economy is endogenously determined by the interaction between the endowment and investment policies. These policies, in turn, may have important effects on the supply of credit facing the economy. A focus of our analysis is to examine the linkages between investment policies and country risk.

As is common in the time consistency literature, there are welfare benefits associated with a credible commitment that will allow the attainment of the first best equilibrium. 2/ We can move towards such an equilibrium by designing investment policies that will increase the openness of the economy, thereby raising the costs of deviations from a

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1/ For an analysis of country risk see, for example, Harberger (1976), Kharas (1981), Eaton and Gersovitz (1981), Sachs (1984), Kletzer (1984), Krugman (1985), Smith and Cuddington (1985), Edwards (1985), Folkerts-Landau (1985), Dooley (1986), Aizenman (1986), Bulow and Rogoff (1986), Aizenman (1987), and Helpman (1987).

2/ For an analysis of time consistency see Calvo (1978) and Kydland and Prescott (1977).

no-default commitment. 1/ In order to examine these policies we will construct a simple two-periods analysis of country risk. Borrowing in period one is used to finance consumption and investment in the various activities. These activities differ in their exposure to international trade, due to varying degrees of reliance on imported inputs or on external markets for sales of output. The country will default in period two if the default penalty falls short of the debt. The international credit market is dominated by risk neutral lenders that will supply sufficient credit to equate the expected yield on their international lendings to the exogenously given risk free interest rate. Domestic agents are small enough to be price takers in the domestic credit market, thus allowing them to treat the interest rate facing them as exogenously given. The default decision against external creditors is made by a centralized decision maker, like the central bank, whose policy is guided by an attempt to maximize the expected welfare of a representative consumer. Thus, the source of country risk is transfer risk. Agents are assumed to be rational and to be fully informed regarding the default decision rule guiding the central bank. We study the factors determining the supply of credit facing the economy, the private sector consumption and investment, and the policies needed to attain the optimal allocation.

Our analysis demonstrates that the supply of credit is determined by the aggregate borrowing and by its decomposition among consumption and the various investment activities. The supply is upward sloping and may include a backward bending portion. The investment in a given sector is determined by the expected incidence of country default and by the relative exposure of the sector to international trade. A rise in country risk is associated with more frequent defaults and consequently with a lower level of investment. The resultant drop in investment is larger in activities with greater reliance on international trade.

The importance of financial policies in the presence of country risk stems from the observation that competitive equilibrium is inefficient in the presence of international debt. The presence of country risk is shown to introduce a distortion. The distortion arises from the fact that individual borrowers treat the rate of interest as given even though from the perspective of the country as a whole the rate of interest changes with the volume of borrowing and investment because of the change in the probability of default. Each small consumer overlooks the change in the probability of default induced by his marginal borrowing and marginal investment. The change in the probability of default creates an externality because of the consequent change in the expected default penalty inflicted on all domestic consumers.

1/ Investment in openness may serve as a credible commitment as long as installed capital is sector specific.

The presence of country risk calls for financial policies. These policies are in the form of a tax on consumption borrowing and a different tax on investment borrowing. The tax internalizes the effect of the activity financed by the borrowing on the probability of default. The optimal tax should be higher the greater the increase in the probability of default resultant from that activity. For example, investment in intermediate goods that must be exported for final assembly will tend to be associated with a lower optimal borrowing tax relative to an investment in the production of final goods. Similarly, investment in export substitutes or non-traded goods will tend to be more taxed than investment in exportable goods. This outcome is consistent with the notion that in the presence of country risk a country will be able more easily to finance export led growth that is biased towards the production of intermediate goods rather than final goods, or inward growth biased towards import substitution.

Unlike the case of a consumption borrowing, the investment borrowing affects the borrowing externality in two opposing directions. First, the marginal borrowing raises the total indebtedness, thus increasing the probability of default. Second, the investment in the traded sector also raises the openness and the productive capacity of the economy, thereby changing the default penalty and the probability of default. The optimal investment borrowing tax balances these two effects. The stronger the openness effect, the lower the optimal investment borrowing tax; and if this effect dominates, the optimal policy is in the form of an investment subsidy. Consequently, in the presence of country risk the marginal use of funds plays a key role in determining the appropriate policies.

A final topic of our analysis is a study of the nature of country risk in the presence of equity finance. We demonstrate that swapping nominal debt with equities may have useful consequences for reducing country risk, but it cannot eliminate the fundamental problems associated with international credit. If the random shocks affect output and the default penalty in the same way we obtain the strong result that equity finance will eliminate defaults up to the credit ceiling. This is done by correlating the repayments with the default penalty. The debt-equity swap, however, is not able to eliminate the resulting need to impose a ceiling on the available credit. Instead, it may allow us to increase the credit ceiling. These results should be viewed as a special case of a more general economic environment: in the presence of several shocks which affect output and the default penalty in different ways, the move to equity finance may be beneficial, but it will not eliminate defaults.

II. The Credit Market Equilibrium

Let us construct a simple framework for the analysis of country risk and investment policy in the presence of default risk. This can be done in a two-periods, multi-sectorial economy. Suppose that the value added in sector i depends on three factors. First, on the realization of a productivity shock. Second, it may be affected by the decision regarding default. For example, if default raises the costs of imported inputs it will tend to depress output. Third, the value added in sector i depends positively on the capital stock, which in turn is determined by past investment. We can summarize the value added in sector i at time t by:

$$(1) \qquad Y_{i;t} = \begin{cases} Y_{i;t}(\Psi;\ n,\ K_{i;t}) & \text{if no default occurs} \\ Y_{i;t}(\Psi;\ d,\ K_{i;t}) & \text{if default occurs} \end{cases}$$

where Ψ is the state of nature, reflecting a productivity shock ($\partial Y_i / \partial \Psi > 0$). The second term stands for the default position of the economy. It can have values of n and d, for default and no-default position, and $K_{i;t}$ is the stock of capital in sector i at time t. For expositional simplicity we assume that the economy is a price taker in the international market, and we normalize all prices of final goods to unity. We also assume a common productivity shock for all sectors, and we assume that the density function of the productivity shock (denoted by $f(\Psi)$) is common knowledge. The GNP in our economy is the sum of the value added in all activities, being given by $Y_t(\Psi;\ s) = \sum_{i=1}^{q} Y_{i;t}(\Psi;\ s,\ K_{i;t})$; where s = n or d (no default or default, respectively) and there are q sectors. 1/

We define the default penalty (denoted by Δ) as the drop in the GNP resultant from the default: $\Delta = Y_t(\Psi;\ n) - Y_t(\Psi;\ d)$. Let us assume that the default penalty is larger in goods states of nature (i.e. $\partial \Delta / \partial \Psi > 0$). 2/

Aggregate indebtedness in the second period (t = 2) is denoted by B, and the interest rate on that indebtedness is r*. Aggregate borrowing is the result of consumption borrowing (denoted by B_c) and investment borrowing in sector i (denoted by I_i, where $1 \leq i \leq q$). For simplicity of exposition we assume that all the investment in period one is financed via external borrowing. In such a case $B = B_c + \sum_{i=1}^{q} I_i$, and assuming no depreciation we obtain that $K_{i;2} = K_{i;1} + I_i$. 3/

1/ Note that the GNP Y_t is a function also of the vector of capital $(K_{1,t}, \ldots K_{2,t}, \ldots, K_{q,t})$. For notational simplicity this vector is suppressed.

2/ This assumption reflects the presumption that in goods states of nature we expect greater volume of international trade, thereby raising the default penalty.

3/ To simplify we neglect the potential role of initial indebtedness by assuming it to be zero. For an analysis regarding a partial default decision in period zero due to initial indebtedness see Krugman (1985).

The default decision in period two can be summarized by the following simple rule: default if the penalty falls short of the payment due:

$$\text{no default if} \quad B\,(1 + r^*) < \Delta$$
$$\text{default if} \quad B\,(1 + r^*) > \Delta$$

Let us denote by Ψ_0 the marginal value of the productivity shock being associated with default (i.e., Ψ_0 is defined by the requirement that $B(1+ r^*) = \Delta$). Consequently, the probability of no default is the probability that the productivity shock exceeds Ψ_0. Let us denote this probability by Π . 1/ Assuming that the international banking sector is dominated by risk neutral agents we can characterize the supply of credit by the combination of B and r^* that solves:

$$(2) \quad 1 + r_f = (1 + r^*)\,\Pi$$

where r_f is the exogenously given risk free interest rate. 2/ We can summarize the supply of credit facing the economy by curve $\overline{SS}$ in Figure 1. The supply schedule is upward sloping for intermediate levels of credit. It may include also a backward bending portion, reflecting the fact that a rise in the interest rate has two opposing effects on expected returns -- for a given probability of no default it increases the expected yield, but at the same time it reduces the probability of payment, depressing the expected yield. If the second effect dominates, we will operate on the backward bending portion of the supply schedule. Direct application of (2) reveals that the elasticity of the supply of credit ($d \log (1 + r^*)/ d \log B$) is determined by the elasticity of the probability of no default with respect to the interest rate, denoted by $-\varepsilon$ (i.e., $\varepsilon = - d \log \Pi/ d \log (1 + r^*)$). It can be shown that $d \log (1 + r^*)/ d \log B = \varepsilon/(1 - \varepsilon)$. 3/ The term ε is a measure of the segmentation of the domestic credit market from the international market, being determined by the nature of the distribution of the default penalty. A lower ε is associated with greater capital market integration, and $\varepsilon = 0$ corresponds to the case where country risk is absent. We restrict the economy to operate along the upward sloping portion of the supply of credit schedule (where a rise in indebtedness is associated with a rise in the interest rate). 4/

1/ Formally, $\Pi = \int_{\Psi_0}^{\infty} f(\Psi)\, d\,\Psi$.

2/ The probability of no default is a function of the following variables $\Pi = \Pi\,(\overset{-}{B}, \overset{+}{r^*}; \vec{K}_2)$; where the signs above the variables stand for the sign of the partial derivatives, and $\vec{K}_2$ stands for the vector of the capital stock in the various activities (in the second period).

3/ Thus, we operate on the upwards sloping portion of the supply of credit as long as $\varepsilon < 1$, and we reach the credit ceiling where $\varepsilon = 1$. for further details on the factors determining the supply of credit in the presence of country risk see Aizenman (1986).

4/ This assumption is consistent with welfare maximization: it can be

We would like to use our framework to evaluate optimal policies in the presence of country risk. We can accomplish this by comparing the optimality conditions determining the consumption and investment from the point of view of the consumer and the centralized planner. A comparison between the planner's and the consumer's solutions reveals that the two differ in that the planner applies the social interest rate, whereas the consumer applies the private one (see the Appendix for the derivations of these results). The social interest rate is defined as the total marginal interest cost associated with the borrowing for consumption and for investment in activity i, given by

$$(1 + r^*)(1 + \frac{d \log (1 + r^*)}{d \log B_c}); \quad (1 + r^*)(1 + \frac{d \log (1 + r^*)}{d \log I_i}); \quad i = 1, \ldots, q$$

respectively. Note that the social planner may face different social interest rates for the various activities. The key difference between the individual agent and the social planner is that the latter is internalizing the marginal changes in the interest rate facing the economy due to marginal borrowing. These changes in turn are determined by the use of these funds, and are reflected in the second term in the social interest rates. The percentage difference between the private and the social interest rate equals the elasticity of the interest rate $(1 + r^*)$ with respect to the borrowing. Logarithmic derivation of (2) gives us that the elasticity with respect to consumption borrowing is $\frac{d \log (1 + r^*)}{d \log B_c} = \frac{\varepsilon}{1-\varepsilon} \frac{B_c}{B} > 0$. This elasticity equals the elasticity of the supply of credit $(\frac{\varepsilon}{1-\varepsilon})$ weighted by the relative importance of the consumption borrowing to the entire volume of borrowing. Applying a similar derivation we can infer that the elasticity with respect to investment borrowing is:

$$\frac{d \log (1 + r^*)}{d \log I_i} = \frac{\varepsilon}{1-\varepsilon} [\frac{I_i}{B} - \frac{I_i}{K_i} \frac{\lambda_i}{\varepsilon}],$$

where λ_i is the elasticity of the probability of no default with respect to the stock of capital in sector i (i.e., $\lambda_i = \partial \log \Pi / \partial \log K_i$). We can view λ_i as a measure of the openness associated with the investment in sector i. An investment project raising the openness will have the consequence of increasing the probability of no default, thereby raising λ_i. Consequently, the magnitude of λ_i is a measure of the importance of this effect. We will further investigate this interpretation in Section III.

The difference between the interest rate of the private consumer and the social planner implies that in the absence of policies the presence of country risk implies a distortion. From the social point of view the equilibrium is associated with "excessive" borrowing for consumption because the private interest rate falls short of the social one. This situation provides the rationale for policies. The

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shown that an equilibrium on the backward bending portion of the supply of credit schedule is inefficient. Ruling out such an equilibrium may require policies in the form of optimal borrowing taxes (see Aizenman (1986)).

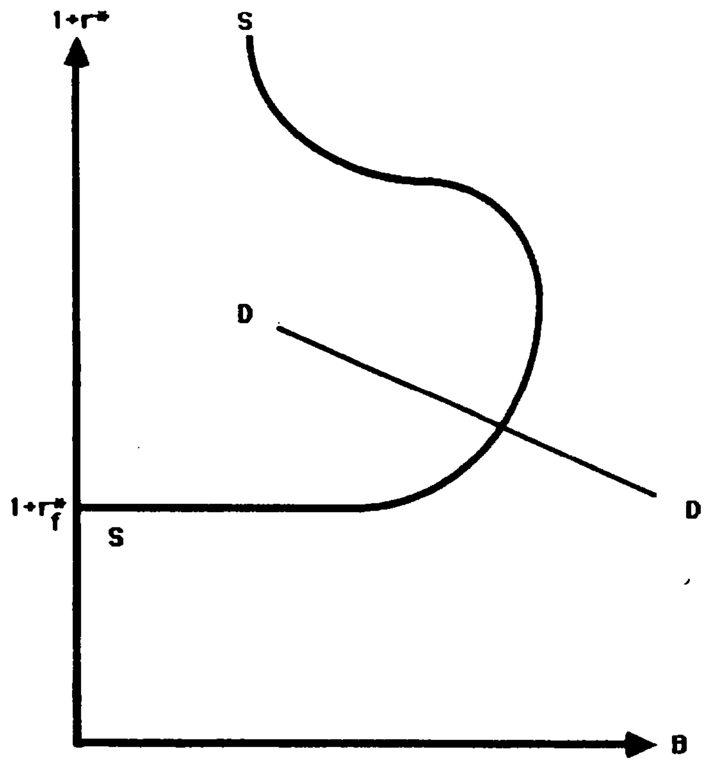

FIGURE 1

distortion arises from the fact that individual borrowers treat the rate of interest as given even though from the perspective of the country as a whole the rate of interest rises with the volume of consumption borrowing due to the rise in the probability of default. Each small consumer overlooks the marginal rise in the probability of default induced by his marginal borrowing. The rise in the probability of default entails a negative externality because of the consequent rise in the expected default penalty inflicted on all domestic consumers. Therefore, the role of policies is to internalize this externality. An optimal consumption borrowing tax (denoted by ρ_c) is needed to yield equality between the social and private interest rates. This tax is defined by the condition that $1+r_c = (1 + r^*)(1 + \frac{d \log (1+r^*)}{d \log B_c})$, where r_c is the domestic interest rate defined by the borrowing tax (i.e., $1 + r_c = 1 + r^* (1 + \rho_c)$). Applying our previous results yields the optimal tax to be

$$\rho_c = \frac{\varepsilon}{1-\varepsilon} \frac{B_c}{B} \frac{1+r^*}{r^*} > 0. \tag{3}$$ 1/

By following a similar approach we can determine that the optimal borrowing tax for investment in sector i (denoted by ρ_i) is

$$\rho_i = \frac{\varepsilon}{1-\varepsilon} \frac{I_i}{B} \frac{1+r^*}{r^*} - \frac{\lambda_i}{1-\varepsilon} \frac{I_i}{K_i} \frac{1+r^*}{r^*} \tag{4}$$

Notice that unlike the case of a consumption borrowing, marginal investment borrowing affects the borrowing externality in two opposing directions. First, the marginal borrowing raises the total indebtedness, thus increasing the probability of default. This effect is captured by the first term in (4), and is similar to the one reported in (3) for consumption borrowing. Second, the investment in the traded sector also raises the openness and the productive capacity of the economy, thereby increasing the default penalty and reducing the probability of default. This effect is captured in the second term, and is proportional to the measure of the investment in openness, λ_i. The optimal borrowing tax balances these two effects. The stronger the investment effect, the lower the optimal investment borrowing tax.

The optimal taxes have a simple diagrammatic interpretation in terms of Figure 2. Let MC stand for the social marginal costs of consumption borrowing. The consumption borrowing tax is defined by the vertical distance between the supply SS and the marginal cost. Note that the location of the supply schedule is determined by the vector of

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1/ Note that as ε approaches one the optimal tax approaches infinity, corresponding to an effective quota on external borrowing implemented by the central bank. This quota has the consequence of ruling out inefficient equilibrium on the backward bending portion of the supply of credit. For more details see Aizenman (1986).

the capital stock in the economy (denoted by $\vec{K}$). Borrowing for investment purposes has the consequence of raising $\vec{K}$ to $\vec{K}'$, thereby shifting the supply and the corresponding marginal cost schedules to SS' and MC', respectively. The optimal investment borrowing tax is defined by the vertical distance between SS' and the new marginal cost schedule. 1/

A relevant implication of country risk is that the marginal use of funds plays a key role in determining the appropriate policies because the role of policies is to internalize the marginal contribution of the activity to the probability of default. If one activity raises this probability by more than another, borrowing for that activity should be taxed at a higher rate. This is the rationale for the differential taxation of borrowing for consumption versus the various investment projects. A possible consequence of our analysis is that investment in openness should be treated favorably relative to investment that does not affect openness or borrowing for consumption. For example, investment in intermediate goods that must be exported for final assembly may be associated with a different contribution towards country risk than investment in the production of a final good. Consequently, the social interest rate and the corresponding borrowing tax/subsidy rates will differ across these activities. To derive this result more formally we should impose further structure on our model. This is done in the next section by specializing the model to deal with an economy where a default results in a rise in the price of imported inputs, and where the various sectors differ in their dependency on importable goods. 2/

III. Investment and Imported Inputs

We would like to construct the simplest example to deal with country risk with endogenous choice of openness. This can be done in a two sectorial economy. The two sectors differ in terms of their reliance on international trade. For example, consider an economy where output in sector i (denoted by X_i) is produced by the following process:

$$X_i = \Psi C_i (K_i)^{\alpha} (M_i)^{\beta_i} \quad ; \quad \alpha + \beta_i < 1; \tag{5}$$

1/ Note that if the investment financed by the borrowing is highly effective in raising the default penalty, the shift to the right of the relevant schedule may result in a policy of subsidizing that investment relative to the initial no policy equilibrium. This will correspond to the case where the MC' schedule is to the right of the original SS.

2/ An example of such an economic environment may be the case of Turkey in the 1970's, whereas a consequence of credit difficulties imports of energy were adversely affected. Our discussion should be viewed as only one example for modeling external dependency. While the focus of the analysis here is on the inputs linkages, similar analysis can apply for output linkages, where various sectors differ in the share of exports.

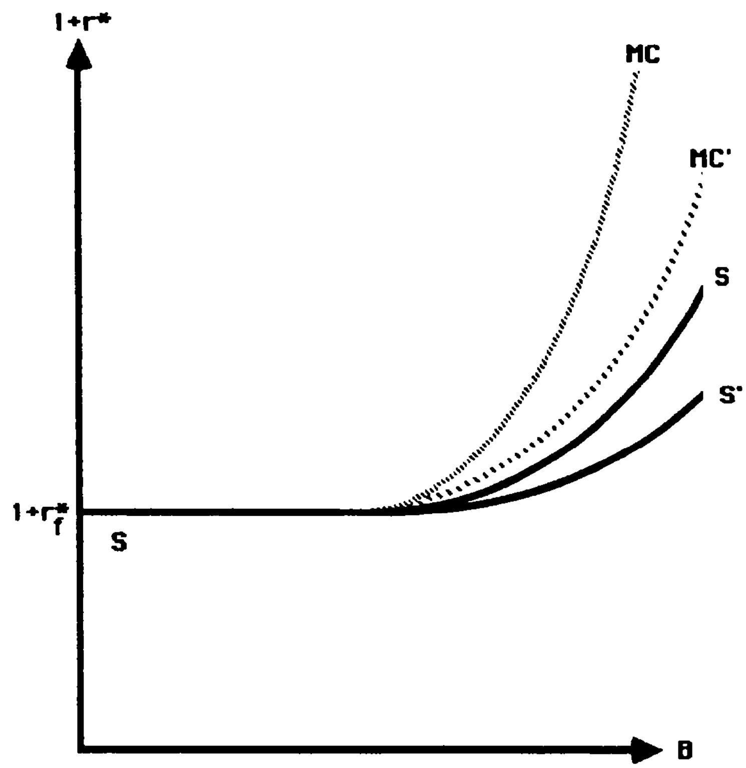

FIGURE 2

where K_i and M_i are the capital and the imported inputs used in sector i, and C_i is a constant. The only difference between the two sectors is that they differ in their dependence on international trade. One of them, suppose sector 1, is more dependent on international trade (i.e. $\beta_2 < \beta_1$). Thus, we can refer to β_i as a measure of the "openness", or the reliance on international trade of activity i. In the short run the stock of capital is exogenously given. We denote the price of the imported input by P_m, and we assume that P_m is determined by the policies of the country. In the absence of default the country faces the international price of P_m, assumed to be unity. A default will have the consequence of triggering a penalty due to a trade embargo. A simple way to capture the penalty is by assuming that it will raise the price of imported inputs by a factor of $p_m > 0$, such that in states of default the effective costs of importables facing the country is exp (p_m).

Producers in each sector maximize profits in two ways. In the first period producers will choose the optimal investment which will determine the capital stock in the second period. Within each period the stock of capital is given, and producers will choose the imported input M in order to maximize profits. As is shown in the Appendix, the default penalty can be approximated by the sum of output in the various sectors in states of no default weighted by a measure of the reliance on trade (the β_i 's) times the increase in imported inputs prices, p_m. Formally, the default penalty can be approximated for small values of p_m by:

$$(6) \qquad \Delta \approx [\beta_1 X_{n;1} + \beta_2 X_{n;2}]\, p_m$$

where $X_{n;1} = c_i\, (\Psi)^{1/(1-\beta_i)} (K_i)^{\alpha/(1-\beta_i)}$ for i = 1,2 (c_i are constants).

The reliance on international trade (as measured by the importance of the imported input, β_i) plays a key role in determining the relative importance of sector i in the aggregate default penalty. A sector that is shielded from international trade would not play a role in the determination of the aggregate default penalty. These observations play a key role in determining the optimal tax on borrowing for investment in sector i. An investment in activity with a larger "openness" index β_i will cause a greater increase in the the default penalty, causing a larger increase in the probability of no default. Thus, we expect sectors with larger exposure to trade to be associated with a larger elasticity of the probability of no default with respect to capital (denoted by λ_i). Let us recall that this λ_i was referred to as a measure of the openness associated with investment in sector i. A larger λ_i was shown to be associated with a smaller investment borrowing tax (see (4)). Applying (6) we show in the Appendix that the value of λ_i equals $s_i \varepsilon\alpha/(1 - \beta_i)$, where s_i is a measure of sector i's share in the aggregate penalty. This measure is proportional to the reliance on imports, β_i. Consequently, we can derive the reduced form of the optimal investment borrowing tax for activity i:

(4') $$\rho_i = \frac{\varepsilon}{1-\varepsilon} \left[\frac{I_i}{B} - \frac{s_i \alpha I_i}{(1-\beta_i) K_i} \right] (1 + r^*)/r^*$$

The optimal tax depends negatively on the openness of activity i to international trade. A smaller openness is associated with smaller values of s_i and β_i, implying a higher investment borrowing tax. An activity with no contribution to the default penalty ($\beta_i = s_i = 0$) should be treated similarly to a borrowing for consumption. 1/

A default is associated with a penalty that results in a lower productivity of capital. To gain further insight regarding the adverse consequences of country risk on investment it is useful to consider a special case. Suppose that investment is conducted by risk neutral agents who equate the expected cost of capital to the expected marginal product, and let us assume that the logarithm of the productivity shock is normally distributed (log $(\Psi) \sim N(0,V)$) with mean zero and a small variance V. In the Appendix we demonstrate that in a competitive equilibrium the stock of capital can be approximated by

(7) $$K_i \approx d_i \left[\frac{1-\beta_i p_m (1-\Pi)/(1-\beta_i)}{1+r_f}\right]^{\frac{1-\beta_i}{1-\beta_i-\alpha}}$$

where d_i is a constant. Equation (7) has a simple interpretation: the stock of capital depends positively on the term in the bracket, which is the ratio of expected net productivity (net of the default penalty) over the expected cost of capital. 2/ The expected cost of credit is the risk free interest rate, and this is the cost element in that equation. Thus, as long as we operate below the credit ceiling, country risk does not change the expected cost of borrowing (being equal to the risk free rate). Instead, country risk operates by reducing the expected marginal product of capital, thereby reducing investment.

It is noteworthy that the adverse consequences of country risk affect the various sectors differentially. Investment drops more in the

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1/ Note that (4') also implies that the condition for subsidizing investment in sector i is that $K_i/B < s_i \alpha/(1-\beta_i)$. Thus, a higher aggregate indebtedness as well as a higher reliance of activity i on international trade will increase the likelihood of subsidizing investment in sector i.

2/ Note that $1 - \Pi$ is the probability of default, and $\beta_i P_m/(1-\beta_i)$ is the percentage drop in output and in the productivity of capital attributed to default. Consequently, $\beta_i P_m(1-\Pi)/(1-\beta_i)$ has the interpretation of the expected drop in output and in the productivity of capital due to default (see (6)).

sectors that are more heavily dependent on international trade. Formally, one can show that a marginal increase in the probability of default will reduce investment in sector i by a factor of $\frac{\beta_i}{1 - \alpha - \beta_i}$, in proportion to the relative openness of sector i. 1/

IV. Debt Versus Equity Finance

Recently we have observed the emergence of schemes intended to swap existing debt with equities. A typical loan in the 1970's and 1980's has been in the form of a nominal interest rate contract, not allowing for contingencies. It is hard to believe that such a contract is optimal, and a purpose of the debt-equity swap is to overcome some of the difficulties associated with loans with limited price contingencies by replacing a non-contingent with a contingent contract. 2/ Indeed, it is somewhat of a puzzle as to why the growing awareness of the welfare benefits of contingencies has occurred only recently. 3/ We turn now to an application of our model, in which we analyze the nature of country risk in the presence of equity finance. The present analysis will demonstrate that swapping nominal debt with equities may have useful consequences for reducing country risk, but it cannot eliminate the fundamental problems associated with international credit.

It is useful to start with the case of a one sector economy (q=1), where for notational simplicity we suppress the sectorial index. Consider an initial equilibrium in our economy with an initial level of indebtedness of B. A debt - equity swap will replace the debt B with claims on an 'equivalent' fraction of the value added. We denote that fraction by τ. A default in the equity scheme will occur if its benefits (in the form of no payments to foreign investors) exceed the default penalty. Thus, a default will occur if and only if $\tau(1-\beta) X_n > \beta X_n p_m$, where the left-hand side stands for the foreign equity income paid in case of no default, and the right-hand side stands for the default penalty. Consequently, the condition assuring no default is that the foreign ownership share τ does not exceed $\beta p_m/(1 - \beta)$. This condition is useful in yielding the maximum equity investment in period one, which is determined by the expected net present value of the foreign equity income extracted for

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1/ Formally, $d \log K_i/d(1 - \Pi) \approx - \beta_i P_m/(1 - \alpha - \beta_i)$.

2/ One standard argument for debt contracts is the costs of monitoring the behavior of the borrower and other informational costs. This argument cannot, however, explain the lack of contingencies that use public information which is exogenous to the borrowers (like the price of oil and other commodities, the real interest rate in the U.S., the growth rate of industrialized nations, etc.).

3/ One of the first attempts to use such contingencies was the rescheduling plan between the IMF and Mexico in the summer of 1986, in which the future supply of credit was made conditional on the price of oil.

$\tilde{\tau} = \beta p_m/(1 - \beta)$ which can be approximated by $\beta p_m\, c(K)^{\alpha/(1-\beta)}/(1+r_f)$. This defines the equivalent of a 'credit ceiling' for the case of equity investment. It is given by a portion $\beta p_m/(1+r_f)$ of the expected output. It can be shown that a useful property of the equity scheme is that it increases the correlation between the income of foreign investors and the default penalty, thereby reducing the incidence of default. In our example, for equity investment below the ceiling defined above we obtain no default because the equity scheme leads to a unitary correlation between the default penalty and the income to foreign owners. It can be also shown that the switch from debt to equity finance has the consequence of increasing the credit ceiling facing the economy. Furthermore, as long as the foreign investment is below the ceiling, an equity scheme will increase the optimal investment. 1/

Our discussion should be viewed only as an example for the benefits of the debt-equity swap. Because we allow the random shock to affect output and the default penalty in the same way we obtain the strong result that equity finance will eliminate defaults up to the credit ceiling. This is done by correlating the repayments with the default penalty. The debt-equity swap, however, is not able to eliminate the impact of country risk and the resulting need to impose a ceiling on the available credit. Instead, it allows us to increase the credit ceiling. It is noteworthy that in the presence of several shocks which affect output and the default penalty in different ways, the move to equity finance will not eliminate incidences of default but it may be beneficial to the degree that it increases the correlation between the default penalty and the repayment. It can be shown that the general optimal contract in the presence of country risk is not an equity finance but rather a loan contract that optimally indexes the repayment to the default penalty, and the credit ceiling subject to such a contract is the expected net present value of the penalty (discounted at the risk free interest rate). 2/

V. Concluding Remarks

The purpose of this study is to draw attention to the linkages between country risk and the openness of an economy, and to demonstrate that in the long run the openness of an economy is endogenously determined by the interaction between endowments and policies. The presence of country risk poses a problem for the smooth operation of international credit markets: the ex-ante first best policy is for countries to pre-commit themselves to no-default policies. Such a

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1/ Formally, the optimal capital stock with equity investment is given by equation (7), for the case where we replace p_m with 0 (assuming that we operate below the credit ceiling).

2/ For further discussion of the optimal contract in the presence of country risk and equity finance see Aizenman (1987).

commitment, however, may not be credible because it may not be the optimal ex-post policy. This suggests a special role for policies leading towards investment in openness - as a way to increase the credibility of a no-default commitment.

The purpose of the Appendix is to summarize the derivation of the results reported in the paper. We start by formulating the problem as a welfare maximization of the representative agent and the central bank. We then derive the optimal credit market and investment policies.

Suppose that the utility of the representative agent is given by (to simplify notation we suppress the consumer index):

$$U = u(C_1) + \delta\, u\,(C_2) \tag{A1}$$

where δ stands for the subjective rate of time preference and C_t is the consumption of traded goods at time t (t = 1, 2). We allow for domestic policies in the form of a borrowing tax and lump-sum transfers. The domestic interest rate for consumption borrowing, r, is defined by

$$r_c = r^*\,(1 + \rho_c) \tag{A2}$$

$$r_i = r^*\,(1 + \rho_i) \tag{A3}$$

where r^* is the interest rate facing the country and ρ_c is the domestic tax on consumption borrowing. Similarly, r_i is the interest rate for investment borrowing in sector i, defined by ρ_i. The budget constraints facing the representative agent are

$$C_1 = Y_1 + B_c \tag{A4}$$

$$C_{2,n} = Y_{2,n} + R - B_c\,(1+r_c) - \sum_{i=1}^{q} I_i\,(1+r_i) \tag{A5}$$

$$C_{2,d} = Y_{2,d} \tag{A6}$$

where subscripts n and d correspond to the cases of no default and default, respectively; and R is a lump-sum transfer (to be specified later). The aggregate investment borrowing and consumption borrowing (B) is obtained as the sum of B_c and I across all the individual borrowers. Equations (A5) and (A6) are the budget constraints for period two for the cases of no default and default, respectively. Domestic agents are assumed to be small enough to be price takers in the domestic credit market. Each faces a given interest rate.

The agent's problem is to allocate consumption and investment so as to maximize his expected utility subject to the budget constraint. Let V denote the value of the expected utility of a representative consumer. It follows that

$$V = u\,(C_1) + \delta\int_0^{\Psi_0} u(C_{2,d})\,d\Psi + \delta\int_{\Psi_0}^{\infty} u(C_{2,n})\,d\Psi \tag{A7}$$

The agent chooses investment I and indebtedness B_c so as to maximize his expected utility, (A7). Because each agent is a price taker in the credit market, Ψ_0 is viewed by the consumer as given. Solving the optimal consumption and investment plan for a representative agent yields the following first-order conditions:

$$\text{(A8)}\quad \begin{aligned} &\text{a. } MU(C_1) = \delta\, E\{MU(C_2)|N.D.\}(1 + r_c) \\ &\text{b. } E\{MU(C_2)|N.D.\}(1 + r_i) = E\{MU(C_2)\ MPK_{2,i}\} \end{aligned}$$

for $i = 1,.., q$.

where $MU(C_t)$ stands for the marginal utility of consumption in period t ($t = 1,2$), $MPK_{2,i}$ stands for the marginal product of capital in period two in activity i, and $E\{Y|N.D.\}$ stands for the expected value of a variable Y conditional on no default (i.e. conditional on $\Psi > \Psi_0$). 1/ Equation (A8) represents two types of intertemporal arbitrage conditions. The first concerns the equality of the marginal utility of consumption at period one to the discounted expected marginal utility of future consumption (conditional on no default) times the interest rate. This is the condition under which the benefit of increasing first period consumption by borrowing equals the future costs associated with repayment. 2/ The second arbitrage condition is with regard to optimal investment borrowing: the expected cost of borrowing (in terms of second period expected marginal utility) should be equated with the expected marginal utility of investment. This is the condition under which the benefit of increasing the investment by borrowing equals the cost associated with repayment.

To gain insight into the potential role of optimal policies let us evaluate the solution of the optimal consumption path by a centralized decision maker. Potential deviations between the planner's and the consumer's solutions will justify policies to support optimality. These policies will be shown to be in the form of optimal borrowing taxes. We assume that the lump-sum transfer R is used to rebate to consumers the proceeds generated by the borrowing taxes. 3/ The planner's problem is to choose consumption borrowing (B_c) and investment borrowing (I_i) that will maximize the welfare of the representative consumer. A key difference between the consumer's and the planner's problems is that the

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1/ Formally, $E\{MU(C_2)|N.D.\} = \delta \int_{\Psi_0}^{\infty} u'(C_{2,n})\, d\Psi$ and $E\{MU(C_2)\ MPK_{2,i}\}$ $= \delta \int_0^{\Psi_0} u'(C_{2,d})(\partial Y_2/\partial K_{2,i})d\Psi + \delta \int_{\Psi_0}^{\infty} u'(C_{2,n})(\partial Y_2/\partial K_{2,i})\, d\Psi$.

2/ Note that since repayment occurs only in states of no default, the expectation operator in (A8 a) is conditional on no default.

3/ Note that this implies that the lump-sum transfer to the consumer is $r^*[\rho_c B_c + \sum_{i=1}^{q} \rho_i I_i]$. Consequently, the budget constraint that is relevant for the policy maker in the absence of default is

$$\text{(A5')}\quad C_{2,n} = Y_{2,n} - \{B_c + I\}\,(1+r^*)$$

centralized planner is not a price taker in the credit market, and he is aware that the choice of borrowing will have an impact on the interest rate via the supply of credit. Consequently, (A7) implies that the condition for optimal borrowing and investment from the planner's perspective is

(A8')

a. $$MU(C_1) = \delta\, E\{MU(C_2)|N.D.\}(1+ r^*)(1 + \frac{d \log (1 + r^*)}{d \log B_c})$$

b. $$E\{MU(C_2)|N.D.\}\ (1 + r^*)(1 + \frac{d \log (1+r^*)}{d \log I_i}) = E\{MU(C_2)\ MPK_{2,i}\}$$

for $i = 1, .., q$.

A comparison between the planner's and the consumer's solutions reveals that the two differ in that the planner applies the social interest rate, whereas the consumer applies the private one. The social interest rate is defined as the total marginal interest cost associated with the borrowing for consumption and for investment activities, given by:

(A9) $$(1 + r^*)(1 + \frac{d \log (1+r^*)}{d \log B_c}) \text{ and } (1 + r^*)(1 + \frac{d \log (1+r^*)}{d \log I_i}),$$

respectively. Note that the social planner face different social interest rates for the various activities. The key difference between the individual agent and the social planner is that the latter is internalizing the marginal changes in the interest rate facing the economy due to marginal borrowing. These changes in turn are determined by the use of these funds, and are reflected in the second term in the social interest rates. The percentage difference between the private and the social interest rates equals the elasticity of the interest rate $(1 + r^*)$ with respect to the borrowing, which in turn define the optimal taxes, as reported in (3) - (4).

We turn now to an overview of the derivation of the equations reported in Section III. Short-run profit maximization with respect to the use of importable M yields the following value for output

(A10) $$X_i = c_i\left[\frac{\Psi}{(P_m)^{\beta_i}}\right]^{1/(1-\beta_i)} (K_i)^{\alpha/(1-\beta_i)}$$

where $c_i = [C_i(\beta_i)^{\beta_i/(1-\beta_i)}]$. Thus, a raise of P_m from 1 to exp (p_m) is associated with a change of output at a rate of:

(A11) $$\exp\,(- \{\beta_i/(1-\beta_i)\}p_m) - 1 \approx - \{\beta_i/(1-\beta_i)\}p_m.$$

Note that a portion β_i of output is spent on the imported input. Thus, the value added is $Y_i = (1 - \beta_i)X_i$, and (A11) implies that the drop in value added in sector i resulting from the default is $(1 - \beta_i)X_{n,i}\{\beta_i/(1-\beta_i)\}p_m = \beta_i X_{n,i} p_m$. Aggregating the drop in the value added across sectors gives us equation (6) in the text.

We turn now to the derivation of the value of λ_i that is applied in (4). Applying (6) and the definition of the marginal value of the productivity shock that is associated with default (Ψ_0) we get that Ψ_0 is the solution to

$$\text{(A12)} \qquad (1 + r^*)\, B = p_m \sum_{i=1}^{2} \beta_i c_i \left[\frac{\Psi_0}{(P_m)^{\beta_i}}\right]^{1/(1-\beta_i)} (K_i)^{\alpha/(1-\beta_i)}$$

Note that $\lambda_i = \partial \log \Pi / \partial \log K_i$. Recalling that $\Pi = \int_{\Psi_0}^{\infty} f(\Psi)\, d\Psi$ we obtain by logarithmic derivation that

$$\text{(A13)} \qquad \lambda_i = - f \frac{\Psi_0}{\Pi} \frac{\partial \log \Psi_0}{\partial \log K_i}.$$

Logarithmic derivation of (A12) yields that:

$$\text{(A14)} \qquad \frac{\partial \log \Psi_0}{\partial \log K_i} = - \frac{\alpha s_i/(1-\beta_i)}{s_1/(1-\beta_1)+s_2/(1-\beta_2)}$$

where s_i is the i's sector share in the aggregate penalty at the marginal default (i.e., $\Psi = \Psi_0$):

$s_i = \dfrac{\beta_i X_{n;i}}{\beta_1 X_{n;1} + \beta_2 X_{n;2}}$ where the X's are obtained by (A10) evaluated at

$\Psi = \Psi_0$ and $P_m = 1$. Applying similar derivation we get also that

$$\text{(A15)} \qquad \varepsilon = - \frac{\partial \log \Pi}{\partial \log B} = - f \frac{\Psi_0}{\Pi} \frac{\partial \log \Psi_0}{\partial \log B} = f \frac{\Psi_0}{\Pi} \frac{1}{s_1/(1-\beta_1)+s_2/(1-\beta_2)}$$

Applying (A15) and (A14) to (A13) yields that $\lambda_i = s_i \varepsilon \alpha/(1-\beta_i)$, which the result applied in the text to derive (4').

We turn now to the derivation of (7). Equation (A10) implies that the marginal product of capital is given by:

$$\text{(A16)} \qquad MPK_i = c_i' \left[\frac{\Psi}{(P_m)^{\beta_i}}\right]^{1/(1-\beta_i)} (K_i)^{\alpha/(1-\beta_i)-1}$$

where $c_i' = \{\alpha/(1-\beta_i)\} c_i$. Denoting by ψ the value of log (Ψ) we obtain that

$$\text{(A17)} \qquad MPK_i = \begin{cases} c_i \exp\{\psi/(1-\beta_i)\}\,(K_i)^{\alpha/(1-\beta_i)-1} & \text{no default} \\ c_i \exp\{(\psi - p_m\beta_i)/(1-\beta_i)\}\,(K_i)^{\alpha/(1-\beta_i)-1} & \text{default} \end{cases}$$

Optimal investment is made so as to yield equality between the expected cost $(1 + r_f)$ and the expected marginal product of capital. Note that for small shocks we can approximate $\exp\{(\psi - p_m\beta_i)/(1-\beta_i)\}$ by $1 + (\psi - p_m\beta_i)/(1-\beta_i)$. Applying this approximation we obtain that the expected marginal product of capital is

$$\text{(A18)} \qquad \begin{aligned} E\left[MPK_{i;2}\right] &\approx c_i'\left[1 - \frac{E(p_m\beta_i \mid \Psi < \Psi_0)}{1-\beta_i}\right](K_i)^{\alpha/(1-\beta_i)-1} \\ &= c_i'\left[1 - (1-\Pi)\,\frac{p_m\beta_i}{1-\beta_i}\right](K_i)^{\alpha/(1-\beta_i)-1} \end{aligned}$$

The optimal stock of capital is obtained by solving the K that equates (A18) to $1 + r_f$, yielding (7) in the text.

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