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The Impact of Inflation Upon Portfolio Choice: A Duality Approach Using U.K. Data

by William Perraudin*

Authorized for Distribution by Mario I. Blejer

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Abstract

By exploiting basic similarities between the theories of portfolio choice and consumption, this paper specifies and estimates a complete system of asset and liability demands on U.K. data over the period 1976-85. The main implication of the results is that inflation is *highly significant to portfolio choice, affecting both the broad* allocation between liabilities, real capital, and financial assets, and the more detailed allocation within these categories. However, it influences this choice in ways that would not greatly deter one from adjusting government deficits for inflation.

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Adjusting measures of the government deficit to account for inflation has recently stimulated considerable interest. A common argument is that depreciation in the real value of government bonds owing to inflation should be subtracted from debt-service charges in calculating the deficit. The resulting figure certainly gives a clearer picture of the government's net demand for real saving from the private and overseas sectors. Of course, it may still be difficult to assess the impact of fiscal policy upon the economy as a whole, given that the various components of the deficit affect aggregate demand to differing degrees. Nevertheless, the inflation-adjusted deficit might, on the surface, appear to represent an advance over the unadjusted balance.

One objection to the simple inflation adjustment of government deficits is, however, that inflation could have significant portfolio effects. This paper employs a fully specified portfolio model to assess the empirical importance of this argument. By exploiting basic similarities between the theories of portfolio choice and of consumption, techniques more commonly used for modeling consumer demand are applied. Among other advantages, this allows the adoption of quite convincing aggregation and separability assumptions while still permitting empirically tractable functional forms for the final estimating equations. The model turns out to fit the data quite well.

The results suggest a number of interesting conclusions. In particular, inflation induces an increase in the demand for both financial assets and liabilities, while desired holdings of real capital are unaffected. Within the category of financial assets, inflation produces significant substitution effects through the rate of return on cash and sight deposits. Some substitution away from government bonds is apparent although this is more than offset by an increase in demand for financial assets as a whole. In summary, inflation appears to have significant portfolio effects but they do not appear likely to compromise the usefulness of inflation-adjusted deficit measures.

I. Introduction

The adjustment of measures of the government deficit for inflation has recently stimulated considerable interest. 1/ A common argument is that the depreciation in the real value of government bonds due to inflation should be subtracted from debt service charges when calculating the deficit. The resulting figure certainly gives a clearer picture of the government's net demand for real saving from the private and overseas sectors. Of course, it may still be difficult to assess the impact of fiscal policy upon the economy as a whole, given that the various components of the deficit affect aggregate demand to differing degrees. Nevertheless, the inflation adjusted deficit might, on the surface, appear to represent an advance over the unadjusted balance.

However, the simple adjustment of deficits for inflation ignores potentially important portfolio effects. For example, if higher inflation induces substitution from money into bonds and real assets, then an increase in inflation could prove expansionary even though the adjusted deficit remained constant. Such effects may be demonstrated in either short-run business cycle models (Mundell (1963)) or in long-run monetary growth models (Tobin (1965), Sidrauski (1967)). Other portfolio shifts could occur if, as seems plausible, inflation affects the risk characteristics of different assets and liabilities. This could be attributable either to changing probabilities of default by liquidity-constrained borrowers or to uncertainty about the time profile of real repayments if, say, the variance of inflation increased with its level. Lastly, inflation illusion, if present, could cause significant portfolio shifts. Modigliani and Cohn (1979) have suggested that investors systematically undervalue equities in the presence of inflation because of a failure to take account of the depreciation in firms' nominal liabilities.

In general, then, it may not be possible to regard an inflation-adjusted deficit in a country with rapidly rising prices as equivalent to a similar unadjusted deficit in an economy where inflation is low or nonexistent. The question of how important are the various portfolio shifts is necessarily empirical. This paper employs a fully specified portfolio model in an attempt to disentangle and evaluate the portfolio effects involved.

II. The Overall Approach

If portfolio demands were independent of inflation, one would expect Fisher's law to hold, that is:

$$r_{nominal} = r_{real} + \left(\frac{\dot{P}}{P}\right)_{expected}$$

1/ References include Siegel (1979), Taylor and Threadgold (1979), Miller (1981), and International Monetary Fund (1986).

where P $\equiv$ consumer price level, and the real interest rate is independent of inflation. One way to investigate the portfolio effects of inflation would be to estimate reduced-form relations of this kind. However, this approach has distinct problems. Both real interest rates and expected inflation are clearly endogenous and so instrumental variables methods would have to be used. Finding a suitable instrument for expected inflation, however, is well-nigh impossible since any shock to nominal interest rates is likely to affect price expectations as well. Fama (1976) sidesteps the problem by adopting a different specification together with the strong assumptions of constant real interest rates and rational expectations. Summers (1984) employs band spectral regression techniques to filter out low frequency movements in the variables. The fact that he is then essentially dealing with steady states justifies his use of actual rather than expected inflation. The problem of finding suitable instruments for expected inflation then disappears.

An alternative and in many ways preferable approach is to estimate directly a complete system of asset and liability demands. 1/ This should permit a differentiation among the several channels through which inflation affects portfolio decisions and should lead to a much richer understanding of what actually occurs within asset markets when the level of inflation changes.

The recent literature on modeling asset demands suggests a number of possible starting points for our study. Friedman and Roley (1979), Conrad (1980), Friedman (1985), Aivazian and others (1983), and Taylor and Clements (1983) all present explicit portfolio models involving utility maximization. An important advantage of assuming such maximization is that it implies restrictions upon the coefficients in the derived demand equations. The job of estimating asset demands, which typically have highly colinear explanatory variables, is much facilitated by such restrictions.

The approach used in this paper comes closest to resembling that of Conrad although the present model differs from his in a good many important respects. While Conrad starts from an indirect utility function in deriving his demand equations, this paper proceeds instead of expenditure functions. Using expenditure functions permits the use of the so-called Almost Ideal Demand System (or AIDS), suggested by Deaton and Muelbauer (1980), in the context of consumer demand theory. This approach will be seen to possess very desirable aggregation

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1/ A variant of this direct approach is applied by Friedman (1980). In his paper, Friedman assesses the elasticity of nominal loan interest rates with respect to inflation by estimating loan supply equations for six categories of lenders in the U.S. market. While his method has the advantage of disaggregating broad types of investment institutions, his failure to estimate more than a single demand equation for each category of institution limits the scope of his study and means that his parameter estimates are inefficient.

properties. Since restrictions based on utility maximization by individual investors will be applied to aggregate demand functions, it is important to be clear about the aggregation assumptions involved. Another major difference from Conrad's paper is that the present study takes full account of expectations formation. Asset demands are functions of expected--not realized--rates of return. Since Conrad estimates his system with realized interest rates and yields, his parameter estimates are all inconsistent.

III. The Model

In deriving the model, it is supposed there exists a utility function defined on the expected future value of current asset and liability stocks.

$$U_t = U(RA_{1t} \ldots RA_{Lt}, RK_{1t} \ldots RK_{Mt}, RL_{1t} \ldots RL_{Nt})$$

where:

$$RA_{it} \equiv (1 + r_{it}^{A}) \cdot A_{it} \qquad i = 1 \ldots L$$

$$RK_{jt} \equiv (1 + r_{jt}^{K}) \cdot K_{jt} \qquad j = 1 \ldots M$$

$$RL_{kt} \equiv (1 + r_{kt}^{L}) \cdot L_{kt} \qquad k = 1 \ldots N$$

Here, r_i^A, r_j^K, and r_k^L are the expected real holding returns on the i^{th} financial asset, j^{th} form of real capital, and k^{th} liability, respectively. As such, they include expected appreciation in the value of the investment. A_{it}, K_{jt}, and L_{kt} represent the real stocks (deflated by a consumer price index) of the i^{th} asset, j^{th} real capital, and k^{th} liability. Liabilities are, by convention, introduced as negative numbers.

There are several ways to justify such a utility function. The simplest assumption one might make is that investors care about expected future wealth and about the expected flows of services that they derive from their asset and liability holdings. If the service flows (housing and liquidity services, for example) are monotonically increasing functions of these holdings, then the utility function can be transformed into a function of expected future stocks. 1/ This approach follows Feige (1964).

1/ Since we are including expectations inside a utility function, we must assume that these expectations are held with subjective certainty.

An alternative justification for the above functional form is provided in Appendix I, where a similar utility function is derived from a general, expected-utility model. Adopting this approach, the shape of the indifference curves may be taken to reflect the risk characteristics of the different assets and liabilities. A further interesting point demonstrated in the Appendix is that if investors care about the second and higher moments of future wealth, Slutsky symmetry should not hold in our final demand equations. It therefore becomes possible, by testing symmetry, to gauge the importance of risk for asset and liability demands. 1/

For convenience it is assumed that the utility function is strictly quasi-concave, twice continuously differentiable, and strictly increasing in all its arguments. The investor's problem is then:

$$\underset{RA_i RK_j RL_k}{\operatorname{Max}} U \quad (RA_1 \ldots RA_L, RK_1 \ldots RK_M, RL_1 \ldots RL_N)$$

subject to:

$$\sum_i A_i + \sum_j K_j + \sum_k L_k = W$$

where W is the stock of real wealth and time subscripts are omitted. Rewriting the budget constraint gives:

$$\sum_i \frac{RA_i}{1 + r_i^A} + \sum_j \frac{RK_j}{1 + r_j^K} + \sum_k \frac{RL_k}{1 + r_k^L} = W .$$

Evidently, the problem is identical to one of consumer demand in which the RA_{it}, RK_{jt}, and RL_{kt} represent goods, and prices are of the form:

$$\frac{1}{1 + r_i^A}, \quad \frac{1}{1 + r_j^K}, \text{ and } \frac{1}{1 + r_k^L} .$$

Taking advantage of this isomorphism, one may define a cost or expenditure function as:

$$C(U, P_A, P_K, P_L) = \underset{RA_i RK_j RL_k}{\min} \sum_i \frac{RA_i}{1 + r_i^A} + \sum_j \frac{RK_j}{1 + r_j^k} + \sum_k \frac{RL_k}{1 + r_k^L}$$

subject to:

1/ For simplicity, the exposition in the main text of the paper proceeds with a simple utility function defined on the RA_i, etc. To understand how risk complicates matters, readers should consult Appendix I.

$$U(RA_1 \ldots RA_L, RK_1 \ldots RK_M, RL_1 \ldots RL_N) = \overline{U}$$

where:

$$P_x = \frac{1}{1 + r_i^X} \ldots \frac{1}{1 + r_S^X} \qquad \text{for} \quad X = A, K, L \quad S = L, M, N$$

Inflation appears in this cost function through r_1^A, the real return on sight deposits and cash, which is, of course, negative the rate of inflation. However, it may also be included, together with a time trend, as direct additional arguments in the utility function. The idea here is that the flows of services derived from the asset and liability stocks might vary systematically with inflation. Alternatively, if the rationale for our utility function given in Appendix I is adopted, then it might be argued that inflation or a time trend could influence the covariance structure of returns implicitly summed up in the shape of the indifference curves. In either case, the resulting cost function can be differentiated to give, by Shephard's lemma, a complete system of compensated demand functions. 1/

To keep the estimation problem tractable, the number of parameters is reduced by assuming quasi-separability of the cost function between financial assets, real capital, and liabilities. 2/ Although in practice one adopts such a separability assumption to simplify estimation, it may be rationalized as reflecting the presence of information costs that induce investors to adopt a form of multistage budgeting. Thus, wealth is allocated first to broad groups of investments and then subsequently within those groups. The form of the cost function is thus:

$$C = C(U, C_A(U, P_A, D), C_K(U, P_K, D), C_L(U, P_L, D), D)$$

where:

$$D \equiv (T, E(\frac{\dot{P}}{P}))$$

$$T \equiv \text{time trend.}$$

One may note, in this context, the virtue of quasi-separability of the cost function compared with the more usual assumption of separability in the indirect utility function. The latter type of separability requires the additional assumption of homothetic sub-indirect utility functions in order to ensure multistage budgeting. 3/ In a sense, therefore, implicit separability is more general.

1/ See Diewert (1974), and Shephard (1953).
2/ The notion of quasi-separability was introduced by Gorman (1976).
3/ See Geary and Morishima (1973).

Separability of an indirect utility or cost function involves a partitioning of the price space in the consumer's demand decision. This may seem unintuitive to those who are used to thinking in terms of traditional direct utility functions. Deaton (1979) demonstrates that quasi-separability implies a corresponding separability of the distance function (that is, the cost function's dual) and hence, a partitioning in the space of goods. To ensure that separability in the indirect utility function implies separability in direct utility and therefore in goods space, requires the additional assumption of homothetic sub-utility functions however. 1/

As a final comment on our separability assumption, one may note, following Gorman (1976) that in practice, quasi-separability amounts to assuming a particular pattern of substitution within the Slutsky matrix. Thus, if i and j are members of groups A and K respectively, then the Slutsky substitution term is of the form:

$$S_{ij}^{AK} = \phi_{AK} \; RA_i \; RK_j \; .$$

Utility maximization and the various assumptions that have been made imply a cost function that, apart from quasi-separability and the usual properties of cost functions, is arbitrary and unknown. To approximate this, one may adopt one of the several flexible forms suggested in the consumer demand literature. These are almost all second-order approximations and, therefore, in principle, possess the same degree of generality. The so-called PIGLOG, or Price-Independent-Generalized-Logarithmic family of flexible forms, has the advantage, however, of highly desirable aggregation properties. Since the aim is to impose the restrictions of individual utility maximization on aggregate demand, it is important to adopt reasonable aggregation assumptions. Within the PIGLOG class, the Almost Ideal Demand System, or AIDS, proposed by Deaton and Muellbauer (1980) is chosen. This is used to represent both the upper and lower level cost functions. The aggregation properties of the twin-level AIDS system are discussed in Appendix II.

The approximated upper and lower level cost functions may therefore be written as:

$$\log C = \alpha_o + \sum_j \alpha_j^* \log C_j + 1/2 \sum_i \sum_j \beta_{ij}^* \log C_j \log C_i + \mu_o U \prod_{k=1}^{K} C_k^{\mu_k}$$

$$\log C_j = \lambda_o^j + \sum_i \lambda_i^j \log \frac{1}{1 + r_i^j} + 1/2 \sum_i \sum_k \gamma_{ik}^{*j} \log \frac{1}{1 + r_i^j} \log \frac{1}{1 + r_k^j}$$

1/ See Lau (1970).

$$+ \xi_o^j U \cdot \prod_{k=1}^{K} \frac{1}{1 + r_k^j} \xi_k^j \quad \text{for} \quad j = A, K, L. \text{ 1/}$$

Using Shephard's lemma, we may take the logarithmic derivative of the upper cost function, with respect to the composite price indices, to find W_j, the compensated desired portfolio share for investment group j.

$$W_j = \frac{\sum_i X_{ij}}{W} = \frac{d \log C}{d \log C_j} = \alpha_j^* + \sum_i \beta_{ij} \log C_i + \mu_j \mu_o U \prod_k C_k^{\mu_k}$$

where:

$$\beta_{ij} \equiv 1/2 \; (\beta_{ij}^* + \beta_{ji}^*)$$

and where X_{ij} is the consumer price deflated holding of asset or liability i within group j.

To eliminate the unobservable variable U, one substitutes from the cost function itself to give:

$$W_j = \alpha_j^* + \sum_i \beta_{ij} \log C_i + \mu_j \log \left(\frac{C}{P^*}\right) \qquad (1)$$

where:

$$\log P^* \equiv \alpha_o + \sum_j \alpha_j^* \log C_j + \sum_i \sum_j \beta_{ij} \log C_j \log C_i$$

C = total wealth.

One should note that the subcost functions C_i that appear in this equation are unobservable, weighted indices of the rates of return within the group. The desired share of a particular asset or liability i within a group j may be found by taking the logarithmic derivative of the relevant subcost function or price index C_j. Thus:

$$W_{ij} = \frac{X_{ij}}{\sum_i X_{ij}} = \frac{\partial \log C_j}{\partial \log \dfrac{1}{1 + r^j}} = \lambda_i^j + \sum_k \gamma_{ik}^j \log \frac{1}{1 + r_k^j}$$

$$+ \xi_i^j \xi_o^j U \prod_k \frac{1}{1 + r_k^j} \xi_k^j \quad \text{for } j = A, K, L,$$

1/ For notational simplicity, the time trend and inflation variable are omitted. These enter into the second-order approximation in exactly the same way as the prices.

where:

$$\gamma^j_{ik} \equiv 1/2\ (\gamma^{j*}_{ik} + \gamma^{j*}_{ki})$$

Again, one may substitute for U to give:

$$W_{ij} = \lambda^j_i + \sum_k \gamma^j_{ik} \log \frac{1}{1 + r^j_k} + \xi^j_i \log \frac{C_j}{P_j} \tag{2}$$

where:

$$\log P_j \equiv \lambda^j_o + \sum_i \lambda^j_i \log \frac{1}{1 + r^j_i} + \sum_i \sum_k \gamma^j_{ik} \log \frac{1}{1 + r^j_i} \log \frac{1}{1 + r^j_k} .$$

Although it is possible, in principle, to estimate the complete model (1) and (2), using methods suggested by Pudney (1980) this would involve a complicated sequence of iterations. Given the fairly large model and the fact that expected rather than realized returns are used, Pudney's suggestion is impracticable. The problem really stems from the presence of C_j in both the upper and lower demand equations. Since the C_j are in themselves unobservable, they have to be substituted out before the system is estimated. One way to get around this problem is to assume $\xi^i_o = 0$ for all i; in other words, that U does not enter into the lower cost functions. This amounts to assuming homothetic quasi-separability. It might then seem that the advantages of using quasi-, as opposed to direct, separability are thereby diminished. Nevertheless, with quasi-separability, it is possible to test the exclusion of U from the subcost functions using limited information methods. Appendix III develops this point further.

Adopting the restriction, the complete demand system becomes:

$$W_j = \alpha^*_j + \Sigma\ \beta_{ij} \log C_i + \mu_j \log (\frac{W}{P^*})$$

$$W_{ij} = \lambda^j_i + \sum_k \gamma^j_{ik} \log \frac{1}{1 + r^j_k}$$

where:

$$\log P^* \equiv \alpha_o + \sum_j \alpha_j \log C_j + \sum_i \sum_j \beta_{ij} \log C_j \log C_i$$

$$= \alpha_o + \sum_j \log C_j\ (\alpha_j + \Sigma\ \beta_{ij} \log C_i) .$$

Following the suggestion of Deaton and Muellbauer (1980), the collinearity of the asset returns can be exploited by approximating P* with the expression:

$$\log P^* \simeq \alpha_o + \sum_j \log C_j W_j$$

$$= \alpha_o + \sum_j W_j \lambda_o^j + \sum_i \sum_j W_j W_{ij} \log \frac{1}{1 + r_i^j} .$$

Absorbing the second term in the constant, P* may be written as:

$$\log P^* \simeq \alpha_o + \sum_i \sum_j W_j W_{ij} \log \frac{1}{1 + r^j} .$$

Since α_o will not be separately identified from the constant term in each equation, this reduces to:

$$\log P = \sum_i \sum_j W_j W_{ij} \log \frac{1}{1 + r_i^j}$$

and constant terms $\alpha_j \equiv (\alpha_j^* - \mu_j \alpha_o)$.

As a last step in specifying the model, dynamics may be introduced through a simple and classic stock adjustment process. Actual demand is taken to equal lagged demand plus a fraction of the discrepancy between desired and lagged holdings. Thus:

$$W_{jt} = W_{jt-1} + \sigma (W_{jt}^* - W_{jt-1})$$

$$W_{ijt} = W_{ijt-1} + \sigma_j (W_{ijt}^* - W_{ijt-1})$$

$$j = A, K, L$$

where * denotes the desired share.

Clearly, this is a strong assumption. It allows demand for a particular good to be affected only by the disequilibrium in that good. In other words, the adjustment matrix is diagonal. As Friedman (1977) points out, portfolio adding-up constraints imply (in the case of demand share equations) that the columns of the adjustment matrix sum to a constant. If each column contains only one non-zero element, then clearly these elements must be equal. The assumption of separability improves matters slightly, however, since the adding-up requirements apply to shares within each of the three subgroups and to the shares explained by the upper level equations. Thus, with three subgroups, there are effectively four adjustment parameters. Adding stochastic error terms to the equations completes the specification of the model.

IV. The Treatment of Expectations

In estimating the demand system, it is essential to take account of the fact that asset and liability demands respond to anticipated rather than realized returns. As is well known, using realized values to proxy the rational expectations of a variable generally leads to errors-in-variables bias, since the realized variable is effectively the rational expectation plus a random error. In the case of a single regressor, this pushes the coefficient toward zero. With more than one right-hand-side variable, the direction of bias is indeterminate. In the present case, the drawbacks in using realized values are even greater since rates of return are endogenous. To illustrate this problem, if some random shock increases the demand for an asset, with given supply, the own-return interest rate will rise, tending to push down the price and bias the coefficient upwards. Although in a large system with many regressors it is impossible to predict the direction of the bias for individual coefficients, it seems likely that own rate of return coefficients will appear larger using realized returned data. When the model was run with realized data, this was indeed found to be the case. For these reasons, Conrad's (1980) reliance on realized return data is highly questionable.

The two most obvious techniques for modeling expectations in this context might be seen as corresponding to limited and full information methods. Following McCallum (1976), a common procedure for treating expectations variables has been to employ realized values instrumented by their own regression on all the independent variables in the models. Assuming rational expectations, these fitted values should be independent of the error terms in the demand model and, hence, consistent estimates may be achieved even though the true expectations variable is unobserved.

As an alternative, one may model the process of expectations formation directly, expanding the demand system to include equations that explain realized values as optimal distributed lags on past rates of return and other lagged variables. Substituting these equations in wherever rates of return appear in the demand equations, one may achieve efficient and consistent estimates by imposing cross-equation restrictions between the demand and the rate of return equations. This amounts to assuming a form of backward-looking rational expectations which is close to the original Muthian sense of the term (see Muth (1961)). Individuals use information optimally, but the only information they have consists of lagged values of rates of return and other variables. A representative example of the use of such methods in another context is Mishkin (1982).

In what follows, the second method is applied. Although instrumental variable techniques were tried, the colinearity of the returns data led in turn to highly colinear instruments. Using these instruments, the model failed to converge. One problem with the method adopted is that expectations formed at the start of the sample period are effectively based on information from the whole sample. A way to overcome this is to run regressions on, say, the twenty quarters preceeding each observation and then to use these regressions to construct a series of fitted values. Unfortunately, this method does not allow one to apply cross-equation restrictions between the demand system and the expectations equations. In any case, it proved impracticable for this model, given that some of the returns series only went back to the first quarter of 1975.

V. Taxation

An important consideration not yet dealt with is the potential significance of taxation. In particular, it is necessary to establish whether the use of pretax rates of return will make the estimates inconsistent. Three issues stand out. First, tax rates may differ across individuals. In principle, with detailed cross-sectional data, account could be taken of this using exact aggregation theory (see Jorgenson, Lau, and Stoker (1982)). Indeed, this would be a good area for further research.

Second, taxes may vary over time. For example, in the United Kingdom, top-rate taxpayers saw a steep decline in their marginal tax rates over the sample period. Although this was true for those in the upper income brackets, tax rates for the average saver seem to have been more stable.

Third, and perhaps most important, tax rates will vary across different instruments. This would not be a problem if taxes were just levied as fixed percentages at real returns, since one could then regard the tax rates as absorbed into the estimates of the price coefficients. As is well known, however (see, for example, Feldstein (1980)), the effective tax rates on real rates of return depend, in a complex way, upon the level of inflation. Using pretax rates of return might be thought to result therefore in seriously inconsistent estimates. Fortunately, since our estimating equations include inflation as a direct variable, one can show that almost any effective tax rate may be absorbed into the price and inflation coefficients combined (see Appendix IV).

To some extent, absorbing the tax coefficients in this way is unsatisfying since the Slutsky substitution parameters are then no longer identified. The procedure, however, at least ensures that our estimates of the effect of inflation and of pretax rates of return on asset demands are reliable and consistent. Full identification of the underlying substitution parameters would require accurate estimates of marginal tax rates. This is an area for further research. The various possible welfare applications of the present model would, after all, require estimates of the true substitution terms.

A further and important point is that, if the parameter estimates contain tax rates, Slutsky symmetry is unlikely to hold in our estimated demand equations. Tests of symmetry therefore become a means of assessing the importance of differing tax rates for asset demands.

VI. The Data

U.K. data is used in the study. This seems appropriate, first because U.K. inflation has varied quite considerably since the mid-1970s. Compounded quarterly, inflation rates have ranged from 3 percent to 45 percent in the last 10 years. Second, U.K. data are unusually complete. The Central Statistical Office has recently begun to publish full, quarterly balance sheets for the main sectors of the U.K. economy. The series have been extended back to the first quarter of 1975. For this study, a data set of 39 observations, from QII-1976 to QIV-1985 is used. The aggregate looked at is the nonbank private sector whose balance sheets include information on 15 financial assets and 12 liabilities, all calculated at market value.

Combining a number of these aggregates, we ended up with six financial assets and two liabilities. Broadly speaking, our asset categories were: (1) sight deposits and cash; (2) time deposits; (3) national savings (nonmarketable government savings bonds and deposits); (4) short-term assets including Treasury bills and trade credit; (5) government bonds; and (6) net foreign assets. Liabilities were divided into: (1) sterling bank lending, and (2) non-sterling bank lending (by U.K. banks).

It seemed important as well to include some form of real capital in this study. This was especially the case since the study's ultimate objective is to trace out the impact of inflation upon real interest rates. The substitutability of real and financial assets had therefore to be considered. The real capital series included were: (1) the capital stock of industrial and commercial companies (including the North Sea sector), and (2) dwellings owned by the personal sector. These aggregates should provide close proxies for the housing and productive capital stocks of the nonbank private sector. Consumer durables and land were omitted from the study due to the lack of comparable data.

In constructing the rate of return series for marketable instruments, an attempt was made to calculate the full return including appreciation. Thus, percentage changes in either investment price indices or representative asset prices were added to coupon and dividend-based rates of return. The list of assets used included: (1) negative the percentage change in the retail price index; (2) the seven-day Clearing Bank deposit rate; (3) an index of National Savings Bank and certificate interest rates; (4) the three-month Treasury bill rate; (5) an index of government bond interest rates; and (6) an index of Bank of England rates of return for foreign assets and liabilities.

The government bond index was constructed by weighting the coupon and price appreciation for three categories of 5-year, 10-year, and 15-year gilt-edged securities. The weights chosen reflected the nonbank private sector's holdings of 0 to 5, 5 to 15, and over 15-year gilts as reported by the Bank of England. Unfortunately, the figures on holdings referred to nominal rather than market holdings, but these were the best available. The index for net foreign assets was constructed by weighting rates of return for foreign assets and liabilities, using Bank of England estimates of the United Kingdom's external balance sheet. Since the figures were annual, we had to interpolate in order to arrive at quarterly series.

For the two liabilities, indices were again constructed. The return on non-sterling bank lending was approximated using an SDR interest rate (constructed, in turn, as a basket of national money market rates) plus the appreciation in the SDR against sterling. This was taken to be a reasonable proxy for the true return, since the primary components in the SDR basket are the major EMS currencies and the dollar. The sterling bank lending category includes a number of disparate assets and liabilities including, for example, bank shares held by the nonbank private sector. As far as possible, the rate of return index takes account of these various components, many of which, in any case, were quite small.

VII. Estimation

For estimation purposes, four of the total twelve demand equations were dropped. This was necessary because both the upper level demand equations and the three sets of lower level equations explain desired shares summing to one, and hence the error terms were dependent, leading to a singular covariance matrix. In principle, with Full Information Maximum Likelihood, it should not matter which equations are dropped. The categories chosen for omission were national savings, housing, and non-sterling bank lending, at the lower level, and liabilities at the upper level. These categories were also selected as the numeraire variables for the purpose of imposing homogeneity. Symmetry of the underlying substitution parameters was expected to hold, given the fairly reasonable aggregation assumptions. The use of pretax rates of return (see Section V), however, and the potential importance of the variance and higher moments of future wealth (see Appendix I) implied that the price coefficients in the estimated demand equations might not be symmetric. Hence, while homogeneity was imposed from the outset and not subsequently examined, symmetry was left as a restriction to be tested.

In the notation used above, symmetry and homogeneity imply, respectively:

$$\gamma^j_{ik} = \gamma^j_{ki} \qquad \text{for all } i, k \in j \text{ and for every } j$$

$$\sum_k \gamma^j_{ik} = 0 \qquad \text{for all } i, j$$

at the lower level; and

$$\beta_{ij} = \beta_{ji} \qquad \text{for all } i, j$$

$$\sum_j \beta_{ij} = 0 \qquad \text{for all } i, j$$

at the upper level.

Since equations had been dropped, the adding-up restrictions were not imposed, but were used instead to identify parameters in the omitted equations. In this notation, the adding-up constraints were:

$$\sum_i \lambda^j_i = 1 \qquad \sum_i \gamma^j_{ik} = 0$$

$$\sum_j \alpha_j = 1 \qquad \sum_j \beta_{jk} = 0 \qquad \sum_j \mu_j = 0 .$$

For estimation, the MINDIS routine within the econometric package RAL was used. This provides nonlinear minimum distance estimates based on methods suggested by Amemiya (1974) and developed by Berndt and others (1974). The resulting estimator is essentially nonlinear three-stage-least squares, reducing, in the case of our model, to nonlinear SURE estimation. Asymptotically, SURE has all the desirable properties of maximum likelihood.

Owing to the size of the model, it was not possible to start with the most general specification and work down (the procedure correctly advocated, in a different context, by Davidson and others (1978)). The approach taken was, therefore, to begin by estimating the expectations formation equations, progressively eliminating variables. Each rate of return was regressed on itself lagged from one to four quarters, inflation lagged one and two quarters, on a constant and on a measure of the gap between actual and trend real GNP. It also proved necessary to insert a dummy variable for the VAT changes in 1979. Initially containing 90 parameters, the expectations equations finished with 27. The likelihood ratio statistic for the 53 excluded parameters was 110, which just satisfied the 5 percent chi-squared value for null hypothesis rejection given small sample adjustment.

Normally, the next step would have been to run the reparametrized rate of return equations jointly with the full demand system. A limit within the MINDIS program prevented this, however. A compromise was therefore struck by running expectations and sub-demand equations together. Using the fitted rates of return as independent variables, the upper and lower demand equations were estimated as a complete system. The procedures used imply that, while the cross-equation restrictions between upper and lower demand systems and between rate of return and lower demand systems were correctly imposed, the restrictions between the two upper demand equations and the rate of return equations were neglected. Although this made no difference to the consistency of the estimates, some small degree of efficiency was sacrificed.

After estimating the full demand system in the manner outlined above, the analysis proceeded by testing for symmetry. The likelihood ratio statistic for symmetry at the lower level proved to be 10, well below the relevant 5% χ^2 value of 18.3. When the further restriction of symmetry in the upper level demand equations was imposed, however, the likelihood ratio rose sharply to 22. This occurred despite the fact that only a single additional parameter was being constrained. While one can still show that this much higher likelihood ratio falls short of a strict 5% rejection level when full allowance is made for small sample bias (see Meisner (1979)), the data nevertheless suggest that upper level symmetry does not hold.

Having imposed lower but not upper level symmetry, the next step was to set to zero parameters with t-ratios less than unity. Some own-price coefficients were kept despite their low significance.

The likelihood ratio test for the joint removal of the omitted parameters was 8, well inside the 5% significance level of 15.5. As a final check we examined the residuals for autocorrelation. If autocorrelation was found then it could be regarded as an indicator either of misspecification in the equations or of problems with the assumption of white noise random errors. In plotted form the residuals showed a reasonable pattern while equation-by-equation Durbin h statistics generally registered acceptable values.

VIII. Empirical Results

In general, the empirical results from the model were pleasing. All the own-price elasticities were the right sign, 21 of the 41 parameters in the demand system had t-ratios greater than two, and all but 3 had ratios greater than unity.

The implications of the parameter values were also quite interesting. Just to recap, in this scheme, inflation influences asset and liability demands through two channels. First, there is a direct substitution effect between cash and sight deposits and other investments. Second, inflation enters the demand equations directly through terms that could be seen as measuring the impact of inflation either upon the covariance structure of returns or upon the various service flows derived from holdings of assets and liabilities. These direct terms would also tend to have significant coefficients if, as suggested in Section VI, effective tax rates depend upon inflation.

Looking at the parameter estimates for the financial asset demand system (see Table 1), one is immediately struck by the insignificance of the direct inflation terms. (The only exception appears in the net foreign assets equation, suggesting that a rise in inflation induces some transfer of funds abroad). In fact, this lack of significance accords neatly with the finding that symmetry holds in the lower level demand equations. It may be recalled that if either risk or differing tax rates are important for asset demands, then symmetry should be expected to break down. On the other hand, significant coefficients on the direct terms may be interpreted as showing the impact of inflation either upon relative riskiness or upon effective tax rates. Thus, the finding that symmetry holds and that the direct terms are insignificant is probably a dual reflection of the fact that neither risk nor inflation-dependent tax rates are major influences upon desired shares within the financial asset category.

While not directly influencing the allocation of financial assets, inflation does lead to sizable substitution effects through the price terms of sight deposits and cash. This substitution consists mainly of a switch from sight deposits and cash into savings deposits. Increased inflation, however, also discourages the demand for marketable government bonds while pushing up the desired share of national savings. The one non-intuitive result of the analysis in this paper is the marked shift away from short-term securities, although interpreting the demand for these short-term instruments is fraught with difficulty since the Bank of England was intervening on a massive scale in this market during much of the sample period.

Tables 2 and 3 present the results for the liability and real capital demand systems. These require rather less comment than the financial asset equations. Once again, inflation has no direct impact. On the other hand, time trends are very important. Price sensitivity seems surprisingly high although this is partly because the dependent variables are larger due to the high degree of aggregation.

Some of the most interesting results come in the upper-level demand equations reported in Table 4. These show the allocation of total net worth between financial assets, liabilities, and real capital. The first point to make is that, unlike in the lower level demand equations, inflation has a highly significant direct influence. In particular, a

Table 1. Lower Level Demand Equations
(i) Financial Assets

		Price Coefficients							
	Constant	Cash and Sight Deposits	Time Deposits	National Savings	Short-term Marketable Securities	Govern-ment Bonds	Net Foreign Assets	Time	Inflation
Cash and sight deposits	.066	-.55	.791	.0889	-.223	-.0364	-.0705	--	--
	(11.67)	(-2.41)	(3.73)		(-3.12)	(-1.42)	(-1.28)		
Time deposits	.0275	.791	-.890	-.477	.622	-.0460	--	.00034	--
	(2.86)	(3.73)	(-2.59)		(2.45)	(-1.15)		(1.99)	
National savings	-.873	.0889	-.477	-.0866	-.0014	.0909	.212	*	*
Short-term marketable securities	--	-.223	.622	-.0014	-.306	-.0331	-.585	--	--
		(-3.12)	(2.45)		(-1.22)	(-1.60)	(-1.20)		
Government bonds	.0997	-.0364	-.0460	0.0909	-.0331	.0246	--	--	--
	(22.0)	(-1.42)	(-1.15)		(-1.60)	(0.559)			
Net foreign assets	-.0661	-.0705	--	0.212	-.0585	--	-.0828	.00269	.282
	(-3.42)	(-1.28)			(-1.20)		(-.742)	(7.0)	(1.63)

Dashes indicate coefficients set to zero.

* Strictly speaking, to satisfy adding up, these coefficients should be non-zero. However, it seems sensible to regard the positive parameters on true deposits and net foreign assets as offset by small effects in all the remaining categories rather than large effects in the omitted equation.

Where appropriate, t-ratios are given in parentheses.

Table 2. Lower Level Demand Equations
(ii) Real Capital

Dependent variable (desired shares in total real capital	Constant	Housing	Productive Capital	Time	Inflation
Housing	.597	-.422	.422	.00332	--
Productive capital	.443	.422	-.422	-.00332	--
	(17.6)		(2.48)	(-5.71)	

Dashes indicate coefficients set to zero.

Where appropriate, t-ratios are given in parentheses.

Table 3. Lower Level Demand Equations
(iii) Liabilities

Dependent variable (desired share in total liabilities)	Constant	Sterling Bank Lending	Non-Sterling Bank Lending	Time	Inflation
Sterling bank lending	-0.024	-.153	.153	-.00203	--
	(-1.456)	(-1.49)		(-5.67)	
Non-sterling bank lending	1.024	.153	-.153	.00203	--

Dashes indicate coefficients set to zero.

Where appropriate, t-ratios are given in parentheses.

Explanatory note: As mentioned at the outset, liabilities are treated as negative assets. Hence, a rise in a liability's yield produces a fall in notional price

$\left(\frac{1}{1 + r_i^L}\right)$ and, thus, an increase in compensated demand.

Since liabilities are negative numbers, an increase in demand represents a decline in the absolute value of the desired liability. Hence, own-price elasticities should be negative (or small and positive) just as for financial assets and real capital.

Table 4. Upper Level Demand Equations

Dependent variable (desired share in total net worth)	Constant	Price Coefficients Assets	Capital	Liabilities	Time	Inflation	Real Wealth
Financial assets	.974	.138	-.137	-.275	.00349	.327	-.067
	(1.23)	(1.01)	(-1.05)		(4.15)	(2.36)	(-1.02)
Capital	-.160	-.157	-.174	-331	-.00195	--	.083
	(-.240)	(-1.33)	(-1.66)		(-2.67)		(1.49)
Liabilities	-0.186	-.019	.037	-.056	-.00154	-.327	.016

Dashes indicate coefficients set to zero.

Where appropriate, t-ratios are given in parentheses.

rise in inflation causes investors to increase their holdings of both financial assets and liabilities, leaving their stock of real capital unchanged.

Once more, we find a neat correspondence between the symmetry test (which fails at this upper level) and the marked significance of the direct inflation terms. This dual result is consistent with either of two competing explanations: inflation-dependent effective tax rates or a systematic link between inflation and the relative riskiness of real capital, financial assets, and liabilities. 1/

Either explanation could be made reasonably convincing. Feldstein (1981) shows for the United States that plausible parameter values for various rates of tax and depreciation imply that inflation markedly reduces the after-tax return on capital. At the same time, one could easily imagine that the chance to offset nominal interest payments against tax would encourage investors to increase their indebtedness and acquire more financial assets and real capital. The net impact of these two effects might well be that the demand for real capital stays unchanged while liabilities and financial asset demands rise.

As an alternative explanation, it is quite possible that higher inflation could alter both the absolute and relative riskiness of real capital, financial assets, and liabilities. Greater overall risk might encourage a general diversification of portfolios, while an increase in the relative riskiness of real capital could mean its desired share remained unchanged. Why might the relative riskiness of desired capital rise? A possible answer, at least for productive capital, is that nominal wage contracts in the presence of high and variable inflation could make returns to capital quite unpredictable. This is especially the case because of the high gearing that such returns are likely to have with respect to changes in the real wage rate.

The last results we need to report are the speeds with which desired shares adjust (given in Table 5). As may be seen, the estimated values are quite reasonable for quarterly data, suggesting that the bulk of adjustment occurs within two years of a shock to the system. Such speeds agree fairly well with those commonly reported in the money demand literature (see Judd and Scadding (1982)).

IX. Concluding Thoughts

The general message of this study is that inflation is highly significant to portfolio choice, affecting both the broad allocation between liabilities, real capital, and financial assets, and the more

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1/ Note that if the only factors affecting the upper level demands were service flows and expected returns, then symmetry ought to hold. Looking at the point estimates of the cross-price parameters given in Table 4, one can see that this is far from the case.

Table 5. Speeds of Stock Adjustment

	Rate of adjustment (percent per quarter)
Upper level desired shares	16.1
Financial asset shares	16.4
Liability shares	13.2
Real capital shares	11.7

detailed allocation within those categories. However, it influences this choice in ways that would not greatly deter one from adjusting government deficits for inflation. There seems to be little impact on the demand for real capital. If one's concern in choosing a deficit measure is to evaluate the effect of a given deficit upon real economic activity, one may therefore safely ignore the possible expansionary or contractionary portfolio effects of inflation, operating through the demand for capital and investment.

Even if the authorities have the narrower aim of funding a particular deficit without raising real interest rates in the bond market, then the estimates in this paper suggest that using inflation-adjusted measures will not lead to overoptimistic funding targets. Indeed, the parameter values in the model imply that inflation actually stimulates the demand for government debt. The expansion in the desired holdings of total financial assets and liabilities easily overwhelms the relatively minor substitution effects away from government bonds within the financial asset category. 1/

Of course, these conclusions are subject to caveats. One issue dealt with very little is the initial decision to look at the balance sheet of the nonbank private sector. It is quite possible that bank portfolio behavior partially offsets changes in the asset demands of the nonbank private sector. Thus, for example, a switch by nonbanks from deposits to government debt might be offset by a fall in bank holdings of government bonds with very little pressure on relative interest rates. The possibility of such offsetting effects suggests that a further step in our research might be to examine bank portfolio behavior.

As a final comment, one may note that the modeling techniques developed in this paper have considerable potential for further applications in both welfare economics and tax policy. Adopting explicit estimates of tax rates, we could identify the substitution parameters and then check for concavity of preferences. If concavity did hold, it would be possible to measure the exact welfare cost of higher inflation upon investors. It would also be possible to devise a Ramsey-type optimal tax structure for U.K. capital markets based on the elasticities of demand for different assets and liabilities. 2/

1/ In other words, investors increase their bank borrowing in order to finance an expansion in their holdings of government debt.

2/ Perraudin (1986) develops a formula for the optimal tax rates on income from risky assets.

Derivation of the Utility Function

Suppose, at t, investors maximize $EU(W_{t+1})$, where $W_{t+1} \equiv$ net wealth at t+1. Using a Taylor series approximation, this may be written as a function of the moments of future wealth conditional on information at t:

$$EU_t = U(E(W_{t+1}), E(W_{t+1}-E(W_{t+1}))^2, \ldots)$$

$$= U(\sum_i E(1+r_{it})X_{it}, \sum_i\sum_j X_{it}X_{jt} \text{ Cov } ((1+r_{it}), (1+r_{jt})), \ldots)$$

where:

$r_{it} \equiv$ realized holding return on i^{th} investment (Note: This differs from the notation used in the main text where r_{it} are expected returns.)

$X_{it} \equiv$ stock of investment held at E.

Rewriting the second and higher moments gives:

$$EU_t = U(\sum_i E(1+r_{it})\, X_{it}, \sum_i\sum_j \frac{E(1+r_{ir})}{E(1+r_{it})} X_{it} \frac{E(1+r_{jt})}{E(1+r_{jt})} X_{jt} \text{ Cov } (1, j), \ldots) .$$

Transforming this utility function yields:

$$U(RX_{it} \ldots RX_{Lt}, \frac{1}{E(1+r_{it})} \ldots \frac{1}{E(1+r_{Lt})}, \underline{COV})$$

where:

$RX_{it} \equiv E(1+r_{it})X_{it}$

$\underline{COV} \equiv$ a vector of conditional second and higher moments of the $(1 + r_{it})$.

To avoid holding the Cov(i, j) fixed over the sample period, one may introduce some amount of variation in the structure of conditional higher moments by supposing that:

$$\underline{COV} \equiv f(E(\frac{\dot{P}}{P}), T)$$

where:

$E(\frac{\dot{P}}{P}) \equiv$ expected inflation

$T \equiv$ time trend.

Thus, we have:

$$U_t = U(RX_{it}, \dots RX_{Lt}, \frac{1}{E(1+r_{it})} \dots \frac{1}{E(1+r_{Lt})}, E(\frac{\dot{P}}{P}), T)$$

This is precisely the same as the utility function in the main text apart from the inclusion of the terms $\frac{1}{E(1+r_{it})}$. If the demand functions implied by this utility function are derived, the resulting equations will include the $\frac{1}{E(1+r_{it})}$ in addition to the usual price and wealth arguments. However, since prices in this problem also happen to be of the form $\frac{1}{E(1+r_{it})}$, the parameters on these additional terms will be absorbed into the estimated price coefficients. This has the interesting implication that the price coefficients will no longer obey Slutsky symmetry. By testing symmetry, one is therefore able to test the importance of the second and higher moments of future wealth as arguments in the initial utility function.

Introducing separability adds a further complication. In this case, the demand equations will not include all the prices so the estimated price coefficients will be subject to omitted variable bias as well as absorbing the influence of some of the $\frac{1}{E(1+r_{it})}$. Once more, the symmetry test may be regarded as a global test of specification. Provided the various biases do not cancel out, if symmetry holds, the $\frac{1}{E(1+r_{it})}$ terms may be considered insignificant.

Aggregation of a Homothetic Quasi-Separable Almost Ideal Demand System

Muellbauer (1975, 1976) develops representative agent aggregation theory for the PIGLOG (summarized in Deaton and Muellbauer (1980)). With homothetic subcost functions, it is straightforward to extend this to the case of the twin-level quasi-separable AIDS. Investors may differ in their level of total net worth, C^I, and also, to a limited extent, in their preferences. The variation in preferences is summed up by a parameter specific to investors, K_I, that serves to deflate net worth.

Thus, if it is true that:

$$W^I_J = \alpha_j + \sum_i \beta_{ji} \log C_i + \mu_j \log \left\{\frac{C^I}{K_I P}\right\}$$

and $$W^I_{ij} = \lambda^j_i + \sum_k \gamma^j_{ik} \log \frac{1}{1 + r^j_k} ,$$

aggregate budget shares are given by:

$$\bar{W}_j = \frac{\sum_I C^I W^I_j}{\sum C^I} = \alpha_j + \sum_i \beta_{ji} \log C_i - \mu_j \log P$$

$$+ \mu_j \left\{\frac{\sum_I W^I \log \left(\frac{W_I}{\overline{KI}}\right)}{\sum_I W_I}\right\}$$

$$\bar{W}_{ij} = \frac{\sum_I C^I W^I_j W_{ij}}{\sum_I C^I W^I_j} = W_{ij} \frac{\sum_I C^I W^I_j}{\sum_I C^I W^I_j}$$

$$= W_{ij} .$$

Assuming that k is independent of wealth, the above system simplifies further, since the expression $- \mu_j \log (k)$ can be absorbed within the constant in each equation.

Clearly, if the subcost functions are not homothetic, W_{ij} will depend on I, and hence the lower stage will prove more difficult to aggregate. This is an area for further study.

Testing the Homotheticity of the Quasi-Separable Cost Function

Testing the homotheticity of the subcost functions would seem to be quite difficult without estimating the whole system. As argued in the text, general estimation is not feasible for our fairly large model. One solution is to assume a simplified upper level structure, and then use limited information methods on the lower stage demand equations.

Thus, assuming:

$$\log C = \alpha_o + \sum_j \alpha_j \log C_j + \mu_o U \log [\prod_j (C_j)^{\mu_j}]$$

$$\log C_j = \lambda_o^j + \sum \lambda_i^j \log \frac{1}{1 + r_i^j}$$

$$+ 1/2 \sum_i \sum_k \gamma_{ik}^{j*} \log \frac{1}{1 + r_i^j} \log \frac{1}{1 + r_i^j} + U \xi_o^j \prod_j \frac{1}{1 + r_i^j} \xi_i^j \, .$$

The upper cost function might seem unfamiliar at first sight but, in fact, it is a restricted version of the translogarithmic approximation proposed by Christensen, Jorgenson, and Lau (1975), and further developed by Jorgenson, Lau, and Stoker (1982).

Taking logarithmic derivatives to obtain the desired share equations yields:

$$W_j = \frac{\partial \log C}{\partial \log C_j} = \alpha_j + \mu_j U \mu_o$$

$$W_{ij} = \lambda_i^j + \sum_k \gamma_{ik}^j \log \frac{1}{1 + r_k^j} + \xi_o^j \xi_i^j U \, . \prod_k \frac{1}{1 + r_k^j} \, .$$

Substituting for U from the upper equation gives:

$$W_{ij} = \lambda_i^j + \sum_k \gamma_{ik}^j \log \frac{1}{1 + r_k^j} + \xi_i^j \xi_o^j \frac{W_j - \alpha_j}{\mu_j \mu_o} \prod_i \frac{1}{1 + r_i^j} \xi_i^j$$

which can be consistently estimated using limited information techniques to see if the ξ_i^j are zero.

Effective Tax Rates and Inflation

In this Appendix, illustrative after-tax rates of return for installed, productive capital and for bonds are calculated.

After-tax real rate of

$$\text{return on installed capital} = \frac{(1-\tau_P)(1-\tau_F)\ R - \lambda\ \frac{\dot{P}}{P}\ (1-\tau_P)\ -\ \frac{\dot{P}}{P}\ \tau_{GC}}{q}$$

After-tax real rate of

$$\text{return on bonds} \quad = (1-\tau_p)r_B - \frac{\dot{P}}{P}$$

where:

$\tau_P \equiv$ personal tax rate

$\tau_F \equiv$ corporate income tax rate

$\tau_{CG} \equiv$ capital gains tax rate

$\lambda \equiv$ change (due to depreciation allowances) in real profits per unit of capital for every one percent change in inflation

$R \equiv$ real pretax profits per unit of capital

$\nu_B \equiv$ pretax nominal interest rate on bonds

$q \equiv$ price of installed capital in terms of consumption goods

$\frac{\dot{P}}{P} \equiv$ anticipated rate of inflation.

Clearly, the after-tax bond return can be written as a linear function of the pretax return and inflation. Assuming that q equals one, the same is true for the return on capital. Hence, as claimed in the text, it is possible to reinterpret the coefficients on real pretax return and on inflation, in the model, as linear combinations of the tax rates and the underlying substitution parameters.

Note: The assumption that q equals one is weaker than it appears since the model explains underlying equilibrium demand toward which actual demand adjusts over time.

Explanatory Note

Table 1 sets out the equations allocating the demand for total financial assets among its constituent, desired shares. Thus, the dependent variables on the left-hand side are the shares of particular assets within the total financial asset category. Price variables in the model are of the form:

$$\log \frac{1}{1 + r_i} .$$

Since this is approximately equal to $-r_i$, one may obtain a rough idea of the effect on desired shares of changes in real rates of return by considering the negative of the price coefficient. A further point to note in interpreting the tables is that since our equations explain desired <u>shares</u>, own-price coefficient can be positive (so long as they are not too large) without contradicting the familiar implication of demand theory that compensated demand curves slope down. The fact that the equations explain shares also, of course, means that negative coefficients, say on real wealth, do not necessarily imply that the desired quantity of that asset declines when wealth goes up, merely that there is a fall in the desired share.

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