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U.S. Federal Deficits and Debt Service

Prepared by Jeroen J.M. Kremers*

Authorized for Distribution by Malcolm D. Knight

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Abstract

This study of regular patterns of the U.S. Federal debt concludes that recent Federal deficits were abnormally large by historical standards. During most of the post-war period, fluctuations of the real debt reflected the following three factors: cyclical fluctuations of real GNP; transitory Federal spending; and a regular feedback to stabilize the interest cost of Federal debt relative to GNP. Feedback to contain debt-related disequilibrium is an important ingredient of sustainable budgetary policy, and forms the focal point of this paper. It appears that the regular feedback of the post-war period ceased to be operative in recent years, as historically very large real interest payments failed to lead to deficit reductions consistent with the pattern of previous decades. The empirical analysis leading to this assessment follows two complementary approaches: co-integration and dynamic modeling.

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I. Introduction and Synopsis

Large actual and projected U.S. Federal deficits have recently stimulated widespread concern. In order to assess their economic significance, the policies generating these deficits must be placed in a dynamic perspective. That is the subject of this paper, which examines regular patterns of the Federal debt during the inter- and post-war period and concludes that recent Federal deficits were abnormally large compared to the pattern of previous decades. It appears that, until a few years ago, government policies in effect stabilized the interest cost of Federal debt relative to GNP, thus indirectly stabilizing the debt/GNP ratio. However, this observed behavior seems to have changed after 1982.

The paper integrates into a common framework two hitherto separate strands of empirical research, one dealing with the short-run determinants of budgetary policy and the other with the longer-run limitations imposed on this policy by the government's intertemporal budget constraint. As an example of the former, Barro (1986a,b) has argued that recent Federal deficits were not exceptional by historical standards, but merely reflected a normal reaction to inflation, recession, and transitory government spending. However, the model supporting his assessment lacks a mechanism for preventing the interest cost of servicing the debt from exceeding the government's revenue-raising capacity. Regarding the latter, Hamilton and Flavin (1986) and Hakkio and Rush (1986) (henceforth HF and HR respectively) have tested and rejected the proposition that budgetary policies of recent decades implied a long-run rate of growth of Federal indebtedness equal to or in excess of the rate of interest, but have not considered the implications for budgetary policy in the short run.

These two strands of analysis can be sharpened, both econometrically and conceptually, by integrating them into a common framework. Indeed, as it is the very nature of macroeconomic stabilization policy to exploit the government's freedom to shift budget deficits intertemporally but within the limits of the intertemporal budget constraint, it seems only natural to analyze both these aspects of policy conjointly. For that purpose, this study utilizes the ratio of real interest payments on the Federal debt to GNP as an indicator for the interest cost of servicing the debt relative to the government's revenue-raising capacity. This ratio was stationary around an average of slightly over 1/2 percent during the inter- and post-war period, implying a restraint on debt growth consistent with, but stronger than that tested by HF and HR. ^{1/} During most of the post-war period, disequilibrium of the interest/GNP ratio was counteracted by a feedback effect on the determination of the Federal debt, linking long-run aspects of the intertemporal budget constraint with the short-run pattern of budgetary policy. However, this feedback ceased to be operative

^{1/} For further comment on the tests conducted by HF, see Kremers (1987).

in recent years, as historically very large real interest payments failed to lead to deficit reductions consistent with the pattern of previous decades. The findings of this study are therefore not consistent with the proposition of Barro (1986a,b), that recent deficits did not represent a shift in budgetary policy. On the contrary, they support the opposite view [cf. von Furstenberg (1983) and Summers (1987)].

Broader implications of the paper can be summarized as follows. First, to the extent that recent U.S. deficits were not anticipated on the basis of regular patterns of previous decades, they may not have been discounted by the private sector until shortly before they actually occurred. Their economic consequences may therefore have been different from those of the largely anticipated deficits in previous years. In particular, unexpectedly large deficits may have been a factor behind high real interest rates [Blanchard and Summers (1984), and Masson and Knight (1986)]. Second, the shift of policy away from the regular pattern of previous years may have raised uncertainty regarding its likely course in the future, similarly with consequences for interest rates. Third and related, empirical studies of the influence of anticipated deficits on interest rates that do not take this policy shift into account may be misspecified and may therefore give rise to unreliable conclusions. Fourth, the fact that budgetary policy recently shifted away from directly stabilizing the interest/GNP ratio does not imply that the Federal government no longer faces an intertemporal budget constraint. Only as time progresses, however, will it become clear to what extent this constraint will again be reflected in the short-run pattern of budgetary policy, and what will be the role of other influences such as the stance of monetary policy [cf. Sargent (1986)]. Finally, the combined effects of government action and market forces have resulted in a stationary interest/GNP ratio for more than half a century now, suggesting that this ratio may be an important indicator for the stance and sustainability of budgetary policy.

The paper is organized as follows. Section II briefly reviews theoretical issues raised by the intertemporal budget constraint, and describes the econometric approach of this paper. The next two sections present empirical results, focusing on long-run aspects of the intertemporal constraint in Section III and implications for the short-run pattern of debt creation in Section IV. Some concluding remarks are made in Section V.

II. The Intertemporal Budget Constraint: Theory and Empirical Analysis

This section first briefly reviews the dynamics of the intertemporal budget constraint. ^{1/} For the purpose of transparency, the introduction of random shocks to the budget is postponed until the second part of the section, where implications of the intertemporal constraint for the short-run pattern of budgetary policy are examined.

^{1/} More extensive discussions can be found in Sargent and Wallace (1981), McCallum (1984), Blanchard et al. (1985), and Spaventa (1986).

In each period the following budget constraint must be satisfied:

$$\Delta B_t = G_t - T_t + r_{t-1}B_{t-1} \quad (1)$$

B_t is the stock of interest-bearing public debt outstanding at the end of period t , G_t is government expenditure net of interest payments, T_t is tax revenue, and Δ is the first-difference operator. The public debt at this stage is taken to consist of one-period bonds issued at par and paying interest at a rate of r_t in period $t+1$. In this section all variables, including the interest rate, are in real terms. Money financing is not considered explicitly, although, following Barro (1979, 1986a,b), it can be thought of as being part of tax revenue T_t .

Recursive forward substitution of (1) from period t to period $n > t$ yields the following expression for public debt in terms of future debt and primary surpluses:

$$B_t = R_{t,n+1}B_{n+1} + \sum_{j=t+1}^n R_{t,j} (T_j - G_j) \quad (2)$$

Discount factors are given by $R_{t,j} = \prod_{h=t+1}^j 1/(1+r_{h-1})$. If the real stock of debt grows asymptotically at a rate smaller than the asymptotic real rate of interest (denoted by r), then the first term in equation (2) converges to zero as n becomes large. In that case budgetary policy respects the present-value budget constraint (3), which maintains that future primary surpluses are necessary to service a currently positive stock of debt:

$$B_t = \sum_{j=t+1}^{\infty} R_{t,j} (T_j - G_j) \quad (3)$$

HF have tested and failed to reject this requirement using annual data from 1960 to 1984, and HR have further supported that result with a variety of tests using quarterly data from 1950 to 1986.

However, this constraint may not prevent the future interest cost of debt service from exceeding the government's revenue-raising capacity. It seems reasonable to assume that in each period the government's capacity to raise revenue for interest debt service (other than by borrowing) is limited by an upper bound on taxation and a lower bound on other public spending, both as a proportion of GNP. ^{1/} In order to determine the implications of the ensuing bounds on primary surpluses, equation (2) can be rewritten in terms of ratios to GNP:

$$B_t/Y_t = Q_{t,n+1}(B_{n+1}/Y_{n+1}) + \sum_{j=t+1}^n Q_{t,j}(T_j - G_j)/Y_j \quad (4)$$

^{1/} This is the assumption made by Barro (1979, 1986), Sargent and Wallace (1981), Blanchard (1983), Blanchard et al. (1985), Masson (1985), and Spaventa (1986). McCallum (1984) has presented a model with debt growth at a rate exceeding that of output and smaller than the rate of interest, but concluded that the incorporation of default and tax evasion motives would presumably further restrict the rate of debt growth to that of output.

Discount factors are given by $Q_{t,j} = \prod_{h=t+1}^j (1+\rho_{h-1})/(1+r_{h-1})$.

Y_t is real GNP, with a rate of growth of $\rho_t = Y_{t+1}/Y_t - 1$.

If the present-value constraint (3) is satisfied, and real GNP grows asymptotically at a rate ρ smaller than r , 1/ then the first term in (4) converges to zero as n becomes large. With bounds on future primary surpluses $(T_j - G_j)/Y_j$, this implies that bounds are required on the ratio of debt to GNP as well. Consequently, the asymptotic rate of growth of debt must be limited to ρ , which, presumably, is smaller than r . If so, the present-value constraint (3) analyzed by HF and HR represents a necessary but not a sufficient condition for an intertemporal constraint that respects the government's revenue-raising capacity.

The present paper deals with a condition that is both necessary and sufficient, namely the boundedness of the ratio of debt (or interest payments--see below) to GNP. 2/ If that condition is satisfied, the intertemporal budget constraint becomes:

$$B_t/Y_t = \sum_{j=t+1}^{\infty} Q_{t,j} (T_j - G_j)/Y_j \quad (5)$$

Within the limits set by this constraint, the government can decide on its plan for the pattern of future primary surpluses. Extension of this constraint to the stochastic case will be discussed below.

Before proceeding to a closer examination of the short-run pattern of primary surpluses, it should be noted that boundedness of the debt/GNP ratio is equivalent to boundedness of the interest/GNP ratio, provided the real rate of interest is bounded as well. However, since boundedness by its very nature is a long-run concept, it is important to determine whether reliable empirical inference can be based on finite-sample evidence. The following application of the theory of co-integrated variables serves to clarify this issue, and offers a background for the discussion of short-run aspects of budgetary policy.

1/ For empirical evidence, see Abel et al. (1986). The intertemporal constraint has little meaning if the rate of GNP growth is larger than the rate of interest, as in that case the government's collateral (i.e., the present value of the government's revenue-raising capacity) is infinite. This issue was raised in a discussion between Feldstein (1976) and Barro (1976), but subsequent studies, including those referred to in the previous footnote, have been based on the premise that the collateral is finite. Nevertheless, even if during a prolonged period of time ρ exceeded r , it would, in the presence of uncertainty regarding future growth, still seem a matter of prudence to keep the debt/GNP ratio within bounds. In that case the main argument of this paper remains valid.

2/ As suggested in the previous footnote, this condition probably remains relevant even if ρ exceeds r during a prolonged period of time.

The idea underlying co-integration is to specify models that capture the behavior of variables drifting apart in the short run, but brought together again by market forces or government intervention, or both, if they continue to be too far apart in the longer run [Granger (1986)]. A variable is defined to be integrated of order d [denoted by $I(d)$] if it must be differenced d times to induce stationarity [denoted by $I(0)$]. ^{1/} A set of variables, each $I(d)$, is said to be co-integrated if a linear combination of them is $I(0)$; that linear combination is characterized by a co-integrating vector.

Natural logarithms of the debt/GNP and interest/GNP ratios are related by the identity: ^{2/}

$$(b-y)_t = (i-y)_t - \ln(r_{t-1}) + \Delta b_t \quad (6)$$

Lowercase letters denote natural logarithms of the corresponding uppercase variables, and $I_t = r_{t-1}B_{t-1}$ denotes interest payments. Provided b_t is $I(1)$ so that Δb_t is $I(0)$, it is clear from (6) that for both the debt/GNP ratio and interest/GNP ratio to be stationary, the real rate of interest must be stationary as well.

However, if the real rate of interest (or Δb_t) is stationary with a component moving at a frequency that is low relative to the finite sample period, then it is possible that, on the basis of that sample, non-stationarity of at least one of the two ratios cannot be rejected, even if in a longer perspective both are stationary. Rejection of non-stationarity of only one of the two ratios can nevertheless be interpreted as finite-sample evidence for the validity of the intertemporal budget constraint, given the assumption that in the long run the real rate of interest and Δb_t remain $I(0)$. But obviously it is important to use tests with high power to reject non-stationarity, which turns out to be the justification for analyzing short- and long-run aspects of budgetary policy in a common framework.

This follows from a point made by Banerjee et al. (1986). Two complementary approaches to testing for co-integration have been proposed in the literature. One requires modeling short-run dynamics and the other does not. The latter consists of the usual tests for (unit root) non-stationarity, applied to deviations of the data from their linear co-integrating combination. In the present context the scalar that co-integrates debt,

^{1/} This is neither precise nor entirely correct--see Granger (1986), Hendry (1986), and Engle and Granger (1987). Note, incidentally, that stationarity is necessary but not fully sufficient for boundedness. Thus, the logic of tests conducted in the literature on this subject, including this paper, must be that boundedness cannot be accepted if non-stationarity cannot be rejected.

^{2/} Stationarity of a variable is equivalent to stationarity of its natural logarithm, provided the latter exists [Granger (1986)].

or interest payments, with GNP will not be estimated but assumed to be given at unity. For example, following this approach the variable to be tested for non-stationarity will be \hat{u}_t , representing disequilibria of the debt/GNP ratio as defined by (7) with $c_1=1$:

$$\hat{u}_t = b_t - c_1 y_t - \hat{c}_0 \quad (7)$$

\hat{c}_0 is the sample-mean of $(b-y)_t$. Notice that, since c_1 is given, this is equivalent to testing non-stationarity of the debt/GNP ratio itself.

As a basis for the other approach to testing for co-integration, Engle and Granger (1987) have proved that co-integration of, say, debt and GNP is a necessary and sufficient condition for the existence of error-correction models for the short-run pattern of debt or GNP, or both [see also Granger (1986)]:

$$\Delta b_t = \alpha_1 \hat{u}_{t-1} + \text{short-run dynamics [I(0)]} \quad (8i)$$

$$\Delta y_t = \beta_1 \hat{u}_{t-1} + \text{short-run dynamics [I(0)]} \quad (8ii)$$

with $|\alpha_1| + |\beta_1| \neq 0$. This approach tests whether the coefficient of at least one of the two feedbacks from past disequilibria is significantly different from zero. As further illustrated in Section IV, these error-correction mechanisms form the link between short-run dynamics as analyzed by Barro and long-run constraints as analyzed by HF and HR. The point made by Banerjee et al. (1986) is that, in finite samples, modeling the short-run dynamics and then applying these significance tests may add considerable power to reject non-stationarity. The discussion above clearly justifies applying both approaches in a complementary fashion.

This section concludes by considering how limitations implied by the intertemporal budget constraint may be reflected in the short-run pattern of budgetary policy. The pattern of primary surpluses is planned subject to intertemporal constraint (5). Under perfect foresight there would be no need for contingency on unexpected future disequilibria of, say, the debt/GNP ratio, because under perfect foresight such disequilibria would not occur. However, in reality random shocks such as planning and expectation errors impinge on the budget, and the system determining the time paths of GNP, the interest rate, and the policy variables must include an element of contingency to ensure consistency with the intertemporal budget constraint. Under forward-looking behavior, error-correction mechanisms can fulfil that role [Salmon (1982), Nickell (1985)]. As indicated by (8), equilibrating feedback can operate directly through the determination of budgetary policy ($\alpha_1 \neq 0$), indirectly through the determination of real GNP ($\beta_1 \neq 0$), or through both these channels.

As argued in Section IV below, feedback effects on the short-run pattern of Federal debt have been operative during most of the post-war period. At a theoretical level, various budgetary policy rules with a

constant rate of error-correction feedback 1/ have been analyzed by Buiter (1984), Sachs and Wyplosz (1984), and Masson (1986). However, Blanchard et al. (1985) have argued that the optimal rate of feedback may depend on the state of the economy. Indeed, as shown in Section IV, cyclical variations in the rate of Federal debt stabilization have occurred during most of the post-war period. Similarly, Masson (1985) has suggested that faster feedback may ensue if debt disequilibrium affects the real rate of interest. This factor will help to explain why finite-sample tests clearly reject non-stationarity of the ratio of real interest payments to GNP, but fail to do so with respect to the ratio of debt to GNP.

III. Empirical Results: Testing for Non-Stationarity

This section is focused on long-run patterns of the debt/GNP and interest/GNP ratios. The evidence presented is twofold: graphical summaries of the data, and Bhargava (1986) and Augmented Dickey-Fuller (1979) tests for unit-root non-stationarity (denoted by BHA and ADF respectively), 2/ corresponding to the first approach to testing for co-integration. The second approach, testing error-correction feedback in a dynamic model, follows in Section IV.

The debt/GNP and real interest/GNP ratios are shown in Figures 1 and 2 respectively. 3/ Interest payments, also shown in Figure 2, have been corrected for inflation in order to remove the fraction corresponding to the nominal component of interest rates [see e.g. Jump (1980), Eisner and Pieper (1984), and Miller (1985)].

The graphical evidence suggests less of a sustained tendency to return to a stationary long-run equilibrium on the part of the debt/GNP ratio than on the part of the real interest/GNP ratio. The latter has fluctuated relatively tightly around an average of slightly over 1/2 percent since the late 1920s.

1/ I.e., with no influences on the growth of debt other than from disequilibrium feedback variables with constant coefficients [e.g. no short-run dynamics in (8i) with a constant $\alpha_1 < 0$].

2/ These and other tests are discussed in Engle and Granger (1987), whose findings suggest that the ADF test is recommended if it is unknown whether the true model is first or higher order. Still, BHA has power in both cases as well. The finite-sample performance of these tests is quite sensitive with respect to the exact characteristics of the underlying true model--reason to consider both tests and see whether they point to the same conclusion. See also Banerjee et al. (1986).

3/ For comparison, it is shown in Figure 1 that the debt/GNP ratio based on the measure for Federal indebtedness of HF has fluctuated similarly to the measure employed here, albeit at a lower level. The main difference between the two ratios consists of the government's gold holdings and the Treasury's operating cash balance, both netted out by HF.

This graphical impression is confirmed by non-stationarity tests. ^{1/} It appears that neither during the combined inter- and post-war period, nor during the post-war period alone, can non-stationarity of the debt/GNP ratio be firmly rejected (Table 1). It is true, however, that ADF statistics testing non-stationarity of the debt/GNP ratio over the entire sample [(1.5) and (1.6)] are close to the 10 percent critical level of about -2.60, suggesting that in a longer perspective the ratio may be stationary.

Moreover, there is clear evidence against non-stationarity of the real interest/GNP ratio, as witnessed by BHA tests for subperiods and for the entire sample from 1929 to 1985, and by an ADF test for the post-war period (Table 2). As regards the exact time period for which non-stationarity can be rejected, two points are noteworthy. First, probably partly because of the small size of the inter-war sample, given the high order of autoregression, it proved to be difficult to estimate a satisfactory ADF equation covering the combined inter- and post-war period. Second, post-war tests appear to be sensitive to the inclusion of 1983-1985. Apparently the short-run dynamics of the interest/GNP ratio varied between these subperiods. However, in view of the long-run character of non-stationarity tests it would be hazardous to make precise statements on the time coverage of their results. Issues of timing are more appropriately addressed within the dynamic approach of the following section.

The conclusion of this section, meanwhile, is that budgetary policies since the 1920s seem to have satisfied the intertemporal budget constraint (5), which takes account of the government's revenue-raising capacity. As explained in the previous section, this result is stronger than that of HF and HR if real GNP grows at a rate smaller than the real rate of interest. The following section examines the underlying short-run dynamics.

IV. Empirical Results: Dynamic Modeling

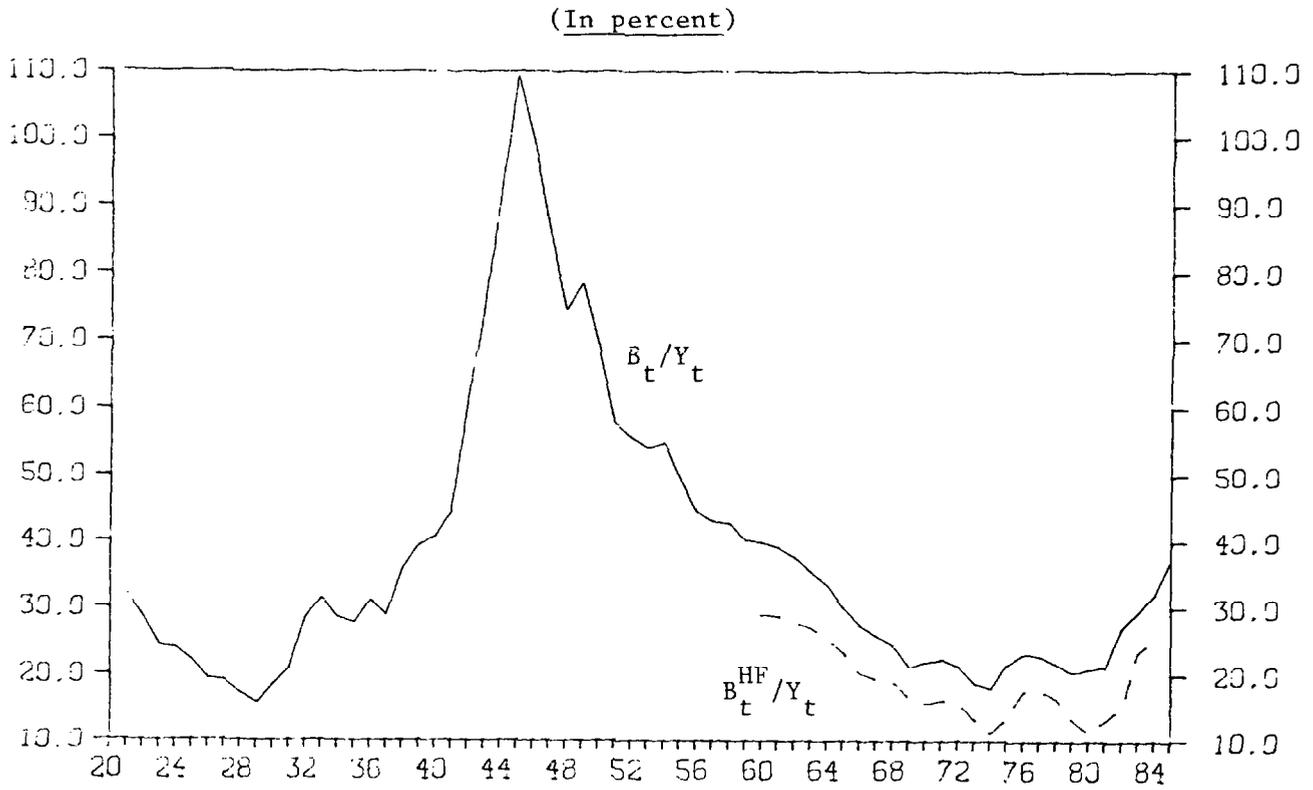
This section serves two purposes. First, significant feedback from Federal debt disequilibrium in a dynamic model of debt creation will reinforce the conclusion of the previous section. Second, this dynamic model will yield insight into the short-run implications of the intertemporal budget constraint and shed light on whether recent Federal deficits were abnormal historically.

A general, stylized error-correction model incorporating the theoretical ingredients mentioned at the conclusion of Section II is the following:

$$\Delta b_t = \alpha_0 - \alpha_1(r_j)(b-y)_{t-1} - \alpha_2 YVAR_t, \quad i \geq 1 \quad (9)$$

^{1/} Throughout the paper, World War II data are excluded from the empirical analysis. All computations use the PCGIVE program by D.F. Hendry and the Oxford Institute of Economics and Statistics.

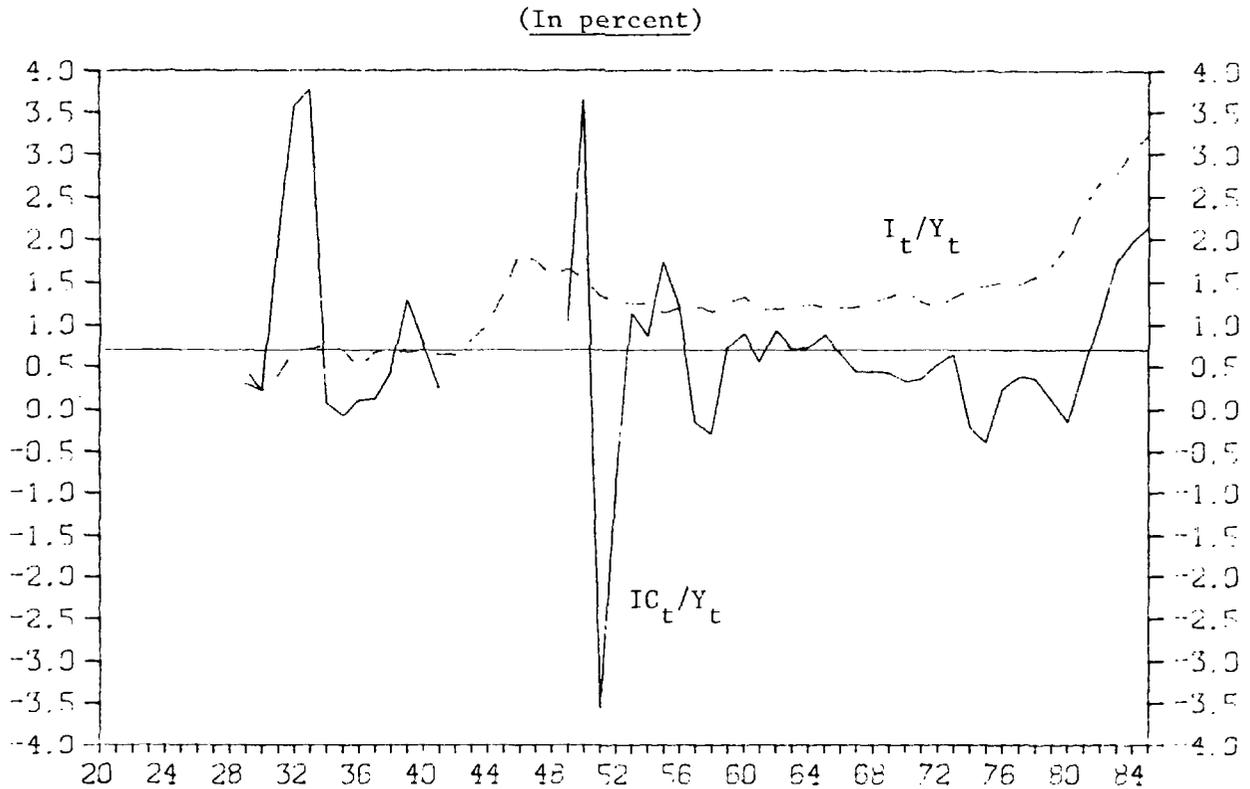
Figure 1. The Debt/GNP Ratio



Note: B_t is the nominal market value of interest-bearing, privately-held Federal debt; B_t^{HF} is the measure of Federal indebtedness analyzed by HF; and Y_t is nominal GNP. Data sources are in Appendix I.



Figure 2. The Interest/GNP Ratio



Note: IC_t is nominal interest payments on the Federal debt (I_t), corrected for inflation according to: $IC_t = I_t - \pi_{t-1,t}B_{t-1}$, where $\pi_{t-1,t} = \ln(P_t/P_{t-1})$ with P_t the January value of the seasonally adjusted CPI; and B_t is the nominal market value of interest-bearing, privately-held Federal debt. Y_t is nominal GNP. Data sources are given in Appendix I. The horizontal line is drawn at the average of the real interest/GNP ratio (0.69 percent).



Table 1. Testing for Non-Stationarity: The Debt/GNP Ratio

Bhargava Tests:

(1.1)	$(\hat{B}/\hat{Y})_t = 0.32$	(1.3)	$(\hat{B}/\hat{Y})_t = 0.31$
	BHA = 0.30		BHA = 0.05
	T = 1920-40,49-82		T = 1953-82
(1.2)	$(\hat{B}/\hat{Y})_t = 0.32$	(1.4)	$(\hat{B}/\hat{Y})_t = 0.31$
	BHA = 0.30		BHA = 0.06
	T = 1920-40,49-85		T = 1953-85

Augmented Dickey-Fuller Tests:

(1.5)	$\Delta(\hat{B}/\hat{Y})_t = 0.02 - 0.09 (B/Y)_{t-1} + 0.33 \Delta(B/Y)_{t-1}$
	(0.01) (0.03) (0.12)
	ADF = -2.70* T = 1923-40,51-82
(1.6)	$\Delta(\hat{B}/\hat{Y})_t = 0.02 - 0.08 (B/Y)_{t-1} + 0.37 \Delta(B/Y)_{t-1}$
	(0.01) (0.03) (0.12)
	ADF = -2.55 T = 1923-40,51-85
(1.7)	$\Delta(\hat{B}/\hat{Y})_t = 0.02 - 0.07 (B/Y)_{t-1} + 0.27 \Delta(B/Y)_{t-1}$
	(0.01) (0.03) (0.21)
	ADF = -2.22 T = 1953-82
(1.8)	$\Delta(\hat{B}/\hat{Y})_t = 0.02 - 0.06 (B/Y)_{t-1} + 0.46 \Delta(B/Y)_{t-1}$
	(0.01) (0.03) (0.17)
	ADF = -1.86 T = 1953-85

Note: B_t is the nominal market value of interest-bearing, privately-held Federal debt; and Y_t is nominal GNP. Data sources are given in Appendix I. BHA is the Bhargava test statistic, with critical levels in Bhargava (1986, Table I). ADF is the Augmented Dickey-Fuller test statistic, with critical levels in Fuller (1976, Table 8.5.2). Statistics rejecting non-stationarity at the 10 percent critical level are marked by one asterisk (*), and those rejecting at 5 percent by two (**). Conventionally computed standard errors are in parentheses. Regressions 5 to 8 have been checked for autoregressive errors up to fourth order.

Table 2. Testing for Non-Stationarity: The Interest/GNP Ratio

Bhargava Tests:

(2.1)	$(IC/\hat{Y})_t = 0.006$	(2.3)	$(IC/\hat{Y})_t = 0.005$
	BHA = 1.65**		BHA = 1.00**
	T = 1929-40,49-82		T = 1953-82
(2.2)	$(IC/\hat{Y})_t = 0.007$	(2.4)	$(IC/\hat{Y})_t = 0.007$
	BHA = 1.53**		BHA = 0.58
	T = 1929-40,49-85		T = 1953-85

Augmented Dickey-Fuller Tests:

$$(2.5) \quad \Delta(IC/\hat{Y})_t = 0.003 - 0.56 (IC/Y)_{t-1} + 0.46 \Delta(IC/Y)_{t-1} \\ (0.001) \quad (0.16) \quad (0.12) \\ - 0.23 \Delta(IC/Y)_{t-2} - 0.20 \Delta(IC/Y)_{t-4} \\ (0.10) \quad (0.04)$$

$$ADF = -3.48** \quad T = 1954-82$$

$$(2.6) \quad \Delta(IC/\hat{Y})_t = 0.002 - 0.22 (IC/Y)_{t-1} + 0.41 \Delta(IC/Y)_{t-1} \\ (0.001) \quad (0.15) \quad (0.14) \\ - 0.29 \Delta(IC/Y)_{t-2} - 0.21 \Delta(IC/Y)_{t-4} \\ (0.12) \quad (0.05)$$

$$ADF = -1.42 \quad T = 1954-85$$

Note: IC_t is nominal interest payments on the Federal debt, corrected for inflation; and Y_t is nominal GNP. Data sources are given in Appendix I. For further explanations, see Table 1. Regressions 5 and 6 have been checked for autoregressive errors up to fourth order.

This specification expresses the idea that real debt grows at the underlying rate of growth of real GNP (ρ , which has been absorbed into α_0), corrected for feedback from past debt disequilibrium [\hat{u}_{t-1} from (8), with the constant long-run equilibrium absorbed into α_0 so that $(b-y)_{t-1}$ remains]. In addition, if $\alpha_2 > 0$ then the rate of debt stabilization is sensitive to the cyclical variable $YVAR_t$ (which has a positive sign when capacity utilization is above normal, and conversely). The possible dependence of the feedback coefficient α_1 on interest rates expresses the idea that debt stabilization may be faster if interest rates are high. Such dependence would be implied by the theoretical argument of Masson (1985), that debt disequilibrium affects real interest rates and thus precipitates debt stabilization.

Three special cases of this general model are the following. First, if $\alpha_1(r_j) \equiv \alpha_1$ and $YVAR_t \equiv \Delta y_{t-\rho}$ then (9) reduces to a standard error-correction model [Hendry and Richard (1983) and Kremers (1985)]:

$$\Delta b_t = \tilde{\alpha}_0 - \alpha_1(b-y)_{t-1} - \alpha_2(\Delta y_{t-\rho}) \quad (9')$$

Second, if $\alpha_1(r_j) \equiv \alpha_1 r_{t-1}$ and $(b-y)_{t-1}$ is replaced by (B_{t-1-1}/Y_{t-1}) then a non-linear error-correction model [cf. Granger (1986)] with feedback from interest/GNP disequilibrium can result:

$$\Delta b_t = \tilde{\alpha}_0 - \alpha_1(I_{t-1}/Y_{t-1}) - \alpha_2 YVAR_t \quad (9'')$$

Third, if $\alpha_1(r_j)$ were identical to zero then a stylized version of the Barro (1979) model of public debt creation would remain. The latter will be used as a starting-point for the dynamic analysis. The background of the Barro model is a neo-classical theory of tax smoothing, but its empirical specification and further elaborations by Barro (1979, 1980, 1986a, 1986b, 1987) allow for countercyclical fluctuations of the Federal budget as well. Using annual data from 1920 to 1982 and deleting war years, the estimate of this model by Barro (1986a, Table 6.1, Set 2) can be closely approximated as follows [see Appendix I for data sources and Appendix II for explanations of the test statistics]:

$$\Delta p \hat{v} b_t = .010 + .98 \hat{\pi}_{t,t+1}^e - 3.96 YVAR_t + .29 GVAR_t \quad (10)$$

(.006) (.09) (.29) (.10)

$$T = 1920-40, 48-82 \quad \bar{R}^2 = .97 \quad \hat{\sigma} = 2.38\% \quad dw = 2.25 \quad \eta_2(4,48)_1^4 = 1.97$$

$$\xi_2(4)_1^4 = 8.88 \quad \xi_2(1)_1^1 = 1.15 \quad \xi_2(1)_2^2 = 3.02 \quad \eta_4(3,56) = 1.07$$

$$SK = .16 \quad EK = -.13 \quad \xi_5(2) = .25 \quad \eta_1(17,35) = 1.08$$

Heteroscedasticity-consistent standard errors are in parentheses [White (1980)]. PVB_t is the nominal par value of interest-bearing, privately-held Federal debt; $\hat{\pi}_{t,t+1}^e$ is inflation anticipated in period t for period $t+1$ (as its estimated coefficient is close to unity, this is essentially a model of real debt growth); and $GVAR_t$ is transitory Federal defense spending.

The parameter estimates do not differ significantly from those of Barro. Because of the fact that the equation for inflation has not been jointly estimated here, the estimated standard error of the coefficient on this variable is slightly smaller [cf. Pagan (1984)], and the equation standard error is slightly larger. The diagnostic tests, most of which were not reported by Barro, are broadly satisfactory. Actual and fitted values are depicted in the upper panel of Figure 3.

As explained in Section II, the second approach to testing for co-integration involves testing the significance of error-correction feedback in dynamic models. Since in Section III firm evidence was found for the stationarity of the real interest/GNP ratio during most of the post-war period, an error-correction feedback from past disequilibria of that ratio will be added to the dynamic model estimated in (10). It appears that feedback was indeed significant from 1953 to 1982, and that its inclusion benefits the equation: 1/

$$\begin{aligned} \Delta p\hat{v}b_t = & .005 + .83 \hat{\pi}_{t,t+1}^e - 3.97 YVAR_t + .29 GVAR_t \\ & (.007) (.12) \quad (.33) \quad (.09) \\ & - 2.15 (IC/Y)_{t-1} D53ff + .023 D53ff \quad (11) \\ & (.52) \quad (.010) \end{aligned}$$

$$T = 1920-40, 50-82 \quad \bar{R}^2 = .97 \quad \hat{\sigma} = 2.22\% \quad dw = 2.11 \quad \eta_2(4,44)_1^4 = .65$$

$$\xi_2(4)_1^4 = 3.53 \quad \xi_2(1)_1^1 = .22 \quad \xi_2(1)_2^2 = 1.18 \quad \eta_4(3,56) = .56$$

$$SK = .16 \quad EK = .60 \quad \xi_5(2) = .93 \quad \eta_1(17,31) = 1.00$$

D53ff is unity from 1953 onwards, and zero before 1953.

The fact that the rate of debt stabilization so clearly depends on the average real rate of interest paid on Federal debt may provide support for the theoretical argument of Masson (1985). In this context it is interesting to note that before 1953, as the Federal Reserve pegged interest rates on Federal debt 2/ and the argument of Masson would therefore seem less relevant, there is no significant feedback from the real interest/GNP ratio. Indeed, from the 1930s to 1952 there seems to have been debt stabilization at a rate independent of the rate of interest:

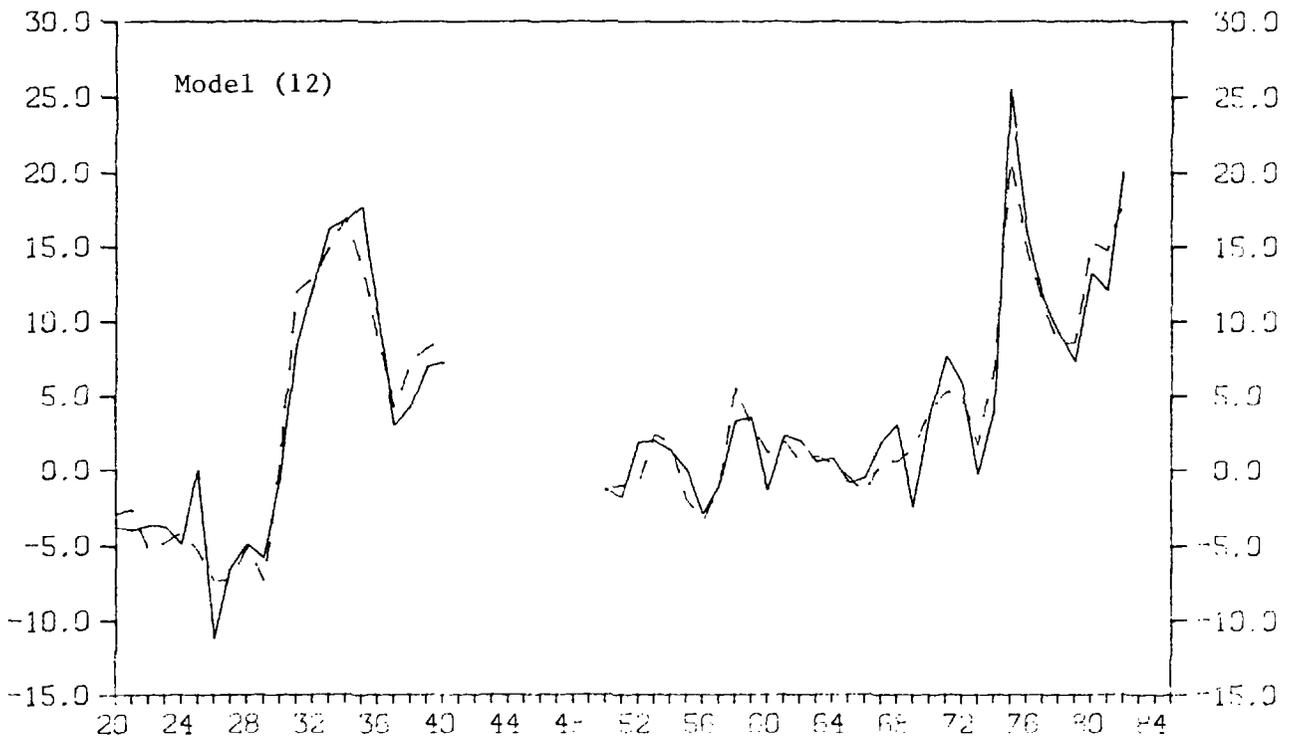
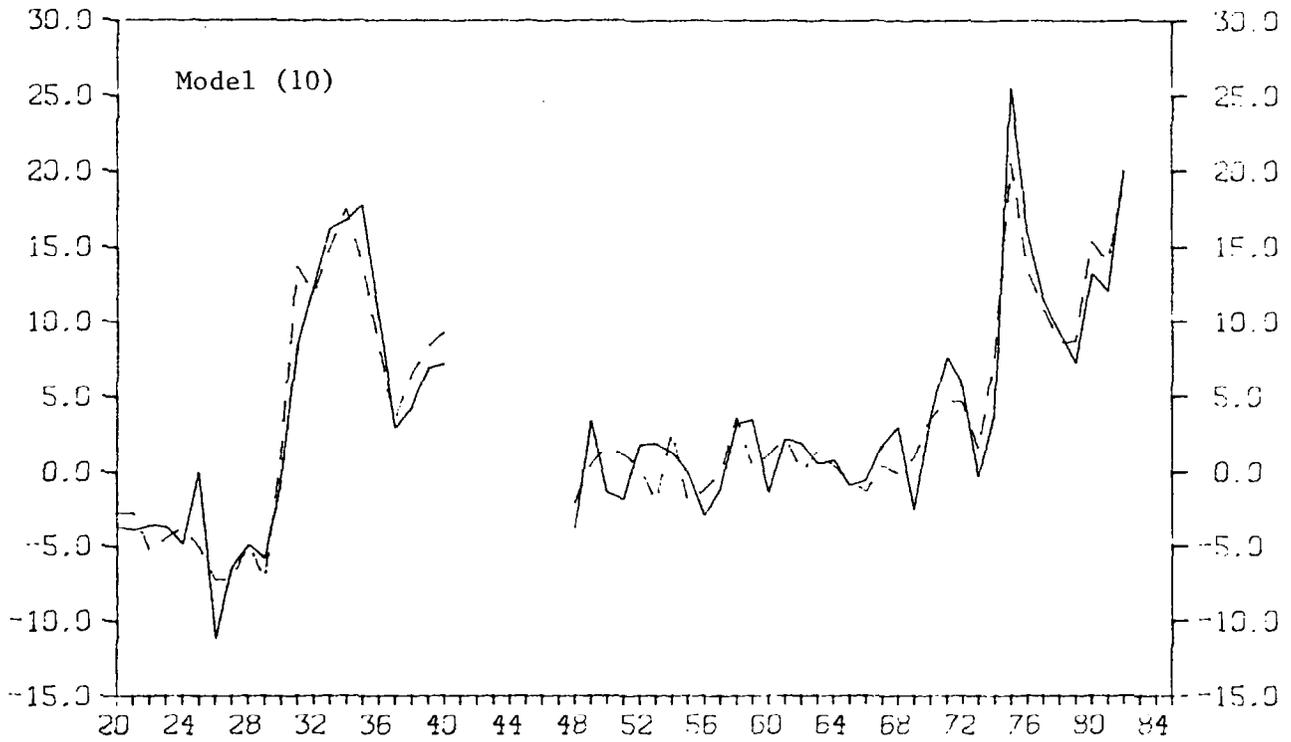
$$\begin{aligned} \Delta p\hat{v}b_t = & .002 + .84 \hat{\pi}_{t,t+1}^e - 3.68 YVAR_t + .27 GVAR_t \\ & (.014) (.14) \quad (.40) \quad (.11) \\ & - 2.40 (IC/Y)_{t-1} D53ff + .026 D53ff \\ & (.54) \quad (.016) \\ & - .08 (B/Y)_{t-1} D3252 + .042 D3252 \quad (12) \\ & (.04) \quad (.023) \end{aligned}$$

1/ The phenomenon that omission of an error-correction term does not show up in diagnostic tests, such as those of (10) above, has been analyzed by Davidson and Hendry (1981).

2/ See for example Ahearn (1963) and Vatter (1963).

Figure 3. Actual and Fitted Debt Growth, 1920-82

(In percent)

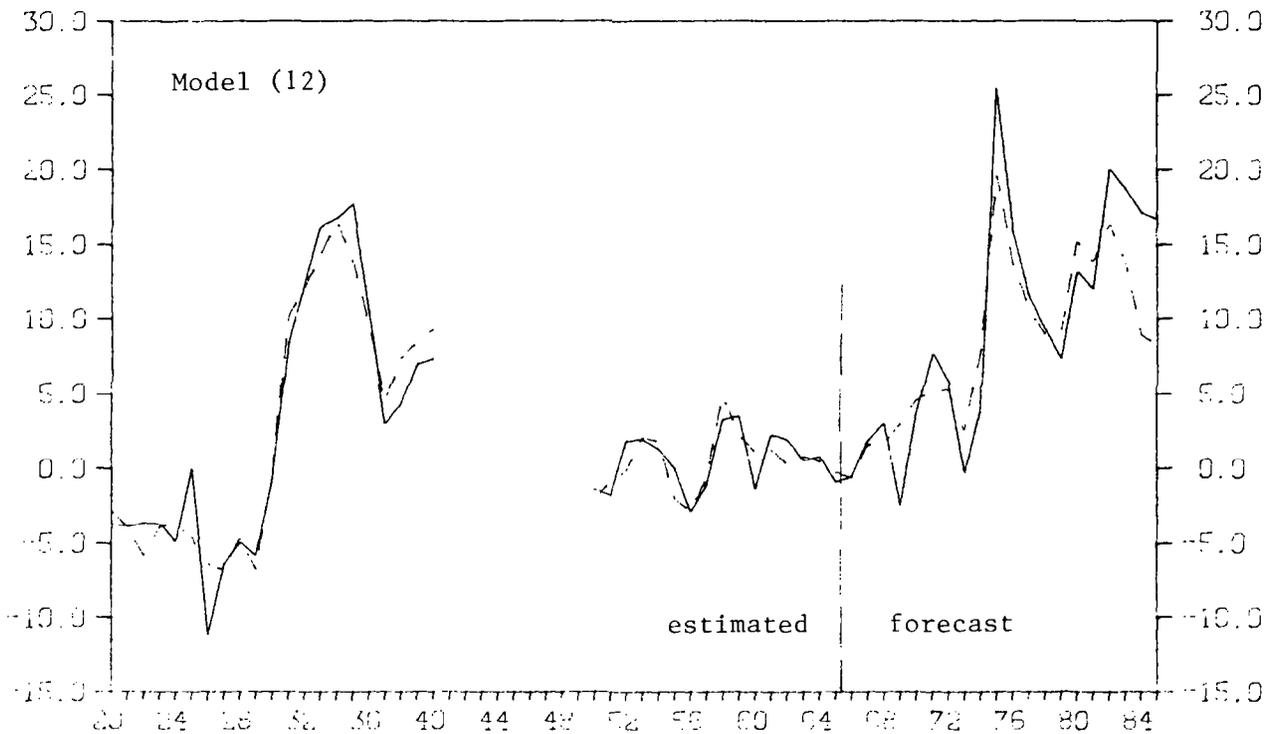
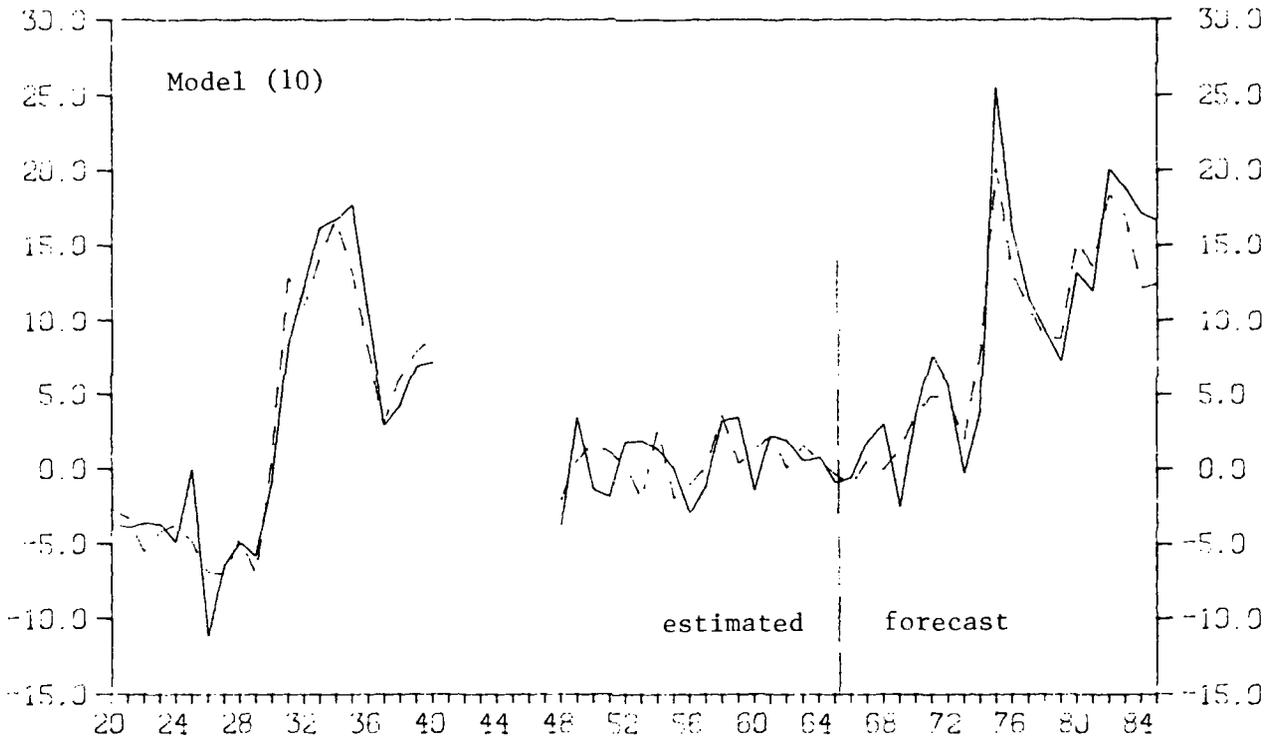


Note: Actual ——— ; Fitted - - - - -



Figure 4. Actual, Fitted, and Forecast Debt Growth, 1920-85.

(In percent)



Note: Actual — ; Fitted/Forecast - - - -



$$\begin{aligned} T = 1920-40, 50-82 \quad \bar{R}^2 = .98 \quad \hat{\sigma} = 2.17\% \quad dw = 2.20 \quad \eta_2(4,42)_1^4 = 1.68 \\ \xi_2(4)_1^4 = 8.67 \quad \xi_2(1)_1^1 = .91 \quad \xi_2(1)_2^2 = 1.45 \quad \eta_4(3,56) = .47 \\ SK = .26 \quad EK = .66 \quad \xi_5(2) = 1.37 \quad \eta_1(17,29) = 1.44 \end{aligned}$$

D3252 is unity from 1932 to 1952, and zero elsewhere. Actual and fitted values of (12) are depicted in the lower panel of Figure 3. Notice that the model including interest and debt stabilization fits Federal debt creation visibly better, particularly in the post-war decades. The fact that the form of stabilization seems to have changed around 1952 helps to interpret the difficulty encountered in Section III to estimate a satisfactory ADF equation for the combined inter- and post-war period.

The model of Barro, estimated in (10), does not clearly indicate that recent Federal deficits were abnormally large by historical standards. The upper panel of Figure 4 shows that a version of (10), estimated using data from 1920 to 1965 only, tracks the pattern of two decades of debt creation quite well. The conditional forecasts for 1983-1985 do not indicate a string of abnormal deficits, 1/ which has led Barro (1987) to describe current Federal deficits as "only a minor crisis." However, the lower panel of Figure 4 shows a different picture: when estimated up to 1965, the model including debt stabilization clearly fits better within-sample, tracks at least as well outside the sample up to 1982, but shows clearly that recent Federal debt growth was larger than consistent with the pattern of previous decades. 2/ It is indeed disequilibrium feedback that ceased to be operative after 1982: when the model is estimated up to 1985 this variable becomes insignificant, while the others remain largely unaffected.

V. Concluding Remarks

This study has encompassed and sharpened empirical insights into the regular pattern of Federal deficit policy obtained by HF, HR, and Barro. An outline of its principal findings and their broader implications has been given in Section I. One main conclusion is that there has recently been a string of deficits that were significantly larger than predicted by the pattern of previous decades. However, there is nothing in this paper to imply directly that these deficits were good or bad. To study the economic consequences of budgetary policies it is, nevertheless, important to have a firm understanding of those policies themselves. The incorporation of the results of this paper into such a study remains a subject for further research.

1/ None of the predicted rates of nominal debt growth differ by more than $2\hat{\sigma}$ from actual debt growth. The values for the explanatory variables during these years are conservative--see Appendix I.

2/ The predicted values for all three years after 1982 lie at least $2\hat{\sigma}$ away from the actual values, and for 1984 and 1985 even more than $3\hat{\sigma}$.

Data Sources

Flow variables correspond to calendar years, and stock variables are end-of-calendar-year.

$$IC_t = I_t - \pi_{t-1,t} B_{t-1}$$

I_t = nominal net Federal interest payments [U.S. Department of Commerce, Survey of Current Business, Table 3.2, Line 22].

$\pi_{t-1,t}$ = $\ln(P_t/P_{t-1})$, with P_t the January value of the seasonally adjusted CPI [1921-1941 and 1949-1983: Barro (1986a, Table 6.4); 1984-1985: U.S. Bureau of Labor Statistics, Monthly Labor Review].

B_t = nominal end-of-year market value of interest-bearing, privately-held Federal debt [1920-1941: PVB_t multiplied by market-to-par ratio from Seater (1981, Table 4); 1942-1984: end-of-December market value from Cox (1985, Table 2); 1985: obtained from W.M. Cox, Federal Reserve Bank of Dallas (value: 1483698)].

PVB_t = nominal end-of-year par value of interest-bearing, privately-held Federal debt [1920-1976: Barro (1979, Table 3); 1977-1985: IMF International Financial Statistics, 88-88aa, or Barro (1986a, Table 6.4) for rates of growth].

Y_t = nominal GNP [1921-1985: U.S. Council of Economic Advisers, Economic Report of the President, Table B.1].

$\hat{\pi}_{t,t+1}^e$ = inflation anticipated in period t for period $t+1$ [January values; 1921-1982: predictions from model for $\pi_{t-1,t}$ above, in Barro (1986a, Table 6.4); 1983-1985: based on Barro (1986b, Table 4) (values: .048, .066, .05)].

$GVAR_t$ = transitory Federal defense spending, scaled by moving average of PVB_t [1921-1982: Barro (1986a, Table 6.5); 1983-1985: based on Barro (1986b, Table 3) (values: -.4, -.49, -.49)].

$YVAR_t$ = business cycle variable based on the rate of unemployment [1921-1982: Barro (1986a, Table 6.5); 1983-1985: based on Barro (1986b, Table 3) (values: .033, .016, .021)].

The Design Criteria of Section IV

This appendix contains a list of diagnostic statistics reported in Section IV, and briefly mentions their sources. The relevant number of degrees of freedom can be found in brackets behind the test symbols. The presence of two numbers indicates that the statistic can be compared against an F-distribution, while a single number suggests a χ^2 -distribution. A consistent framework for these model design criteria can be found in Hendry (1985).

Autocorrelation

$\eta_2(n,.)_1^j$ = Lagrange Multiplier test for autocorrelation from lags i to j ($j-i+1=n$), F-form of Harvey (1981).

$\xi_2(n)_1^j$ = Lagrange Multiplier test for autocorrelation from lags i to j ($j-i+1=n$), χ^2 -form of Godfrey (1978).

Heteroscedasticity

$\eta_4(n,.)$ = Lagrange Multiplier test for n -th order Autoregressive Conditional Heteroscedasticity, F-form, Engle (1982).

Normality Residuals

SK = Skewness.

EK = Excess Kurtosis.

$\xi_5(2)$ = χ^2 -test for normality residuals, Jarque and Bera (1980).

Parameter Constancy

$\eta_1(n,.)$ = Chow test for n forecasts.

$\xi_1(n)$ = χ^2 -test for n forecasts. As explained by Kiviet (1986), this test can be used as a general model specification test, whereas η_1 is more a measure of numerical parameter stability.

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