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Options on Foreign Exchange and Exchange Rate Expectations

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Abstract

This paper tests alternative assumptions concerning the time series behavior of foreign exchange rates. Data for about 20,000 individual trades on foreign exchange options for dollar exchange rates against six major currencies carried out from February 1983 to June 1985 are analyzed. The tests carried out suggest that, judging from the predictions of a model of options prices based on the assumption that exchange rates follow a diffusion process, market participants paid too high a price for call options that would have been profitable only if the dollar depreciated substantially within a short time period. An alternative model which allows for discrete jumps in exchange rates is found to be more consistent with the data.

Contents

Summary

Empirical models of exchange rate determination have not been successful in explaining a large component of observed changes in exchange rates in recent years. Numerous structural models have been proposed and tested and a variety of techniques for specifying expectations employed. However, tests of such models share a common assumption concerning the statistical characteristics of the "error terms" that should be "left behind" after the structural model has identified the systematic component of exchange rate behavior. This assumed error term is typically a random variable drawn from a symmetric probability distribution that is unchanged over time. If this assumption is inappropriate, estimates of the structural parameters are probably misleading.

Indeed, theoretical work in economics has increasingly suggested that error terms may not have the typically assumed structure. Actual or potential changes in policy regimes may lead to shifts in the random distribution of errors of any structural model that does not explicitly and correctly model policy behavior. Since policy reactions are typically absent or modeled naively it may be inappropriate to assume that the error structures generated by such models remain unchanged over time. In this paper an attempt is made to see whether market participants expect the large discrete changes in exchange rates that are implied by changes in policy regimes. It is argued that foreign exchange options contracts would reflect such expectations. An option contract allows the holder to pay a price today for the right to buy or sell a currency at a fixed price for some time interval. It is shown that the price of such options depends on the statistical process that the buyer and seller of the option expect the exchange rate to follow over time.

The empirical work is based on data for about 20,000 individual trades carried out at the Philadelphia Stock Exchange from the opening of this market in February 1983 to June 1985. The main result is that a model of options prices based on the usual assumption that discrete changes in exchange rates are not expected is not consistent with the data on options prices. This model suggests that market participants placed too high a value on options that were profitable only if exchange rates changed by large amounts over short time periods. The data on options prices is better explained if it is assumed that market participants expect exchange rates to experience large discrete jumps, perhaps because of policy changes. In particular the empirical tests suggest that the possibility of a sudden depreciation of the U.S. dollar with respect to the deutsche mark and the Japanese yen was taken quite seriously in financial markets between 1983 and 1985. These results suggest that the normal assumptions regarding error terms in the structural models typically used to explain exchange rate changes may be inappropriate. If this is the case it is possible that valid models that relate exchange rate determination to fundamental economic variables may have been rejected prematurely.

I. Introduction

Empirical models of exchange rate determination have not been successful in explaining a large component of observed changes in exchange rates in recent years. A large number of structural models have been proposed and tested including those that emphasize monetary variables, portfolio balance conditions and more recently real exchange rate behavior. Moreover, a variety of techniques for specifying expectations have also been employed. However, tests of such models share a common assumption concerning the statistical characteristics of the "error terms" that should be "left behind" after the structural model (including the specification of expectations) has identified the systematic component of exchange rate behavior. This assumed error term is typically a random variable with a symmetric probability distribution around a zero mean; this random structure is assumed to be constant over time, and independent of the values assumed by the error term at other points in time.

Theoretical work in economics, however, has increasingly suggested that error terms may have a different structure. In particular, actual or potential changes in policy regimes may lead to discontinuities in the random distribution of errors of any structural model that does not explicitly and correctly model policy behavior. Since policy reactions are typically absent or modeled naively it is argued that it is inappropriate to assume that the errors generated by such models will have the structure that is assumed. 1/

The difficulty in directly observing the effects of expected regime changes that happen infrequently has led us to look for clues in the foreign currency options market. While prices quoted in most international financial markets tend to reflect a measure of the expected future value of the exchange rate (probably obscured by unobservable risk premiums), option contracts provide a different type of information. Option prices reflect the markets' view about the type of time series process followed by the exchange rate given prices (rather than expectations of those prices) such as spot exchange rates and interest rates on securities denominated in different currencies.

An additional advantage is that models of option prices are independent of the structural characteristics of the economy, that is, factors such as investors' degree of risk aversion and the process by which asset prices are determined in the economy. This is true because those factors are entirely summarized by the current exchange rate and security prices. Thus, option prices provide a unique opportunity to identify the kind of exchange rate stochastic process implicit in market participants' behavior.

1/ Theoretical work that suggests that exchange rate dynamics may be influenced by expected regime changes includes Krugman (1979) and Flood and Garber (1982) with reference to exchange system changes; Dornbusch (1982) with reference to policy shifts.

Our empirical results show that the option price formula based on the assumption that exchange rates follow a process that admits no "jumps" or discontinuities in its path (the Black-Scholes model) differs systematically from recorded prices. In particular, market prices of options that will be valuable only if exchange rates increase by a large amount (options said to be "out of the money") are systematically higher than model predictions. These are options that required a substantial depreciation of the US dollar to be of value at maturity.

Our preferred explanation of this bias (the "high" prices of options that are out of the money) is that, during this sample period 1/ market participants considered that a large, abrupt depreciation of the dollar was possible, perhaps as the consequence of a change in policy regime. In fact, there is a good deal of casual evidence that points in this direction. The U.S. dollar had been appreciating since 1980 and was reaching a level increasingly regarded as "overvalued." Actually, some three months after the sample-end the dollar did fall very rapidly following statements of concern by authorities in the Group of Five industrial countries. (We plot the exchange rate of the U.S. dollar versus the six currencies for which foreign exchange options are traded in Chart 1.) Furthermore, surveys of exchange rate expectations show that in 1983-84 forecasters consistently predicted a depreciation of the U.S. dollar (that did not take place until March 1985). 2/

It is possible to provide an intuitive explanation of why the possibility of an abrupt depreciation of the dollar would produce a bias such as the one observed in the predictions of the Black-Scholes model. When the option contract is out of the money, only a large rise in the price of foreign exchange (depreciation of the dollar) will prevent that contract from being valueless by expiration time. If the exchange rate follows a continuous process through time the chances of such large depreciation are virtually nil, and the predicted prices of these options would correspondingly be very low. Of course, an exchange rate that followed a continuous process with sufficiently high volatility would explain high prices for out of the money options, but in such a case all predicted options prices would be higher. Therefore, a higher volatility parameter would not avoid the exercise price bias, but instead it would lead to overprediction of contract prices which are "at the money" or "in the money". If market participants expected that a significant probability of a sudden large depreciation of the dollar existed, they would be willing to pay higher prices for out-of-the-money options relative to in-the-money options than would be predicted by models based on the assumption that such "jumps" are not possible.

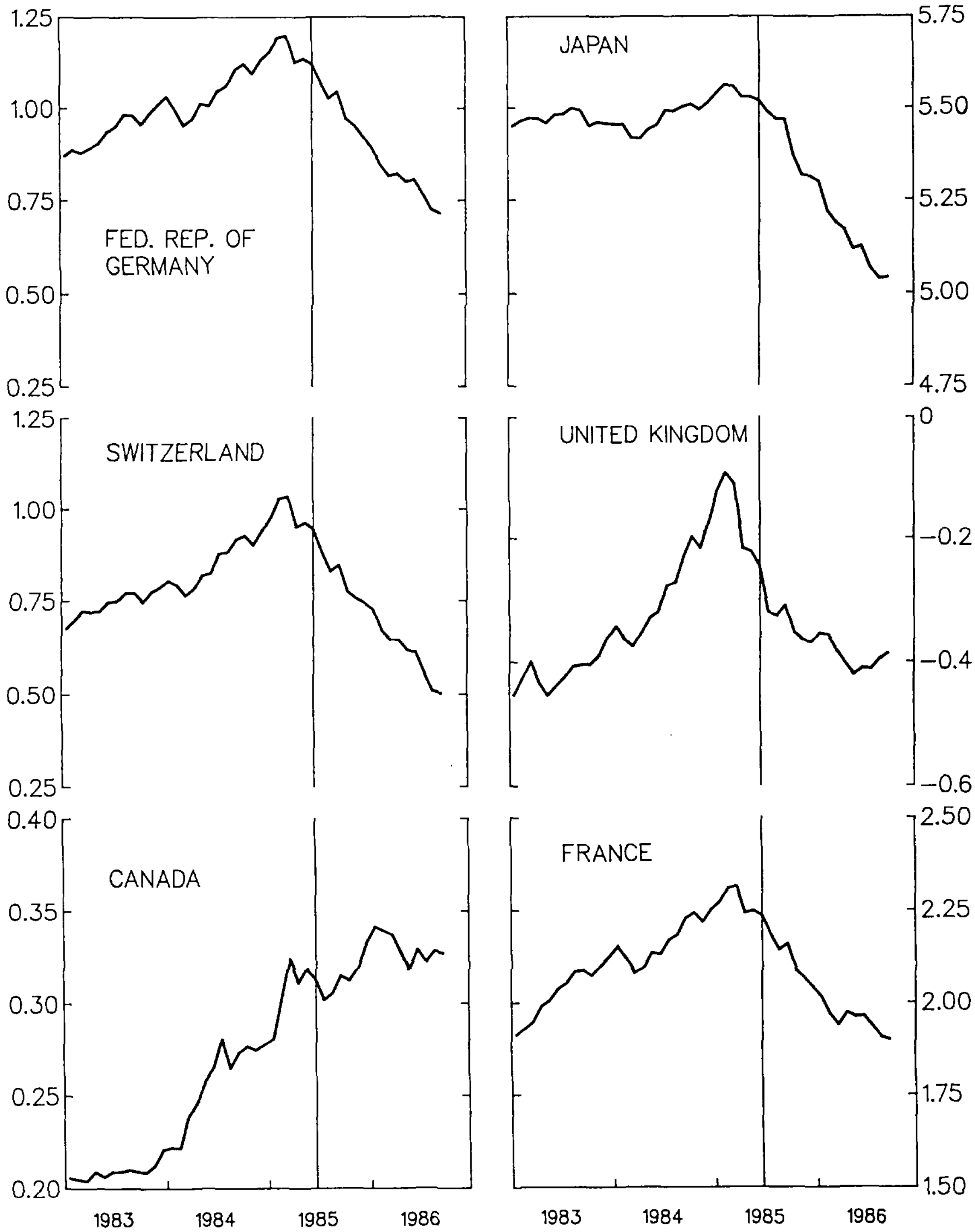
In order to gain a better perspective on this exercise-price bias, a second model of option pricing was developed and estimated. In this

1/ February 1983 to June 1985.

2/ See Frankel and Froot (1986) and Dominguez (1986).

CHART 1
U.S. DOLLAR EXCHANGE RATES, JANUARY 1983 – SEPTEMBER 1986

(Local currency per U.S. dollar, in logarithmic scale)



Sample period:
February 1983 – June 1985

Sample period:
February 1983 – June 1985

framework, the exchange rate is assumed to evolve through time in the following way: it displays a constant rate of change most of the time but, at discrete intervals, it takes a "jump" of a given size in its value. 1/ The times of arrival of jumps are stochastic, and there is a constant probability that an arrival will take place at any instant. 2/ Although the assumptions on which this model is based are probably too restrictive, this model may capture the effect of the probability of discontinuities in the exchange rate process. In fact, in turns out that, for most currencies, this "pure jump" model does not exhibit the exercise price bias displayed by the Black-Scholes model.

II. The Black-Scholes Model

Option valuation theory is based on the proposition that options are not an independent asset in the sense of representing an independent risk. For example, consider the case of a stock option. The fate of the firm's investments will determine the movements in the stock price and, indirectly, the value of the option; only the random events that change the stock price will affect the option price. This suggests that it should be possible to obtain an option pricing formula that is solely a function of the price of the underlying financial asset (the stock price), and that is independent of the way in which that price is determined, that is, of the kind of asset market pricing that rules financial markets.

The same proposition applies to the pricing of options on foreign exchange in which the foreign currency constitutes the underlying asset. In fact, this valuation theory can be applied only in certain cases, that is, when the exchange rate (or the price of the underlying asset) follows a stochastic process through time of some particular class. In even more special cases, the differential equation that is obtained has a closed form solution and can, therefore, be readily used as an option price formula. 3/

The first and most widely used model of option price valuation is due to Black and Scholes (1973), and has been applied to foreign exchange options by Garman and Kolhagen (1983) and Grabbe (1983). It assumes that the exchange rate (the stock price in Black-Scholes) follows a diffusion process. Loosely speaking, a diffusion process can be described as the continuous-time version of a random walk with a drift. Under this assumption, in any interval of time, the rate of increase of the exchange rate is equal to the sum of two components: first, a given non-stochastic

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1/ This model is the application to foreign exchange options of the "pure jump" model designed by Cox and Ross (1975) for stock options. For details see Appendix 2.

2/ That is, the arrivals are realizations of a Poisson process.

3/ Black and Scholes solved it by putting it in a form analogous to the heat equation. A good intuitive exposition of option valuation theory can be found in Cox and Ross (1976).

rate, and, second, a random term that is the continuous-time analog to a random walk. The latter term must, therefore, be independent both of the contemporaneous level of the exchange rate and of the rate of increase of the exchange rate in any other non-overlapping interval. The diffusion process can be written in the following way:

$$dS/S = \mu dt + \sigma dZ \tag{1}$$

where S is the exchange rate, μ is the (instantaneous) mean rate of growth (called the drift of the process), σ^2 is the variance of the process (more precisely, $\sigma^2\Delta t$ is the variance over the interval Δt), and Z is a random variable that follows a normal Weiner process with zero mean and unit variance (the increase in a Weiner process over any time interval is a normal variate and is independent of its increase over any other nonoverlapping interval, no matter how small). 1/

If μ is equal to zero, the diffusion process (1) is the continuous-time analog of a random walk on the log of the exchange rate. It is fortunate that the Black-Scholes model includes the random walk as a particular case because that model is considered to be a good approximation to the exchange rate. 2/ As we will see, the value of μ is, in fact, irrelevant for the option price formulae, so that the empirical analysis carried out here is consistent with a family of models, one of which is the random walk.

When the exchange rate follows a diffusion process and risk-free interest rates on the two currencies are constant over the time to expiration of the option 3/ it is possible to obtain an option price formula along the Black-Scholes lines. This formula expresses the price of a call option as a function of the current spot price, the two interest rates, the time to expiration of the option, and its exercise price. The only unknown parameter of this formula is σ, the volatility of the exchange rate process:

$$c = \exp(-r^*T)SN(x+\sigma\sqrt{T}) - \exp(-rT)EN(x) \tag{2}$$

where

$$x = \frac{\ln(S/E)+(r-r^*-\sigma^2/2)T}{\sigma\sqrt{T}}$$

1/ Discussions of Ito processes are available in many stochastic process textbooks. An intuitive treatment is given in Cox and Miller (1968).

2/ However, the empirical work that favors the random walk model tests only against local alternatives to that specification. See Meese and Singleton (1982) or Takagi (1986).

3/ The same formula obtains if interest rates are themselves diffusion processes. However, in that case, the parameter σ does not represent the volatility of the exchange rate but is a function of the variances and covariances of the exchange rate and the two interest rates.

Above, c is the price of the call option, r* and r are the (instantaneous) rates of interest on foreign and domestic bonds, respectively, S is the spot exchange rate (price of foreign exchange), E is the exercise price, T is the time to maturity of the contract, and N(z) is the value of a standard normal distribution function evaluated at z.

Two comments on (2) are important here. First, it is remarkable that only σ, the instantaneous standard deviation or volatility of the exchange rate enters (2) while μ, the deterministic part of the exchange rate process does not enter (2). The reason is that the rate of appreciation or depreciation does not affect the functional relationship between the option price and the exchange rate (although it will obviously affect the value of the exchange rate itself and also interest rates). 1/ This means that the option price formula is identical for different values of the trend rate of change in the exchange rate. Therefore, if different investors had different assessments about the expected exchange rate, they would still agree on the option price as long as they agreed in the volatility of the process.

The second comment on (2) is that it strictly applies only to European options. This is a problem because the options traded in the Philadelphia Stock Exchange are of the American type. In the case of dividend-protected 2/ stock options, it has been demonstrated (Merton (1973)) that a call option will never be exercised before its expiration, and therefore European and American options should be valued the same. Unfortunately, this property does not apply to foreign exchange options, and (2) should be strictly interpreted as a lower bound of the equilibrium American call option price. However, calculations made by Shastri and Tandon (1986) show that this potential early exercise bias tends to be of insignificant magnitude in the case of call options, in particular when the domestic interest rate is at least as high as the foreign one. By contrast, American and European put option prices differ more significantly and much more frequently. For this reason, we restricted the empirical work to call options.

1/ However, μ affects the rate of change of the option price over time. This is immediate from the fact that the option price is a function of the exchange rate. More precisely, the expected rate of change in the option price will satisfy:

$$\frac{\mu_c - r}{\sigma_c} = \frac{\mu + r^* - r}{\sigma}$$

where μ_c and σ_c are the (instantaneous) expected rate of return and standard deviation of the option price, c.

2/ A dividend-protected stock option is one in which the exercise price decreases on the ex date by an amount equal to the dividend.

III. Empirical Results: Black Scholes Model

The most significant empirical result is that the Black-Scholes model displays a substantial exercise-price bias. 1/ The value of the volatility parameter implicit in the Black-Scholes model was estimated, for each foreign currency option separately, as that value which minimized the sum of squared deviations from the price formula. That is, σ is the coefficient obtained from a non-linear regression of the option prices on the formula (2). Then, the residuals of such regression (the difference between market prices and Black-Scholes prices), expressed as percentage deviations from market prices, were plotted against the ratio of the exercise price to the current spot price. 2/ See Chart 2. Points in plots correspond to the average values of the variables in the corresponding ranges.

The exercise price bias displayed by the Black-Scholes model is very significant in the cases of deutsche mark, Canadian dollar, Swiss franc, and Japanese yen options. The model underpredicts prices of options that are out of the money, and the underprediction increases as the ratio of the exercise price to the current spot price rises. For example, consider the deutsche mark option prices obtained using the value of σ that minimizes squared residuals. The divergence between market prices and predicted prices is less than ten percent of the market price for options that are approximately at the money. But for options for which the exercise price to spot price ratio is between 12.5 and 15 percent, market prices are almost twice the predicted prices, and, for options for which the exercise price to spot price ratio is over 23 percent, market prices are almost fifty times the predicted prices.

In contrast, the Black-Scholes model appears to fit well the data on French franc options, since the corresponding plot shows no systematic relationship between prediction errors and the position of the option in or out of the money. The British pound exchange rate appears to be a special case as well. Although the Black-Scholes option prices do not display the same exercise price bias (or at least not in the same magnitude), it is harder to argue that there is no bias altogether, because the plot is far from a flat line. This special behavior of the French franc and the pound sterling options is an interesting result; we will speculate later on possible reasons.

The observed bias does not arise from an imprecise estimate of the parameter σ. Because the option price depends positively on the volatility of the exchange rate, an increase in that parameter produces higher predicted option prices. Thus, plots for different values of volatility,

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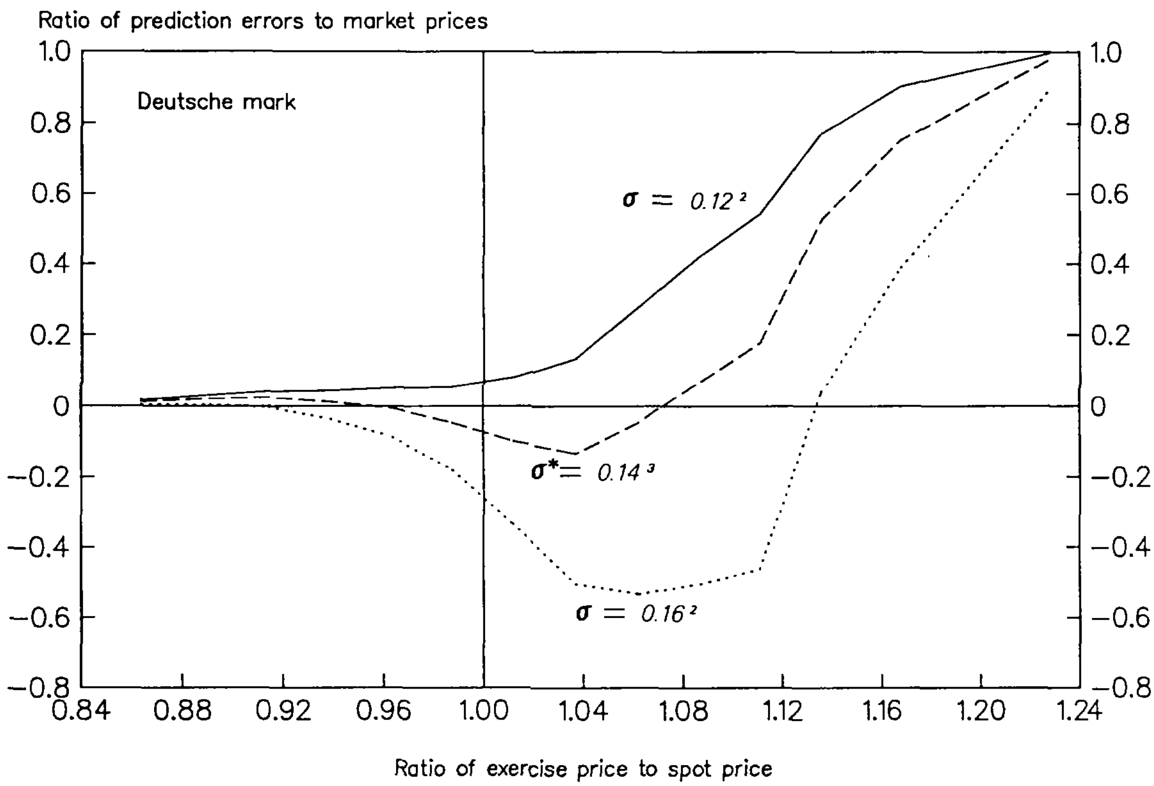
1/ The data used in the estimation, described in more detail in Appendix 1, comprise call option trades in the Philadelphia Stock Exchange market between February 1983 and June 1985.

2/ Actually, against the more standard measure: $E \exp(-rT)/S \exp(-r^{*}T)$.

CHART 2 [1]

CALL OPTIONS: BLACK–SCHOLES EXERCISE PRICE BIAS, FEBRUARY 1983 – JUNE 1985



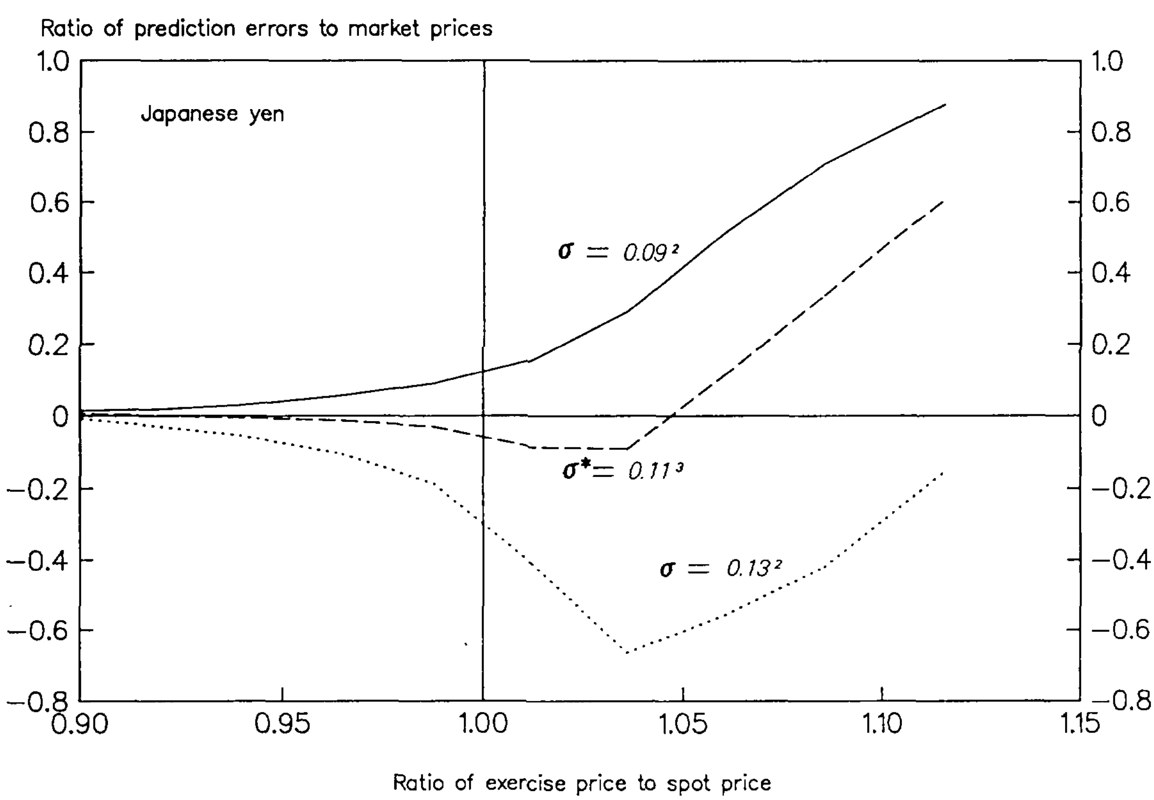
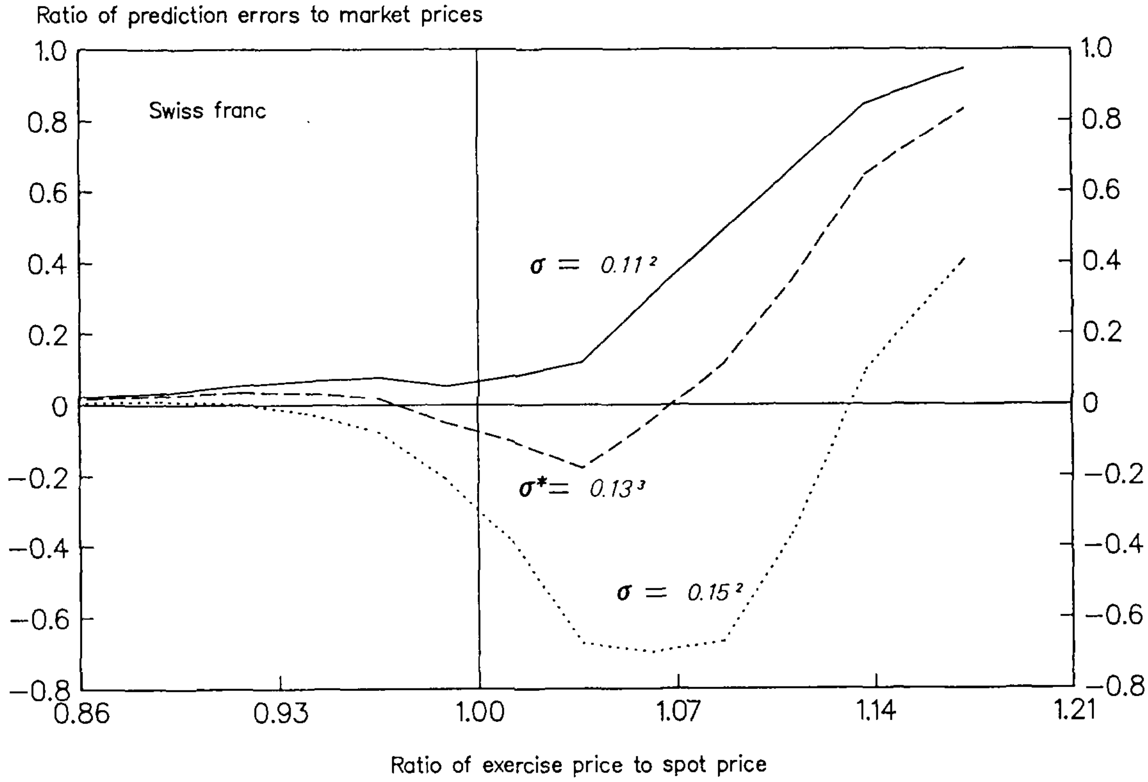


CHART 2 (Continued)

CALL OPTIONS: BLACK–SCHOLES EXERCISE PRICE BIAS, FEBRUARY 1983 – JUNE 1985



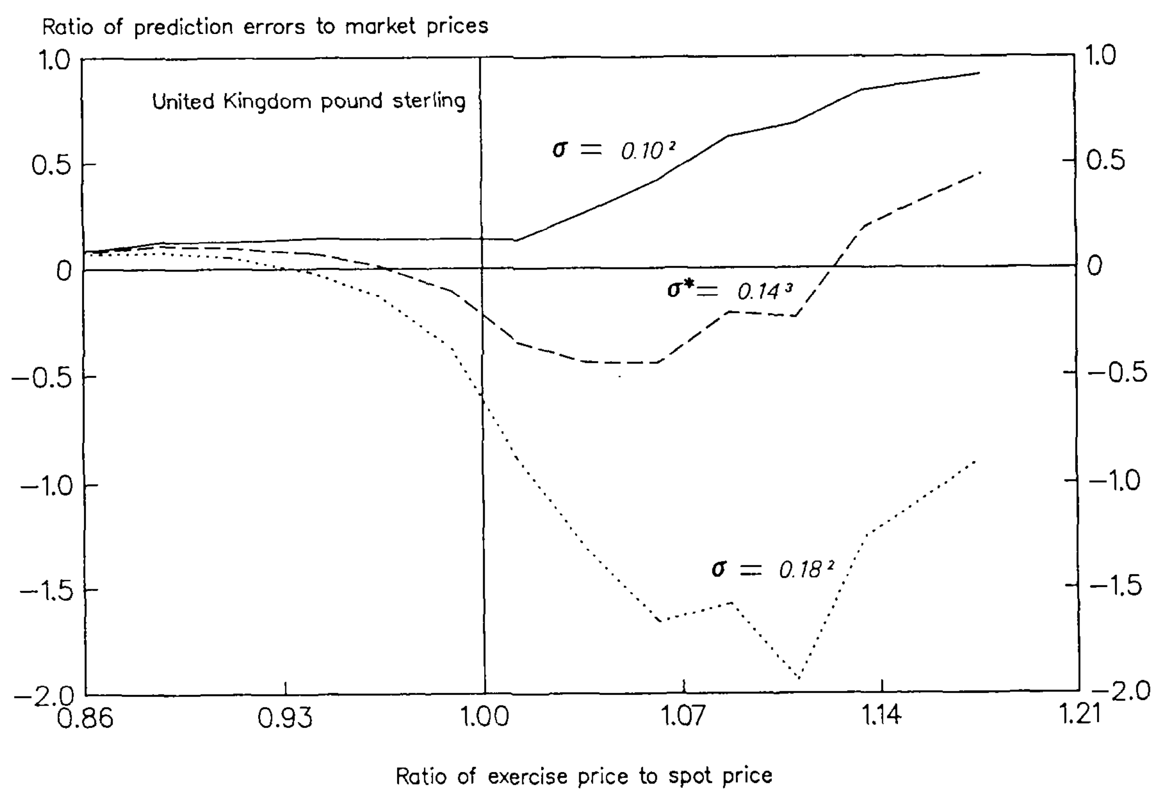
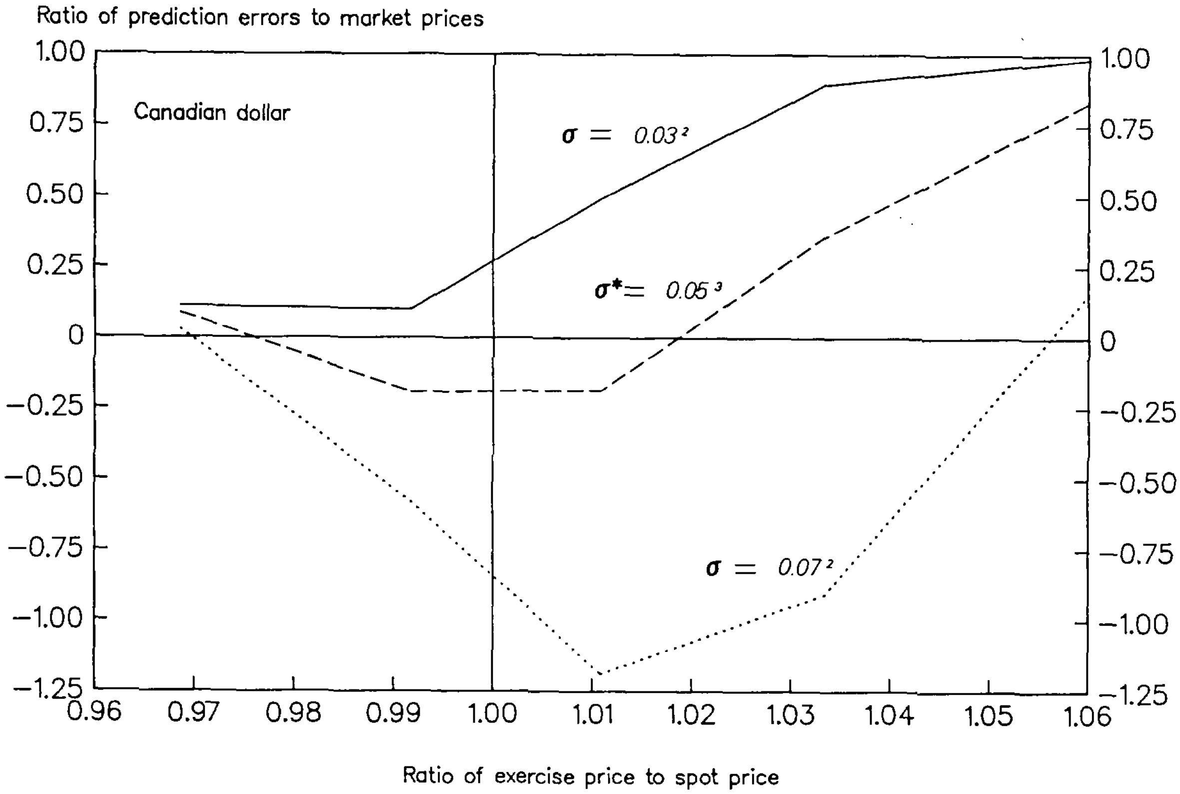
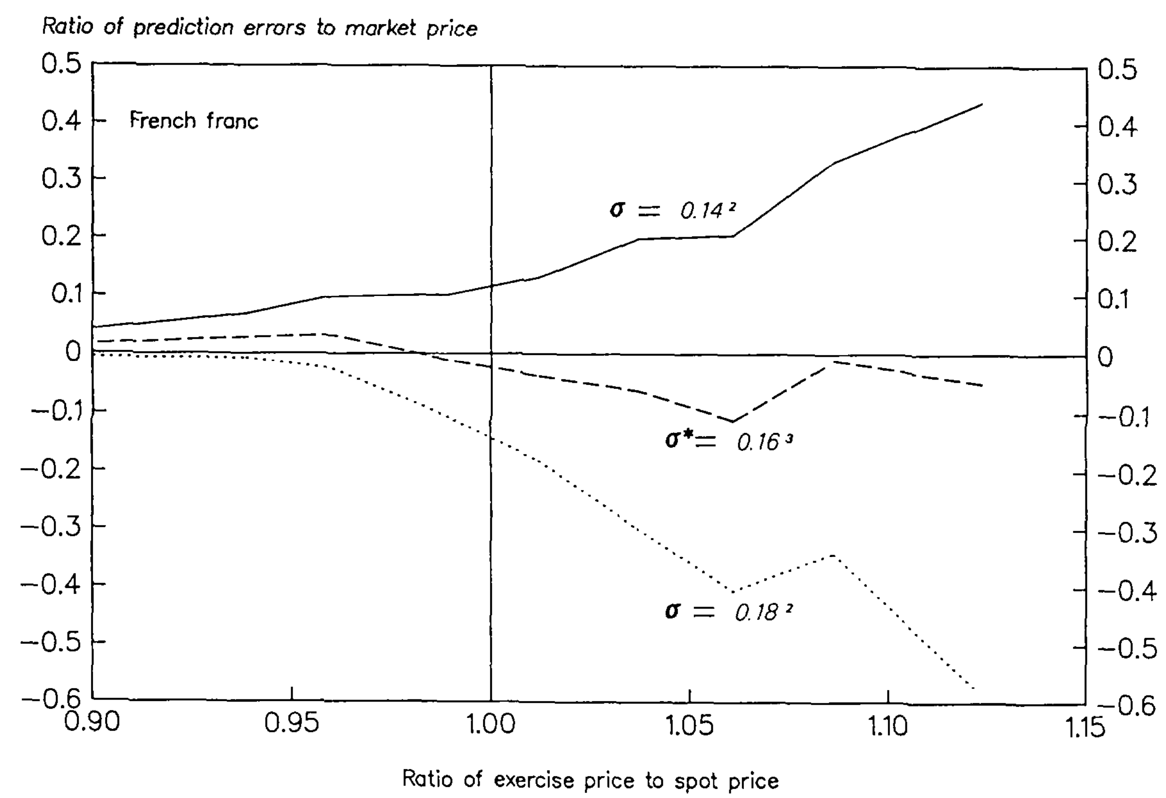


CHART 2 (Concluded)

CALL OPTIONS: BLACK–SCHOLES EXERCISE PRICE BIAS, FEBRUARY 1983 – JUNE 1985





[1] See Appendix I for source and description of data, and variables and methodology used.
[2] σ = diffusion parameter in Black Scholes' option pricing formula.
[3] σ^* = parameter that minimizes squared residuals.

σ, can never cross over. Furthermore, we found that the plots for different values of σ were remarkably parallel. This means that increasing the assumed volatility of the process produces smaller errors at one end of the in-out of the money spectrum, but only at the expense of producing higher errors at the other end.

Moreover, we found no evidence that instability in the value of the volatility parameter over the sample period could serve as the explanation for the bias in the Black-Scholes model. Instability in σ could be the cause of the bias if, for example, most observations of out-of-the money options corresponded to a period in which volatility was relatively high, etc. 1/ Indeed, prediction errors appeared to change signs at roughly the middle of the sample period. For that reason, the sample was divided in half--in terms of the number of observations--at August 1984, and the volatility parameter was estimated separately. Although estimated volatilities are indeed higher in the second half of the sample period, the difference is not very large (the values obtained for the two samples fall within the bounds used in Chart 2). More importantly, with the exception of British pound options, the same general pattern of exercise-price bias remains present in both subsamples. (This is depicted in Chart 3.)

There is no evidence of any systematic bias in the Black-Scholes model between prediction errors and time to maturity. As can be seen in Chart 4, the prediction errors do not appear to be correlated in any way with the time to maturity of the option contracts. A time to maturity bias can also be evidence of model misspecification, as illustrated in Rubinstein (1985).

If the explanation of the exercise-price bias advanced in this paper--the influence of potential jumps in exchange rates--is correct, we should observe an increase in the exercise-price bias of the diffusion model as the time to maturity shortens. This is so because the contrast between the predictions of a diffusion and a jump model would be greater for options with a shorter time to expiration.

In order to test that proposition, we plotted the prediction errors against the time to maturity, only for those options for which the ratio of the exercise price to the spot exchange rate exceeded 1.05 (1.01 for Canadian dollar options). Our presumption is confirmed in the case of *deutsche mark, yen, and Swiss franc options. For those three currencies,* the underpricing of out-of-the-money options becomes larger as the time to maturity becomes shorter.

In summary, the empirical analysis indicates that, with the exception of French franc options, the Black-Scholes model is not a good representation of foreign exchange option prices. It also poses the question of

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1/ Note that the plots in Chart 2 are of a mixed cross section-time series nature. Each point corresponds to an average of contract prices that may be dated at widely scattered points in time.

why market participants traded out of the money options at such "high" prices. We suspect that the answer is that market participants do not perceive the exchange rate as a diffusion process, and now turn to the study of a model based on a different type of stochastic process.

IV. The "Pure Jump" Model

In the diffusion process (1) the random component follows a random walk over time displacing the exchange rate in a continuous way; by contrast, the pure jump process represents the other polar case: the random term is nonzero only at scattered points in time, and, at those points, it produces discrete jumps in the value of the exchange rate. This implies that, in the pure jump process, the exchange rate follows a deterministic path except for the discrete changes in its value that arrive at random intervals.

The assumption that an asset price follows a jump process captures the possibility that the economic "news" that affects the asset price tends to arrive in discrete lumps rather than in a smooth flow. In the case of the exchange rate, the possibility of jumps appears to be an important feature describing their time paths, particularly considering events such as market intervention, realignments, shifts in monetary or fiscal policy, commodity price changes, etc.

In this sample period, it is natural to assume that such abrupt changes were expected to be linked to a depreciation of the U.S. dollar (an increase in S, the price of foreign exchange). This is so because after an almost continuous appreciation since 1980, the U.S. dollar was reaching a point increasingly considered as "overvalued". In fact, some three months after the sample-end, the Group of Five industrial countries decided to coordinate efforts to obtain a lower value for the U.S. dollar, a policy change that was followed by a sharp drop in dollar exchange rates.

The pure jump model is derived from the assumption that the exchange rate follows the following process:

$$dS/S = \mu dt + \phi d\pi \tag{3}$$

where ϕ is the jump amplitude and π is a random variable with a Poisson distribution such that:

$$d\pi = 1 \text{ with probability } \lambda dt$$

$$d\pi = 0 \text{ with probability } 1-\lambda dt$$

The difference between the diffusion process (1) and the jump process (3) is that the nature of their stochastic parts is opposite. While the random term of the diffusion process smoothly pushes the exchange rate in

CHART 3 [1]

CALL OPTIONS: BLACK-SCHOLES EXERCISE PRICE BIAS, FEBRUARY 1983 - AUGUST 1984, AND SEPTEMBER 1984 - JUNE 1985

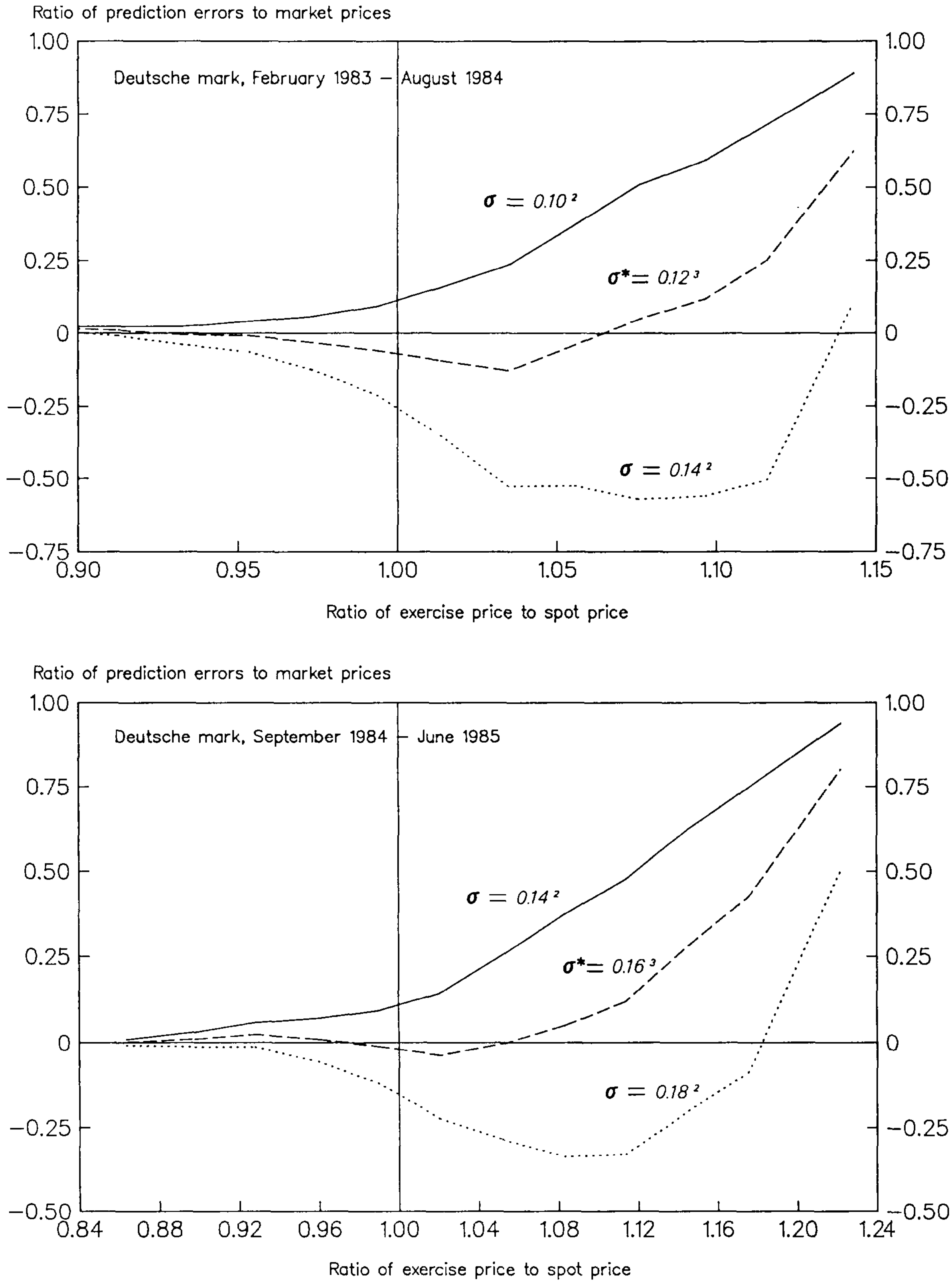
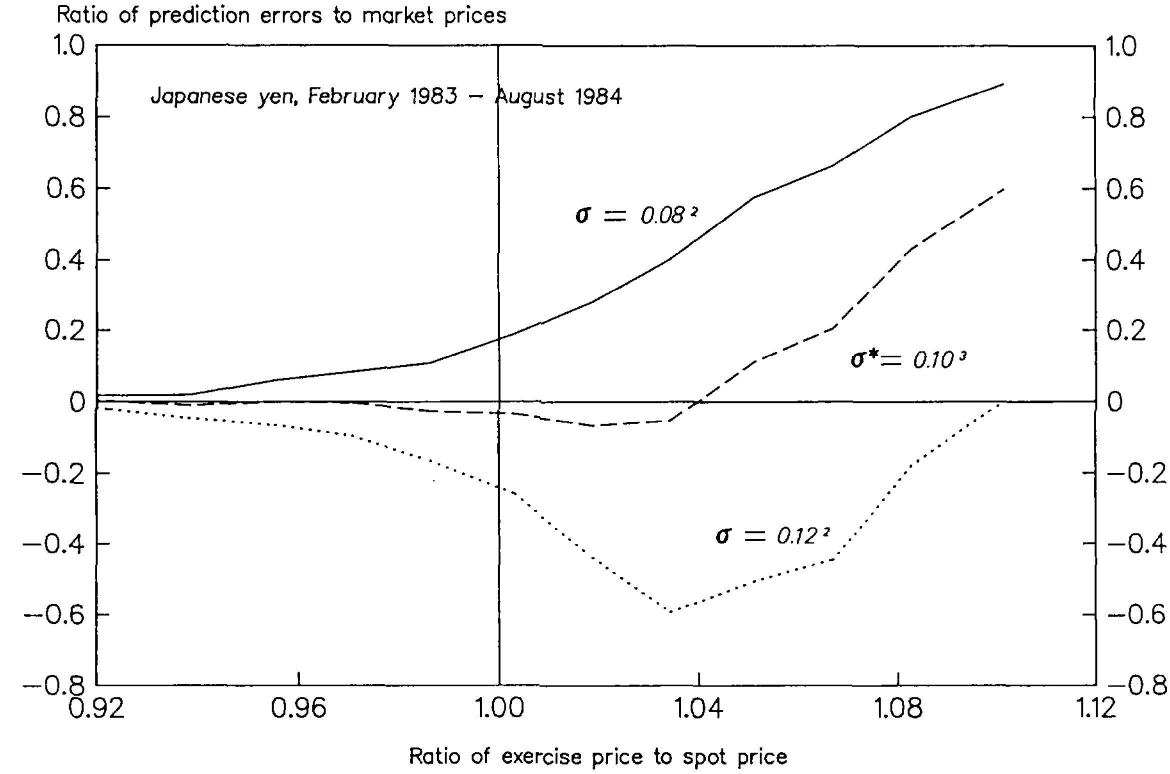


CHART 3 (Continued)

CALL OPTIONS: BLACK-SCHOLES EXERCISE PRICE BIAS, FEBRUARY 1983 – AUGUST 1984, AND SEPTEMBER 1984 – JUNE 1985



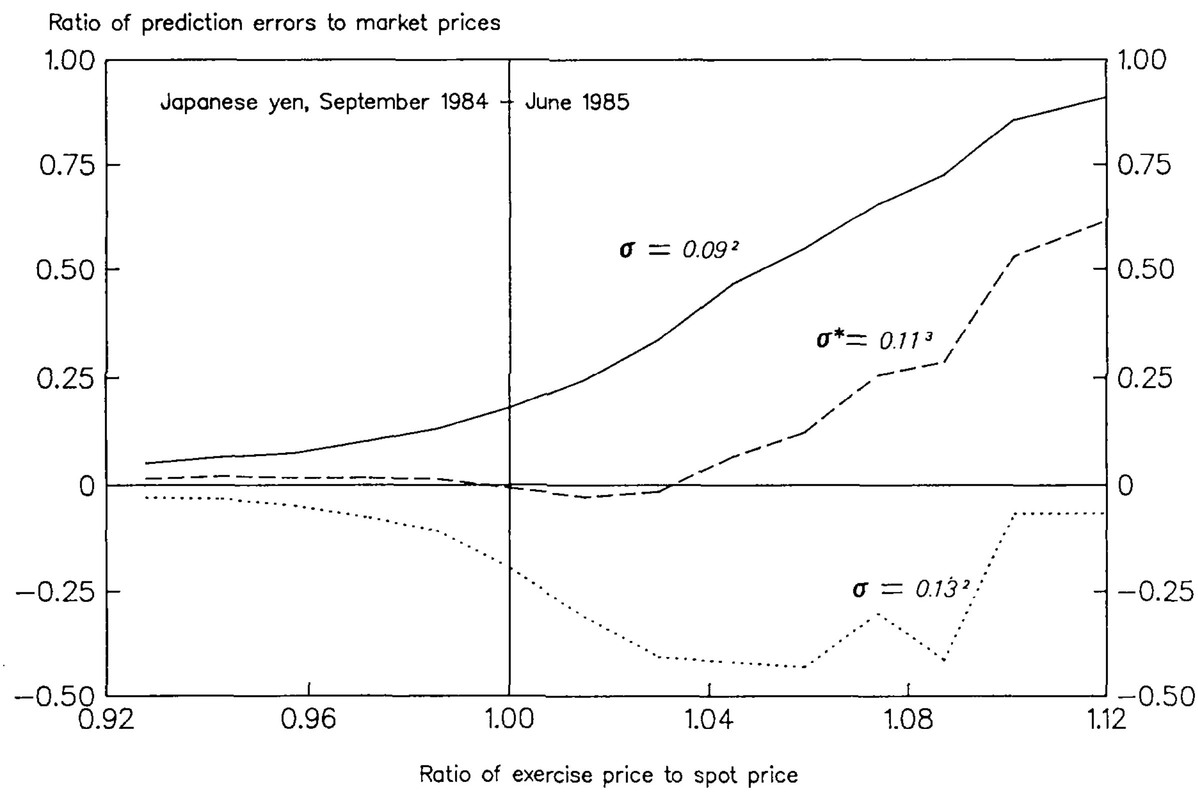
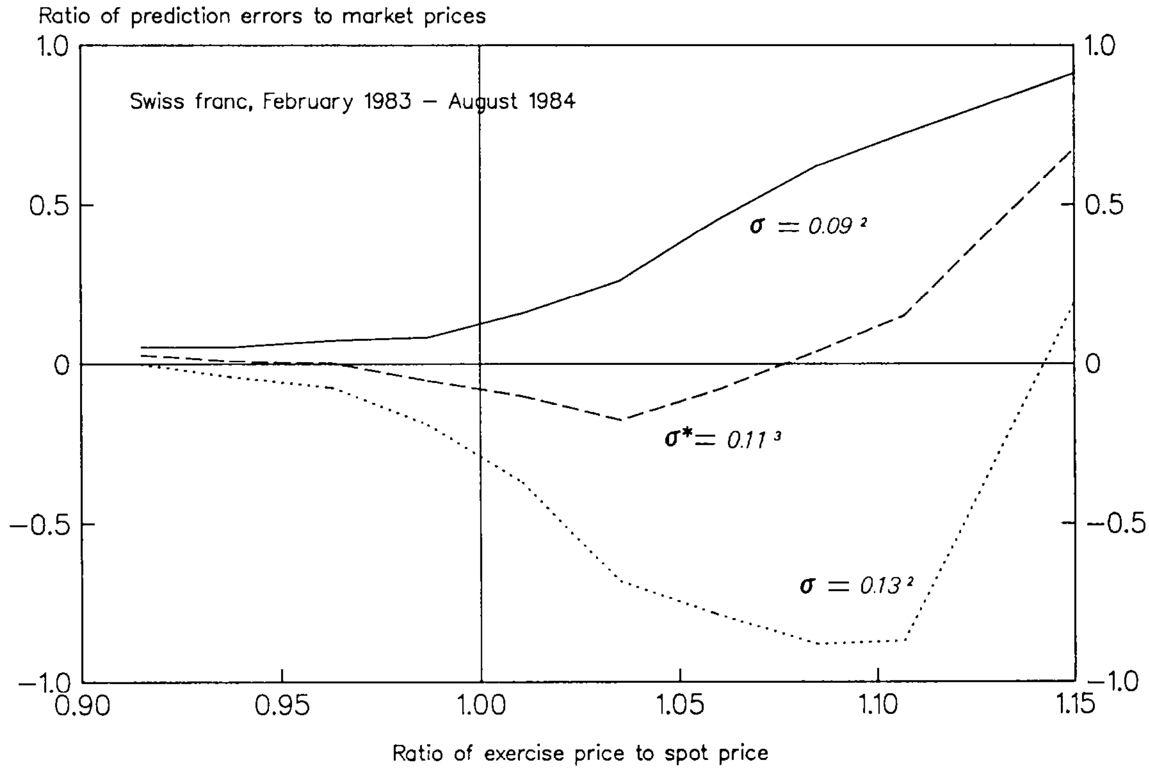
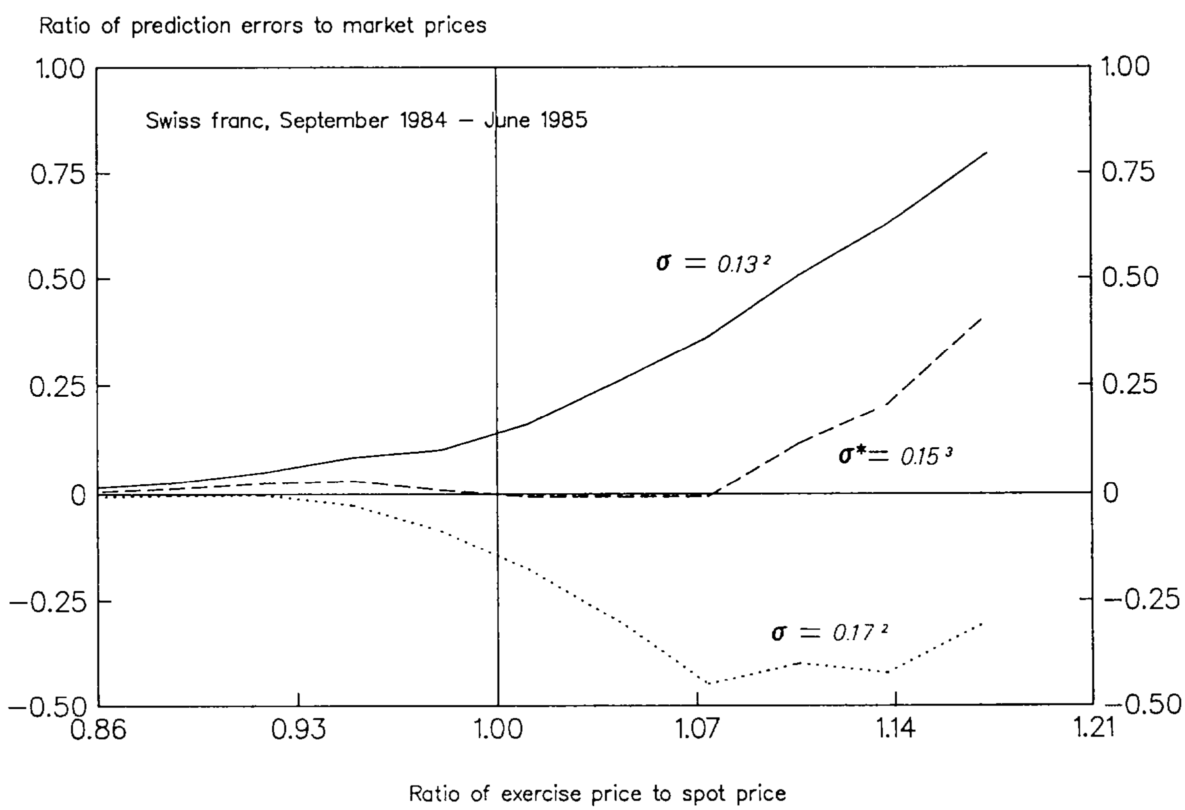


CHART 3 (Continued)

CALL OPTIONS: BLACK–SCHOLES EXERCISE PRICE BIAS, FEBRUARY 1983 – AUGUST 1984, AND SEPTEMBER 1984 – JUNE 1985





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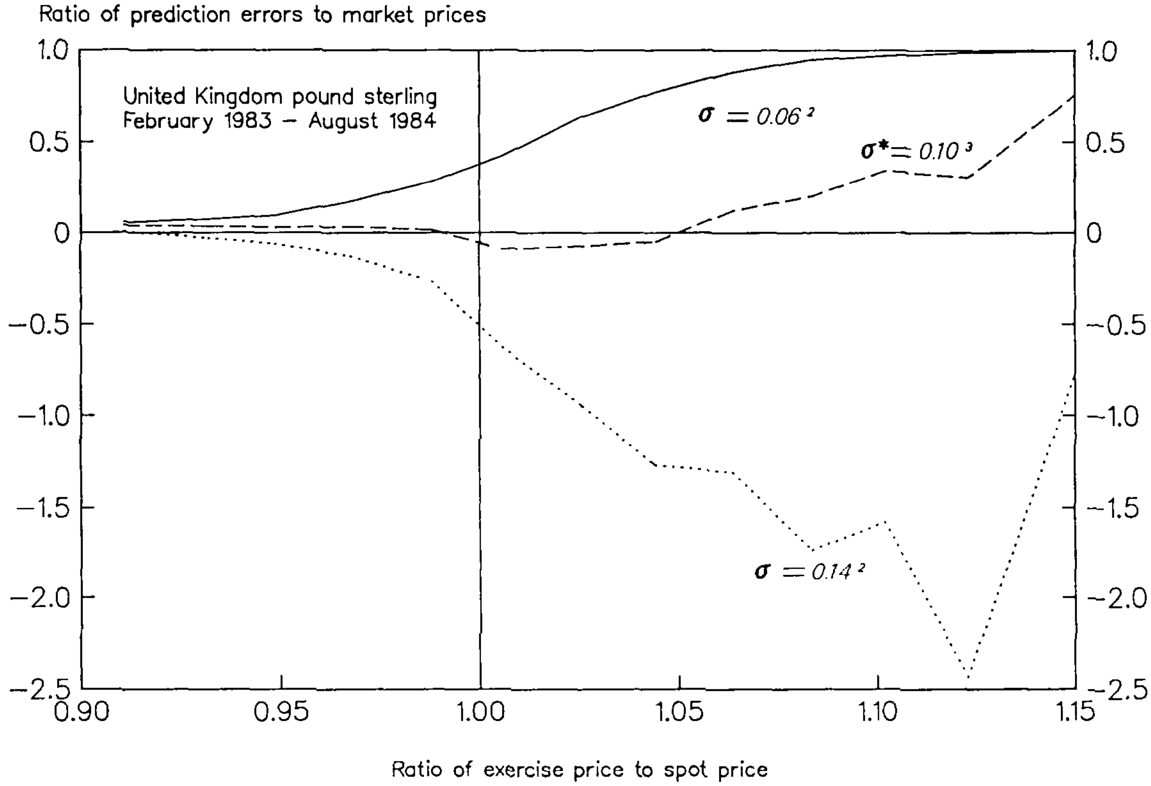

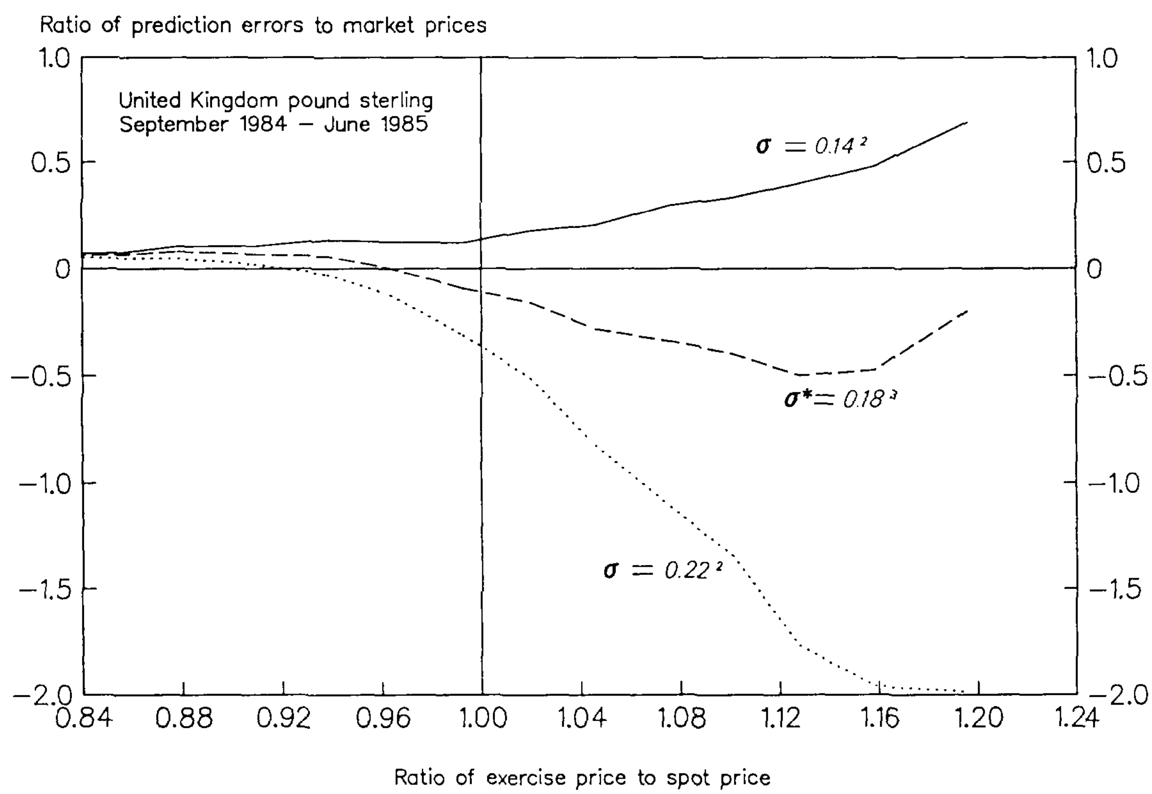

CHART 3 (Concluded)

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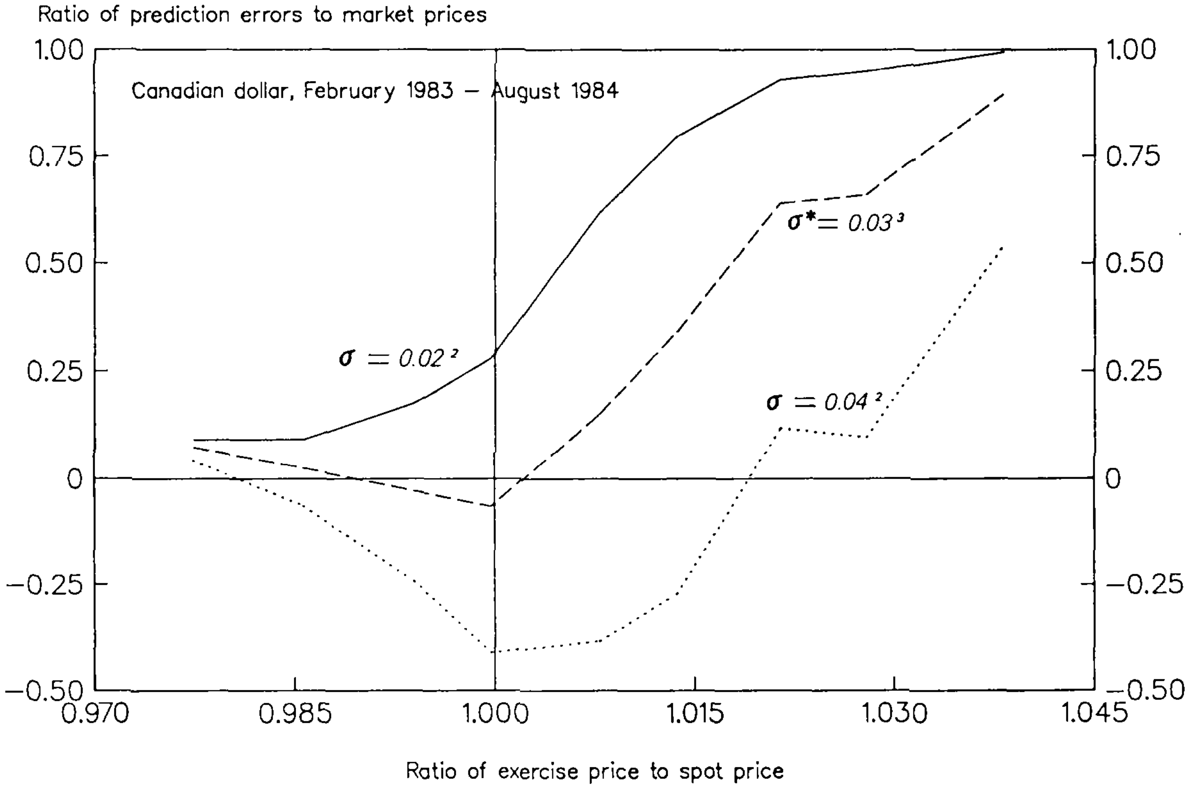
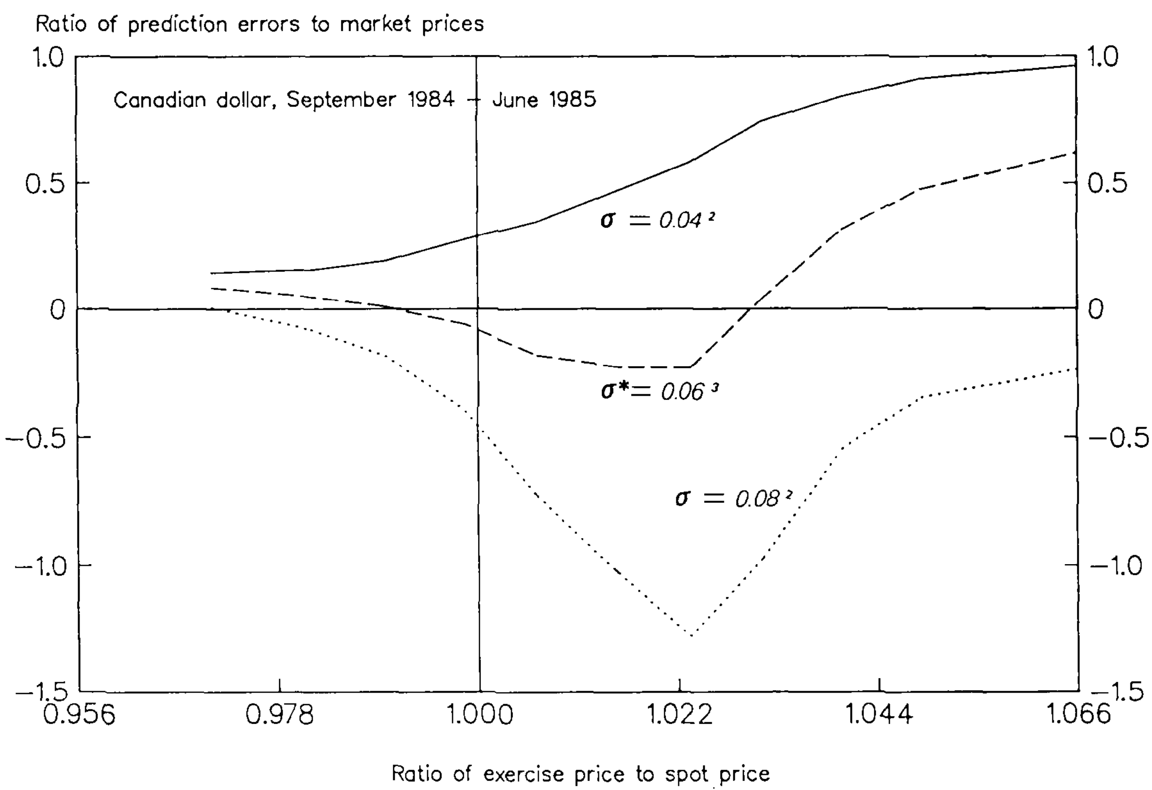




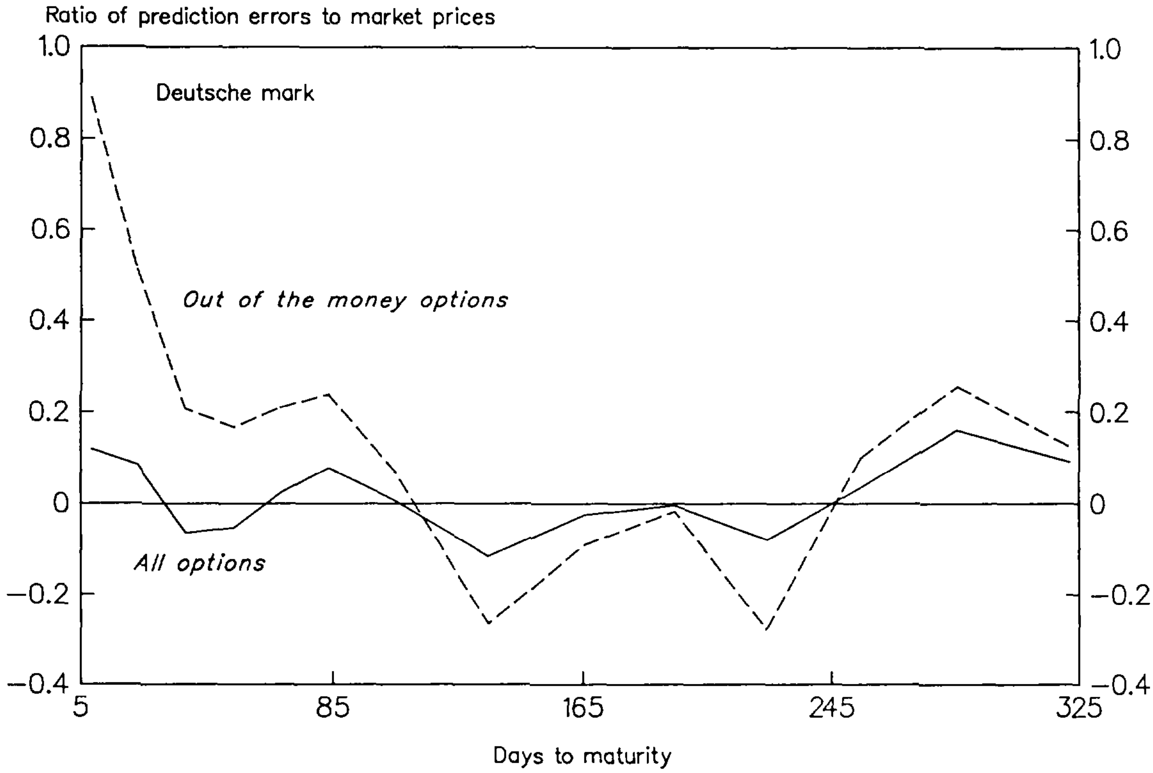
1 See Appendix I for source and description of data, and variables and methodology used.

2 σ = diffusion parameter in Black-Scholes' option-pricing formula.

3 σ^* = parameter that minimizes squared residuals.

CHART 4 [1]

CALL OPTIONS: BLACK–SCHOLES TIME TO MATURITY BIAS, FEBRUARY 1983 – JUNE 1985



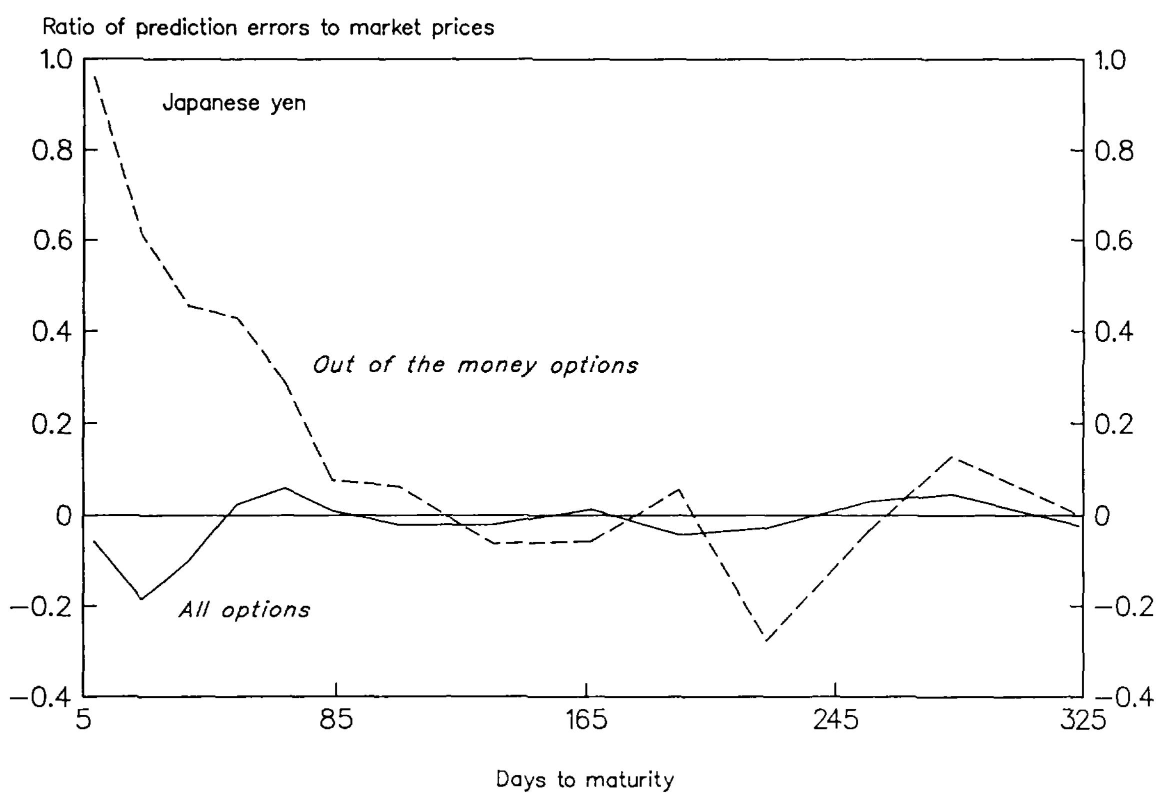
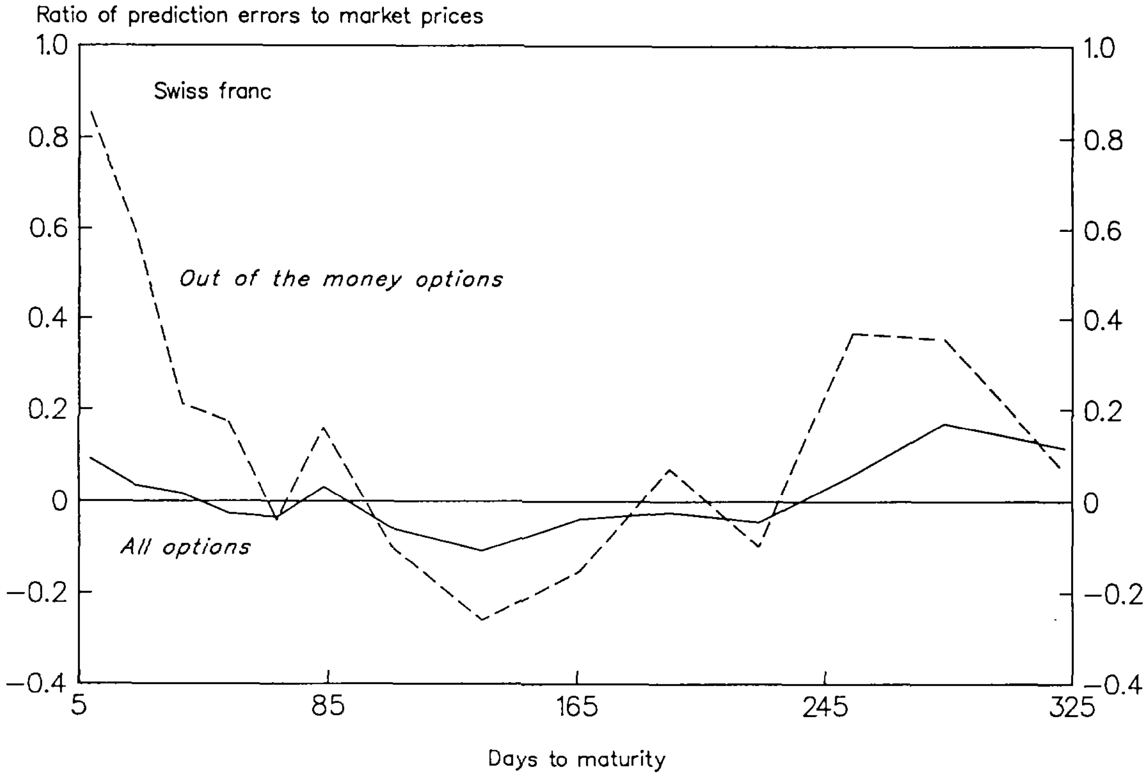


CHART 4 (Continued)

CALL OPTIONS: BLACK–SCHOLES TIME TO MATURITY BIAS, FEBRUARY 1983 – JUNE 1985



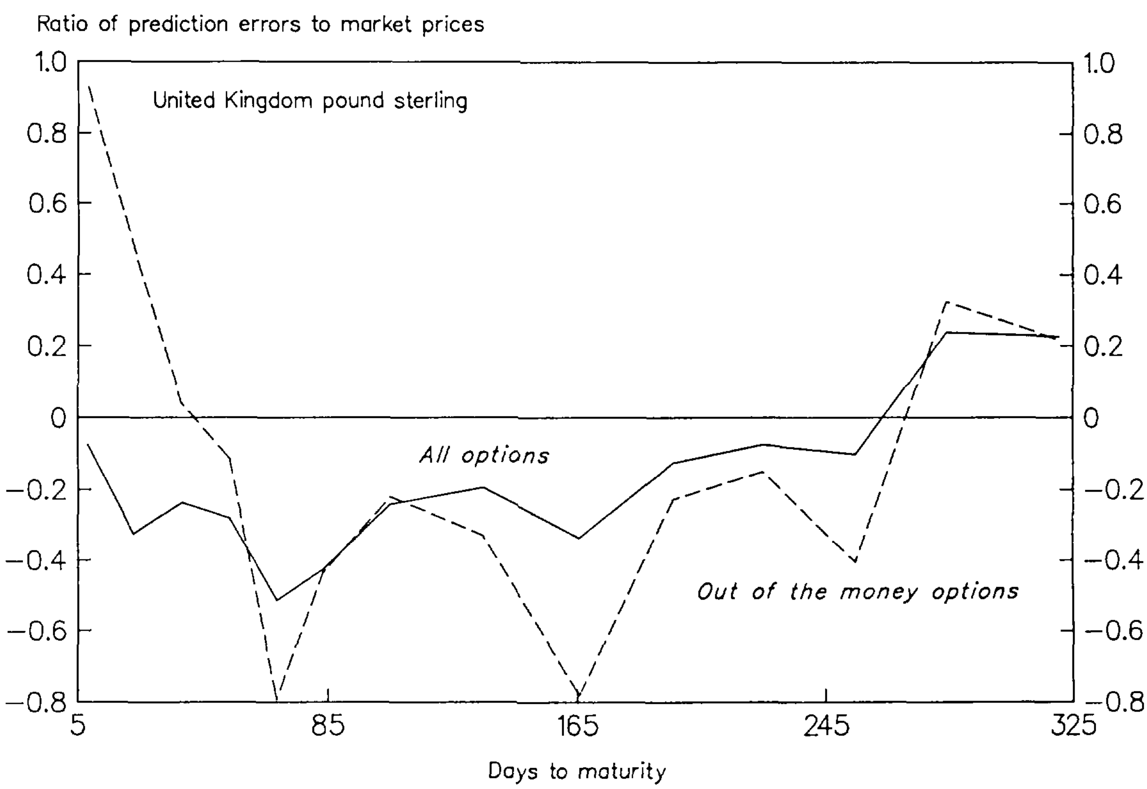
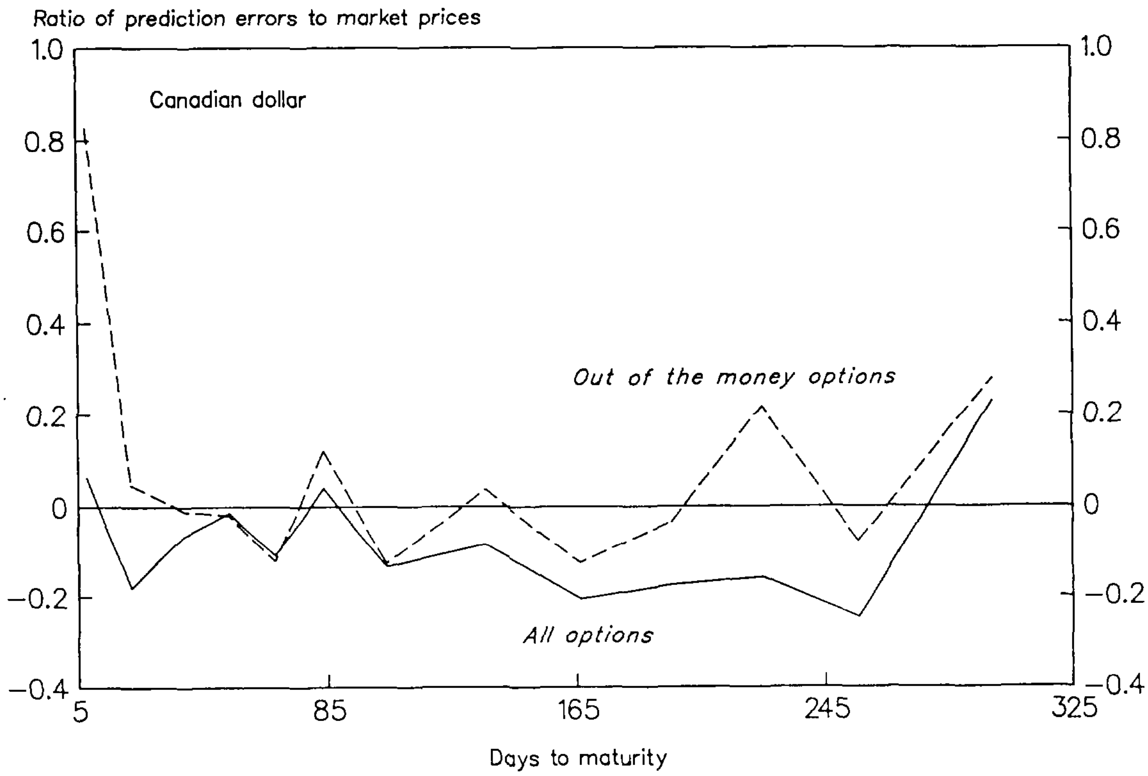
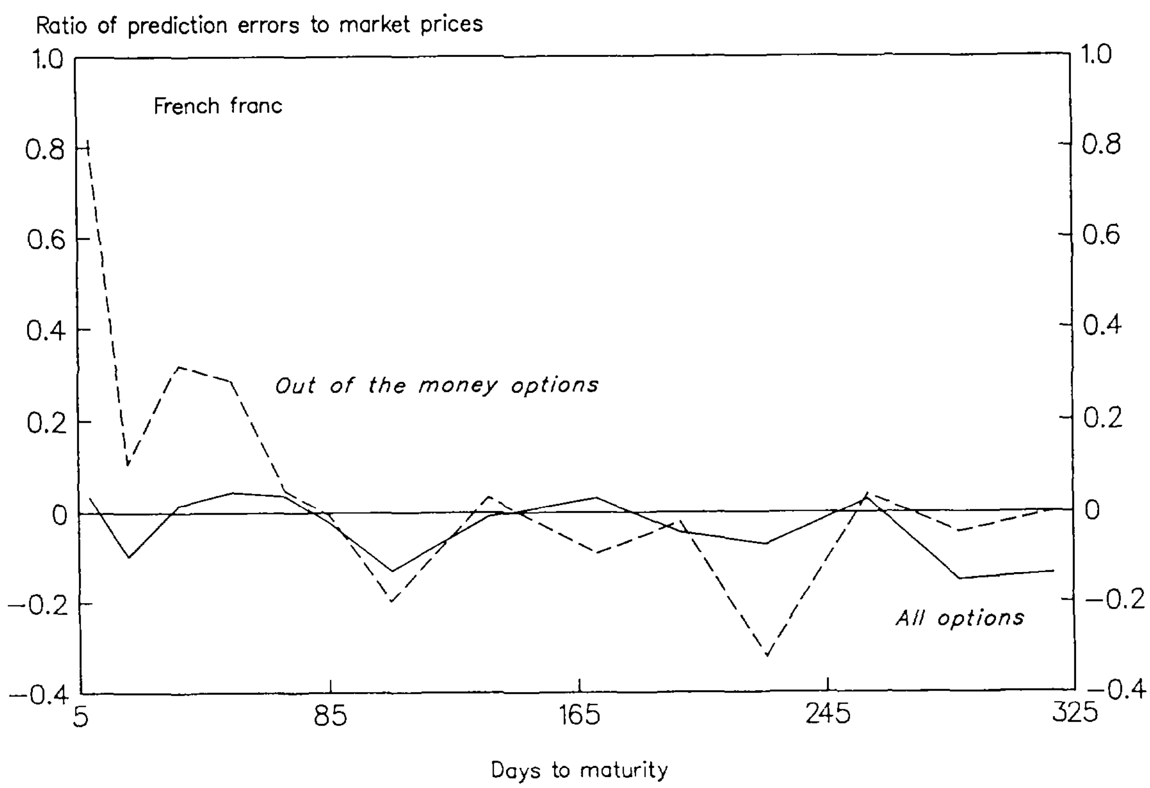


CHART 4 (Concluded)

CALL OPTIONS: BLACK–SCHOLES TIME TO MATURITY BIAS, FEBRUARY 1983 – JUNE 1985





[1] See Appendix I for source and description of data, and variables and methodology used.

one direction or the other, the random term of the jump process has no influence on the exchange rate over short intervals almost surely but, over longer time intervals, it generates discrete jumps of constant size ϕ in the exchange rate. 1/ An interesting comparison between the two processes can be obtained from the fact that, in the limit, if the size of the jump is reduced towards zero and its frequency increased towards infinity, the jump process converges to a diffusion process.

If the exchange rate follows a jump process the value of out-of-the-money options will be relatively higher than if the exchange rate follows a diffusion process. 2/ Consider the value of a call option at maturity; if the exercise price is higher than the actual spot exchange rate, the option will obviously have zero value; if, on the contrary, the exercise price is lower than the spot price, the value of the option will equal the difference between the two. For an out-of-the-money option it is only the probability of the exchange rate reaching values higher than the exercise price that gives the option a positive price. Now consider the price of the option at a moment close to maturity; if the exchange rate follows a diffusion process, the chances of its rising above the exercise price may be very small, but if it is a jump process, all that might be required is that it takes one jump before maturity. It follows that investors would price the option correspondingly higher. The pure jump model has, therefore, the potential to do a better job in predicting market prices of out-of-the-money options.

As shown in Appendix 2, when the exchange rate follows a jump process with positive jumps, the call option price is given by:

$$c = Se^{-r^*T}x_1 - Ee^{-rT}x_2 , \tag{4}$$

where

$$x_1 = e^{-y} \sum_{n}^{\infty} \frac{y^i}{i!}$$

$$x_2 = e^{-(y/\phi)} \sum_{n}^{\infty} \frac{(y/\phi)^i}{i!}$$

1/ The Poisson distribution implies that the probability that the number of jumps j equals some value i in an interval of size T is given by:

$$\Pr(j=i) = \frac{e^{-\lambda T}(\lambda T)^i}{i!}$$

2/ That is, for example, if at-the-money options are priced the same by both models, the pure jump model will price out-of-the-money options higher.

$$n \quad = \quad \max \left[\ 0;\ \text{min integer} > \frac{\ln(E/S)-\mu}{\ln\phi}\ \right]$$

$$y \quad = \quad \frac{(r - r^* - \mu)T\phi}{\phi - 1}$$

Above, μ and ϕ are the two parameters that describe the stochastic process followed by the exchange rate. μ is the (instantaneous) rate of depreciation of the exchange rate and ϕ is equal to one plus the jump size; in the empirical work, we will search for specifications with $\mu<0$ and $\phi>1$, reflecting the trend appreciation of the U.S. dollar and the potential fall in its value. The value of the call option decreases for higher values of μ, and increases for higher values of ϕ. The latter derivative is intuitively clear, the larger the size of the potential jump in the price of foreign exchange, the larger the value of the option to acquire foreign exchange. To obtain some intuition on the sign of the effect of μ, one should consider that μ represents part of the (non-stochastic) interest differential favoring foreign currency denominated bonds, which is associated negatively with the value of the option.

The variable n represents the minimum number of jumps that the exchange rate would need to experience for the option to be in the money. 1/ Note that when $n=0$, $x_1=x_2=1$, and the option price does not depend on the parameters μ or ϕ. That is, when no jumps are needed for the option to be in the money at maturity, this price formula is no longer affected by the existence of potential jumps in the exchange rate process. Indeed, in that situation, the pure jump price formula becomes very close to a Black-Scholes formula with very low volatility. This fact reinforces our intuition that potential jumps in the process are a possible explanation of the exercise price bias of the Black-Scholes model. If market participants expected the exchange rate to follow a pure jump process, they would price options for which $n=0$ (which, unless μ is large, includes almost all the options that are in the money) in a way similar to the Black-Scholes formula but they would price options that are out of the money significantly higher because of the potential large changes in the exchange rate.

V. Empirical Results: Pure Jump Model

The relevant question is whether the pure jump model is able to overcome the exercise-price bias suffered by the Black-Scholes model. If so, we would conclude that expectations of potential jumps in the exchange

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1/ This can be immediately seen by noting that, after an interval of time T, the value of the exchange rate will be $Se^{\mu T}\phi^n$, where S is the initial value of the exchange rate, and n is the number of jumps it experiences in the period.

rate process are the cause of the bias in the Black-Scholes model predictions. The answer turns out to be in the affirmative, at least for options on most currencies.

As displayed in Chart 5, the pure jump model does a better job in predicting prices of options that are out of the money; the residuals are not very large on average, and the plots tend to be quite flat in that region, except for the extreme values of E/S, for which there is still some underprediction of market prices. However, for options that are close to the money, the pure jump model tends to underprice. As we noted above, when no jumps are needed to put the option in the money, the pure jump model converges to a Black-Scholes model with very low volatility, and this turns out to generate underpredictions unless the option is deep in the money. We can attribute this to the fact that the pure jump model is not rich enough to describe the exchange rate process; in particular, the nonstochastic evolution of the exchange rate during periods in which there are no jumps may be a problem. Unfortunately, the pure jump model is, as far as we know, the only applicable model that includes jumps in the exchange rate value for which a closed-form pricing formula exists. 1/ However, it is the successful performance of the model with respect to out of the money options which is more relevant here.

The pure jump model is most successful when applied to deutsche mark and yen options. Also, the estimated values of the parameters are within plausible ranges. μ, the annual rate of appreciation of the dollar in the absence of jumps, is between 10 and 15 percent with respect to the deutsche mark, and between 12 and 17 percent with respect to the yen. For reference, the annual appreciation of the U.S. dollar during 1983-84 (when we could argue there were no depreciation jumps) was about 15 percent with respect to the mark, and 8 percent with respect to the yen. The values of the parameter ϕ imply that depreciations of between 7 and 9 percent with respect to the deutsche mark, and of between 4 and 5 percent with respect to the yen would take place in the event of an arrival of a jump. These parameter values are not those which minimize the sum of squared residuals, but instead they minimize a weighted average of residuals so as to increase the relative importance of out of the money options that do not constitute a large proportion of the total number of observations. This estimation procedure was used because the purpose of this estimation was to check whether the pure jump model was capable of avoiding the exercise price bias rather than to fit the best possible parameter values. In addition, at least in the case of these two currencies, the parameter values obtained following this procedure were very close to the values that minimized total squared residuals.

1/ Merton (1976) has developed a mixed diffusion-jump model for stock options. However, that model is based on the assumption that the jumps in the stock price result from information specific to the firm and, therefore, uncorrelated with any of the other available investments. This assumption is, obviously, not applicable to exchange rate jumps.

With respect to the other currencies, the model appears to be less successful, although it almost always represents an improvement over the Black-Scholes model if judged by the criterion of exercise price bias. In some instances, the general pattern of the residuals does not appear to differ very significantly from the Black-Scholes model (Swiss franc, 1st half); in others, the model overpredicts deep out of the money options (Swiss franc, 2nd half). In addition, some parameter values are not entirely plausible, such as expected jumps of 18 percent in the U.S.-Canadian dollar exchange rate for the 2nd half of the sample.

Overall, these results support the conclusion that the possibility of large, sudden changes in exchange rate parities account for the high prices of out the money options. Judging from actual developments in foreign exchange markets, the success of the pure jump model appears to be an expected result for deutsche mark and Japanese yen options. For British pound and Swiss franc options, the pure jump model represents an improvement from Black-Scholes in terms of exercise price bias, but the results are somewhat murkier.

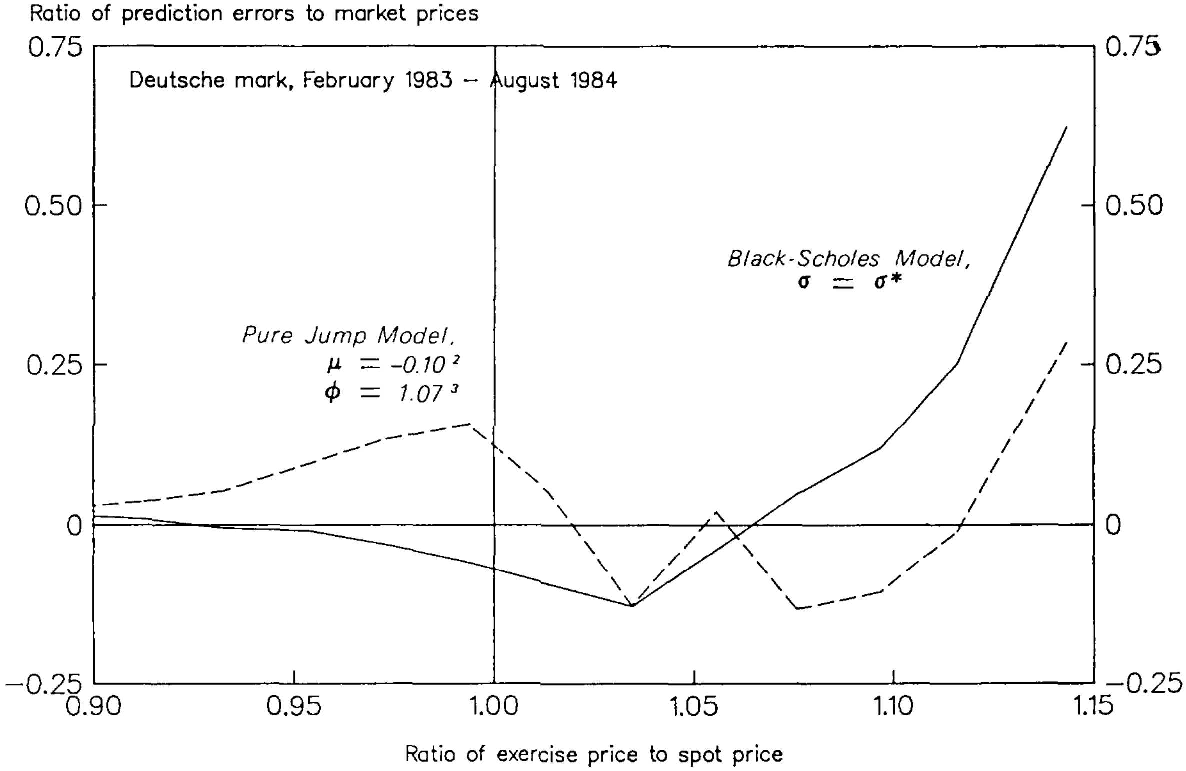
The results of the application of the model to Canadian dollar and French franc options are somewhat puzzling. Because the U.S.-Canadian dollar exchange rate has been quite steady over several years, there does not seem to be a strong case a priori for the jump model. However, out-of-the-money options on the Canadian dollar are also underpredicted by the Black-Scholes model and better explained by assuming a pure jump model. Although there should not be serious objections to these results for the first half of the sample, because the implicit jump size is only 2 percent, the results for the second half are certainly less appealing, since the jump size becomes 18 percent. In the case of the French franc, there is no exercise-price bias in the Black-Scholes model. France being a member of the EMS, one would expect this exchange rate to be a good candidate for the pure jump model, because expectations on the dollar-franc exchange rate should be consistent with expectations on the dollar-mark exchange rate. A possible explanation may lie in an empirical regularity that has been pointed out by students of the EMS such as Giavazzi and Giovannini (1986). They have observed that on occasions in which the mark appreciates with respect to the dollar it also strengthens with respect to the French franc (and other EMS currencies). Therefore, if this is the case, the potential jumps in the mark-dollar rate may not affect the franc-dollar parity. It is also true that the French franc results should be taken more cautiously because options on that currency started to be traded only in September 1984, and the sample size is much smaller than that of other currencies for the same time period (350 transactions versus a typical 2000 for the other currencies).

VI. Conclusions

The development of the foreign currency options market provides the opportunity of undertaking a broader investigation of exchange rate expectations. While other financial transactions or even survey data typically

CHART 5 [1]

CALL OPTIONS: BLACK–SCHOLES AND PURE JUMP EXERCISE PRICE BIAS, FEBRUARY 1983 – AUGUST 1984, AND SEPTEMBER 1984 – JUNE 1985



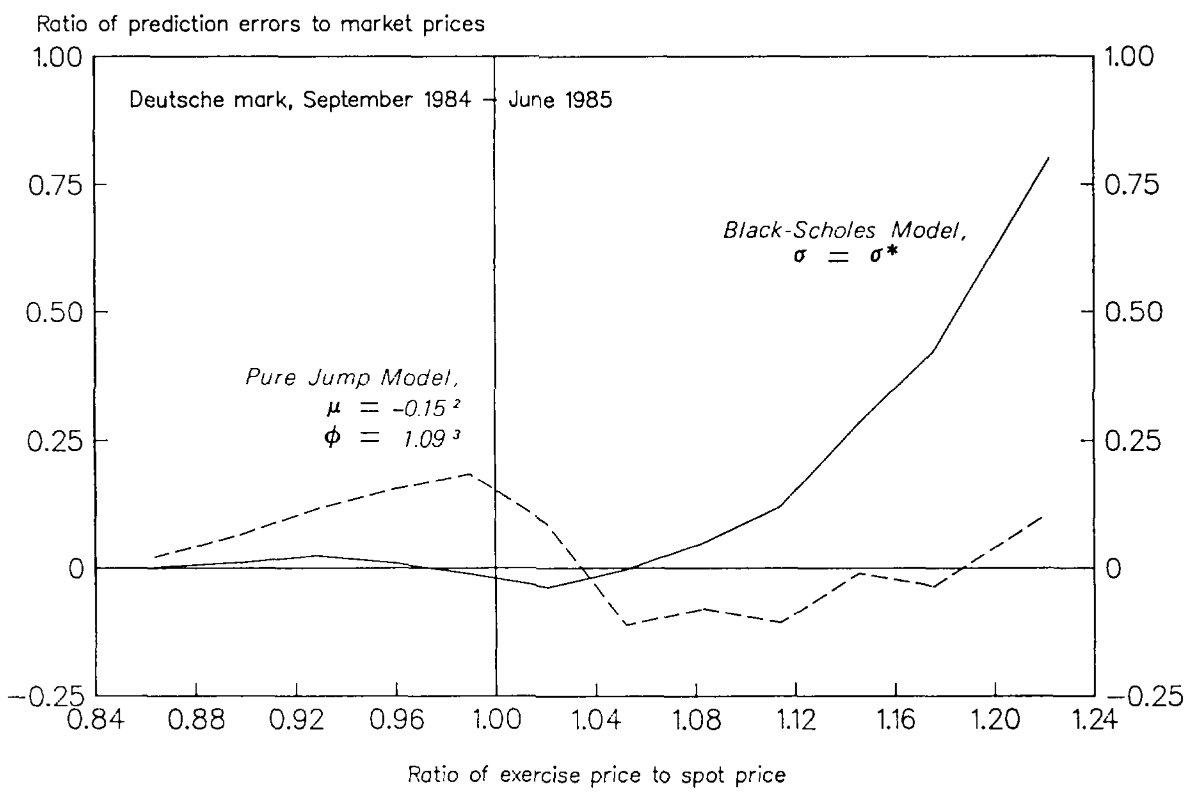
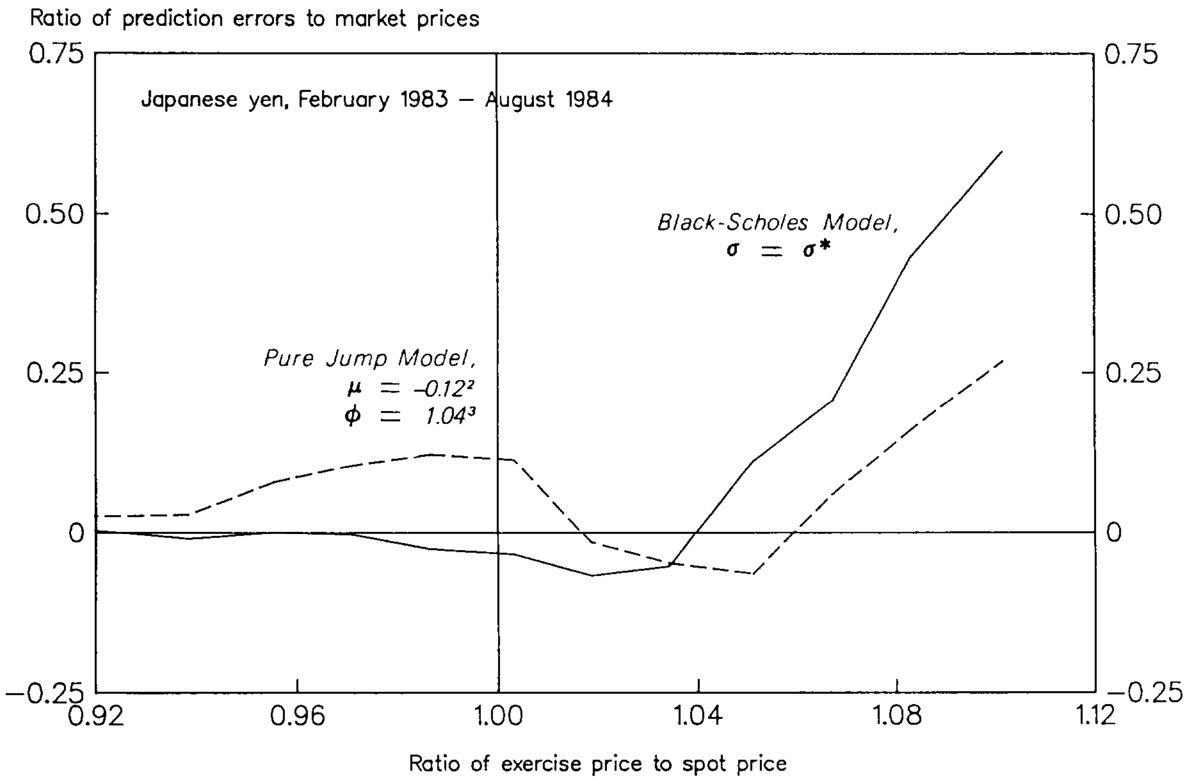


CHART 5 (Continued)

CALL OPTIONS: BLACK–SCHOLES AND PURE JUMP EXERCISE PRICE BIAS, FEBRUARY 1983 – AUGUST 1984, AND SEPTEMBER 1984 – JUNE 1985



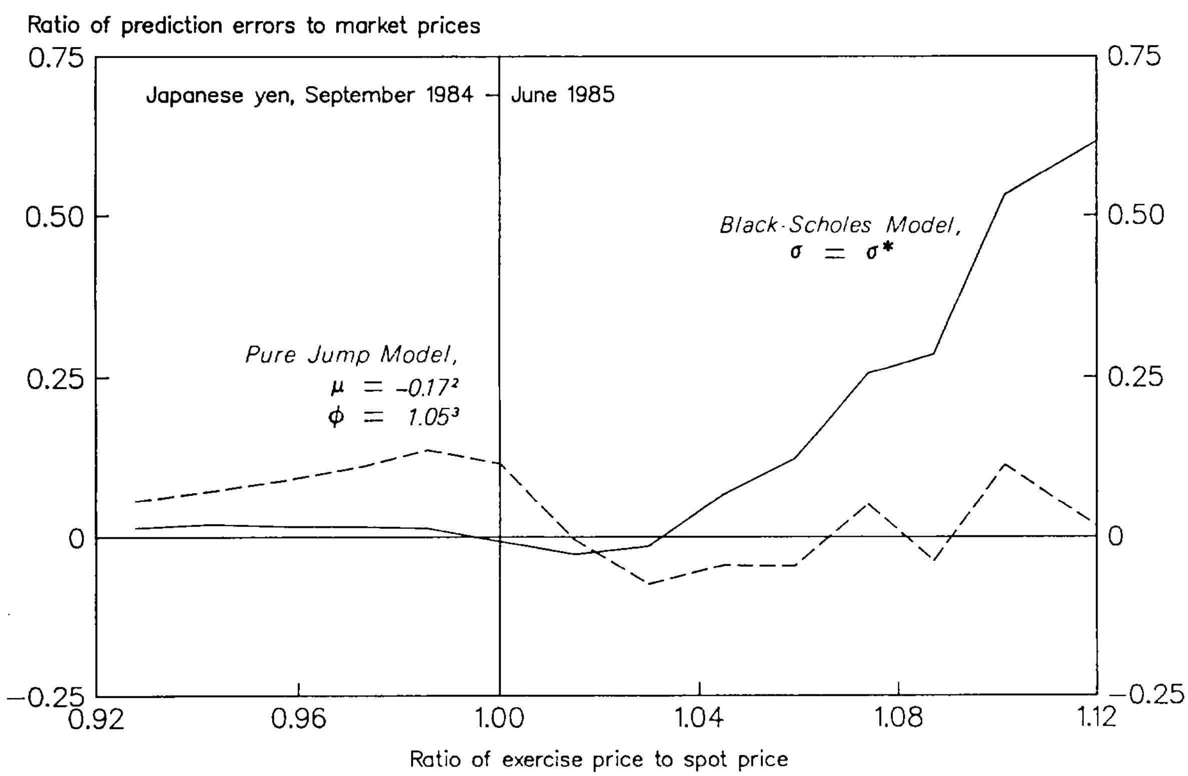
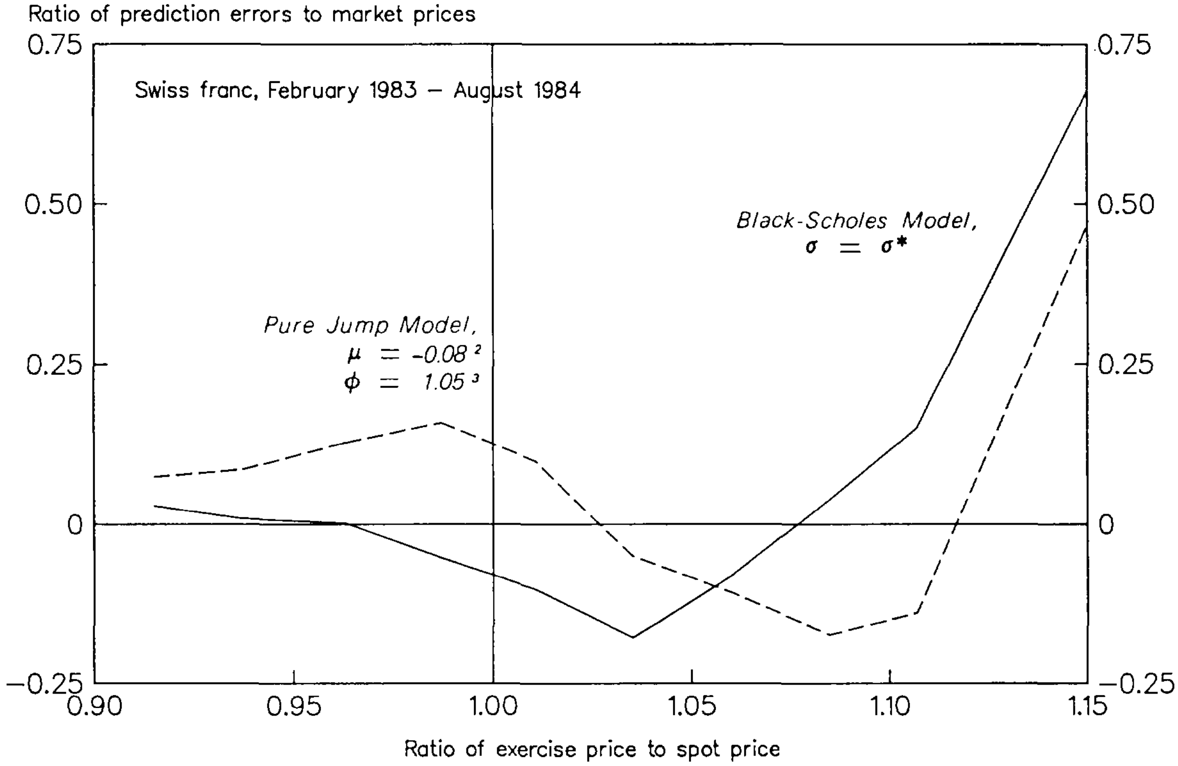


CHART 5 (Continued)

CALL OPTIONS: BLACK–SCHOLES AND PURE JUMP EXERCISE PRICE BIAS, FEBRUARY 1983 – AUGUST 1984, AND SEPTEMBER 1984 – JUNE 1985



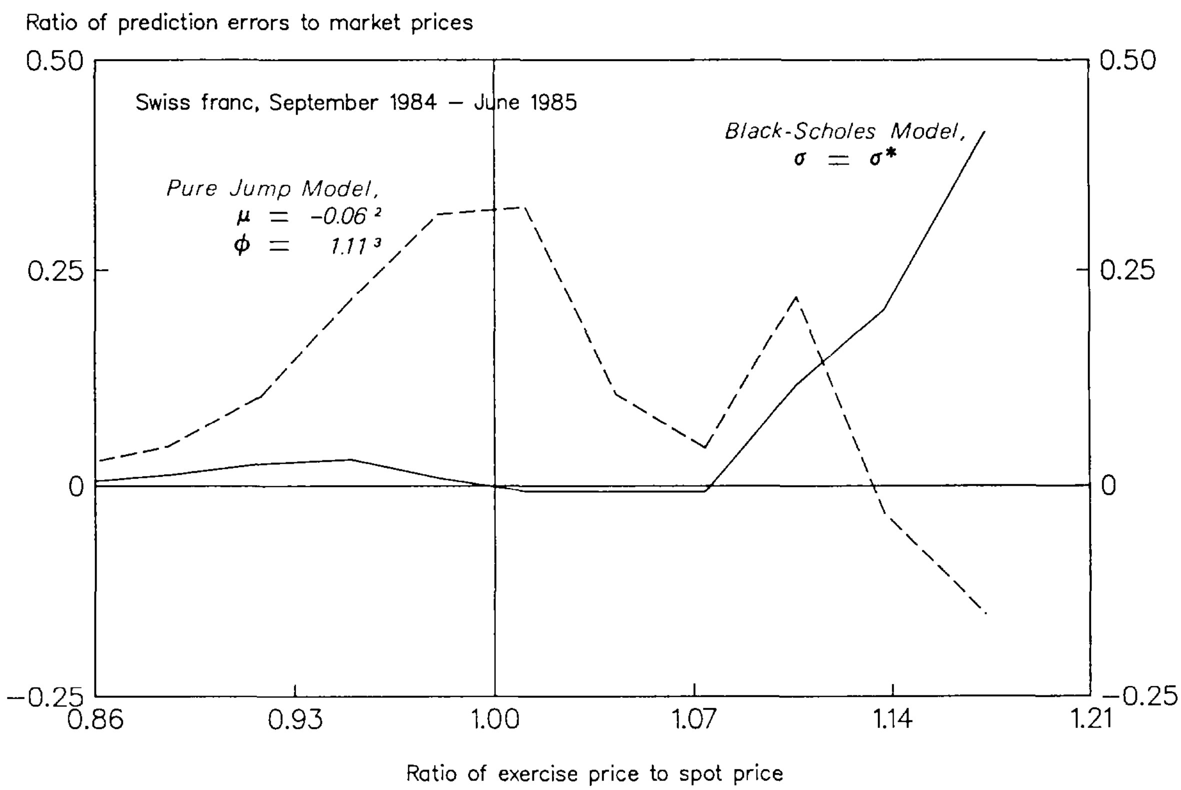
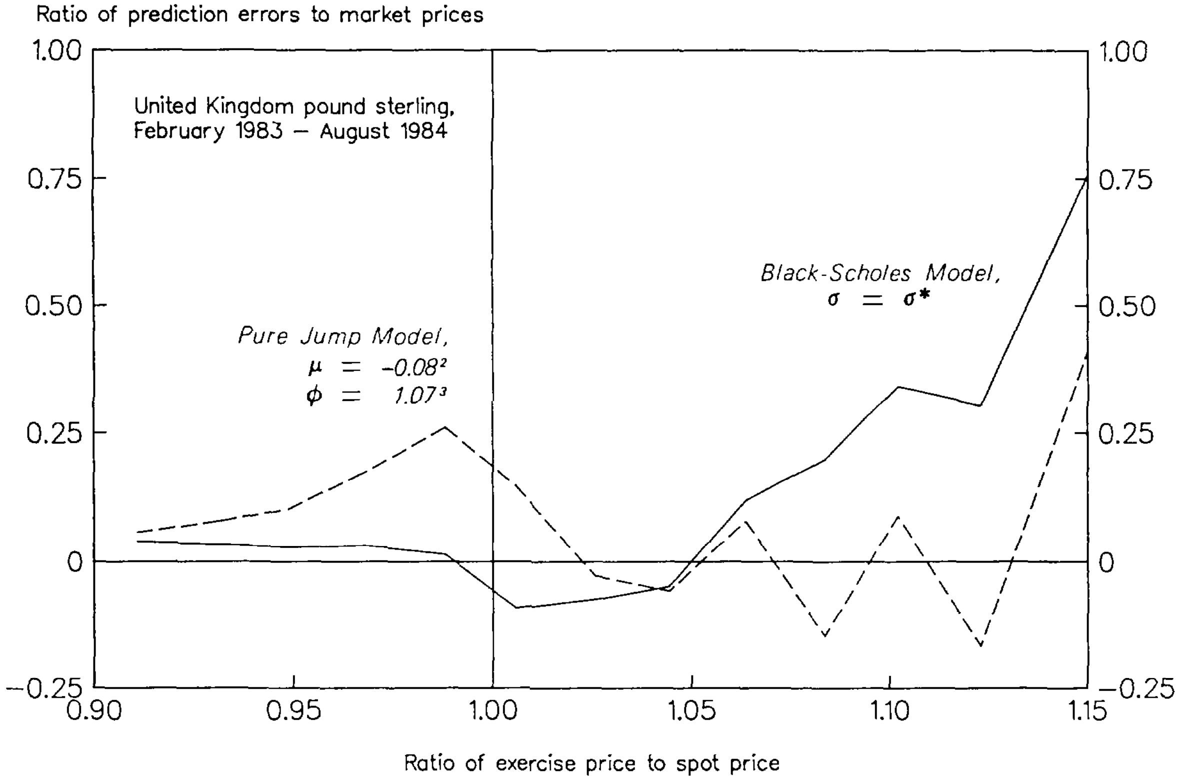


CHART 5 (Continued)

CALL OPTIONS: BLACK–SCHOLES AND PURE JUMP EXERCISE PRICE BIAS, FEBRUARY 1983 – AUGUST 1984, AND SEPTEMBER 1984 – JUNE 1985



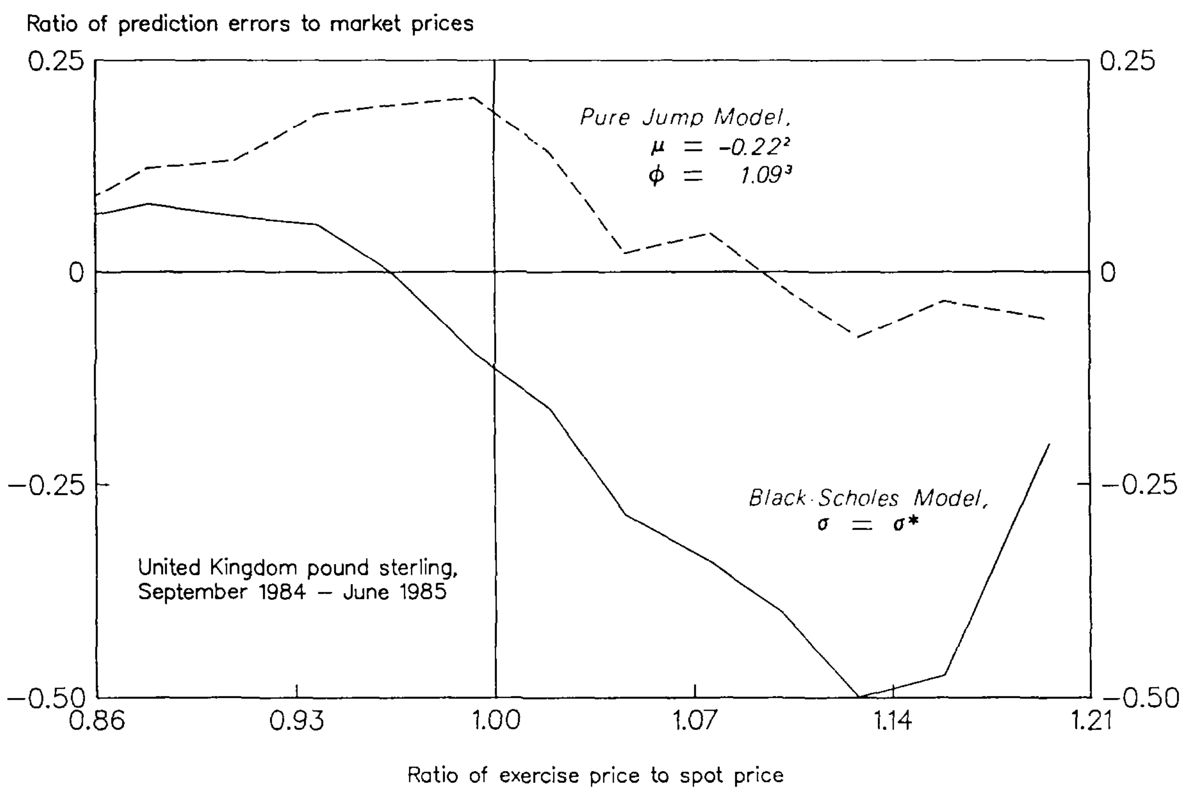
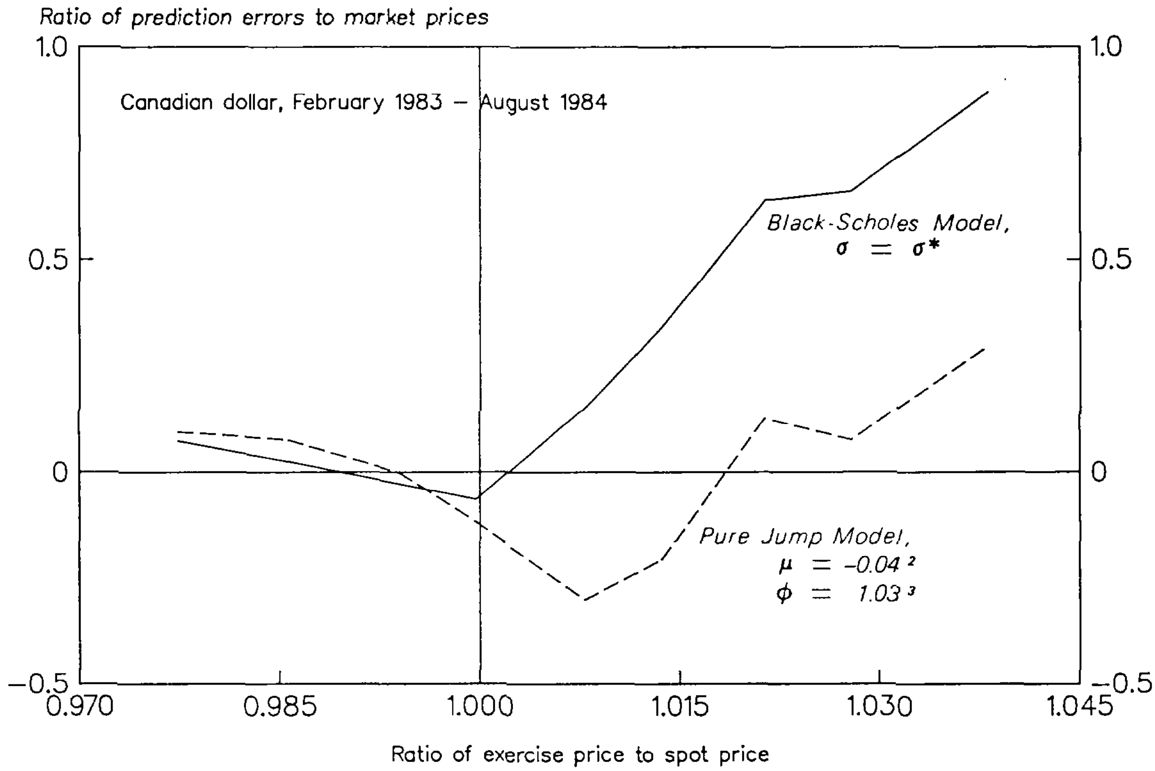


CHART 5 (Continued)

CALL OPTIONS: BLACK–SCHOLES AND PURE JUMP EXERCISE PRICE BIAS, FEBRUARY 1983 – AUGUST 1984, AND SEPTEMBER 1984 – JUNE 1985



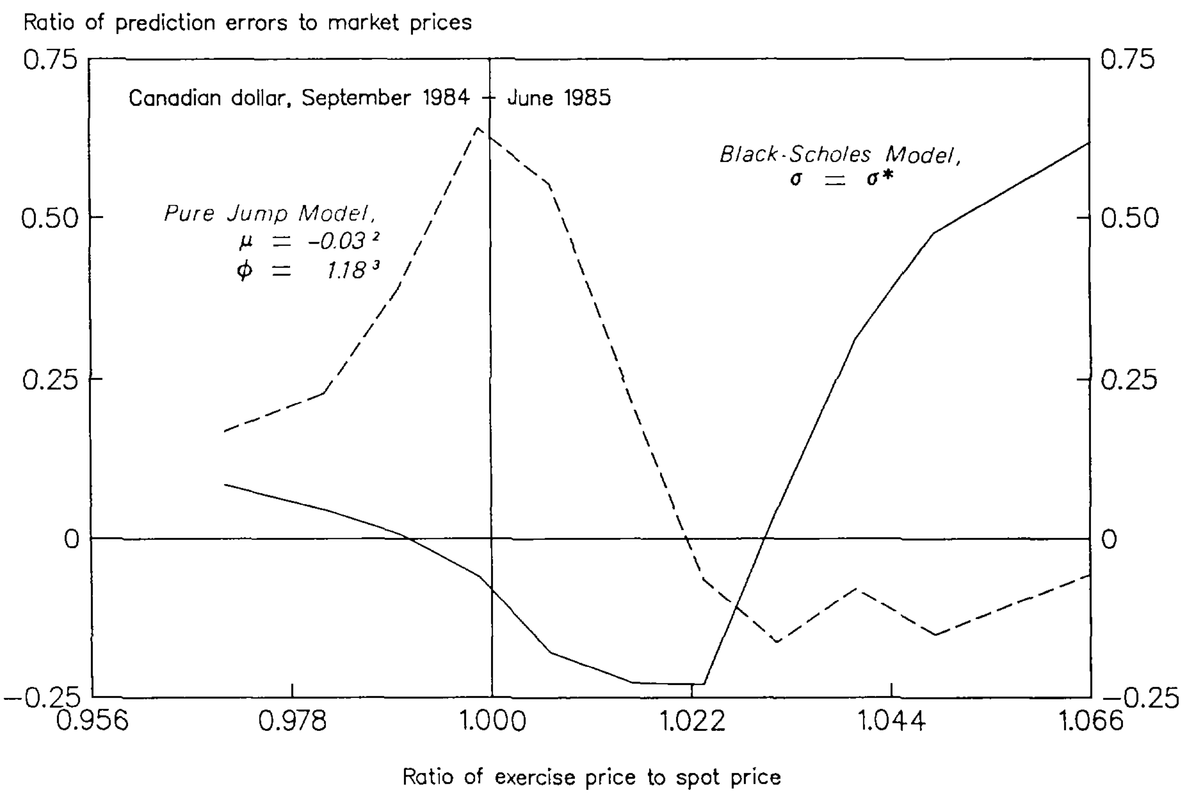
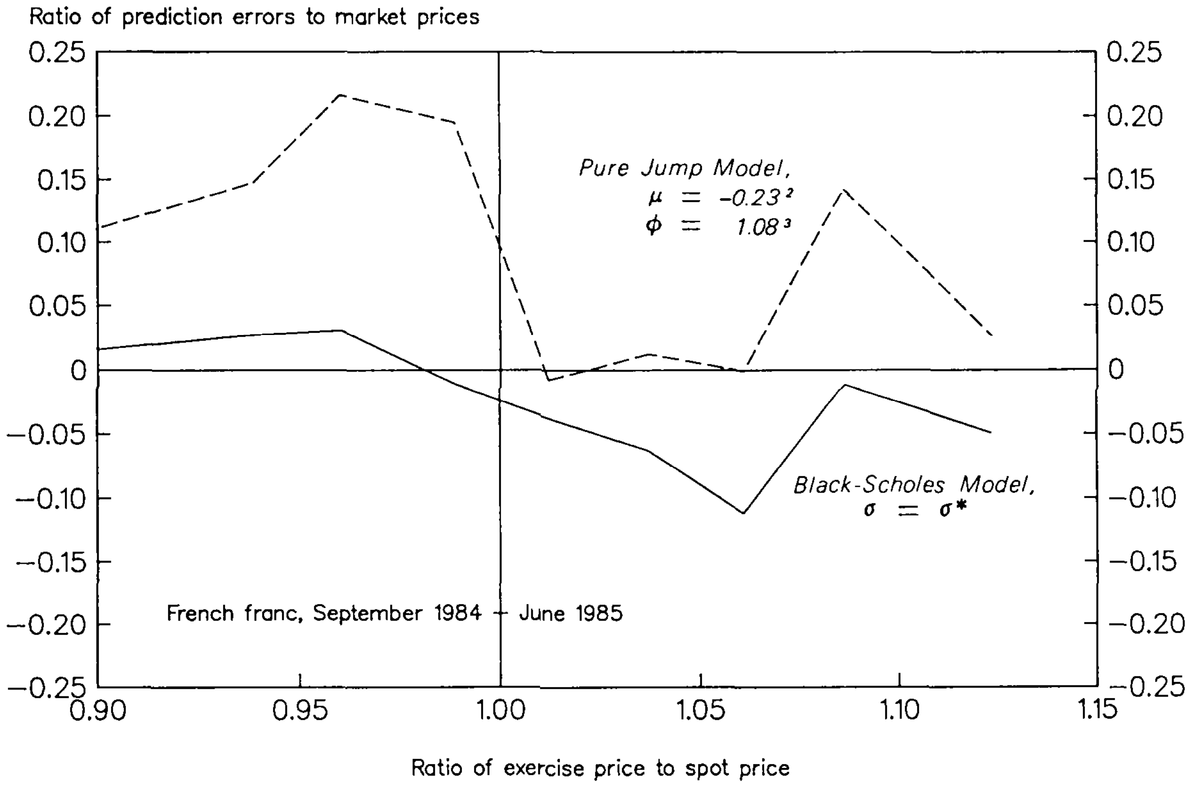


CHART 5 (Concluded)

CALL OPTIONS: BLACK-SCHOLES AND PURE JUMP EXERCISE PRICE BIAS, FEBRUARY 1983 - AUGUST 1984, AND SEPTEMBER 1984 - JUNE 1985



[1] See Appendix I for source and description of data, and variables and methodology used.
[2] μ = drift parameter for Pure Jump Model.
[3] ϕ = jump parameter for Pure Jump Model.

provide information only on an expected value parameter, option prices provide information concerning the expected volatility over time of exchange rates.

The main empirical result of this paper is the underprediction of prices of out-of-the-money options by the Black-Scholes model--that assumes that the exchange rate follows a sort of random walk with drift process. This means that, by Black-Scholes standards, market participants paid too high prices for options to buy foreign exchange at prices above the prevailing spot rate. We think that a reason for those high prices was that market participants did not believe that the exchange rate followed a diffusion process, but instead some kind of process that includes the possibility of sudden large changes or discontinuities.

An alternative model, originally developed for the valuation of stock options, provides a good framework to test this idea. This is the pure jump model, in which the time path of the underlying stock is subject to jumps that arrive at random intervals. With the exception of French franc options, the pure jump model clearly outperforms the Black-Scholes model in accounting for the large out-of-the-money option prices. For the deutsche mark and yen options, the pure jump model appears to be quite successful, suggesting that the possibility of a sudden depreciation of the U.S. dollar with respect to the deutsche mark and the Japanese yen was taken quite seriously in financial markets between 1983 and 1985.

Data and Methodology

The data base 1/ is composed of all the trades on foreign exchange options carried out at the Philadelphia Stock Exchange (PHLX) since the opening of this market on February 28, 1983 to June 27, 1985. For the analysis only the last daily trade on each call contract was used, a subset of 19,817 trades out of the just over 140,000 (call and put) options included in the data base. Only call contracts were used in order to minimize potential early exercise biases. A contract is defined by the currency, exercise price and expiration date. Therefore, on the same date, we may, for example, have several option contracts on deutsche marks, each with a different exercise price or expiration date. The last trade of the day on each of these contracts is included in the sample. 2/

The option prices correspond to actual trade prices and not to bid-ask quotes. The time to maturity of the contract, its exercise price, and the closest previous spot exchange rate quotation from the interbank market are also provided in the data set. Interest rates were obtained from the DRI data base on eurocurrency rates. To obtain the yield over the time to expiration of each contract, we used weighted geometric averages of the smallest possible number of interest rates on different maturities. In theory, a superior procedure to obtain the foreign interest rate is to use futures contracts on foreign exchange, because they have the same maturity dates as the foreign exchange options. (The ratio of future to spot exchange rates must be equal to the yield differential between domestic and foreign currency denominated assets by closed interest parity.) However, given the volatility of spot rates, any slight misalignment of spot and future rates may produce very large mistakes in this measure, which makes it unreliable.

The plots represent the divergence between market prices and model prices for different groups of options. In the plots included in Charts 2, 4 and 5, options are grouped according to their position in or out of the money; in the plots included in Chart 3, options are grouped according to their time to maturity.

For Charts 2, 4 and 5, the procedure is the following. First, the option prices predicted by the models are computed using the corresponding parameter values. Second, options are grouped into 15 equal size intervals, according to the value of Ee^{-rT}/Se^{-r^*T}, that is, the ratio of the exercise price to the spot rate adjusted for interest differentials (this is a standard measure of the extent to which an option is in or out of the

1/ The PHLX-OSU Currency Options Data Base compiled by James Bodurtha, Jr. We thank Prof. Bodurtha for providing us with these data.

2/ This sample was further filtered by taking away the contracts that presented an anomalous value for some variable and the extreme outliers in the Black-Scholes estimation. The total of discarded observations amounted to less than one percent of the sample.

money). If an interval comprises less than ten observations, it is merged with the next to the left. The average value of $Ee^{-rT}/Se^{-r^{*}T}$ within each interval forms the variable represented in the horizontal axis of the plots. This means that as we move to the right along the horizontal axis, we find groups of options that are further out of the money. Finally, the percentage prediction error is computed as (market price-model price)/market price. The average value of this variable for each group of options forms the variable represented in the vertical axis of the plots. For the plots included in Chart 3, the procedure is the same, with the exception that the variable in the horizontal axis is the number of days remaining to the date of maturity of the contract.

The Pure Jump Model Applied to Foreign Exchange Options

This application of the pure jump model to the case of foreign exchange options mimics the steps followed by Cox and Ross (1975) in their development of the stock option pricing formula for this process.

The price of foreign currency is assumed to evolve according to the following process:

$$\frac{dS}{S} = \mu dt + (\phi-1)d\pi$$

where $d\pi = 1$ with probability λdt

$d\pi = 0$ with probability $1-\lambda dt$

and we will consider the case in which $\mu < 0$ and $\phi > 1$.

Let c be the price of an European call option. We want to find a formula that depends only on the current exchange rate and time $c = c(S,t)$. The evolution of c over time will be different in the points in which there is a jump in the value of the exchange rate from the points in which there is none. At the point in which there is a jump, the change in the price of the option will be:

$$\frac{dc}{c} = \frac{c(\phi S,t) - c(S,t)}{c(S,t)}$$

and at the points with no jumps:

$$\frac{dc}{c} = c_S \mu \frac{S}{c} dt + c_t \frac{1}{c} dt$$

where c_S and c_t indicate the partial derivatives of the option price function with respect to the price of foreign exchange and time, respectively. We want to form a portfolio, with shares α_c of the option and α_S of foreign bonds (pure discount) that has no uncertainty. For this purpose, it is only necessary to hedge against the jumps, because the evolution of the exchange rate is nonstochastic otherwise. For the value of the portfolio not to be affected by jumps, the shares must satisfy the following condition:

$$\alpha_c \frac{c(\phi S,t) - c(S,t)}{c(S,t)} + \alpha_S(\phi-1) = 0 \tag{1}$$

Let r* denote the instantaneous rate of return on foreign currency-denominated bonds; then, the dollar-denominated return is $r^* + dS/S$, which equals $r^* + \mu$ in points with no jumps. Since this portfolio has no risk, it must yield the riskless dollar interest rae r. This implies the following additional restriction:

$$\alpha_c(c_S \mu \frac{S}{c} + c_t \frac{1}{c} - r)dt + \alpha_S (\mu+r^*-r)dt = 0. \tag{2}$$

Equations (1) and (2) can now be used to substitute for the corresponding values of α_S and α_c. Skipping some tedious algebra, we obtain:

$$-c_t = c_S \mu S + \frac{\mu+r^*-r}{1-\phi} c(\phi S,t) + \frac{\phi r-\mu-r^*}{1-\phi} c(S,t) \tag{3}$$

Equation (3) above is a mixed difference-differential equation. It is subject to the following terminal condition, that expresses the value of the option at expiration date:

$$c(S,T) = \max [0,S_T-E] \tag{4}$$

Equation (3) is very difficult to solve by any mathematical method. The ingenious approach of Cox and Ross uses economics to obtain a solution, and it only involves elementary algebra. The argument is as follows. Equation (3) holds for any economy, independently of preferences, market structure, etc., since none of those factors are necessary to derive it; then, if it is possible to find its solution for any particular economy, this solution will also apply to any other economy. It is easier to solve (3) for the case of an economy composed of risk-neutral agents; in this case, all assets must render the same expected rate of return, which simplifies the analysis considerably. Note that this is only a strategy to integrate equation (3); the assumption of risk neutrality is not being made, and the solution that will be obtained applies to any economy.

Let j denote the number of jumps of the exchange rate over [0,T]. By the properties of the Poisson distribution, it is true that:

$$\text{Prob}(j=i) = \frac{e^{-\lambda T}(\lambda T)^i}{i!}$$

Since the rate of change of the price of foreign exchange in [0,T] is given by:

$$\frac{S_T}{S_0} = \phi^j e^{\mu T}$$

it follows that:

$$E\left[\frac{S_T}{S_0}\right] = \sum_{i=0}^{\infty} \phi^i e^{\mu T} e^{-\lambda T}\frac{(\lambda T)^i}{i!} = e^{\mu T} e^{-\lambda T} \sum_{i=0}^{\infty} \frac{(\lambda\phi T)^i}{i!} =$$

$$= e^{\mu T}e^{-\lambda T}e^{\lambda\phi T}$$

Since, by equality of the expected rates of return:

$$E\left[\frac{S_T}{S_0}\right] = e^{(r-r^*)T}$$

$$\lambda = \frac{r - r* - \mu}{\phi - 1} \tag{5}$$

The expected return on holding the call option between time 0 and the time to expiration T is:

$$\frac{1}{c(S,0)} E\left[\max\left(0; S_T - E\right)\right] = \frac{1}{c(S,0)} \sum_{i=n}^{\infty} \left(e^{\mu T} S \phi^i - E\right) \frac{(\lambda T)^i}{i!} e^{-\lambda T}$$

And, by equality of expected rates of return, this rate must also be equal to e^{rT}. Above, n is the minimum number of jumps necessary for S_T to become larger than the exercise price E. That is:

$$n = \text{smallest integer} > \frac{\ln(E/S) - \mu(T-t)}{\ln(\phi)} \tag{6}$$

By substituting out λ, and after some algebraic transformations, we reach the option price formula for the pure jump process:

$$c(S,t) = S(t) e^{-r*(T-t)} \sum_{i=n}^{\infty} e^{-y} \frac{y^i}{i!} - E e^{-r(T-t)} \sum_{i=n}^{\infty} e^{-(y/\phi)} \frac{(y/\phi)^i}{i!} \tag{7}$$

where

$$y = \frac{\phi(r - r* - \mu)(T-t)}{\phi - 1}$$

In order to check that (7) is actually the solution to (3), the proper derivatives must be computed. There is, however, one complication. At points in which small changes in S or in t do not change n (as defined in (6)), c_S and c_t are straightforward to compute and it can be seen that (7) satisfies (3). However when S and t are such that:

$$n = \frac{\ln(E/S) - \mu(T-t)}{\ln(\phi)}$$

an increase or a decrease in S or t will affect c(S,t) differently by changing or not the number of terms in the summations in (7). In more technical words, the left and right derivatives are not the same. This problem is solved by recalling that c_S and c_t come into (3) describing the evolution of S in periods in which there are no jumps. Therefore, if μ is negative, the proper derivative c_S is the left derivative because S is decreasing over time. Also, the proper derivative c_t is the right derivative because we are moving forward in time.

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