

## IMF Working Paper

© 1999 International Monetary Fund

This is a *Working Paper* and the author(s) would welcome any comments on the present text. Citations should refer to a *Working Paper of the International Monetary Fund*. The views expressed are those of the author(s) and do not necessarily represent those of the Fund.

WP/99/75

INTERNATIONAL MONETARY FUND

Research Department

### Simple Monetary Policy Rules Under Model Uncertainty

Prepared by Peter Isard, Douglas Laxton, and Ann-Charlotte Eliasson\*

May 1999

#### Abstract

Using stochastic simulations and stability analysis, the paper compares how different monetary rules perform in a moderately nonlinear model with a time-varying nonaccelerating-inflation-rate-of-unemployment (NAIRU). Rules that perform well in linear models but implicitly embody backward-looking measures of real interest rates (such as conventional Taylor rules) or substantial interest rate smoothing perform very poorly in models with moderate nonlinearities, particularly when policymakers tend to make serially correlated errors in estimating the NAIRU. This challenges the practice of evaluating rules within linear models, in which the consequences of responding myopically to significant overheating are extremely unrealistic.

JEL Classification Numbers: E52, E31, E17

Keywords: monetary policy rules, NAIRU uncertainty, macroeconomic models

Author's E-Mail Address: pisard@imf.org; dlaxton@imf.org; Ann-Charlotte.Eliasson@hhs.se

---

\*This paper will be published in *International Finance and Financial Crises: Essays in Honor of Robert P. Flood Jr.*, ed. by Peter Isard, Assaf Razin, and Andrew K. Rose (Washington: International Monetary Fund and Boston: Kluwer Academic Publishers), forthcoming 1999. An earlier draft was prepared for the January 15–16, 1999 conference at the International Monetary Fund in celebration of the contributions of Robert Flood. The first two authors are with the Research Department of the IMF. The third author is at the Stockholm School of Economics. The views expressed are those of the authors and do not necessarily reflect the views of the International Monetary Fund. We thank Jeffrey Fuhrer, Lars Svensson, Robert Tetlow, and Volker Wieland for useful discussions and Susanna Mursula and Sarma Jayanthi for extensive research assistance.

	Contents	Page
I.	Introduction and Overview .....	4
II.	A Model of the Unemployment-Inflation Process .....	13
	A. The Short-Run Phillips Curves .....	14
	B. The Dynamics of Inflation Expectations .....	20
	C. The Dynamics of the Unemployment Rate .....	22
III.	The Policy Rules and Stochastic Simulation Framework .....	23
	A. The Simple Policy Rules .....	23
	B. The Policy Objective Function .....	26
	C. The Monetary Authority's Behavior .....	27
	D. The Stochastic Simulation Experiments .....	28
IV.	Simulation Results .....	28
	A. Optimal Calibrations of IFB1 Rules .....	28
	B. Perspectives on Several of the Simple Rules Proposed in the Literature .....	37
V.	Conclusions .....	42
Tables		
1.	Phillips Curves and the Time-Varying NAIRU .....	19
2.	Dynamics of Inflation Expectations .....	21
3.	Dynamics of the Unemployment Rate .....	24
4.	Optimal Calibrations of IFB1 Rules .....	30
5.	Optimal Calibrations of IFB1 Rules Under a Backward-and-Forward-Looking Components Model of Inflation Expectations .....	31
6.	Sensitivity of Optimal Calibrations of IFB1 Rules to Different Assumptions .....	33
7.	Performance Characteristics of Optimally-Calibrated IFB1 Rules .....	35
8.	Performance Characteristics of Selected Rules .....	41
Figure		
1.	The Convex Phillips Curve .....	16
Text Boxes		
1.	Research on Monetary Policy: 1989 vs. 1999 .....	6
2.	Alternative Lines of Research on Monetary Policy Rules .....	9
Appendix I. Optimal Forecasting Rules for Bounded Random Walks .....		44
Appendix II. Stability Analysis of Taylor Rules and LWW Rules .....		46

Appendix Tables

B1.	Slope of Phillips Curve and Impact Effects of Surprises in the Unemployment Gap. ....	51
-----	--	----

Appendix Figures

A1.	Optimal Forecast Trajectories of a Bounded Random Walk .....	45
B1.	Regions of Uniqueness, Explosiveness and Indeterminacy .....	48
B2.	Regions of Uniqueness, Explosiveness and Indeterminacy .....	52
B3.	Regions of Uniqueness, Explosiveness and Indeterminacy .....	54

References .....	56
------------------	----

## I. INTRODUCTION AND OVERVIEW

This paper employs stochastic simulations and stability analysis to compare the performances of several types of simple monetary policy rules in a small model of the U.S. economy. The model, which is estimated with quarterly data for the post-1968 period, exhibits a moderate degree of nonlinearity, assumes that inflation expectations have a model-consistent component, and treats the non-accelerating-inflation rate of unemployment (NAIRU) as a time-varying and unobservable parameter. The simulation framework assumes that policymakers update their estimates of the NAIRU period by period, using their information about the macroeconomic model, and in a manner that implicitly recognizes the tendency to make serially-correlated errors in estimating the NAIRU.

The simulations and stability analysis demonstrate that several classes of rules that have been shown to perform well in linear models of the U.S. inflation process perform very poorly in our moderately-nonlinear model. These include conventional Taylor rules, as advocated by Taylor (1993, 1998a) and others; a class of forward-looking rules with a high degree of interest rate smoothing, as proposed by Clarida, Gali, and Gertler (1998); and a first-difference rule for the interest rate, as proposed by Levin, Wieland, and Williams (1998). One of the main conclusions is that rules that implicitly embody backward-looking measures of real interest rates (such as conventional Taylor rules) or too much interest rate smoothing can be too myopic to meet the stability conditions for macro models with moderate nonlinearities, particularly in a world in which policymakers tend to make serially-correlated errors in estimating the NAIRU. This finding in turn suggests, as a second main conclusion, that the propensity of economists to analyze the properties of monetary policy rules within the confines of linear models is difficult to defend as a research strategy. Linear models, in which bad policy rules affect the variances but not the means of inflation and unemployment, are fundamentally inappropriate for policy analysis because they fail to capture the fact that policymakers who allow economies to overheat significantly can fall behind “shifts in the curve” and fail to provide an anchor for inflation expectations, with first-order welfare consequences.

The paper also reports simulation results for inflation-forecast-based rules without explicit interest rate smoothing<sup>2</sup> and explores how their optimal calibrations vary with the degree of NAIRU uncertainty, the shape of the Phillips curve, and the nature of inflation expectations. These results support the view that optimally-calibrated simple rules can deliver attractive macroeconomic performances in small empirical macro models. Indeed, a third main conclusion is that optimally-calibrated *linear* rules in which the interest rate is a function of the inflation forecast—when applied to model-consistent inflation forecasts—can stabilize our *nonlinear* model with approximately the same welfare outcome as a strategy that explicitly optimizes the policy loss (welfare) function. However, echoing the theme of Flood and Isard

---

<sup>2</sup>Losses associated with interest rate variability are reflected in the policy objective function, however, and thereby influence the optimal calibration of the policy reaction function.

(1989), we stress that policymakers face considerable difficulties in attempting to identify the true macroeconomic model and the optimal calibration for any proposed policy rule.

In recognizing the contributions of Bob Flood, it is useful to reflect on the extent to which research on monetary policy strategies has shifted focus over the past decade (Box 1). Ten years ago, the academic discussion centered on the time inconsistency problem and the issue of monetary policy credibility.<sup>3</sup> The rules-versus-discretion debate continued to rage, and new thinking had emerged on the roles of institutional mechanisms other than rules (e.g., independent central banks and conservative central bankers), along with reputation, as vehicles for mitigating credibility problems.<sup>4</sup> In this setting, Bob Flood became intrigued by the observation that central banks found it appealing to adopt simple policy rules, such as target growth rates for monetary aggregates, but to periodically modify the rules. This led to Flood and Isard (1989), which was interpreted as a contribution to both normative and positive economics.

Flood and Isard (1989) started from the premise that an optimal fully-state-contingent rule for monetary policy is not a relevant possibility in a world in which knowledge about the macroeconomic structure and the nature of disturbances is incomplete. Since simple rules (including partially-state-contingent rules) and discretion cannot be unambiguously ranked, a mixed strategy of combining a simple rule with discretion can be preferable both to rigid adherence to the rule and to complete discretion.<sup>5</sup> The paper showed formally that a mixed strategy under which the authorities adhered to a simple rule in “normal circumstances,” but overrode the rule when there were relatively large payoffs from doing so, could increase social welfare (relative to either the case of complete discretion or the case of rigid adherence to the rule) by providing a mechanism for both enhancing credibility during normal times and allowing for flexibility when it was most needed. It was also suggested that, by establishing well-designed institutional mechanisms, society could motivate the monetary authority to avoid both the overuse and the underuse of its override option. By 1990–91 such mixed strategies were referred to as “rules with escape clauses.”<sup>6</sup>

---

<sup>3</sup>Attention to the credibility problem was stimulated by Kydland and Prescott (1977) and Calvo (1978).

<sup>4</sup>Barro and Gordon (1983a, 1983b), Backus and Driffill (1985), Canzoneri (1985), and Rogoff (1985), among others, were instrumental in advancing the analysis of these issues.

<sup>5</sup>The arguments are further articulated in Flood and Isard (1990), which corrects a technical error identified by Lohmann (1990).

<sup>6</sup>The terminology emanated from Persson and Tabellini (1990).

---

**Box 1. Research on Monetary Policy: 1989 vs. 1999**

**The 1989 setting**

- concern with time inconsistency
- focus on rules and other institutional arrangements for mitigating credibility problems
- interest in robustness of rules—i.e., in simple rules that perform well across the spectrum of plausible macro models

**Key points in Flood and Isard (1989)**

- Fully-state-contingent rules are not relevant possibilities in practice.
- Since partially-state-contingent rules and discretion cannot be unambiguously ranked, it seems attractive to consider mixed strategies that combine a simple rule with discretion (an “escape clause”) and to establish institutional arrangements that provide incentives for policymakers not to overuse or underuse discretion.
- In evaluating a simple policy rule, it is not valid to base counterfactual historical simulations on the assumption that rational market participants would have expected the authorities to completely adhere to the rule when policymakers, had they actually been confronted with the counterfactual history, would have sometimes had incentives to deviate from the rule.

**The 1999 setting**

- consensus that simple rules cannot and should not be mechanically followed by policymakers
  - notion that research can nevertheless be useful for identifying the types and calibrations of rules that are relatively attractive as guidelines for policy
  - extensive reliance on stochastic simulation analysis with some attention to the robustness issue, little attention to modeling the process that the authorities use to update their information on key model parameters, and little explicit allowance for the fact that rational market participants might not find an announced rule fully credible
-

Along with the conceptual analysis that had emerged ten years ago, a second strand of literature was oriented toward simulating and comparing the performances of different types of simple monetary policy rules in empirically-estimated models of macroeconomic behavior. A primary objective of this literature, spearheaded by McCallum (1988), was to find a simple rule that performed reasonably well across the spectrum of plausible models. Although the search for robustness seemed appropriate in the context of model uncertainty, Bob Flood recognized a serious flaw in the methodology that was typically used to evaluate how well the rules performed. In particular, as Flood and Isard (1989) pointed out, it is not generally valid to base counterfactual historical simulations on the assumption that rational market participants would have expected the authorities to adhere rigidly to a given monetary rule when policymakers, had they actually been confronted with the counterfactual history, would have sometimes had incentives to deviate from the rule. While some economies have experienced prolonged periods of stable non-inflationary growth guided by transparent and predictable monetary policy behavior, no economy is insulated from occasional strong unanticipated shocks (such as the oil price shocks of the 1970s, or the current global financial crisis) that create situations in which the pursuit of short-run economic objectives would require a departure from any simple rule that the monetary authorities might have been following, and would therefore call into question the credibility of the rule.

Compared with the situation a decade ago, the academic literature today has become more extensively dominated by simulation studies, with credibility issues no longer at center stage. The shift in emphasis has obviously been facilitated by advances in computational technology, but it also reflects changes in the practice of monetary policy along with the widespread success that the industrial countries have had in subduing inflation during the 1990s. Monetary authorities in a number of industrial countries today are pursuing strategies of inflation targeting, broadly defined to encompass objectives for both the inflation rate and output/employment. To help guide the formulation of monetary policy strategies in both the inflation targeting cases and other countries, economists at central banks and elsewhere have been generating a large volume of research that simulates and compares the performances of selected forms of simple policy rules in different macroeconometric models.<sup>7</sup>

For the most part, contributors to the current stream of research on monetary policy rules implicitly accept the “escape clause” notion that monetary authorities should have a

---

<sup>7</sup>Recent examples of such papers were included in the programs of the NBER Conference on Monetary Policy Rules (January 15–17, 1998), the Federal Reserve Bank of San Francisco Conference on Central Bank Inflation Targeting (March 6–7, 1998), the Riksbank-IIES Conference on Monetary Policy Rules (June 12–13, 1998), the 1998 Symposium on Computational Economics at Cambridge University (June 29–July 1, 1998), and the Reserve Bank of New Zealand Conference on Monetary Policy Under Uncertainty (June 29–July 3, 1998). Earlier contributions to the inflation targeting literature include the conference volumes Leiderman and Svensson (1995), Haldane (1995), Federal Reserve Bank of Kansas City (1996), and Lowe (1997).

certain degree of flexibility to deviate from simple rules. In particular, few economists today seriously suggest that central banks should adhere mechanically to simple policy rules. In a world in which the structure of macroeconomic relationships and the distribution of shocks is imperfectly known *ex ante*, central banks need to be prepared to adjust their reaction patterns, and to exercise discretion intelligently,<sup>8</sup> when macroeconomic behavior deviates substantially from the model on which previous reaction patterns were conditioned.

That being said, however, there remains considerable interest in analyzing how different types and calibrations of well-defined policy reaction functions would perform in hypothetical macroeconomic models, reflecting sentiment that such analysis can provide useful insights for monetary policy. Most central bankers and academic economists also believe that it is important for monetary policy to be transparent, and many have argued that the adoption of policy rules as guidelines can be helpful for communication, accountability, and credibility. Svensson (1999) argues that this is particularly true for “targeting rules” that correspond to the first-order conditions of policy optimization problems.<sup>9</sup>

The recent literature on monetary rules has taken several directions (Box 2). Some researchers have sought to derive optimal rules (first-order conditions) for relatively simple macro models.<sup>10</sup> Others have compared the performances of different simple rules in macro models with optimizing agents.<sup>11</sup> Still others have looked for simple rules that exhibit robustness in performing relatively well across a spectrum of plausible macro models.<sup>12</sup>

---

<sup>8</sup>A crucial prerequisite for exercising discretion intelligently, of course, is that the monetary authorities must understand the time-consistency issue and continuously evaluate the extent to which their behavior may be affecting the credibility of their announced objectives.

<sup>9</sup>In this context, Svensson argues that the strategy of inflation targeting can be viewed as a way of committing to minimize a particular loss function by adopting a rule (first-order condition) that involves target variables or forecasts of target variables and by implementing communication practices that allow the public to evaluate the monetary authority’s performance and hold it accountable.

<sup>10</sup>For example, Wieland (1998), Rudebusch and Svensson (1998), Svensson (1998).

<sup>11</sup>For example, McCallum and Nelson (1998a, 1998b), Henderson and Kim (1999).

<sup>12</sup>For example, McCallum (1988), Levin, Wieland, and Williams (1998).



---

## **Box 2. Alternative Lines of Research on Monetary Policy Rules**

### **Alternative research objectives**

- identify properties of optimal rules (first-order conditions) for particular macro models
- analyze performances of simple rules in macro models with optimizing agents
- look for simple rules that exhibit robustness in performing well across a spectrum of plausible macro models
- analyze how the optimal calibration of simple rules varies with key characteristics of macro models

### **Characteristics of macro models used in recent research**

- most of the models embody Phillips curves
  - some assume backward-looking inflation expectations; others embody a forward-looking model-consistent component of inflation expectations
  - most of the models are linear, such that policy-rule evaluation with quadratic loss functions focuses (almost) exclusively on the variances of inflation, output/unemployment, and in some cases the policy instrument (i.e., the nominal interest rate)
  - some studies allow explicitly for uncertainty about key model parameters—in particular, the NAIRU—but most of these studies simply treat the implications of this uncertainty as white noise rather than extending the model to allow the authorities to update their estimates of parameters period-by-period in a model-consistent manner that mimics the policymaking process and recognizes that policymakers in reality tend to make serially-correlated errors
  - almost all studies either implicitly assume that the candidate policy rules are fully credible or treat the degree of credibility as exogenous
-

A fourth approach, as reflected in the first set of simulation experiments reported in this paper, looks for insights from the somewhat different tack of exploring how the optimal calibration of simple policy rules varies with key characteristics of the macro model. This approach provides perspectives that may be useful in suggesting how policymakers should adapt the overall aggressiveness of their policy reactions, and the relative strengths of their reactions to inflation and unemployment, to the specification and parameters of the macroeconomic “model” they confront,<sup>13</sup> including such characteristics as the degree of NAIRU uncertainty, the degree of nonlinearity in the model, and the nature of inflation expectations.

The main conclusion from our stochastic simulations, however, relates to the robustness properties of rules that have been shown to perform well in linear models of the U.S. inflation process. In particular, we find that rules that perform well in linear models but implicitly embody backward-looking measures of real interest rates (such as conventional Taylor rules) or high degrees of interest rate smoothing, can fail to provide a nominal anchor for inflation expectations in models with moderate nonlinearities.

The analysis is developed by focusing on a small macro model in which certain key characteristics can be varied. The model resembles most of the others that have been used to analyze monetary policy rules insofar as it embodies Phillips curves as a “fixed” characteristic;<sup>14</sup> beyond that, the treatment of inflation expectations, the shape of the Phillips curve, and the degree of uncertainty about the NAIRU are variable characteristics.

The relevance of models that rely on the Phillips curve paradigm has been a topic of active debate in recent years.<sup>15</sup> Casual inference from the failure of inflation to accelerate in the United States through mid-year 1998, despite unemployment rates in the vicinity of 4 ½ percent, suggests that the U.S. NAIRU may have declined over time to well below the 6 percent neighborhood in which it was thought to reside several years ago. Recent empirical work supports the view that the NAIRU for the United States has declined over the past

---

<sup>13</sup>Data uncertainties, which also have implications for the optimal strength of policy reactions, are not addressed in this paper; see Orphanides (1998) for a recent exploration of this topic. On the general importance of changing the strength of policy responses when the (perceived) macro model changes, see Amano, Coletti, and Macklem (1998).

<sup>14</sup>See McCallum and Nelson (1998a, 1998b) and Henderson and Kim (1999) for studies of monetary policy rules in models with optimizing agents rather than postulated Phillips curves.

<sup>15</sup>See, for example, the Symposium comprised of Blanchard and Katz (1997), Galbraith (1997), Gordon (1997), Rogerson (1997), Staiger, Stock, and Watson (1997), and Stiglitz (1997), as published in the Winter 1997 issue of the *Journal of Economic Perspectives*.

decade, but it also suggests that a 95 percent confidence interval around the current value of the NAIRU may be as wide as 3 percentage points.<sup>16</sup>

Such time variation and imprecision in estimates of the NAIRU have led some economists to conclude that it is time to abandon the Phillips curve paradigm.<sup>17</sup> We regard this position as premature in the absence of a stronger consensus on an alternative analytic framework. It may also be noted that most industrial-country central banks continue to rely on the Phillips curve framework and to condition the nature and strength of their policy reactions on such analytic frameworks. That being said, however, a central premise of this paper is that monetary policy analysis based on the Phillips curve paradigm can be strengthened considerably by taking account of the nature of *ex ante* uncertainty about the NAIRU and by updating estimates of the NAIRU regularly and in a model-consistent manner—that is, by explicitly modeling the process through which the monetary authorities rationally update their estimates of the NAIRU period by period, based on new observations of unemployment and inflation along with their information about the structure of the model. This approach recognizes that errors in estimating the NAIRU tend to be serially correlated rather than white noise.<sup>18</sup>

In the tradition of most other recent simulation studies of policy rules, the model variants we use in this paper assume that adherence to the policy rule is fully credible; in other papers we have simulated the performances of simple policy rules under a crude but empirically-based representation of imperfect and endogenous credibility.<sup>19</sup> We demonstrate, however, that analysis based on the full credibility assumption is internally consistent in the following limited sense: When the macro model is well defined and known to the policymaker, when inflation expectations are either backward looking or model consistent, and when institutional arrangements motivate the policymaker to optimize over a long horizon, then the realized means and variances of inflation and unemployment are essentially independent of the loss-function parameter to which the credibility problem has traditionally been ascribed, so the announced calibration of a simple rule is time consistent. While this might be taken to justify the assumption that adherence to the announced calibration of the rule is fully credible, it does

---

<sup>16</sup>Staiger, Stock, and Watson (1997).

<sup>17</sup>For example, Galbraith (1997).

<sup>18</sup>Other recent analyses of the implications of NAIRU uncertainty (or output gap uncertainty) include Wieland (1998) and Smets (1998), who use simple linear models with backward-looking expectations to demonstrate that uncertainty about the NAIRU (or about the Phillips-curve parameters on which NAIRU estimates depend) provides a motive for cautious policy reactions.

<sup>19</sup>Isard and Laxton (1998), Isard, Laxton, and Eliasson (1998).

not generally imply—in a stochastic world with nonlinear behavior—that the prospect of achieving the inflation target embodied in the monetary policy rule is fully credible.

These considerations suggest that if policymakers are motivated by appropriate institutional arrangements cum reputation, credibility problems can be primarily attributed to the shortcomings of the analytic frameworks on which policies are based—that is, to the limitations of the authorities’ understanding of macroeconomic behavior. By the same token, they emphasize that institutional arrangements (commitment mechanisms) alone cannot make announced policy objectives fully credible when the authorities have imperfect information about the nature of macroeconomic behavior. Thus, although a number of simulations studies have now shown that simple policy rules, when optimally calibrated, are capable of generating an impressive degree of macroeconomic stability in well-defined macro models,<sup>20</sup> economists should not be quick to take comfort in these results. Such findings need to be weighed against the realization that policymakers confront difficulties in trying to arrive at optimal calibrations of policy rules when the “true” macro model is not well defined, and even more so, against analysis suggesting that several rules that have been advocated on the basis of good performances in linear models perform very poorly in models with moderate nonlinearities.

The remainder of the paper is structured as follows. Section II presents the equations and estimated parameters of the base-case model, along with details on the other model variants. The model describes a closed economy and is estimated with quarterly data for the United States. It includes: short-run Phillips curves that link observed inflation rates (for the CPI and the CPI excluding food and energy) to both the expected rate of inflation and the gap between the NAIRU and the observed unemployment rate; an equation describing the behavior of survey data on inflation expectations; a description of the dynamics of the unemployment rate as a function of the real interest rate; and a model-consistent process for generating and updating estimates of the NAIRU. We consider several model variants (linear and nonlinear short-run Phillips curves paired with forward- and backward-looking inflation expectations), each of which is consistent with the long-run natural rate hypothesis. The incorporation of uncertainty is limited to simple additive uncertainty about the NAIRU and ex ante uncertainty about various exogenous shocks in an environment where all model parameters and frequency distributions of shocks are known and dynamic learning occurs only through the process of updating estimates of the NAIRU.

Section III describes the simple policy rules, the loss function, the monetary authority’s behavior, and the stochastic simulation framework. We distinguish between two classes of inflation forecast based (IFB) rules: IFB1 rules, in which a forward-looking measure of the real interest rate—in particular, a measure that embodies a model-consistent inflation forecast—is adjusted in response to both the deviation of inflation from target and a measure

---

<sup>20</sup>Williams (1999) finds that even in models with hundreds of state variables, parsimonious specifications of simple policy rules appear to be very effective in achieving stabilization objectives. See also Rudebusch and Svensson (1998).

of the unemployment gap; and IFB2 rules, in which the same measure of the real interest rate is adjusted in response to deviations of an inflation forecast from target as well as the unemployment gap. Most of our simulations involve IFB1 rules. The Monte Carlo experiments employ a conventional quadratic loss function in searching for the optimal calibrations of the simple policy rules, but we focus in addition on a longer list of performance indicators, including the standard deviations of the unemployment, inflation, and the nominal interest rates and, for the nonlinear model variants, also the means of the unemployment and inflation rates.

Section IV reports the simulation results, which are presented in two subsections, each addressing a different set of issues. Subsection IV. A describes and compares the optimal calibrations of IFB1 rules under different well-defined model variants. In particular, it explores how the optimal calibrations of these rules depend on policy preferences, the degree of NAIRU uncertainty, the extent to which inflation expectations are backward looking, and the shape of the Phillips curve.

Subsection IV.B then addresses several specific rules that have been proposed in the literature and compares their performances with the performances of optimally calibrated IFB1 rules. The additional rules on which we focus are: (i) Taylor's (1993, 1998a) conventional Taylor rule; (ii) an inflation-forecast-based rule with interest rate smoothing, as estimated for the United States by Clarida, Gali, and Gertler (1998); (iii) the IFB2 rule analyzed by Isard and Laxton (1998); and (iv) a first-difference rule for the interest rate, as proposed by Levin, Wieland, and Williams (1998). The stochastic simulation results, supplemented by stability analysis (Appendix II), demonstrate that in a world in which inflation expectations have a forward-looking model-consistent component, monetary policy guided by a myopic rule that incorporates a backward-looking measure of the real interest rate, such as a conventional Taylor rule, can be destabilizing in our moderately nonlinear model. Similarly, rules with high degrees of interest rate smoothing, such as certain calibrations of forward-looking Clarida, Gali, and Gertler (CGG) rules and the first-difference rule proposed by Levin, Wieland, and Williams (LWW), can lead to instability in our model.

Section V summarizes the key messages of the paper.

## **II. A MODEL OF THE UNEMPLOYMENT-INFLATION PROCESS**

Our model is a somewhat extended version of the framework developed in Laxton, Rose, and Tambakis (1999). It includes four estimated equations: two Phillips curves (one focusing on the CPI, the other on the CPI excluding food and energy), an equation describing the dynamics of inflation expectations, and an equation describing the dynamics of the unemployment rate. The inclusion of two Phillips curves allows us to exploit a larger data set when drawing inferences about the NAIRU. The model estimates are based on quarterly data for the United States over the period since 1968:Q1. The model is closed with a monetary

policy reaction function and a model-consistent procedure for updating estimates of the NAIRU (both described in Section III). In the “base-case” version of the model, the Phillips curve specifications are convex and inflation expectations include a forward-looking model-consistent component. Other model variants include linear Phillips curves and entirely-backward-looking inflation expectations.

### A. The Short-Run Phillips Curves

The convex versions of our Phillips curves are broadly similar to the specification used in Debelle and Laxton (1997):

$$\pi_t = \lambda \bar{\pi}_t^e + (1-\lambda) \pi_{t-1} + \gamma(u_t^* - u_t)/(u_t - \phi_t) + \epsilon_t^\pi \quad (1)$$

$$\pi_t^x = \lambda^x \bar{\pi}_t^e + (1-\lambda^x) \pi_{t-1}^x + \gamma(u_t^* - u_t)/(u_t - \phi_t) + \epsilon_t^{\pi^x} \quad (2)$$

where

$$\bar{\pi}_t^e = \left( \sum_{i=1}^N E_{t-i} \pi_{t+4-i} \right) / N \quad (3)$$

Here  $\pi_t$  denotes the rate of consumer price inflation during quarter  $t$ , measured at an annual rate;  $\pi_{t+4}$  denotes the rate of inflation over the year through quarter  $t+4$ ;  $E_t \pi_{t+4}$  is the public’s expectation in quarter  $t$  of the rate of inflation over the year through quarter  $t+4$ ;  $\pi_t^x$  denotes the annualized rate of change during quarter  $t$  of the consumer price index excluding food and energy;  $u$  is the unemployment rate; and  $\lambda$ ,  $\lambda^x$ , and  $\gamma$  are parameters to be estimated. ( $u^*$  and  $\phi$  will be defined below.)

The model of how expectations influence inflation dynamics is meant to reflect a bargaining framework that is capable of generating significant persistence in the inflation process.<sup>21</sup> The implicit underlying assumption is that a standard contract has an  $N$ -quarter

---

<sup>21</sup>The model contains important backward- and forward-looking components, as derived from the bargaining framework in Fuhrer and Moore (1995a, 1995b), but the functional form is less restrictive and is more consistent with empirical evidence that suggests that there is a small weight on the “rational” or forward-looking component of the US inflation process—for example, see Fuhrer (1997).

horizon, with one- $N$ th of the contracts respecified every quarter.<sup>22</sup> Thus, equation (3) defines  $\bar{\pi}_t^e$  as an average of one-year ahead inflation expectations that economic agents held during the  $N$  quarters in which currently-prevailing contracts were written. Inflation dynamics are also assumed to depend on the lagged inflation rate, which can be viewed as a summary indicator of the strength of incentives to incur the costs of revising price or wage contracts before their specified expiration dates.

Note that the coefficients on the first two right-hand-side terms in equations (1) and (2) are constrained to sum to unity, consistent with the long-run natural rate hypothesis. We refer to the sums of these terms as the core rates of inflation,  $\pi^c$  and  $\pi^{cx}$ .

$$\pi_t^c = \lambda \bar{\pi}_t^e + (1 - \lambda) \pi_{t-1} \quad (4)$$

$$\pi^{cx} = \lambda^x \bar{\pi}_t^e + (1 - \lambda^x) \pi_{t-1} \quad (5)$$

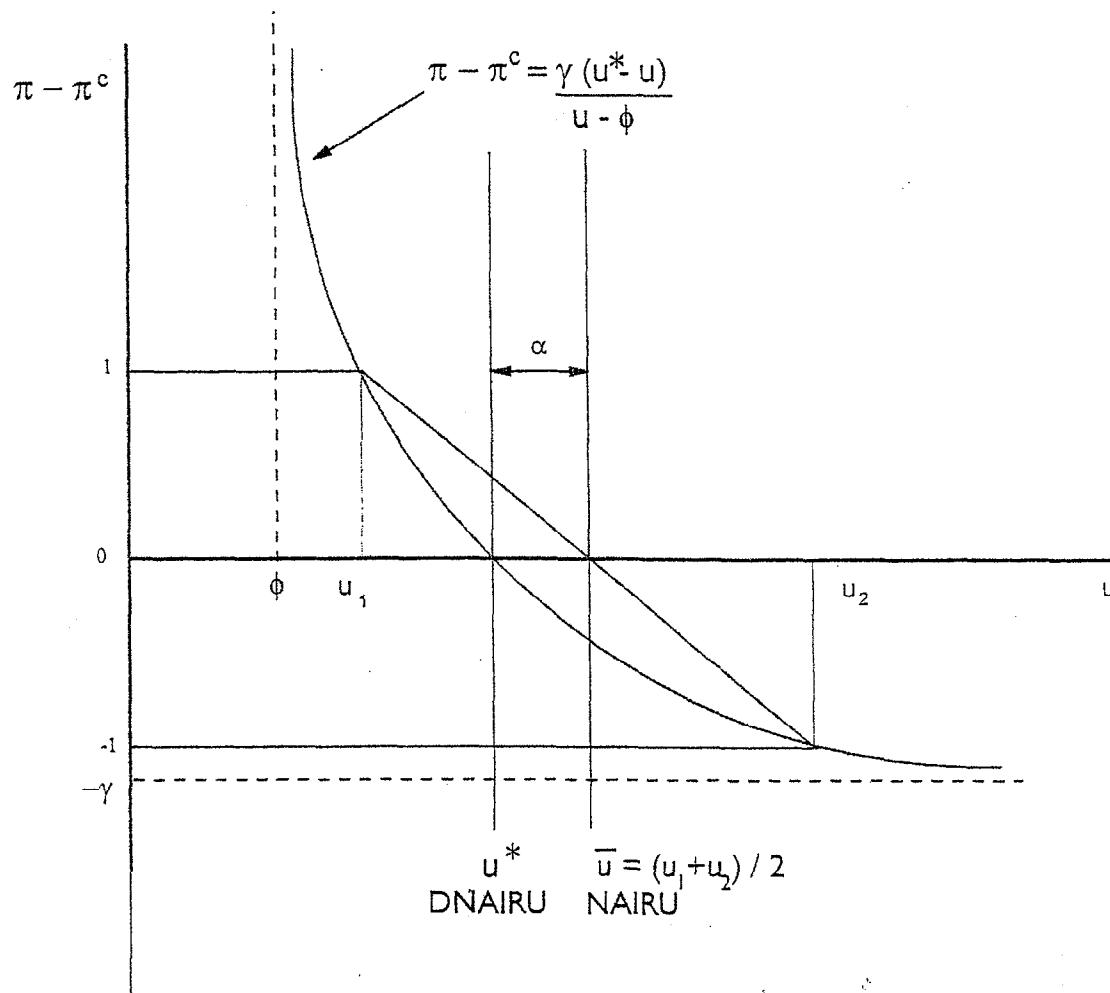
Figure 1 plots the difference between observed inflation and core inflation (vertical axis) against the unemployment rate (horizontal axis). For purposes of the discussion here, we interpret core inflation as synonymous with expected inflation, so the figure can be viewed as an expectations-augmented Phillips curve. Consistent with the specification in equation (1), the short-run Phillips curve is convex with horizontal asymptote at  $\pi - \pi^c = -\gamma$  and vertical asymptote at  $u = \phi$ . Following Laxton, Meredith, and Rose (1995),  $\phi$  can be interpreted as a “wall parameter,” reflecting short-run constraints on how far rising aggregate demand can lower unemployment before capacity constraints become absolutely binding and inflationary pressure becomes unbounded. The magnitude of  $u^*$  corresponds to the unemployment rate at which actual inflation and expected inflation coincide, such that there would be no systematic pressure for inflation to rise or fall *in the absence of stochastic shocks*. This corresponds to the non-accelerating-inflation rate of unemployment in a *deterministic* world. We refer to  $u^*$  as the DNAIRU (deterministic NAIRU).<sup>23</sup>

---

<sup>22</sup>The base-case model variant assumes  $N=12$ , but to explore the sensitivity of the results to the length of contracts we have also conducted simulations with  $N=4$ .

<sup>23</sup>In equations (1) and (2), the estimated value of  $\gamma$  is 3.20. The estimation and stochastic simulations are based on the assumption that  $\phi_t = \max[0, u_t^* - 4]$ , and  $u_t^*$  is always strictly greater than 4 in the actual and hypothetical data we address.

**Figure 1. The Convex Phillips Curve**





An important point is that the DNAIRU is not a feasible stable equilibrium in a *stochastic* world with a convex Phillips curve. The average rate of unemployment consistent with non-accelerating-inflation in a stochastic world, denoted by  $\bar{u}$  and referred to as the NAIRU, must be greater than the DNAIRU when the Phillips curve is convex. This can be illustrated in Figure 1 by assuming that actual inflation turned out to be uniformly distributed between plus and minus one percentage point of core (or expected) inflation, which would imply an average rate of unemployment of  $\bar{u} = 0.5 (u_1 + u_2)$ . It can easily be seen that with a wider distribution of the actual inflation rate around core inflation, the average rate of unemployment would be even greater. The fact that the difference between the NAIRU and DNAIRU—and hence the average rate of unemployment—depends, in a nonlinear world, on the degree to which the authorities succeed in mitigating the variance of inflation has important implications for monetary policy.

Following Debelle and Laxton (1997) and others, the Phillips curve equations are estimated jointly with an equation that describes a time-varying DNAIRU.<sup>24</sup> We assume here that the latter follows a bounded random walk and arbitrarily set a floor at 4 percent and a ceiling at 8 percent, such that

$$u_t^* = \begin{cases} 8 & \text{if } u_{t-1}^* + \epsilon_t^{u^*} \geq 8 \\ 4 & \text{if } u_{t-1}^* + \epsilon_t^{u^*} \leq 4 \\ u_{t-1}^* + \epsilon_t^{u^*} & \text{otherwise} \end{cases} \quad (6)$$

where  $\epsilon_t^{u^*}$  is drawn from a normal distribution with mean zero. We also extend the formulation of the estimation problem beyond the approach used in previous studies by adding the assumptions that the business cycle component of unemployment is a stationary (and presumably highly autocorrelated) process,  $\epsilon_t^u$ , and that the difference between the NAIRU and the DNAIRU is a constant:<sup>25</sup>

$$u_t = \bar{u}_t + \epsilon_t^u \quad (7)$$

---

<sup>24</sup>For discussions of the potential pitfalls associated with conventional tests for asymmetries in the Phillips curve, see Clark, Laxton, and Rose (1996) and Laxton, Rose, and Tambakis (1999).

<sup>25</sup>The latter would be implied by constant adherence to a given policy rule. Note that equation (7) is relevant for interpreting history and updating estimates of the NAIRU, but that apart from assuming stationarity, we do not require specific assumptions about the distribution of the  $\epsilon_t^u$  terms, which are not drawn directly in the simulation analysis.

$$\alpha = \bar{u}_t - u_t^* \quad (8)$$

If we rewrite equations (1) and (2) for heuristic purposes as

$$\pi_t - \pi_t^c = \delta_t[1/(u_t - \phi_t)] - \gamma[u_t/(u_t - \phi_t)] + \epsilon_t^\pi \quad (9)$$

$$\pi_t^x - \pi_t^{cx} = \delta_t[1/(u_t - \phi_t)] - \gamma[u_t/(u_t - \phi_t)] + \epsilon_t^{\pi^x} \quad (10)$$

where  $\delta_t = \gamma u_t^*$  is a time-varying parameter, equations (6)-(10) provide a nonlinear estimation problem that can be solved using the Kalman filter technique.<sup>26</sup>

Table 1 reports the estimation results. The estimated parameters of the Phillips curves have the correct signs and are statistically significant. The gap between the NAIRU and the DNAIRU is estimated to be two-tenths of a percentage point. To remain consistent with these parameter estimates, the Phillips curves that are used in analyzing the linear variants of the model are calibrated as:<sup>27</sup>

$$\pi_t = \pi_t^c + 0.80(u_t^* - u_t) + \epsilon_t^\pi \quad (1a)$$

$$\pi_t^x = \pi_t^{cx} + 0.80(u_t^* - u_t) + \epsilon_t^{\pi^x} \quad (2a)$$

---

<sup>26</sup>Kuttner (1992, 1994) has applied this idea to measuring potential output. In using information about the error terms in each of the two Phillips curves, our procedure for estimating the NAIRU and DNAIRU essentially gives equal weight to the data on the CPI and the CPI excluding food and energy.

<sup>27</sup>The simulations set  $\phi_t = u_t^* - 4$ , so the convex term in the unemployment rate in equations (1) and (2), based on the estimates reported in Table 1, can be expressed as  $3.20F(g)$ , where  $g = u^* - u$ . The linear approximations in equations (1a) and (2a) replace  $F(g)$  with  $[F'(0)](u^* - u) = (3.20/4)(u^* - u) = 0.80(u^* - u)$ .

Table 1. Phillips Curves and the Time-Varying NAIRU

Estimated equations: 1/

$$\pi_t = 3.20(u_t^* - u_t)/(u_t - \phi_t) + 0.41\bar{\pi}_t^e + (1-0.41)\pi_{t-1} \quad (9.39) \quad (6.97)$$

$$\pi_t^x = 3.20(u_t^* - u_t)/(u_t - \phi_t) + 0.64\bar{\pi}_t^e + (1-0.64)\pi_{t-1}^x \quad (9.39) \quad (9.59)$$

$$\bar{u}_t = 0.20 + u_t^* \quad (0.30)$$

$$u_t^* = u_{t-1}^* + \epsilon_t^{u^*}$$

**Variables:**

$\pi_t$	Percent change in consumer price index, quarterly at annual rate.
$\pi_t^x$	Percent change in CPI excluding food and energy.
$\bar{\pi}_t^e$	Expected inflation rate based on equation (3) with N=12.
$\phi_t$	Time-varying “wall parameter”—see text.
$u_t^*$	Unobserved DNAIRU as estimated from the Kalman filter.
$\bar{u}_t$	Estimated NAIRU.

1/ t-values in parentheses.

## B. The Dynamics of Inflation Expectations

Within the sample period over which the Phillips curves are estimated, inflation expectations are based on the mean responses from the Michigan survey of expectations about one-year-ahead changes in the consumer price index. For purposes of the simulation analysis, however, we require a model of how expectations evolve. An important issue is the extent to which expectations are forward looking and model consistent.

The approach here, following Laxton, Rose, and Tambakis (1999), is based on an investigation of three alternative equations for explaining the historical survey data. The different specifications are described in Table 2. The first two include a forward-looking model-consistent component,  $\pi 4^{mc}$ , which was constructed from a proxy for the model—namely, as the fitted values of an auxiliary equation that predicts observed inflation over the year ahead using four lagged values each of the unemployment rate, a long-term interest rate, the survey measure of inflation expectations, and the inflation rate.

As can be seen from Table 2, the constrained model has almost the same fit as the basic unconstrained model and slightly outperforms the overfitted model with inflation lags. The latter result indicates that, conditional on the presence of the forward-looking proxy, the estimation prefers the lagged dependent variable to lagged data on observed inflation. This suggests that expectations are not inherently backward looking; the lags of inflation are useful in explaining the expectations data only to the extent that they help predict the future, as reflected in their contribution to  $\pi 4^{mc}$ . The estimates also point to substantial inertia in inflation expectations, as reflected in a relatively high coefficient on the lagged dependent variable.

In light of the estimates reported in Table 2, most of our simulation analysis reflects the following base-case assumption about inflation expectations:

$$E_t \pi 4_{t+4} = 0.261 \pi 4_t^{mc} + 0.739 E_{t-1} \pi 4_{t+3} + \epsilon_t^{\pi^e} \quad (11)$$

As alternatives, we also consider a fairly traditional forward-and-backward-looking components model with equal weights of 0.5 on the forward-and-backward-looking components,<sup>28</sup> along with two extreme cases in which inflation expectations are entirely

---

<sup>28</sup>Estimates of equations based on the Michigan survey measures of inflation expectations suggested a weight of .6 on the model consistent component, but there was significant evidence of residual autocorrelation in the estimated equations.

Table 2. Dynamics of Inflation Expectations

Estimated equation:

$$E_t \pi_{t+4} = \alpha + \delta_1 \pi_{t+4}^{mc} + \delta_2 E_{t-1} \pi_{t+3} + \rho_1 \pi_{t-1} + \rho_2 \pi_{t-2} + \rho_3 \pi_{t-3} + \rho_4 \pi_{t-4}$$

$E_t \pi_{t+4}$  Expected inflation, CPI, one-year-ahead measure from the Michigan survey  
 $\pi_{t+4}^{mc}$  Fitted values from auxiliary regression to forecast one-year-ahead CPI inflation  
 $\pi_t$  Inflation, CPI, quarterly measure at annual rate

Estimation period 1968:Q1 to 1997:Q2

Coefficient	Basic Model Unconstrained	Constrained ( $\delta_2 = 1 - \delta_1$ )	Overfitted, With Inflation Lags
$\alpha$			0.470 (2.3)
$\delta_1$	0.258 (3.8)	0.261 (3.8)	0.346 (4.5)
$\delta_2$	0.726 (10.5)		
$\rho_1$			-0.092 (1.7)
$\rho_2$			-0.072 (1.2)
$\rho_3$			0.076 (1.2)
$\rho_4$			0.011 (0.2)
$\bar{R}^2$	0.743	0.745	0.732
DW statistic	2.33	2.35	2.27
Residual variance	1.08	1.08	1.03

Note: Figures in parentheses are t-statistics.

backward looking and entirely forward looking. These correspond, respectively, to the following specifications:

$$E_t \pi 4_{t+4} = 0.5 \pi 4_{t+4}^{mc} + 0.5 \pi 4_{t-1} + \epsilon_t^{\pi^e} \quad (11a)$$

$$E_t \pi 4_{t+4} = \pi 4_{t-1} \quad (11b)$$

$$E_t \pi 4_{t+4} = \pi 4_{t+4}^{mc} \quad (11c)$$

Specification (11), which we consider more realistic than the other three, presents a case in which shocks to inflation expectations can be more persistent than under the traditional forward-and-backward-looking components model, thereby presenting a more difficult challenge for monetary policy.

### C. The Dynamics of the Unemployment Rate

We draw again on Laxton, Rose, and Tambakis (1999) in modeling the behavior of the unemployment rate, which reflects the influence of monetary policy as transmitted through aggregate demand. The estimated equation has the form

$$u_t = c_t + \sum_{i=1}^2 \eta_i u_{t-i} + \sum_{i=1}^3 \phi_i r_{t-i} + \epsilon_t^u \quad (12)$$

where

$$r_t = rs_t - E_t \pi 4_{t+4} \quad (13)$$

is our measure of the real interest rate.<sup>29</sup> The time-varying “constant,”  $c_t$ , is assumed to follow a random walk to capture the combined effects of any changes in the trend levels of unemployment and the real interest rate.

Table 3 reports the fit of equation (12), which is also estimated using a Kalman filter. The lag structure is written in its final form, following testing down from specifications with longer lags.<sup>30</sup> The results reflect two stylized facts concerning the monetary authority’s ability to control the economy. First, there are important lags between changes in interest rates and their effects on aggregate demand. Second, there is persistence in movements in the unemployment rate, implying that shocks to aggregate demand propagate into future periods. The coefficients on the unemployment lags imply some augmenting propagation, but with relatively speedy reversion to the mean.<sup>31</sup>

### III. THE POLICY RULES AND STOCHASTIC SIMULATION FRAMEWORK

#### A. The Simple Policy Rules

We focus on several classes of simple policy rules. Part of the motivation for focusing on simple forms of policy reaction functions is pragmatic; particularly in the nonlinear variants of our model, the task of deriving the optimal rule associated with conventional specifications of policy loss functions would be horrendous. In addition, simple classes of rules are transparent and relatively appealing to policymakers.

Most prominent among the simple policy rules that have received attention in the recent literature are conventional Taylor rules. Under Taylor rules the monetary authorities

---

<sup>29</sup>Fuhrer and Moore (1995b) argue that longer-term interest rates are more relevant for explaining aggregate demand and unemployment. The implications of such an alternative representation of the monetary transmission mechanism might be interesting to explore as an extension of the analysis in this paper.

<sup>30</sup>See Laxton, Rose, and Tambakis (1999) for details on the estimation.

<sup>31</sup>The qualitative results and main conclusions of this paper do not hinge on the precise nature of the unemployment dynamics, although they clearly depend on a positive response of unemployment to the real interest rate, as well as on the existence of both lags in the response of unemployment to policy actions and a persistent component of unemployment. It might be interesting, in future work, to consider modifications of the model in which the response of unemployment to the interest rate was forward looking. It might also be interesting to treat the parameters of the unemployment equation as an additional element of uncertainty—along with the level of the NAIRU—that policymakers take into account when choosing the “optimal calibration” for a policy rule.

Table 3. Dynamics of the Unemployment Rate

---

Unemployment rate equation	$u_t = c_t + \sum_{i=1}^2 \eta_i u_{t-i} + \sum_{i=1}^3 \varphi_i r_{t-i} + \epsilon_t^u$ $c_t = c_{t-1} + \epsilon_t^c$
Real federal funds rate	$r_t = rs_t - E_t \pi_{t+4}$

---

$u_t$             unemployment rate  
 $rs_t$             federal funds rate  
 $E_t \pi_{t+4}$         expected inflation over the next year from the Michigan survey  
 $c_t$             time-varying parameter estimated using Kalman-filtering methods

Estimation period: 1968:Q1 to 1998:Q2

---

Coefficient	Estimate	T-ratio
$\eta_1$	1.027	11.6
$\eta_2$	-0.258	2.9
$\varphi_1$	0.010	0.6
$\varphi_2$	0.034	1.8
$\varphi_3$	0.023	1.2
Residual variance	0.0285	
Log likelihood	-28.96	

---



adjust the short-term nominal interest rate in response to both the deviation of the current inflation rate from target and either the deviation of current output from potential output or the deviation of unemployment from the NAIRU.<sup>32</sup> The conventional specification of Taylor rules, when expressed in terms of the unemployment gap, is:

$$rs_t = r^* + \pi 4_t + w_\pi(\pi 4_t - \pi^{TAR}) + w_u(\bar{u}_t - u_t) \quad (14)$$

where:  $rs_t$  is the nominal interest rate setting at time  $t$ ;  $\pi 4_t$  and  $u_t$  represent the rates of inflation and unemployment;  $\pi^{TAR}$  denotes a target rate of inflation;  $\bar{u}_t$  is the authorities' estimate of the NAIRU based on observed data through period  $t-1$ ;  $r^*$  is a constant corresponding to the equilibrium real interest rate; and  $w_\pi$  and  $w_u$  are parameters.

Note that, in the second term on the right-hand side of (14), the inflation rate over the four quarters through period  $t$  appears as a backward-looking measure of the expected rate of inflation. As discussed below, in the context of our moderately nonlinear model of the U.S. economy, the precise form of the rule suggested by Taylor (1993, 1998a, 1998b) is a very myopic rule that in some situations is not sufficient to ensure stability in the inflation process.<sup>33</sup>

The second class of rules that we examine—which can be regarded as a class of inflation forecast based (IFB) rules that we refer to as IFB1 rules— replaces the second term in equation (14) with a model-consistent measure of inflation expectations. Specifically, IFB1 rules can be written in the general form:

$$\tilde{r}_t = r^* + \tilde{E}_t\{w_\pi(\pi 4_t - \pi^{TAR}) + w_u(\bar{u}_t - u_t)|\Omega_t\} \quad (15)$$

where

$$\tilde{r}_t = rs_t - \tilde{E}_t\{E_t\pi 4_{t+4}|\Omega_t\} \quad (16)$$

---

<sup>32</sup>Interest in this formulation received considerable impetus from Taylor (1993), who defined his rule in terms of the output gap. Recent studies of the performance of Taylor rules can be found, for example, in Levin, Wieland, and Williams (1998) and Taylor (1998a).

<sup>33</sup>Appendix II discusses the stability conditions for models that are based on linear and nonlinear Phillips curves and explains why the conventional Taylor rule ensures stability in models with linear Phillips curves but does not ensure stability in nonlinear models of the inflation process.

Here  $\tilde{r}_t$  is the monetary authority's ex ante measure of the real interest rate on which aggregate demand and unemployment depend;  $E_t \pi_{t+1}$  denotes the public's expectations at time  $t$  of the inflation rate over the year ahead; and  $\tilde{E}_t\{\cdot | \Omega_t\}$  denotes a model-consistent forecast at time  $t$  based on the authorities' information set  $\Omega_t$ , which includes information about the model along with the observed values of the inflation rate through quarter  $t$  and all other economic variables through quarter  $t-1$ .

Note that the IFB1 rule is specified in the form of a rule for real interest rate adjustment. Although monetary policy operates by setting the nominal interest rate, in our model (and most others) the extent to which monetary policy adjustment stimulates or restrains aggregate demand and employment depends on the real interest rate. It would thus make no sense to propose that policy be guided by a nominal interest rate rule that could not be explicitly translated into an economically reasonable rule for the real interest rate.

The IFB1 rule involves a forward-looking measure of the real interest rate. A third class of rules that has received attention in the literature, which we refer to as IFB2 rules and make a number of references to in parallel with our discussion of IFB1 rules, is defined through a simple modification of the bracketed expression in equation (15) in which the period- $t$  inflation rate is replaced by an inflation forecast.

By focusing on the deviation from target of the authorities' inflation forecast, inflation-forecast-based rules have the appealing feature of inducing the authorities to condition their interest rate settings on current information about the determinants of future inflation, given their information/assumptions about the structure of the model.<sup>34</sup> As we will demonstrate, the conditioning of monetary policy reactions on forward-looking inflation forecasts, rather than backward-looking inflation measures, appears to be critical to stability in moderately nonlinear models in which inflation expectations have a model-consistent component and policymakers confront an historically normal degree of uncertainty about the NAIRU.

## B. The Policy Objective Function

The literature on optimal policy rules has traditionally relied on quadratic loss functions that are separably additive in the deviation of inflation from target, the unemployment (or output) gap, and sometimes also the change in the nominal interest rate; see, for example, Rudebusch and Svensson (1998) and Wieland (1998). To remain consistent with this literature, we adopt an objective function in which the period- $t$  loss has the following general form

---

<sup>34</sup>Different types of IFB rules have been shown to deliver reasonable economic performances over a wide range of disturbances; see, for example, Amano, Coletti, and Macklem (1998), Haldane and Batini (1998), and Rudebusch and Svensson (1998).

$$L_t = (\pi_t - \pi^{TAR})^2 + \theta[u_t - (u_t^* - \beta)]^2 + v(rs_t - rs_{t-1})^2 \quad (17)$$

where  $\theta$ ,  $\beta$ , and  $v$  are parameters and  $u_t^*$  is the DNAIRU (deterministic NAIRU). For  $\beta = 0$  this corresponds to the specification that it has been popular to use in recent simulation studies of policy rules. More generally, it also allows us, somewhat in the spirit of Barro and Gordon (1983a, 1983b) and Rogoff (1985), to consider cases in which the authorities' preferences with regard to unemployment are not symmetric around the DNAIRU but center on an unemployment rate below the DNAIRU (i.e., cases with  $\beta > 0$ ), and to note how our simulation results are affected by credibility issues in these cases.<sup>35</sup>

### C. The Monetary Authority's Behavior

The monetary authority adjusts a short-term nominal interest rate period by period in accordance with a prespecified simple policy rule. We assume that its action in quarter  $t$  is timed to come soon after the announcement of the observed inflation rate for quarter  $t$ , when the period- $t$  values of other macroeconomic variables have not yet been observed.

The monetary authority is assumed to have full information about the structure of the model and the ex ante distributions of the exogenous shocks. After observing the period- $t$  inflation rate, the central bank is assumed to update its estimates of the DNAIRU and NAIRU (i.e., resolve the Kalman filter problem defined by equations (6)-(10)) and set the period- $t$  interest rate based on an information set  $\Omega_t$  that includes: the complete specification of the true model, including the process that generates the DNAIRU and NAIRU as well as the bounds on the DNAIRU; the history of all observable variables (including the survey measures of inflation expectations) through period  $t-1$ , along with the inflation rate for period  $t$ ; and the probability distributions (but not the realizations) of the shocks for period  $t$  and all future periods.

Under these strong informational assumptions, the central bank knows that the exogenous shocks have independent normal distributions with zero means, and it also knows the standard deviations of the shocks (which we calibrated to reflect the unexplained variances of the dependent variables during the historical periods over which the model equations were estimated). For purposes of implementing its policy rule, it needs to solve for the expected rate of inflation that defines the level of the real interest rate in equation (16), which requires it to solve its forward-looking macro model for the expected future time paths of all the

---

<sup>35</sup>The  $\beta$  parameter also provides a basis for analyzing the pros and cons of central bank transparency. As Faust and Svensson (1998) emphasize, transparency about the central bank's objectives, by improving the accuracy of the public's information about  $\beta$  (or alternatively, about the difference between  $u_t^* - \beta$  and the long-run DNAIRU,  $u^*$ ), can make it possible for the public to distinguish more accurately between the intended components of macroeconomic outcomes and the central bank's control errors, thereby making the central bank's reputation and credibility more sensitive to its actions.

endogenous variables, since inflation expectations have a forward-looking model-consistent component. We make the assumption that the central bank follows a certainty equivalence procedure in solving the model—in particular, that it assumes that all future shocks will be equal to their expected values of zero.<sup>36</sup> The forecasting rule that it uses to project the path of the DNAIRU (a bounded random walk) is described in Appendix I. In solving the model, the central bank determines, *inter alia*, a set of projections for the entire future time paths of both its policy instrument and the rate of inflation.

#### **D. The Stochastic Simulation Experiments**

Our first set of stochastic simulation experiments is oriented toward identifying the optimal calibrations of IFB1 rules, based on a grid search, under different model variants and parameterizations of the loss function. Additional simulations focus on evaluating the performances of selected calibrations of other classes of simple rules.

With regard to the first objective, we simulated the performance of the economy for a range of reaction function weights ( $w_\pi$ ,  $w_u$ ), with  $w_\pi$  running over a grid from 0.1 to 2.0 in intervals of 0.1 and  $w_u$  running from 0.0 to 2.0 in intervals of 0.1. Focusing first on the base-case model, we computed the value of the loss function under several parameterizations in order to evaluate how the monetary authorities' preferences would influence the optimal parameters in the IFB1 rule. We then considered several alternative variants of the model in order to see how specific modeling assumptions influence the optimal calibration of the parameters in the reaction function. In each case the hypothetical path of the economy was simulated 64 times over a horizon of 100 quarters, starting in a position with the inflation rate at its target (specifically, 2.5) and the unemployment rate at the long-run NAIRU (specifically, 6.0), and using a common set of the 64 different random drawings of the timepaths of the various shocks that enter the model. This generated 6,400 observations for evaluating the cumulative (undiscounted) loss in each case, and provided information both on the optimal calibration of the rule (conditional on the loss function parameters, particular model variant, and the grid over which we searched), its implications for a range of performance indicators in addition to the cumulative loss, and the sensitivity of the cumulative loss to the calibration of the rule.

### **IV. SIMULATION RESULTS**

#### **A. Optimal Calibrations of IFB1 Rules**

This section describes the optimal calibrations of IFB1 rules, as defined by equations (15) and (16), under different model variants and for a range of loss-function parameters.

---

<sup>36</sup>The stochastic simulations were performed using a robust and efficient Newton-Raphson simulation algorithm that is now available in portable TROLL—for a discussion of the properties of this algorithm see Juillard and others (1998).

We first focus on the case in which the behavior of inflation expectations follows our preferred model, as specified in equation (11). Table 4 reports optimal calibrations of IFB1 rules under base-case assumptions about the shape of the Phillips curve, the degree of NAIRU uncertainty, and the length of wage-price contracts. The first seven rows consider combinations of three different settings of  $\theta$  (the loss attached to unemployment variance relative to inflation variance) and three different settings of  $\beta$  (the strength of the short-run temptation to push unemployment below the DNAIRU) when a positive loss is attached to interest rate variability ( $v=0.5$ ).<sup>37</sup> Several points may be noted.

First, even for cases in which no loss is attached to unemployment variance (top row), the optimal calibration of the IFB1 rule places a positive weight on the unemployment gap. Thus, in setting the nominal interest rate relative to a model-consistent measure of expected inflation, authorities who condition their interest rate settings on information about both inflation and unemployment can achieve a more desirable path for future inflation than authorities who ignore information about unemployment. Second, as the relative loss attached to unemployment variance increases, so do the *relative* weights on unemployment in the optimal calibrations of these rules (compare, e.g., rows 1, 2, and 5). Third, as  $\beta$  increases and the “target” unemployment rate ( $u^* - \beta$ ) declines, the optimal *relative* weight on unemployment increases (compare rows 2, 3, and 4 and rows 5, 6, and 7). We regard these results as intuitively very plausible and likely to prove fairly robust across both models and different classes of simple policy rules. It may be noted, however, that when we double the relative loss associated with interest rate variability by raising the setting of  $v$  from 0.5 to 1.0, the optimal calibration of  $w_u$  declines in all cases and rounds to 0.0 in cases with  $\theta=0$ .

A fourth result is that the optimal weights on inflation and unemployment are inversely related to the loss attached to interest rate variability; compare rows 3 and 8. The lower is the loss associated with interest rate variability, other things equal, the more aggressive are the optimal responses to unemployment gaps and deviations of inflation from target.

As a check on the sensitivity of these results to our assumption about the dynamics of inflation expectations, Table 5 reports results comparable to those in Table 4 in all respects except for the assumption about inflation expectations. In Table 5, inflation expectations are assumed to reflect the traditional forward-and-backward looking components model defined by equation (11a). It may be seen that the four points noted about Table 4 are equally evident in Table 5.

---

<sup>37</sup>The setting  $v=0.5$  corresponds to the base-case value used by Rudebusch and Svensson (1998). Note also that the setting of  $\beta$  is irrelevant when  $\theta = 0$ .

Table 4. Optimal Calibrations of IFB1 Rules 1/

(The case of a convex Phillips curve, historically-normal NAIRU uncertainty, and 12-quarter contracts.)

	Loss Function Parameters 2/			Optimal Weights 3/	
	$\theta$	$\beta$	$\nu$	$w_u$	$w_\pi$
1.	0	Irrelevant	0.5	0.2	0.8
2.	1	0	0.5	0.3	0.8
3.	1	1	0.5	0.4	0.8
4.	1	2	0.5	0.4	0.7
5.	2	0	0.5	0.4	0.7
6.	2	1	0.5	0.5	0.7
7.	2	2	0.5	0.6	0.7
8.	1	1	0	1.5	1.0

1/ Inflation expectations are described by equation (11).

2/ Loss function is  $L_t = (\pi_t - \pi^{TAR})^2 + \theta[u_t - (u_t^* - \beta)]^2 + \nu(rs_t - rs_{t-1})^2$ .

3/ Reaction function is  $rs_t - \tilde{E}_t\{E_t\pi_{t+4}|\Omega_t\} = r^* + \tilde{E}_t\{w_\pi(\pi_{t+4} - \pi^{TAR}) + w_u(\bar{u}_t - u_t)|\Omega_t\}$ .

Table 5. Optimal Calibrations of IFB1 Rules Under a Backward-and-Forward-Looking Components Model of Inflation Expectations 1/

(The case of a convex Phillips curve, historically-normal NAIRU uncertainty, and 12-quarter contracts.)

	Loss Function Parameters 2/			Optimal Calibrations 3/	
	$\theta$	$\beta$	$v$	$w_u$	$w_\pi$
1.	0	Irrelevant	0.5	0.2	0.8
2.	1	0	0.5	0.3	0.8
3.	1	1	0.5	0.4	0.8
4.	1	2	0.5	0.4	0.8
5.	2	0	0.5	0.4	0.8
6.	2	1	0.5	0.5	0.7
7.	2	2	0.5	0.6	0.7
8.	1	1	0	1.8	1.1

1/ Inflation expectations are described by equation (11a).

2/ Loss function is  $L_t = (\pi_t - \pi^{TAR})^2 + \theta[u_t - (u_t^* - \beta)]^2 + v(rs_t - rs_{t-1})^2$ .

3/ Reaction function is  $rs_t - \tilde{E}_t\{E_t\pi_{t+4}|\Omega_t\} = r^* + \tilde{E}_t\{w_\pi(\pi_{t+4} - \pi^{TAR}) + w_u(\bar{u}_t - u_t)|\Omega_t\}$ .

Table 6 characterizes the sensitivity of the optimal calibrations to the degree of NAIRU uncertainty and alternative assumptions about the model. Each of the five panels corresponds to a particular choice of the loss function parameters. Within each panel, the top row corresponds to the base-case results reported in Table 4.

We noted above that the optimal weight on the unemployment gap depends on the relative loss attached to unemployment variance. Here we add an intuitive result on how the optimal reaction function parameters vary with the degree of NAIRU uncertainty. In particular, rows 2 and 3 report results for the cases in which the magnitudes of NAIRU uncertainty are, respectively, half as much and twice as great as the base-case level. These results confirm that the optimal *relative* weight on the unemployment gap is inversely related to the degree of NAIRU uncertainty.

Another finding is that the optimal weights increase (implying more aggressive policy reactions) as the length of standard wage and price contracts shortens and thereby reduces the degree of inertia in the backward-looking component of inflation expectations. This can be seen by comparing rows 1 and 5 or rows 4 and 6. Consistently, for cases in which market participants are assumed to have completely backward-looking expectations, the optimal calibrations of the IFB1 rule involve weaker policy responses to unemployment gaps than for analogous cases with partially forward-looking expectations; compare rows 1 and 7.

We expected to also find that, other things equal, the optimal policy reaction is more aggressive in model variants with convex Phillips curves than in model variants with linear approximations to the same Phillips curves.<sup>38</sup> This is simply because the greater the degree of Phillips-curve convexity, the higher is the inflation and/or unemployment variance that tends to be generated by shocks to the economy, other things equal. The results in Table 6 support these priors insofar as the optimal weights on unemployment are higher in the cases with convex Phillips curves than they are in the cases with linear Phillips curves (compare rows 1 and 4 and rows 5 and 6). In reflecting on this result, it may be noted, in addition, that the differences in weights across the two models is moderated by two factors: first, the forward-looking IFB1 rules, which implicitly take account of the nonlinearities in the model, are highly successful in avoiding large boom and bust cycles; and second, our convex Phillips curves are approximately linear in the region of the NAIRU.

Table 7 reports a number of relevant performance characteristics associated with a selected subset of the optimally-calibrated IFB1 rules shown in Table 6. The performance characteristics include the cumulative undiscounted losses (over 6,400 simulated observations, with a scale factor); the average rates of inflation and unemployment; the standard deviations of the inflation and unemployment rates; and the standard deviations of both the level of, and the change in, the nominal interest rate (the policy instrument). Several points may be noted.

---

<sup>38</sup>Recall the discussion in Section III.A on how the linear Phillips curves are calibrated.



Table 6. Sensitivity of Optimal Calibrations of IFB1Rules to Different Assumptions

Model Characteristics				Optimal Calibrations 5/	
Phillips Curve 1/	Inflation Expectations 2/	NAIRU Uncertainty 3/	Contract Length 4/	$w_u$	$w_\pi$
1. Base Case: $(\theta, \beta, v) = (1, 1, 0.5)$					
Convex	F	Normal	12	0.4	0.8
Convex	F	Low	12	0.8	0.8
Convex	F	High	12	0.0	0.7
Linear	F	Normal	12	0.0	0.8
Convex	F	Normal	4	1.1	0.9
Linear	F	Normal	4	0.7	0.9
Convex	B	Normal	12	0.0	0.9
2. No Unemployment Loss and Preference for u = DNAIRU: $(\theta, \beta, v) = (0, 0, 0.5)$					
Convex	F	Normal	12	0.2	0.8
Convex	F	Low	12	0.6	0.9
Convex	F	High	12	0.0	0.8
Linear	F	Normal	12	0.0	0.9
Convex	F	Normal	4	0.9	0.9
Linear	F	Normal	4	0.6	0.9
Convex	B	Normal	12	0.2	1.0
3. High Unemployment Loss and Preference for u = DNAIRU: $(\theta, \beta, v) = (2, 0, 0.5)$					
Convex	F	Normal	12	0.4	0.7
Convex	F	Low	12	0.8	0.8
Convex	F	High	12	0.1	0.7
Linear	F	Normal	12	0.1	0.8
Convex	F	Normal	4	0.9	0.8
Linear	F	Normal	4	0.7	0.8
Convex	B	Normal	12	0.0	0.9

Table 6 (concluded). Sensitivity of Optimal Calibrations of IFBI Rules to Different Assumptions

Model Characteristics				Optimal Calibrations 5/	
Phillips Curve 1/	Inflation Expectations 2/	NAIRU Uncertainty 3/	Contract Length 4/	$w_u$	$w_\pi$
4. High Unemployment Loss and Strong Preference for $u < D$ NAIRU: $(\theta, \beta, \nu) = (2, 2, 0.5)$					
Convex	F	Normal	12	0.6	0.7
Convex	F	Low	12	1.0	0.7
Convex	F	High	12	0.1	0.6
Linear	F	Normal	12	0.1	0.8
Convex	F	Normal	4	1.1	0.8
Linear	F	Normal	4	0.7	0.8
Convex	B	Normal	12	0.2	0.8
5. No Loss on Interest Rate Volatility: $(\theta, \beta, \nu) = (1, 1, 0)$					
Convex	F	Normal	12	1.5	1.0

1/ The convex Phillips curves are described by equations (1) and (2). The linear Phillips curves are described by equations (1a) and (2a).

2/ F denotes partially forward-looking expectations as characterized by equation (11). B denotes completely backward-looking expectations defined by equation (11b).

3/ Normal level of NAIRU uncertainty reflects sample period variances of  $\epsilon^\pi$ ,  $\epsilon^{\pi^*}$ , and  $\epsilon^{u^*}$  in equations (1), (2), and (6). Low (high) NAIRU uncertainty corresponds to variances of  $\epsilon^\pi$ ,  $\epsilon^{\pi^*}$ , and  $\epsilon^{u^*}$  that are half (twice) as large as in the normal case.

4/ Length of standard price and wage contracts measured in calendar quarters; value of  $N$  in equation (3).

5/ Reaction function is  $r_{S_t} - \tilde{E}_t\{E_t\pi_{t+4}|\Omega_t\} = r^* + \tilde{E}_t\{w_\pi(\pi_{t+4} - \pi^{TAR}) + w_u(\bar{u}_t - u_t)|\Omega_t\}$ .

Table 7. Performance Characteristics of Optimally-Calibrated IFB1 Rules 1/

Model Characteristics				Performance Characteristics						
Phillips Curve	Inflation Expectations	NAIRU Uncertainty	Contract Length	Cumulative Loss	Inflation Outcomes		Unemployment Outcomes		Standard Deviation of Nominal Interest Rate	Standard Deviation of Change in Nominal Interest Rate
					Mean	Standard Deviation	Mean	Standard Deviation		
$(\theta, \beta, v) = (1, 1, 0.5)$										
Convex	F	Normal	12	6.90	2.65	2.17	6.03	0.54	2.17	1.06
Convex	F	Low	12	2.71	2.57	1.13	6.01	0.36	1.20	0.58
Linear	F	Normal	12	6.80	2.51	2.17	6.00	0.55	2.17	1.04
Convex	F	Normal	4	8.46	2.71	2.44	6.04	0.59	3.01	1.22
Linear	F	Normal	4	8.31	2.51	2.44	6.00	0.60	3.03	1.20
Convex	B	Normal	12	7.13	2.66	2.04	6.03	0.54	2.90	1.52
$(\theta, \beta, v) = (2, 0, 0.5)$										
Convex	F	Normal	12	6.02	2.64	2.17	6.02	0.52	2.04	1.01
Convex	B	Normal	12	6.26	2.66	2.04	6.03	0.54	2.90	1.52
$(\theta, \beta, v) = (2, 2, 0.5)$										
Convex	F	Normal	12	14.20	2.64	2.17	6.02	0.52	2.04	1.02
Convex	B	Normal	12	14.52	2.65	2.04	6.03	0.52	2.77	1.47

1/ Reaction function is  $rs_t - \tilde{E}_t\{E_t\pi_{t+4}|\Omega_t\} = r^* + \tilde{E}_t\{w_\pi(\pi_{t+4} - \pi^{TAR}) + w_u(\bar{u}_t - u_t)|\Omega_t\}$ .

First, for the linear model variants, the simple policy rules succeed in hitting the inflation target to a very close approximation (i.e., with an error averaging less than .015 percentage points over 6400 observations) and achieving an average unemployment rate equal to the long-run DNAIRU (i.e., the center of the band within which the DNAIRU randomly walks).

Second, as noted in Section II, for model variants with convex Phillips curves (and simulations that start with inflation on target and the unemployment rate at the long-run DNAIRU), it is infeasible to hit the inflation target on average in a stochastic environment without generating an average unemployment rate above the long-run DNAIRU. For these cases, the policy rule calibrations that are optimal under the quadratic loss function—which trades off deviations of inflation from its target against deviations of unemployment from an implicit target at or below the DNAIRU—result in both above-target average inflation rates and above-DNAIRU average unemployment rates.

Third, the realized means and standard deviations of inflation and unemployment are essentially independent of the loss-function parameter  $\beta$ , other things equal; compare, for example, the results in the second and third panels of Table 7. Thus, under the assumptions that the macro model is well defined and known to the authorities, that inflation expectations are either backward looking or model consistent, and that the authorities are motivated to optimize over a long horizon, there may be some basis for taking comfort in the traditional assumption that adherence to a well-performing policy rule would be fully credible. As noted earlier, however, except in completely linear models such an assumption does not imply that the prospect of achieving the announced inflation target is fully credible.

Fourth, for all cases shown in Table 7—and more generally, for most cases with historically normal (or low) levels of NAIRU uncertainty—the optimal calibrations of the IFB1 rules succeed in keeping average inflation within .25 percentage points of target and average unemployment within .05 percentage points of the long-run DNAIRU. Moreover, the associated standard deviations of inflation, unemployment, and the nominal interest rate might also be regarded as attractively small. This highly attractive performance is an interesting result in light of Schaling's (1998) argument that for models with convex Phillips curves, optimal policy reaction functions are nonlinear in the observable state variables. In particular, our simulation results provide the additional perspective that when policymakers are assumed to have complete information about the structure of the model, linear IFB rules that embody model-consistent measures of real interest rates and thereby implicitly take account of the nonlinearities in the model may provide acceptably-close approximations to "optimal rules." By contrast, as is emphasized in the next section, complete information about the structure of the model would have little value if policymakers were committed to follow a conventional Taylor rule.

A fifth result, evident from simulation results not reported in this paper, is that the performances of IFB1 rules are slightly dominated by the performances of IFB2 rules, which we have described verbally in Section III.A above and characterize formally by equation (20)

in Section IV.B below. But the shift from a Taylor rule to an IFB1 rule achieves a much larger gain in macroeconomic stability than the shift from an IFB1 rule to an IFB2 rule.

While the results reported in this section suggest that simple rules are capable of performing very well as monetary policy guidelines when macro models are well defined, such a conclusion does not necessarily extend to the model-uncertain environments in which monetary policy is actually conducted. At best, such inference would rest on a presumption that the monetary authorities can identify a rule calibration that comes close to achieving the performance of the optimal calibration. Moreover, a point that we find even more alarming is that economists seem to have a poor track record at identifying rules with good stabilizing properties. In particular, as the next section illustrates, a number of rules that economists have chosen to advocate on the basis of their performances in linear models of the U.S. economy perform very poorly in models with moderate nonlinearities, particularly when policymakers tend to make serially-correlated errors in estimating the NAIRU.

### **B. Perspectives on Several of the Simple Rules Proposed in the Literature**

This section focuses on the stabilizing properties of several types of simple rules that have been proposed in the literature, and compares their performances with the performance of an optimally-calibrated IFB1 rule. The additional rules on which we focus are: (i) the conventional Taylor rule advocated by Taylor (1993, 1998a), (ii) an inflation-forecast-based rule with interest rate smoothing, as estimated for the United States by Clarida, Gali, and Gertler (1998), (iii) an optimally-calibrated IFB2 rule, as analyzed previously in Isard and Laxton (1998) and Isard, Laxton, and Eliasson (1998), and (iv) a first-difference rule for the interest rate, as proposed by Levin, Wieland, and Williams (1998).

One of the key findings of our simulation analysis, supported by the stability analysis presented in Appendix II, is that in a world in which inflation expectations have a forward-looking model-consistent component, monetary policy guided by a myopic rule that incorporates a backward-looking measure of the real interest rate, such as a conventional Taylor rule, can be destabilizing in our moderately nonlinear model. A second key finding is that rules with high degrees of interest rate smoothing, such as the forward-looking rule estimated by Clarida, Gali, and Gertler (CGG) and the first-difference rule proposed by Levin, Wieland, and Williams (LWW), can also lead to instability in our moderately nonlinear model.

The specification of conventional Taylor rules has been described by equation (14) above. The CGG rules that we consider can be written as:

$$rs_t = (1-\rho)c + \rho rs_{t-1} + (1-\rho)\tilde{E}_t\{\beta\pi_{t+4} + \gamma(\bar{u}_t - u_t) \mid \Omega_t\} \quad (19)$$

where  $(\rho, \beta, \gamma)$  are the parameters to be chosen.<sup>39</sup> Note that in CGG rules the interest rate is adjusted in reaction to an inflation forecast rather than a backward-looking measure of inflationary pressures as embodied in the conventional Taylor rule.<sup>40</sup>

The IFB2 rule that we consider here has the following form:

$$\tilde{r}_t = r^* + \tilde{E}_t \{ w_\pi (\pi_{t+3} - \pi^{TAR}) + w_u (\bar{u}_t - u_t) | \Omega_t \} \quad (20)$$

where

$$\tilde{r}_t = rs_t - \tilde{E}_t \{ E_t \pi_{t+4} | \Omega_t \} \quad (21)$$

As in the IFB1 rule,  $\tilde{r}_t$  is the monetary authority's ex ante measure of the real interest rate on which aggregate demand and unemployment depend;  $E_t \pi_{t+4}$  denotes the public's expectations at time  $t$  of the inflation rate over the year ahead; and  $\tilde{E}_t \{ | \Omega_t \}$  denotes a model-consistent forecast at time  $t$  based on the authorities' information set  $\Omega_t$ , which includes information about the model along with the observed values of the inflation rate through quarter  $t$  and all

---

<sup>39</sup>Clarida, Gali, and Gertler (1998) consider specifications based, alternatively, on output gaps and unemployment gaps. The CGG rule is derived by combining the following two equations

$$\begin{aligned} rs_t^* &= r^* + \tilde{E}_t \{ (\beta - 1)(\pi_{t+1} - \pi^{TAR}) + \gamma(\bar{u}_t - u_t) | \Omega_t \} \\ rs_t &= (1 - \rho)rs_t^* + \rho rs_{t-1} \end{aligned}$$

where  $rs_t^*$  represents a target nominal interest rate. Thus, the constant term in equation (19) can be decomposed into  $c = r^* - (\beta - 1)\pi^{TAR}$ .

<sup>40</sup>Clarida, Gali, and Gertler (1998) also suggest that it is more consistent with actual Fed behavior for the interest rate reaction function to depend upon the one or two-quarter ahead forecast of the unemployment gap rather than the contemporaneous unemployment gap. It would be interesting to further explore the stabilizing properties of CGG rules to see if they change significantly when the rule is based on forecasts of the unemployment gap rather than contemporaneous measures.

other economic variables through quarter  $t-1$ .<sup>41</sup> However, in the IFB2 rule the real interest rate is adjusted in response to a forecast of the annualized inflation rate three quarters ahead rather than the contemporaneous year-on-year inflation measure that is used in the conventional Taylor rule and IFB1 rule. While the assumption of a three-quarter-ahead inflation forecast is somewhat arbitrary, some experimentation suggested that such a specification was capable of producing reasonable macroeconomic stability, and with offsetting unanticipated shocks to inflation within a horizon of two or three years.<sup>42</sup> Nevertheless, the issue may deserve more consideration in future work.<sup>43</sup>

There are three potentially important differences between the CGG rule and the IFB2 rule. First, the IFB2 rule embodies information about the real interest rate on which aggregate demand depends. Consequently, with prespecified reaction-function parameters, the behavior of nominal interest rates under the IFB2 rule appears to be more sensitive to assumptions about the manner in which inflation expectations are formed by the private sector. Second, the CGG rule employs a fourth-quarter-ahead forecast of the year-on-year inflation rate while the IFB2 rule employs a third-quarter-ahead forecast of quarterly inflation (measured at an annual rate). Third, the IFB2 rule does not incorporate interest rate smoothing.

CGG provide estimates of the “Volcker-Greenspan” calibrations that best fit the post-October 1979 and post-1982 data for the United States; these estimates, in rounded numbers, are  $(\rho, \beta, \gamma) = (0.7, 2.0, 0.1)$  and  $(\rho, \beta, \gamma) = (0.8, 1.6, 0.9)$ , respectively. CGG implicitly suggest that the “Volcker-Greenspan” calibrations have attractive stabilizing properties, and that a CGG rule with these calibrations could have avoided the stagflation that occurred in the late 1960s and 1970s. We test this conjecture by using our stochastic simulation framework to

---

<sup>41</sup>It may be noted here that the literature has distinguished between IFB rules that embody rule-consistent inflation forecasts, as does our IFB2 rule, and IFB rules defined in terms of constant-interest-rate inflation forecasts.

<sup>42</sup>We did not undertake an extensive search for the optimal inflation forecast horizon but note that three quarters is roughly half the time that is generally believed to be required for interest rates to have their full effects on the economy. By comparison, Clarida, Gali, and Gertler (1998) use a four-quarter-ahead inflation forecast, while Haldane and Batini (1998) and Rudebusch and Svensson (1998) explore the performances of IFB rules with a range of forecast horizons.

<sup>43</sup>Svensson (1999) suggests that a two-year horizon might be preferable on conceptual grounds, reflecting his notion (perhaps inferred from models with substantial inflation inertia) that inflation forecasts at shorter horizons have significant predetermined components. This suggestion seems to reflect a preference for rules that (approximately) correspond to the first-order conditions from policy optimization problems, along with the notion that such first-order conditions boil down to simpler expressions of the relationship between the interest rate and an inflation forecast when the inflation forecast is not largely predetermined.

explore how well their rule would work in our model under the Volcker-Greenspan calibrations and a historically-normal degree of ex ante NAIRU uncertainty.

The final rule that we consider is a first-difference rule for the nominal interest rate, as explored by Levin, Wieland, and Williams (1998). This LWW rule can be written as:

$$rs_t - rs_{t-1} = w_\pi \sum_{i=0}^{N-1} (\pi_{t-i} - \pi^{TAR})/N + w_u(\bar{u}_t - u_t) \quad (22)$$

where we consider both  $N=12$  and  $N=4$ .

Table 8 summarizes the performance characteristics of the selected policy rules in the context of our base-case model and base-case loss function parameters. Among other things, the base-case model embodies an historically normal degree of NAIRU uncertainty and recognizes that such uncertainty creates a tendency for policymakers to make serially-correlated errors in estimating the unemployment gap. In this context, both the conventional Taylor rule and the LWW first-difference rule are too myopic to satisfy the stability conditions for our moderately nonlinear model in which inflation expectations have a model-consistent component.

Some intuition for these results is provided by the following points. First, the stability conditions depend essentially on the inflationary consequences of excess demand. Second, when an economy gets into a region of significant overheating, the inflationary consequences of a marginal increase in excess demand can be significantly greater with a convex Phillips curve than with a linear Phillips curve. Third, serially-correlated errors in estimating unemployment gaps increase the likelihood of getting into states with significant excess demand. Fourth, when inflationary expectations have a forward-looking model-consistent component and policy reactions are not sufficiently forward looking, attempts to achieve a target average rate of inflation can put the economy through boom and bust cycles where the busts are more pronounced than the booms. And fifth, as an alternative and more extreme outcome, reliance on backward-looking or sluggish policy rules can leave policymakers so far behind “shifts in the curve” that they fail to provide an anchor for inflation expectations.

Appendix II provides an extended discussion of the stability properties of Taylor rules and LWW rules. It also demonstrates that LWW rules bear a close resemblance to price level targeting.

Instability was encountered in some of simulations with CGG rules. Although as Table 8 indicates, reasonably good performances are delivered, on average, for those cases of random shock drawings in which the simulations do not explode, it may be noted that the first calibration of the CGG rule failed to prevent instability in 8 out of 64 simulations, and the second in 22 of 128 simulations.



Table 8. Performance Characteristics of Selected Rules

(The case of a convex Phillips curve, historically-normal NAIRU uncertainty, and 12-quarter contracts under loss function parameterization  $(\theta, \beta, \nu) = (1, 1, 0.5)$ .)

Rule	Performance Characteristics						
	Cumulative Loss	<u>Inflation Outcomes</u>		<u>Unemployment Outcomes</u>		Standard Deviation of Nominal Interest Rate	Standard Deviation of Change in Nominal Interest Rate
		Mean	Standard Deviation	Mean	Standard Deviation		
Conventional Taylor rules	too myopic to satisfy stability conditions (see text)						
IFB1 rule 1/  $(w_u, w_\pi) = (0.4, 0.8)$	6.90	2.65	2.17	6.03	0.54	2.17	1.06
CGG rules 2/  based on cases that do not explode (see text)							
$(\rho, \beta, \gamma) = (0.8, 1.6, 0.9)$	8.13	2.90	2.33	5.99	0.53	1.74	0.30
$(\rho, \beta, \gamma) = (0.7, 2, 0.1)$	6.98	2.74	2.24	6.02	0.56	1.85	0.45
IFB2 rule 3/  $(w_u, w_\pi) = (0.0, 1.5)$	6.32	2.57	2.06	6.02	0.50	1.93	1.12
LWW first-difference rule	too myopic to satisfy stability conditions (see text)						

1/ Reaction function is  $rs_t - \tilde{E}_t\{E_t\pi_{t+4}|\Omega_t\} = r^* + \tilde{E}_t\{w_\pi(\pi_{t+4} - \pi^{TAR}) + w_u(\bar{u}_t - u_t)|\Omega_t\}$ .

2/ Reaction function is  $rs_t = (1-\rho)c + \rho rs_{t-1} + (1-\rho)\tilde{E}_t\{\beta\pi_{t+4} + \gamma(\bar{u}_{t+1} - u_{t+1}) | \Omega_t\}$ .

3/ Reaction function is  $rs_t - \tilde{E}_t\{E_t\pi_{t+4}|\Omega_t\} = r^* + \tilde{E}_t\{w_\pi(\pi_{t+3} - \pi^{TAR}) + w_u(\bar{u}_t - u_t)|\Omega_t\}$ .

These results for CGG rules, along with the analysis of LWW rules, emphasize that it would be very dangerous to constrain policymakers to always exercise gradualism in adjusting interest rates. It may be quite appropriate to associate losses with interest rate variability and to take these losses into consideration when calibrating the strength of policy reactions to estimated unemployment gaps and deviations of inflation from target. But to adhere mechanically to either an LWW rule or a CGG rule with a high degree of interest rate smoothing would be a recipe for disaster.<sup>44</sup>

By contrast, the optimally-calibrated IFB1 and IFB2 rules succeed in preventing instability in all 64 simulations and deliver relatively attractive outcomes for the means and variances of inflation and unemployment. By incorporating model-consistent measures of inflation expectations and the real interest rate, which implicitly takes account of the nonlinearities in macroeconomic behavior, these rules are highly successful in avoiding boom and bust cycles within our well-defined macro model, and in delivering average rates of inflation and unemployment respectively close to the inflation target and the long-run DNARU.

## V. CONCLUSIONS

The various simulation results reported in this paper illustrate the prospective dangers of adopting a simple policy rule as an automatic pilot for monetary policy. These dangers are reflected, among other places, in the observation that economists seem to have a poor track record in identifying rules with good stabilizing properties. In particular, several prominent types of simple monetary policy rules—rules that have been shown to perform well in linear models of the U.S. economy and that have accordingly received prominent attention in the economics literature—are too myopic to deliver macroeconomic stability in our moderately nonlinear model in which inflation expectations have a forward-looking model-consistent component.

While recognizing that simple policy rules should not be followed mechanically, many economists argue that the adoption of simple rules as guidelines can be helpful for communication, accountability, and credibility. This is reflected, for example, in the widespread attention that inflation targeting strategies have received during the 1990s. Moreover, even for advocates of discretionary monetary policies, simulation experiments and analytic studies of the properties of simple rules can provide valuable insights. In this context, two key messages of this paper are that it is very important for policymakers to calibrate their nominal interest rate adjustments on the basis of forward-looking measures of real interest rates, and that it is also important to be aware that excessive caution (interest rate smoothing)

---

<sup>44</sup>In a separate forthcoming paper we argue that the high estimates of  $\rho$  (and associated high  $t$ -statistics) that are obtained when CGG rules are fitted to historical data are probably reflections of specification error.

in policy reactions can be much more costly in a nonlinear world than is apparent from simulation experiments with linear models. These messages are especially relevant for a world characterized by NAIRU uncertainty in which policymakers tend to make serially-correlated errors in estimating the unemployment gap. Our analysis suggests that relying heavily on guidance from a conventional Taylor rule would lead to a repeat of the policy errors of the 1970s, independently of how the Taylor rule was calibrated.

A third message of the paper is that the propensity of economists to analyze the properties of simple policy rules within the confines of linear models is difficult to defend as a research strategy. In linear models, bad policy rules affect the variances of unemployment and inflation, but not the means: bad rules do not have first-order welfare consequences. By contrast, nonlinear models recognize that policy rules that allow economies to overheat significantly can leave policy “behind shifts in the curve,” with consequences that are much more dire than simply increasing the variances of unemployment and inflation. While the nonlinearity on which this paper has focused is a moderate one associated with convex Phillips curves that are highly linear in the region of the DNAIRU (deterministic NAIRU), in reality policymakers also confront other possibly-important nonlinearities, such as nonlinearities in the response of inflation expectations (policy credibility) to the authorities’ track record in hitting announced inflation targets (including asymmetry in the speeds with which credibility can be lost and regained), asymmetric hysteresis in the dynamics of unemployment, and floors on nominal interest rates.

### Optimal Forecasting Rules for Bounded Random Walks

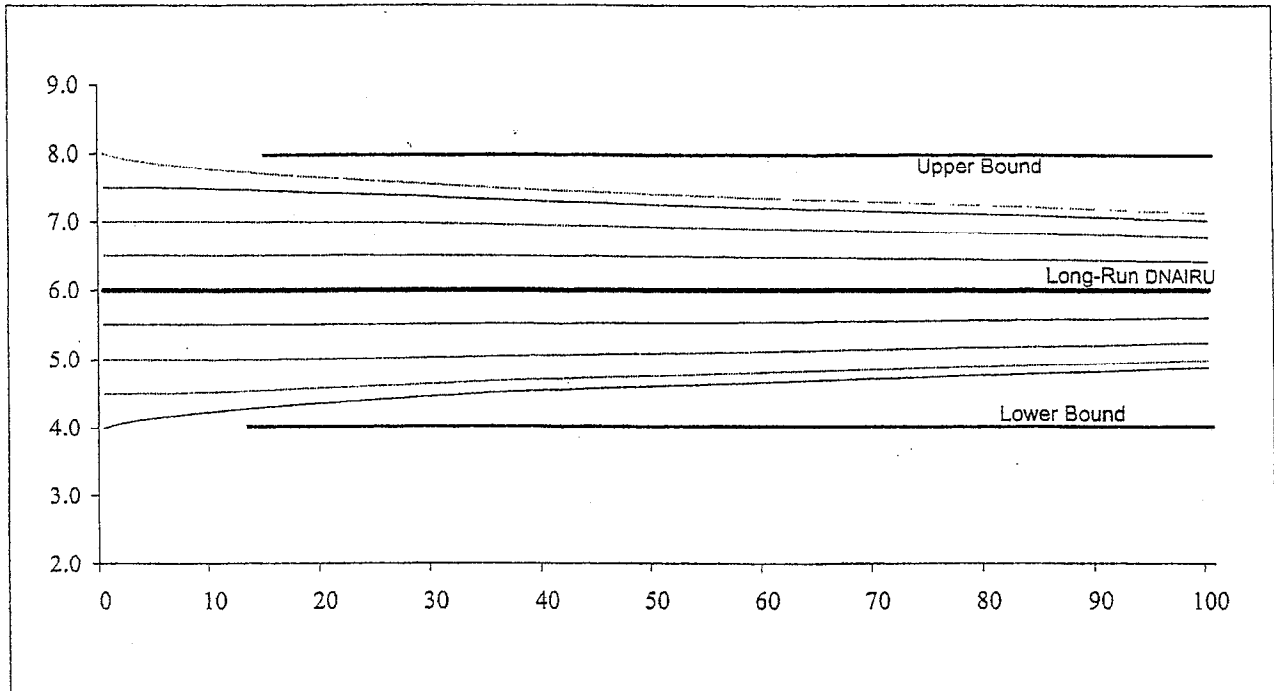
As summarized by equation (6), our simulations assume that the DNAIRU follows a bounded random walk centered at 6 percent, with a floor and ceiling of 4 percent and 8 percent, respectively. The central bank is assumed to know the process that generates the DNAIRU, including the variance of the random walk and its upper and lower bounds. In each period the central bank updates its estimates of the historical path of the DNAIRU, based on knowledge of the structural model, the ex ante distributions of the exogenous shocks, and the history of all observable variables. For purposes of implementing its IFB1 or IFB2 rules, it needs to solve its forward-looking macro model, which, among other things, requires it to forecast the timepath of the DNAIRU.

The optimal forecasts for a bounded random walk depend on the upper and lower bounds, the variance of the random walk process, and the most recently observed or estimated value of the time series. Figure A1 shows optimal forecasts over horizons of 100 periods, based on different estimates of the initial value of the DNAIRU, a standard deviation of 0.12, and the lower and upper bounds of 4 percent and 8 percent. As can be seen in the figure, the optimal forecasts revert very gradually toward the long-run DNAIRU of 6 percent. Note, however, that the expected speed of convergence is positively related to the distance between the estimated initial value of the DNAIRU and its long-run value.

In solving for the optimal forecast paths for the DNAIRU, we rely on numerical derivations. In particular, the paths shown in Figure A1 were constructed by averaging over 500 outcomes for each initial estimated value of the DNAIRU. For purposes of conducting the stochastic simulation experiments reported in the text, we created a grid of candidate optimal forecasts, corresponding to a grid of initial values that varied between 4 percent and 8 percent in increments of .001 percent. We then assumed that the central bank's forecast, in period  $t$ , of the path of the DNAIRU from period  $t$  to  $t+100$  corresponded to the optimal path associated with its period- $t$  estimate of the DNAIRU for period  $t-1$ .

The bounded random walk has a number of advantages over other stochastic processes that might be assumed in modeling the DNAIRU. First, it allows for quasi-permanent, or highly persistent, shifts in the underlying DNAIRU. Second, it allows us to differentiate between long-run concepts like the natural rate of unemployment, which is usually presumed to be fairly stable over time, and short-run concepts like the DNAIRU, which potentially is considerably more variable as a reflection of mismatches resulting from stochastic variability in the offer curves of workers and firms. For purposes of simplification, we have based our analysis on the assumption of a constant expected long-run DNAIRU of 6 percent, but it would be relatively straightforward in principle to vary the expected long-run DNAIRU as a function of unemployment compensation schemes, demographics, and so forth.

**Figure A1: Optimal Forecast Trajectories of a Bounded Random Walk  
(Standard Deviation of DNAIRU = .12)**



**Sources:**

Each line in the figure is derived from taking the average values over 500 simulations of a bounded random walk with a standard deviation of 0.12, a lower bound of 4.0 percent and an upper bound of 8.0 percent.

### Stability Analysis of Taylor Rules and LWW Rules

Research during the last few years has suggested that conventional Taylor rules, when appropriately calibrated, have reasonably attractive stabilization properties within a range of different macro models. Along similar lines, Levin, Wieland, and Williams (LWW:1998) have recently shown that simple rules linking the change in the interest rate to the variables that enter conventional Taylor rules have desirable properties in four different macro models.

Such favorable impressions of Taylor rules and LWW rules have not gone unchallenged, however. Christiano and Gust (CG: 1999) have been quick to point out that the stabilization properties of these rules have been evaluated almost exclusively in sticky-price IS-LM models with similar structures. CG demonstrate that in a limited-participation-rate model developed by Christiano, Eichenbaum and Evans (1998), Taylor rules or LWW rules may be dangerous, especially if the rules place too high a weight on output (or unemployment) relative to inflation.

This appendix argues that Taylor rules and LWW rules are also likely to be highly destabilizing in plausible specifications of sticky-price IS-LM models. In particular we emphasize that in contrast to their favorable performances in linearized versions of such models where overheating does not have first-order welfare consequences, Taylor rules and LWW rules tend to perform very poorly in nonlinear rational-expectations models in which myopic or sluggish policy responses can fail to provide an anchor for inflation expectations.

For each of the two classes of rules, we start by reporting the Blanchard-Kahn (1980) saddle-point stability conditions for a linear forward-looking model developed by Fuhrer and Moore (FM: 1995a, 1995b).<sup>45</sup> We show that both classes of rules produce saddle-point stability over an enormous range of parameter values. We then go on to argue that the stability of these rules in linear sticky-price IS-LM models with rational expectations breaks down in models that do not presume global linearity.<sup>46</sup> In particular, we show that in our model with moderate nonlinearities, such rules can give rise to extreme instabilities in inflation expectations and would risk a repeat of the monetary policy errors of the 1970s. As a corollary, the assumption that such myopic policy rules would be fully credible in these models

---

<sup>45</sup>We chose this model because it was more easily accessible than the other models considered by Levin, Wieland, and Williams (1998). We are indebted to Jeffrey Fuhrer for taking the time to help us replicate some of his earlier results. The results reported in this appendix have been derived from the parameter estimates reported in Fuhrer and Moore (1995b).

<sup>46</sup>Our line of argument in this appendix is broadly similar to the one presented in Christiano and Gust (1999), who show not only that poorly parameterized simple rules can give rise to poor simulation properties in their particular model, but also that choosing the parameters of rules on the basis of one particular class of models can give rise to indeterminacy or explosiveness in other models.

is untenable, especially in cases where the monetary authorities place a high weight on output (or unemployment) relative to inflation.

### Conventional Taylor Rules Generalized for Interest Rate Smoothing

Figure B1 reports the combinations of parameter settings that lead to unique, explosive, and indeterminate solution paths in the Fuhrer-Moore (1995b) model under a conventional Taylor rule that has been generalized to allow for interest rate smoothing. This rule can be written as:

$$rs_t = \rho rs_{t-1} + (1-\rho)[w_\pi(\pi_4)_t + w_y(y_t)] \quad (\text{B.1})$$

where:  $rs_t$  is the nominal interest rate setting at time  $t$ ;  $\pi_4$  is the average inflation rate over the previous four quarters;  $y_t$  represents the output gap in the Fuhrer-Moore model; and  $\rho$ ,  $w_\pi$ ,  $w_y$  are parameters.<sup>47</sup> Note that the interest rate reaction function has been coded so that the parameters  $w_\pi$  and  $w_y$  represent asymptotic long-run responses of interest rates to the year-over-year inflation rate and the output gap.<sup>48</sup>

A striking feature of Figure B1 is that for a very wide range of parameter values—and independently of the speed with which monetary policy reacts to inflation and output gaps (i.e., independently of  $\rho$ )—the model has a stable and unique solution. Indeed, the stability properties of the generalized Taylor rule in this linear rational expectations IS-LM model are extremely simple. The only condition necessary for stability and uniqueness is that the long-run response of the interest rate to year-over-year inflation must be greater than one. Provided this condition is met, even a Taylor rule that reacts much more aggressively to output than to inflation will provide an anchor for inflation expectations in the Fuhrer-Moore model.

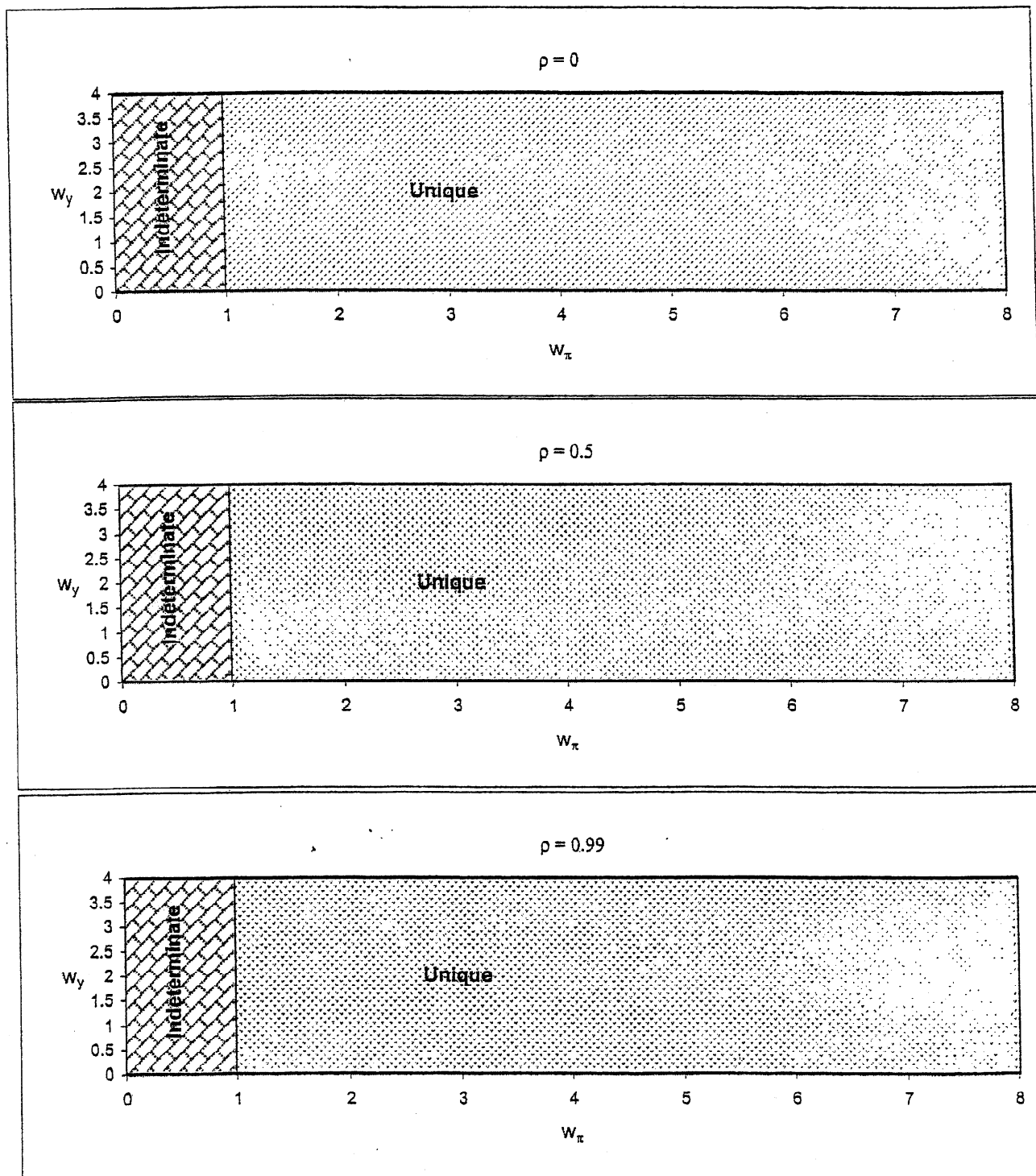
What is it that explains the “excessive stability” generated by Taylor rules in these sticky-price linear rational expectations models? What gives rise to stable macroeconomic behavior even when the monetary authorities respond in a very myopic way to inflation developments, or place an extremely high weight on real objectives relative to inflation objectives? Two assumptions appear to be critical here. The first is the assumption that the

---

<sup>47</sup>It is convenient here to follow Taylor (1993) in defining the rule in terms of the output gap rather than the unemployment gap. For notational convenience we have dropped the constant term in the equation by assuming that the equilibrium real interest rate and long-run inflation target are zero.

<sup>48</sup>For example, the long-run effects of a permanent unitary change in the output gap is equal to the short-run effect,  $(1-\rho)w_y$ , divided by  $(1-\rho)$ .

**Figure B1: Regions of Uniqueness, Explosiveness and Indeterminacy**  
 (Generalized Taylor Rule in Fuhrer and Moore (1995) Model)  
 Policy Reaction Function:  $rs_t = \rho rs_{t-1} + (1-\rho)[w_\pi(\pi_t) + w_y(y_t)]$





economy can be characterized by global linearity.<sup>49</sup> The second is the premise that no matter how myopic policy responses are in the short run, the private sector forms its expectations under the assumption that the monetary policy rule will be adhered to forever.

For the nonlinear model considered in this paper, which is cast in terms of unemployment gaps rather than output gaps, even moderately myopic policy rules like the conventional Taylor rule can result in explosive behavior if the economy is subjected to a significant degree of overheating. This reflects a combination of factors. Recall, first, that even moderate convexity in the Phillips curve implies that at some point the short-run unemployment-inflation tradeoff must worsen considerably when unemployment falls significantly below the NAIRU, and beyond this point a further marginal easing of monetary policy results mainly in inflation with only a very small incremental reduction in unemployment. Second, to the extent that policymakers tend to make serially correlated errors in estimating unemployment and output gaps, as reflected in our model, the probability of experiencing a significant degree of overheating is heightened. Third, when inflation expectations have a model-consistent component and rational agents possess information about the policy rule and the nonlinear nature of the expansionary effects of monetary policy, attempting to adhere to a conventional Taylor rule with a high weight on imprecise measures of unemployment gaps relative to a backward-looking measure of inflation could be conducive to wide swings or explosiveness in inflation expectations.

To sharpen quantitative perspectives, and to emphasize as well that in the region of small unemployment gaps our nonlinear model exhibits similar behavior to models that impose global linearity, consider the following. From equation (1), the direct effects of unemployment gaps ( $u^*-u$ ) on the annualized inflation rate in the nonlinear model are given by the functional form  $\gamma(u^*-u)/(u-\phi)$ , where we also assume  $\phi = u^* - 4$ . To simplify, define  $g = (u^*-u)$  such that the direct effects of the gap on inflation can be written as  $F(g) = \gamma g/(4-g)$ . Differentiating this last term with respect to  $g$  leads to the expression  $F'(g) = [\gamma(4-g) + \gamma g]/(4-g)^2$  and shows how the slope of the Phillips curve will depend on the initial state of excess demand conditions in the labor market. For example, if we linearize the Phillips curve at  $g = 0$ , the point where there is zero labor market tightness, and also substitute in our empirical estimate of  $\gamma = 3.2$ , we can see that the slope of the Phillips curve is 0.8. This estimate would suggest that a surprise of 0.1 percentage point in labor market tightness would have a direct effect on the contemporaneous annualized inflation rate of 0.08 percentage point, and hence would raise the year-on-year inflation rate by 0.02 percentage points. Such an order of magnitude of the

---

<sup>49</sup>Under the global linearity assumption, the estimated slope of the Phillips curve (based on post war U.S. data) suggests that unemployment gaps or output gaps have small effects on the inflation process. These small effects imply that it can be very costly, in the context of these models, to reduce inflation once high inflation expectations have become entrenched. The estimated slope also means that for given inflation expectations, the marginal effect on inflation of an increase in excess demand is small, even when the level of excess demand is high.

direct impact effect of marginal changes in labor market tightness on year-on-year inflation is consistent with numerous studies of the U.S. inflation process based on linear models that suggest that the direct impact effects on inflation of changes in unemployment or output gaps are extremely small. Indeed, such findings have led some commentators in U.S. policymaking circles to sometimes describe the in-quarter and 1 to 2-quarters-ahead year-on-year inflation rate as “predetermined,” or as effectively independent of small surprises in the degree of labor market tightness.

Although our nonlinear model encompasses this prediction in the neighborhood of zero excess demand, it also suggests that the direct impact effects of a marginal increase in labor market tightness can be much more significant when there is already substantial tightness in the labor market. To illustrate, Table B1 shows the slope of the Phillips curve, and the direct impact effects on the year-on-year inflation rate of a 0.1 percentage point change in the unemployment gap, at various initial levels of the unemployment gap. Note that the direct impact effects accelerate as the unemployment gap widens.

To appreciate how easily the inflation process can become explosive in our nonlinear model when policy follows a conventional Taylor rule, it needs to be recognized that the full effects on inflation of shocks to the unemployment gap can be much larger than the direct effects. This reflects the fact that inflation expectations have a model-consistent component and depend on the entire timepath of the unemployment gap that rational agents come to expect, given their information about the nature of the policy rule. When our Phillips curve is linearized around the NAIRU, our model satisfies the Blanchard-Kahn conditions for a Taylor rule calibrated with the weights originally suggested by Taylor (1993):  $w_{\pi} = 1.5$ ,  $w_u = 1.0$ , and  $\rho = 0$ .<sup>50</sup> However, if we linearize the Phillips curve around a point of excess demand at which the unemployment gap exceeds 1.6 percentage points, the Blanchard-Kahn conditions are no longer satisfied. In this case, the Taylor rule is sufficiently myopic in terms of responding to inflationary pressures that monetary policy fails to provide an anchor for inflation expectations and the solutions of the model become explosive.

As is evident in Figure B1, one of the striking features of the stability conditions for the Fuhrer-Moore model is that they appear to be independent of the degree of interest rate smoothing. This points to a general problem with linear models of the inflation process, which imply that slow monetary policy responses to information about future inflation developments only have second-order welfare consequences. Figure B2 reports the regions of stability for our nonlinear model in terms of both the slope of the Phillips curve and the degree of interest rate smoothing. For  $\rho = 0$ , the critical slope of the Phillips curve, about 2.2, corresponds to an unemployment rate of 1.6 percentage points below the NAIRU, as noted in Table B1. Notice

---

<sup>50</sup>Taylor (1993) suggested a weight of 0.5 on the output gap, which we translate into a weight of 1.0 on the unemployment gap under the Okun’s Law assumption that the unemployment gap varies approximately half as widely as the output gap over the business cycle.

Table B1. Slope of Phillips Curve and Impact Effects of Surprises in the Unemployment Gap

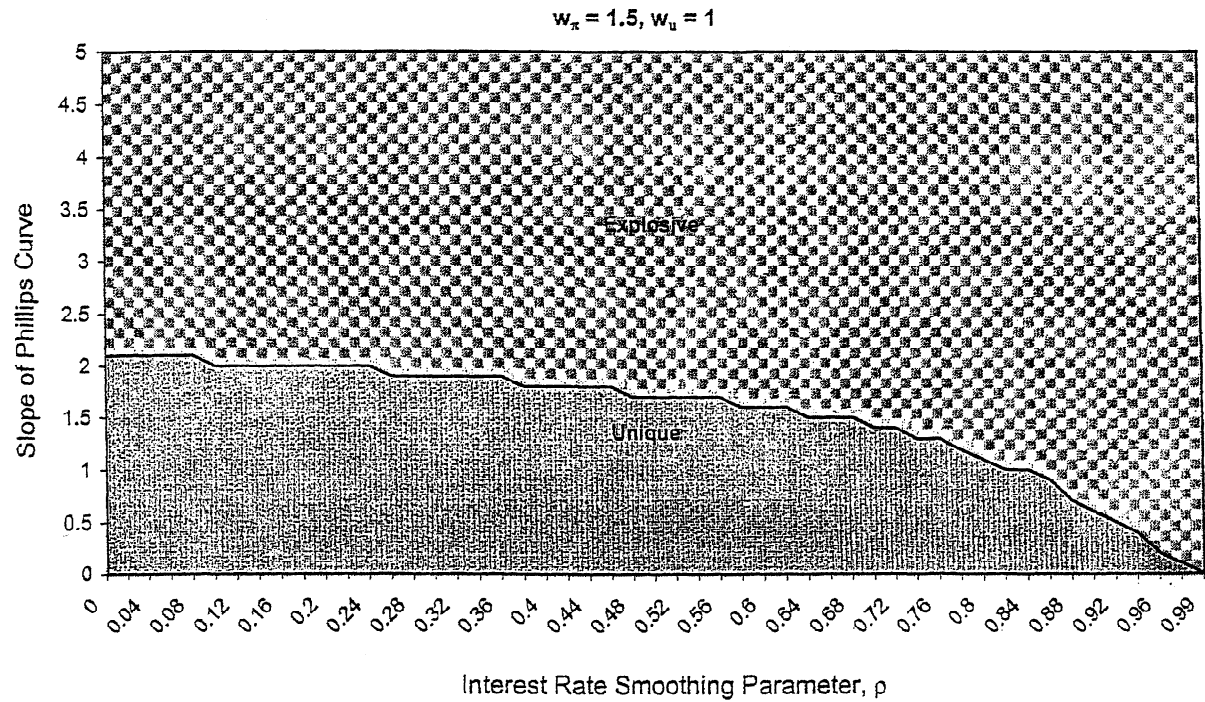
Initial Unemployment Gap	Slope of Phillips Curve	Impact Effect of Surprise in the Unemployment Gap 1/
$g = u^* - u$	$F'[g]$	$0.1F'[g]/4$
-3.00	0.26	0.01
-2.00	0.36	0.01
-1.00	0.51	0.01
0.00	0.80	0.02
+1.00	1.42	0.04
+1.6	2.22	0.06
+2.00	3.20	0.08
+3.00	12.80	0.32

1/ Response of year-on-year inflation to an unanticipated shift of 0.1 percentage point in the unemployment gap.

Figure B2: Regions of Uniqueness, Explosiveness and Indeterminacy

(Generalized Taylor Rule in the NonLinear Model)

$$\text{Policy Reaction Function: } rs_t = prs_{t-1} + (1-\rho)[w_\pi(\pi_{4t}) + w_u(\underline{g}_t)]$$



that in the nonlinear model, the region of stability shrinks further if interest rate smoothing is imposed on an already myopic policy rule.

### LWW Interest Rate Change Rules

Figure B3 reports the regions of stability for the class of interest rate change rules suggested by Levin, Wieland, and Williams (LWW: 1998). In this case, the general form of the reaction function is:

$$rs_t = rs_{t-1} + (w_\pi(\pi n_t) + w_y(y_t)) \quad (\text{B.2})$$

where  $\pi n_t$  is an  $n$ -quarter moving average of inflation measured over the previous  $n$  quarters. The top and middle panels of Figure B3 consider the two optimal rule parameterizations reported by Levin, Wieland and Williams (1998), where  $n$  is equal to 4 quarters and 12 quarters; the longer lag structure on inflation was found to be optimal in a linearized version of the FRB-U.S. model, while the shorter lag structure was found to be optimal in the other linear models that they included in their study. In this case again, even where there is extreme interest rate smoothing and monetary policy responds to very backward-looking measures of inflation, the linear model is stable for an incredibly wide range of weights on inflation and output. The lower panel of Figure B3 considers an even more extreme case of myopic reaction functions, where the reaction function now depends on a six-year moving average of past inflation. Here there is some evidence of instability in the model; but in contrast to the type of results found by Cristiano and Gust (1999), in this case explosiveness can arise from setting too low a weight on output.

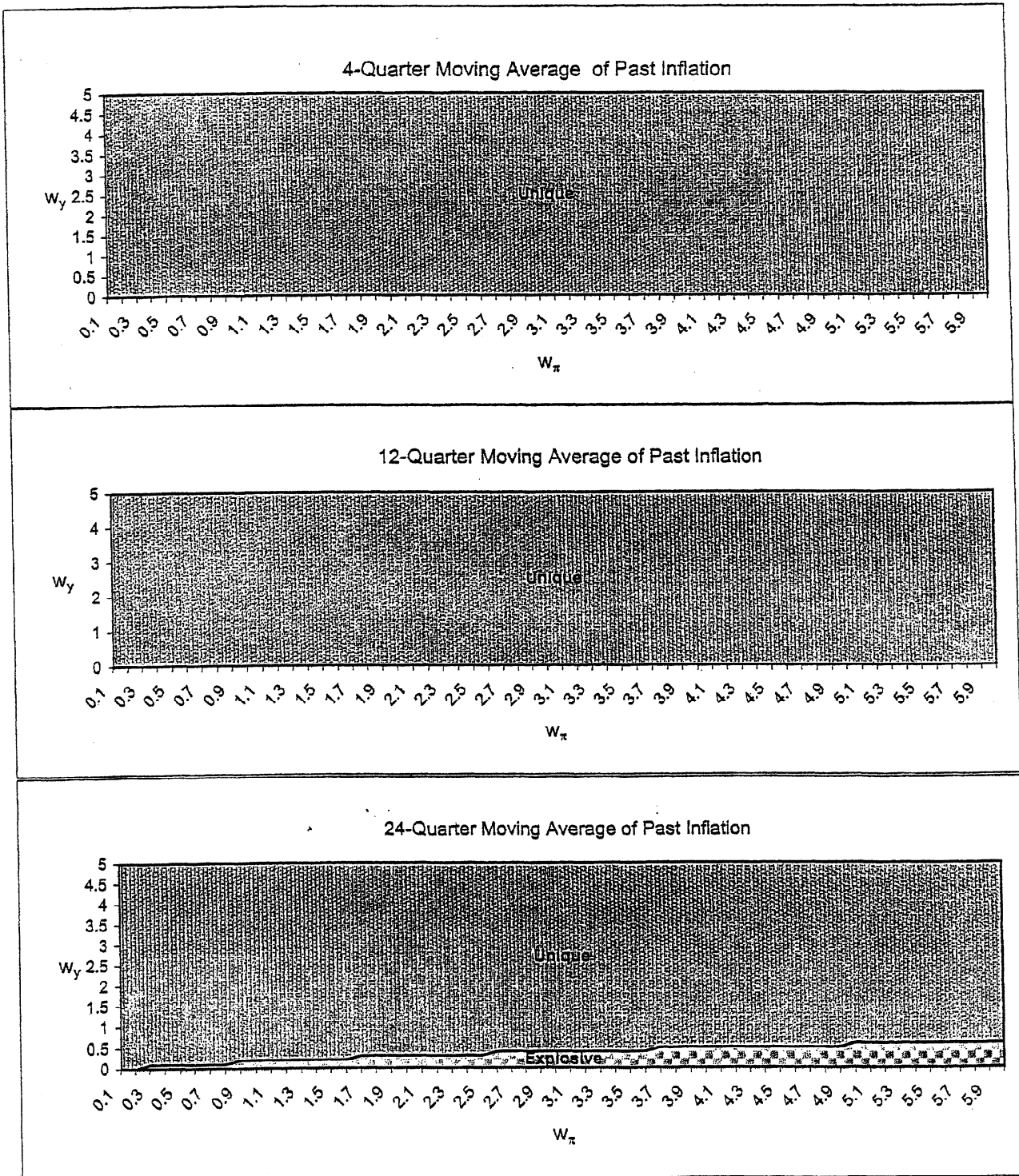
The LWW rule has extremely poor stabilizing properties in the nonlinear model developed in this paper. First, for the model that was estimated the rule is so myopic and backward-looking that it fails to provide an anchor for inflation expectations. Second, even if one recalibrates the model to reduce the effects of overheating very substantially, an optimal parameterization of the LWW rule still gives rise to significant boom and bust cycles.

It does not seem to be widely recognized that interest rate change rules such as equation B.2 are exactly equivalent to targeting a trend change in the price level when  $w_y = 0$ , and result in approximate price level targeting for small values of  $w_y$ . To see this, consider a simple case in which the interest rate change depends solely on the quarterly change in the price level ( $P$ ) expressed at an annual rate:

$$rs_t = rs_{t-1} + w_\pi \pi_t \quad (\text{B.3})$$

**Figure B3: Regions of Uniqueness, Explosiveness and Indeterminacy**  
(LWW Rule in the Fuhrer and Moore Model)

Policy Reaction Function:  $rs_t = rs_{t-1} + w_\pi(\pi 4_t) + w_y(y_t)$



where  $\pi_t = 4(P_t - P_{t-1})$ . As initial conditions, assume that inflation is on target and the real interest rate is at its equilibrium value (i.e., in period 0,  $rs_0 = rs^*$  and  $\pi_0 = \pi^* = \pi^e$ , where  $*$  denotes equilibrium).

Now assume that a demand or supply shock raises the inflation rate in period 1 to some arbitrary value  $\pi_1$ . It is interesting, and perhaps even surprising, that monetary policy governed by equation B.3 would attempt to move the price level back to the original baseline path. This will be the case, for example, if long-run neutrality holds (as LWW claim for each of the models they consider), because long-run neutrality implies that the real interest rate must return back to its initial value. But if the real interest rate returns back to control, the nominal interest rate must also eventually return back to control in some period  $T$  since, by assumption, the rule is successful in moving inflation back to its initial level of  $\pi^*$ .

If we now sum equation B.3 between periods 1 and  $T$  we obtain

$$rs_T - rs_0 = w_\pi \sum_{i=1}^T \pi_i \quad (\text{B.4})$$

So  $rs_T - rs_0 = 0$  implies

$$\sum_{i=1}^T \pi_i = 0. \quad (\text{B.5})$$

Thus, under the assumption that long-run neutrality holds, a policy rule in the form of equation B.3 essentially amounts to a price-level targeting rule, since any shock that generates positive inflation must be offset at some point by negative inflation rates. This result obviously carries over to cases in which the contemporaneous inflation rate in equation B.3 is replaced by some finite moving average lag structure on past inflation; and even when the rule is extended to include a term in the output gap, as in the general form of LWW rules described by equation B.2, it continues to bear a close resemblance to price-level targeting. Thus, it should not be surprising that such myopic LWW rules can generate extremely poor business cycle properties in models with strong inflation persistence and convexity in the Phillips curve.

## References

- Amano, Robert, Don Coletti, and Tiff Macklem, 1998, "Monetary Rules When Economic Behavior Changes," forthcoming in Reserve Bank of New Zealand Conference Volume on *Monetary Policy Under Uncertainty*.
- Backus, David, and John Driffill, 1985, "Inflation and Reputation," *American Economic Review*, Vol. 75, No. 3, pp. 530–38.
- Barro, Robert J., and David B. Gordon, 1983a, "Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy*, Vol. 91 (August), pp. 589–610.
- , 1983b, "Rules, Discretion, and Reputation in a Model of Monetary Policy," *Journal of Monetary Economics*, Vol. 12, pp. 101–121.
- Blanchard, Olivier J., and Charles M. Kahn, 1980, "The Solution of Linear Difference Models Under Rational Expectations," *Econometrica*, Vol. 48, pp. 1305–11.
- Blanchard, Olivier, and Lawrence F. Katz, 1997, "What We Know and Do Not Know About the Natural Rate of Unemployment," *Journal of Economic Perspectives*, Vol. 11, No.1 (Winter), pp. 51–72.
- Calvo, Guillermo, 1978, "On the Time Consistency of Optimal Policy in a Monetary Economy," *Econometrica*, Vol. 46, pp. 1411–28.
- Canzoneri, Matthew, 1985, "Monetary Policy Games and the Role of Private Information," *American Economic Review*, Vol. 75, pp. 1056–1428.
- Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans, 1998, "Modeling Money," NBER Working Paper 6371 (January).
- Christiano, Lawrence J., and Christopher J. Gust, 1999, "Taylor Rules in a Simple Limited Participation Model," Working Paper (January).
- Clarida, Richard, Jordi Gali, and Mark Gertler, 1998, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," NBER Working Paper No. 6442 (March).
- Clark, Peter B., Douglas Laxton, and David Rose, 1996, "Asymmetry in the U.S. Output-Inflation Nexus: Issues and Evidence," *IMF Staff Papers*, Vol. 43 (March), pp. 216–50.



- Debelle, Guy, and Douglas Laxton, 1997, "Is the Phillips Curve Really a Curve? Some Evidence for Canada, the United Kingdom, and the United States," *Staff Papers*, International Monetary Fund, Vol. 44 (June), pp. 249–82.
- Faust, Jon W., and Lars E.O. Svensson, 1998, "Credibility and Transparency: Monetary Policy with Unobservable Goals," NBER Working Paper 6452 (March).
- Federal Reserve Bank of Kansas City, 1996, *Achieving Price Stability: A Symposium* (Kansas City, Missouri).
- Flood, Robert, and Peter Isard, 1989, "Monetary Policy Strategies," *Staff Papers*, International Monetary Fund, Vol. 36, pp. 612–32.
- , 1990, "Monetary Policy Strategies—A Correction," *Staff Papers*, International Monetary Fund, Vol. 37, pp. 446–8.
- Fuhrer, Jeffrey C., and George R. Moore, 1995a, "Inflation Persistence," *The Quarterly Journal of Economics*, Vol. 109, pp. 127–59.
- , 1995b, "Monetary Policy Trade-offs and the Correlation Between Nominal Interests Rates and Real Output," *American Economic Review*, Vol. 85, pp. 219–39.
- Fuhrer, Jeffrey C., 1997, "The (Un) Importance of Forward-Looking Behavior in Price Specifications," *The Journal of Money Credit and Banking*, Vol. 29 (August), pp. 338–50.
- Galbraith, James K., 1997, "Time to Ditch the NAIRU," *The Journal of Economic Perspectives*, Vol. 11, No.1 (Winter), pp. 93–108.
- Gordon, Robert J., 1997, "The Time-Varying NAIRU and its Implications for Economic Policy," *The Journal of Economic Perspectives*, Vol. 11, No.1 (Winter), pp. 11–32.
- Haldane, Andrew G., ed., 1995, *Targeting Inflation*, Bank of England, London.
- , and Nicoletta Batini, 1998, "Forward-Looking Rules for Monetary Policy," NBER Working Paper No. 6543 (May), forthcoming in Taylor, 1999.
- Henderson, Dale, and Jinill Kim, 1999, "Exact Utilities Under Alternative Monetary Rules in a Simple Macro Model with Optimizing Agents," draft (January).
- Isard, Peter, and Douglas Laxton, 1998, "Monetary Policy with NAIRU Uncertainty and Endogenous Credibility: Perspectives on Policy Rules and the Gains from Experimentation and Transparency," forthcoming in Reserve Bank of New Zealand Conference Volume on *Monetary Policy Under Uncertainty*.

- , and Ann-Charlotte Eliasson, 1998, "Inflation Targeting with NAIRU Uncertainty and Endogenous Policy Credibility," draft (September).
- Juillard, Michel, Douglas Laxton, Peter McAdam, and Hope Pioro, 1998, "An Algorithm Competition: First-Order Iterations Versus Newton-Based Techniques," forthcoming, *Journal of Economic Dynamics and Control*, Vol. 22, pp. 1291–1318.
- Kuttner, Kenneth N., 1992, "Monetary Policy With Uncertain Estimates of Potential Output," Reserve Bank of Chicago, *Economic Perspectives* (January–February), pp. 2–15.
- , 1994, "Estimating Potential Output as a Latent Variable," Research Department, Federal Reserve Bank of Chicago, *Journal of Business and Economic Statistics*, Vol. 12, No. 3, pp. 55–79.
- Kydland, Finn E., and Edward C. Prescott, 1977, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, Vol. 85, pp. 473–92.
- Laxton, Douglas, David Rose, and Demosthenes Tambakis, 1999, "The U.S. Phillips Curve: The Case for Asymmetry," forthcoming, *Journal of Economic Dynamics and Control*.
- , Guy Meredith, and David Rose, 1995, "Asymmetric Effects of Economic Activity on Inflation: Evidence and Policy Implications," *Staff Papers*, International Monetary Fund, Vol. 42, No. 2 (June), pp. 344–74.
- Leiderman, Leonardo, and Lars E.O. Svensson, eds., 1995, *Inflation Targets*, Centre of Economic Policy Research, London.
- Levin, Andrew, Volker Wieland, and John Williams, 1998, "Robustness of Simple Monetary Policy Rules Under Model Uncertainty," forthcoming in Taylor, 1999.
- Lohmann, Susanne, 1990, "Monetary Policy Strategies—A Correction: Comment on Flood and Isard," *Staff Papers*, International Monetary Fund, Vol. 37 (June), pp. 440–45.
- Lowe, Phillip, ed., 1997, *Monetary Policy and Inflation Targeting* (Sydney: Reserve Bank of Australia).
- McCallum, Bennett T., 1988, "Robustness Properties of a Rule for Monetary Policy," *Carnegie-Rochester Series on Public Policy*, Vol. 29, pp. 173–203.
- , and Edward Nelson, 1998a, "Performance of Operational Policy Rules in an Estimated Semi-Classical Structural Model," forthcoming in Taylor, 1999.

- , and Edward Nelson, 1998b, “Nominal Income Targeting in an Open-Economy Optimizing Model,” prepared for Sveriges Riksbank-IIES Conference on Monetary Policy Rules, Stockholm (June 12–13).
- Orphanides, Athanasios, 1998, “Monetary Policy Evaluation With Noisy Information,” Working Paper, Federal Reserve Board (October).
- Persson, Torsten, and Guido Tabellini, 1989, “Macroeconomic Policy Credibility and Politics” (unpublished; Stockholm and Los Angeles: Institute for International Economic Studies and California State University, Los Angeles, April).
- Rogerson, Richard, 1997, “Theory Ahead of Language in the Economics of Unemployment,” *The Journal of Economic Perspectives*, Vol 11, No.1 (Winter), pp. 73–92.
- Rogoff, Kenneth, 1985, “The Optimal Degree of Commitment to an International Monetary Target,” *Quarterly Journal of Economics*, Vol. 100, pp. 1169–89.
- Rudebusch, Glenn, and Lars Svensson, 1998, “Policy Rules for Inflation Targeting,” NBER Working Paper 6512 (April), forthcoming in Taylor, 1999.
- Schaling, Eric, 1998, “The Nonlinear Phillips Curve and Inflation Forecast Targeting—Symmetric versus Asymmetric Monetary Policy Rules,” Working Paper (July).
- Smets, Frank, 1998, “Output Gap Uncertainty: Does It Matter for the Taylor Rule?,” forthcoming in Reserve Bank of New Zealand conference volume on *Monetary Policy Under Uncertainty*.
- Staiger, Douglas, James H. Stock, and Mark W. Watson, 1997, “The NAIRU, Unemployment and Monetary Policy,” *Journal of Economic Perspectives*, Vol. 11, No. 1 (Winter), pp. 33–49.
- Stiglitz, Joseph., 1997, “Reflections on the Natural Rate Hypothesis,” *Journal of Economic Perspectives*, Vol. 11, No. 1, pp. 3–10.
- Svensson, Lars E.O., 1998, “Inflation Targeting as a Monetary Policy Rule,” NBER Working Paper No. 6790; forthcoming in Taylor, 1999.
- , 1999, “Monetary Policy Issues for the Eurosystem,” forthcoming in *Carnegie-Rochester Conference Series on Public Policy*.
- Taylor, John B, 1993, “Discretion Versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy*, Vol. 39 (December), pp. 195–214.

- , 1998a, “A Historical Analysis of Monetary Policy Rules,” NBER Working Paper No. 6768 (October); forthcoming in Taylor, 1999.
- , 1998b, “The Robustness and Efficiency of Monetary Policy Rules as Guidelines for Interest Rate Setting by the European Central Bank,” prepared for Sveriges Riksbank-IIES Conference on Monetary Policy Rules, Stockholm (June 12–13).
- , 1999, *Monetary Policy Rules* (Chicago: University of Chicago Press), forthcoming.
- Wieland, Volker, 1998, “Monetary Policy and Uncertainty about the Natural Unemployment Rate,” Finance and Economics Discussion Paper No. 22, Federal Reserve Board.
- Williams, John C., 1999, “Simple Rules for Monetary Policy,” draft (February).