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**Increasing Dependency Ratios, Pensions, and Tax Smoothing<sup>1</sup>**

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**Abstract**

The implication of increasing dependency ratios for pay-as-you-go, defined-benefit pension programs are examined. Modifications aimed at smoothing contributions while maintaining benefits intact are analyzed for both open and closed economies.

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## SUMMARY

A pay-as-you-go, defined-benefit (PAYG-DB) pension program has a recognized advantage over a fully funded, defined-contribution (FF-DC) pension program in the early years of introduction of the program. Under the PAYG-DB program, it is possible to let those who retire receive pension benefits right away because funds are available through the contributions of those who work. In fact, for a while the system is likely to accumulate funds because at the beginning the contributions will exceed the payments. This is impossible under FF-DC programs because, after their introduction, the funds accumulated in the individual accounts are very small. This advantage is reversed when a pension program is abruptly terminated, or in some more realistic situations that are similar to terminating a program. These situations occur, for instance, when population growth rates decline or when people begin to live longer.

A remedy often suggested to deal with the above-mentioned situations in PAYG-DB programs is to accumulate assets large enough for them to be able to act as “shock absorbers” to help smooth the contributions over time and generations, without having to change the level of benefits. This option, however, may have its own shortcomings. The paper discusses these shortcomings under different assumptions.

## I. INTRODUCTION

In recent years, pension programs, especially those publicly financed and administered, have become the focus of attention of policymakers, economists, and the public at large. The most common programs are of the pay-as-you-go, defined-benefit (PAYG-DB) type.<sup>1</sup> These programs have attracted public attention in recent years partly because several of them are running deficits or are actuarially imbalanced. For some of them, the present values of their future liabilities are enormous.<sup>2</sup> The tax contributions to the programs are broadly related to earning. However, the pensions are paid at amounts that are only partially and loosely related to earnings and to contributions. Because of these programs' frequent redistributive feature, the contributions are viewed by many contributors as payroll taxes rather than as forced or voluntary savings.<sup>3</sup>

The current deficits and the actuarial imbalances of the public pension programs are expected to be aggravated by, or are the result of, aging population not anticipated when the programs were created. This trend increases the dependency ratio (i.e., the number of retirees per worker) and with it both the deficits and the actuarial imbalances. Attempts to financially stabilize the pension programs can be aimed at raising the contribution rates (i.e., the payroll taxes) and/or at cutting the benefits to the retirees either by increasing the retirement age or by reducing the size of the pensions, that is, the replacement ratio. The latter option is often politically difficult and some may consider it even unfair. It could be argued that it is unfair because the retirees "honestly earned" their rights to receive the defined-pension benefits by paying their dues (payroll taxes) when they were working. That is, in a way, the government would be reneging on an explicit contract. A contribution (tax) increase is also unpopular and, additionally, it generates deadweight losses and other problems. The deadweight loss is traditionally associated with economic (Pareto) inefficiency in a competitive environment, as measured by the familiar Harberger (1964) triangle.<sup>4</sup> The impact of higher taxes may be larger in noncompetitive environments where high payroll taxes may raise the cost of labor and lead to high rates of unemployment as it may have happened in European Union

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<sup>1</sup>See Mackenzie, Gerson, and Cuevas (1997, Table 1).

<sup>2</sup>See, inter alia, Chand and Jaeger (1996).

<sup>3</sup>Perhaps, this is why Mackenzie, Gerson, and Cuevas (1997) who surveyed the literature concluded "...it is not possible to generalize across countries about the impact of the public system on saving" (p. 1). For if public pensions were perceived as purely forced or voluntary saving, then they should definitely have had a pronounced, close to one-to-one adverse effect on private saving.

<sup>4</sup>See Harberger (1964).

countries.<sup>5</sup> High social security taxes have also been blamed for a growing underground economy.<sup>6</sup> Therefore, a tax hike may be deemed undesirable.

In contrast, a fully funded, defined-contribution (FF-DC) pension plan, whether privately managed or publicly administered, is by its own nature always actuarially balanced: the expected present value of the pension benefits to any particular individual is equal to the accumulated present value of her contributions. The pension benefits over the retirement age cannot exceed the value of the accumulated assets. This is one of the reasons why FF-DC pension plans have gained some popularity among economists in recent years. Of course, FF-DC pension plans have also several shortcomings (relative to a PAYG-DB plan); see, *inter alia*, Heller (1998) and Hemming (1998). Among these is the inability of an FF-DC plan to pay benefits to the retirees immediately after the plan is introduced; such payments are, of course, not only possible under a PAYG-DB plan but also are an inherent element of the latter plan.<sup>7</sup>

On the other hand, the FF-DC method has a relative advantage over the PAYG-DB method in the opposite case of terminating a pension program. Under the former plan, the working generation is then still left with all the accumulated assets to support itself upon retirement; while under the latter plan, the working generation makes contributions to finance the benefits to the retired generation but is left with no funds for its own retirement.

At first glance, this advantage of the FF-DC plan over the PAYG-DB may seem purely academic, since it is unlikely that a pension program would be abruptly abolished. However, this advantage may become more real in circumstances which produce effects which are financially similar to those occurring when a pension program is terminated. These circumstances are slower population growth rates and/or increased life expectancy. Both of these phenomena result in increases in the dependency ratio.<sup>8</sup> In these cases too, the working generation of today (denoted as "the working generation" from now on) who chooses, for instance, to have fewer children under a PAYG-DB pension plan will continue to finance the pensions paid to the current retired generation; however, it will not give birth to enough children so that they can finance its pension at the existing contribution (tax) rate.

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<sup>5</sup>See Daveri and Tabellini (1997), Zee (1997), and Mendoza, Razin, and Tesar (1994).

<sup>6</sup>See Schneider (1997 and 1998).

<sup>7</sup>In practice, PAYG-DB programs require a minimum contribution period to qualify for pension benefits but the point is that, in principle, it is possible, if the government so wishes, to pay pension benefits right away to retirees because the funds are available through the contributions being made by those who are working.

<sup>8</sup>For instance, Chand and Jaeger (1996, Table 1) report that dependencies ratios in the G-7 countries are projected to rise from 0.45-0.55 in 1995 to 0.7-0.9 in the year 2050.

In this paper, we examine some implications of aging populations and increasing dependency ratios for PAYG-DB pension programs. Some of these implications have not attracted much attention. In Section II, we examine modifications in these programs that might work as “shock absorbers” to compensate over time for variations in the dependency ratio. The benefits and costs of these modifications are discussed for closed economies or large open economies (Section III) and for small open economies that may enjoy a given world rate of interest (Section IV). Some concluding remarks are provided in Section V.

## II. A MODIFIED PAYG-DB PENSION PROGRAM

As was already pointed out, if the birth rate declines and/or life expectancy rises, then in order to maintain intact the pension benefits to the currently working generation, it will be necessary to raise the taxes on the next working generation. But as Barro (1979) convincingly argued, because taxes usually impose an excess burden (deadweight loss) and this excess burden grows disproportionately with the level of the tax rates, it is efficient to smooth taxes over time.<sup>9</sup> That is, the total excess burden over time (and over different generations) would fall if the marginal excess burdens of taxation (and the marginal tax rates) were equated over time. Therefore, it is Pareto-efficient to shift some of the burden of the higher taxes from the working generation of tomorrow to the working generation of today. Furthermore, such a shift could be justified on equity grounds as well. It can be argued that the working generation of today chose voluntarily to have fewer children in order to improve its own economic well-being. So there may be some justification for imposing on this generation at least part of the extra burden of financing its own retirement benefits rather than shifting all this burden to the next and smaller working population.

Thus, suppose that a pure PAYG-DB pension plan is modified in order to introduce the aforementioned tax smoothing efficiency advantage. Specifically, suppose that a government that forecasts a declining population growth rate raises right away the taxes (contributions) on the working generation and does not wait to raise taxes until the next (smaller) generation enters the labor force. In this way, the contributions made by the working generation will exceed the amount needed to pay for the pensions of the current retirees. A special (reserve) pension fund is established to cover the amount by which the pension benefits to be paid to the working generation when it retires exceed the contributions made then by the next working generation. This provides some partial funding for otherwise pure PAYG pension plans.

It should be stated from the onset that the accumulation in the special pension fund will increase national saving. The private sector will not undo what the special fund does; private saving will not fall by the amount accumulated in this fund. The Ricardian equivalence hypothesis of a Barro (1974)-type economy does not apply in this case. The reason is as follows: The special fund serves essentially to make a transfer between generations (from the working generation of today to the working generations of tomorrow) and not within

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<sup>9</sup>See, also Barro (1995).

generations. Then, in the absence of a perfect intergenerational link by an operational bequest motive,<sup>6</sup> such a transfer will affect the intertemporal budget constraint (or the wealth) of each generation and, consequently, the allocation of consumption over time. Assuming that consumption is a normal good (which is the only plausible assumption in this case), the transfer will reduce the consumption of the working generation and increase national saving.

But if the increase in national saving is substantial and leads to a larger stock of capital, it could affect the rate of return to capital, and, if the rate of return falls substantially, would the accumulation in the special pension fund suffice to cover the costs of the young generation retirement? We consider this question in the next two sections.

### III. A CLOSED (OR LARGE OPEN) ECONOMY

In a closed economy, national saving is equal to investment. An increase in national saving thus amounts to an equivalent increase in the stock of capital. With diminishing marginal productivity of capital, an increase in national saving lowers the rate of return to capital. Denote the existing stock of capital by  $K$ . Suppose that the special pension fund makes an increment of  $I$  to its assets, thereby the national stock of capital increases to  $K+I$ . Denote by  $p$  the share of the pension fund in the existing stock of capital. Thus, the pension fund will have at its disposal an amount of

$$A = (pK + I)(1 + r) \quad (1)$$

to pay for the pension benefits of the working generation when it retires;  $r$  is the rate of return to capital. The question is how large an increase in  $A$  is indeed generated by an increase in  $I$ , that is, how much more assets will be accumulated by the fund in the future if it makes an additional investment of one dollar today. Is it even possible that the fund might have a lower asset value in the future if it increases its assets today? Note that the question posed here is different from the question of what happens to the total income from capital when the stock of capital rises, because the pension fund holds only a fraction of the existing stock of capital; some of the loss that accrues because of the lower interest falls on other holders of capital as well.

Employing a constant-elasticity-of-substitution (CES) production function, one can show (see Appendix I) that<sup>7</sup>

$$\frac{dA}{dI} = 1 + r - \frac{rp(1 - S_k)}{\sigma}, \quad (2)$$

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<sup>6</sup>See, for instance, Razin and Sadka (1995).

<sup>7</sup>This derivatives is evaluated at  $I=0$ .

where  $\sigma$  is the (constant) elasticity of substitution and  $S_K$  is the share of capital in national income.

In order to evaluate the gain from a one-dollar increment in the assets of the special fund in the present, one should really look at the present value of the increase in the future value of its assets, that is: one should look at

$$PV \equiv \frac{1}{1+r} \frac{dA}{dI} = 1 - \frac{rp(1 - S_K)}{(1+r)\sigma} . \quad (3)$$

Naturally, if the fund does not hold any fraction of the existing stock of capital (namely,  $p=0$ ), it is not affected by the decline in the rate of return to capital and  $PV=1$ , that is, a one-dollar addition to its assets in the present raises the present value of its future assets by exactly one dollar. But if it holds some fraction of the existing capital, it gains, in present terms, less than one dollar for each dollar it invests today. This, by itself, is not surprising but what is disturbing is that, if the special pension fund is large, the gain resulting from a one-dollar investment could be significantly smaller than one dollar.

The term  $rp(1-S_K)/\sigma(1+r)$  on the extreme right-hand side of (3) expresses the loss to the special fund, that is: the amount by which the present value of investing one dollar falls short of one dollar. As expected, the loss increases as the elasticity of substitution between capital and labor ( $\sigma$ ) is smaller. Assuming a 30-year interval between the mean working year and the mean retirement year and an **annual** real rate of return to capital of 4 percent, then  $r$  in equation (3) comes to about 2.25. Assuming further that the capital share in national income is about 0.25,<sup>8</sup> then

$$PV = 1 - 0.52p/\sigma,$$

so that the loss is about  $0.52p/\sigma$ . Table 1 presents the magnitude of the loss for alternative values of the parameters  $p$  and  $\sigma$ . For a very high value of  $p$  and/or a very low value of  $\sigma$  (namely, when the  $p/\sigma > 1/0.52=1.92$ ), the loss may exceed one, that is: investing one dollar adds nothing to the fund!

For more plausible parameter values, as described in Table 1, the loss of course is smaller but still sizable. For instance, in the original article by Arrow, Chenery, Minhas, and Solow (1961) that developed the CES production function, the value of  $\sigma$  in manufacturing was found to be significantly (from a statistical point of view) below one, about 0.8-0.9.<sup>9</sup> Similarly, Atkinson (1998), relying partly on Poterba (1997), emphasizes that the share of capital in national income rose moderately in most of the G-7 countries as a result of increases

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<sup>8</sup>See, for instance, Auerbach and Kotlikoff (1987, Chapter 4).

<sup>9</sup>See also Nerlove (1967).



in the real rates of interest. Conversely, the share of capital in national income would decline when the capital-labor ratio rises and the real rate of interest falls. This means that  $\sigma$  is below one. For values of  $\sigma$  between 0.8 to 0.9 and for values of  $p$  between 0.3 to 0.5, which are quite plausible,<sup>10</sup> investing one dollar yields a present value of only 0.67-0.83 dollar. Furthermore, these present values may become even smaller if the availability of extra revenue tempts the government to increase its expenditures on public consumption or to finance popular, but economically questionable, public investment projects, as it seems to have happened in some countries.

Table 1. The Present Value of the Loss from Investing One Dollar

	$p = 0.1$	$p = 0.2$	$p = 0.3$	$p = 0.4$	$p = 0.5$
$\sigma = 0.5$	0.104	0.204	0.312	0.416	0.520
$\sigma = 0.6$	0.087	0.173	0.260	0.347	0.433
$\sigma = 0.7$	0.074	0.149	0.223	0.297	0.371
$\sigma = 0.8$	0.065	0.130	0.195	0.260	0.325
$\sigma = 0.9$	0.058	0.116	0.173	0.231	0.289
$\sigma = 1.0$	0.052	0.104	0.156	0.208	0.260

The conclusion is that, in the face of significant demographic (or other) changes, attempting to modify a PAYG-DB pension scheme so as to smooth contributions (taxes), in order to minimize the excess burden over time, may be quite costly to the pension program itself. The benefits of tax smoothing may be overwhelmed by the costs of lower rates of return. The conclusion is that taxes on the working generation may have to be raised by an amount which is significantly higher than the amount that the special pension fund will have at its disposal in the future and that can be used in order to alleviate the burden on the next working generation. Thus, the advantage of tax smoothing must be weighed against the aforementioned loss to the pension program that stems from declining rates of return to capital, since the latter loss also implies higher taxes and excess burden.

A similarly disturbing conclusion that stems from the decline in the real rate of return to capital when its stock rises can be drawn for some other related problems. For instance, the estimates reported by Chand and Jaeger (1996) suggest that the *negative* net asset positions of public pension funds in major industrial countries can reach about 200 percent of GDP in about 50 years, with France, Germany, Italy, and Japan standing out with net negative asset positions of about 400 percent of GDP. Suppose that countries in such positions attempted to

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<sup>10</sup>See Chand and Jaeger (1996).

raise their saving rates in order to increase over time their capital stocks by, say, a multiple of three-four times their GDPs so as to replenish their pension funds. Assuming a capital-output ratio of about three to four, this means that they roughly double their stock of capital. What increase in GDP would such an increase in capital generate? For an elasticity of substitution in the vicinity of one and a capital share of 0.25, the increase in output will be less than 20 percent.<sup>11</sup>

The above conclusion was drawn for a closed economy. But it applies with straightforward modifications, to an open economy that can invest its national saving abroad, provided that it is sufficiently large so as to exert a downward pressure on international rates of return to capital when it increases its national saving. The effect on the global rate of return may well be less pronounced in this case than in the closed economy case, but the basic point is that one may no longer assume that the real rate of return to capital is exogenously given. Thus, the decline in the rate of return must be taken into account when weighing whether to increase the accumulated assets in the pension fund.

#### IV. A SMALL OPEN ECONOMY

A small open economy will be a different situation. It can in theory invest part or all of its national saving abroad without exerting any downward pressure on the real rates of return in the international capital markets. By doing so, the special pension fund needs not experience any of the loss incurred in the closed economy case.

This remedy, however, may be more theoretical than practical for many economies. For instance, it is hardly conceivable that the transition and third-world economies will or can indeed invest abroad. Some are burdened by high transitional unemployment rates which make it politically unfeasible for their governments to allow pension funds to invest abroad thus “creating jobs abroad rather than at home.” Many have high debts and are short of foreign exchange, so that they can hardly afford to invest abroad.

In the small open economies that could invest abroad without affecting the international capital markets (for instance, the small EU economies or Chile), the pension funds would be exposed to *real* exchange rate risks. The extent of this risk depends, by and large, on the particular exchange rate policy that the government follows.<sup>12</sup>

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<sup>11</sup>To see this, note that when  $\sigma$  in the vicinity of one (namely, when the production function is roughly of the Cobb-Douglas type), then the capital share is also the elasticity of output with respect to capital. In this case, doubling the stock of capital (with no significant exchange in the labor input) raises output by a multiple of  $2^{0.25} \approx 1.19$ .

<sup>12</sup>The rate of exchange may, of course, be influenced also by the size of the capital export induced by the investment abroad.

For instance, if the government adopts a managed exchange rate policy aimed at maintaining a purchasing power parity (PPP), then the real exchange rate risk may be largely eliminated. To see this, recall that a PPP policy means that the managed rate of devaluation of the domestic currency is equal to the inflation differential between home and abroad. That is, for any given period, the exchange rates (in units of domestic currency per one unit of foreign currency) at the beginning and at the end of the period, denoted by  $e_0$  and  $e_1$ , respectively, are related to one another by

$$\frac{e_1}{e_0} = \frac{(1 + \pi)}{1 + \pi^*}, \quad (4)$$

where  $\pi$  and  $\pi^*$  are the domestic and foreign rate of inflation. Suppose, then that a pension fund invests one unit of the domestic currency abroad at the beginning of the period. That is, it invests  $1/e_0$  units of the foreign currency. It will receive  $(1/e_0)(1+r^*)$  units of the foreign currency at the end of the period, where  $r^*$  is the foreign *nominal* rate of interest during that period. This amounts to  $(1/e_0)(1+r^*)e_1$  in units of the domestic currency. In *real* terms, the pension fund gets only  $(1/e_0)(1+r^*)e_1/(1+\pi)$ . Employing the PPP equation (4), we find that the pension fund receives a *real* rate of return of  $(1+r^*)/(1+\pi^*) - 1$  which is the foreign rate of return. The latter is unaffected by the exchange rate. Thus, the pension fund is not exposed to *real* exchange rate risks. However, it is highly questionable whether a government could or would strictly adhere to a PPP exchange rate policy in the wake of a large capital export generated by the pension fund starting to invest abroad. It would be practical to assume, in this case, an initial devaluation of the domestic currency, inflicting some loss to the pension funds. And when the pension fund starts to cash in on its investment abroad in order to pay pensioners, the resulting capital import may appreciate the domestic currency, again inflicting some loss to the pension fund.

Another example of exchange rate policy is when the government does not manage the exchange rate and lets it float freely in international capital markets. Then, the pension fund will be fully exposed to exchange risks. In this case, the rate of devaluation of the domestic currency during any given period is equal to the nominal interest differential (between home and abroad). A person who invests one unit of the domestic currency at the beginning of the period gets  $1+r$  units of the domestic currency at the end of the period, where  $r$  is the domestic *nominal* rate of interest. Alternatively, she can convert the unit of domestic currency into  $1/e_0$  units of the foreign currency and invest abroad in which case she receives  $(1/e_0)(1+r^*)$  units of the foreign currency or  $(1/e_0)(1+r^*)e_1$  units of the domestic currency at the end of the period. Capital market arbitrage then implies that

$$1+r = \frac{(1 + r^*)e_1}{e_0},$$

or

$$\frac{e_1}{e_0} = \frac{1 + r}{1 + r^*} . \quad (5)$$

In this case, the real rate of return that the pension fund earns on its investment abroad is

$$\frac{(1 + r^*)e_1}{(1 + \pi)e_0} - 1 . \quad (6)$$

This is equal, by (5), to  $(1+r)/(1+\pi) - 1$ , which is the real rate of return that it earns on its investment at home. As is evident from (6), the pension fund is fully exposed to the exchange rate risk.

## V. CONCLUSION

A PAYG-DB pension program has a recognized advantage over an FF-DC pension program at the initial stage when a pension program is just started, namely those who retire can start right away to receive pension benefits. This is, of course, impossible under an FF-DC program. This advantage is exactly reversed when a pension program is terminated or, more realistically, when fertility declines and/or life expectancy rises. A possible remedy for the PAYG-DB program in these circumstances would be to create a special, "shock absorber" fund to help smooth the contributions over time and generations, while maintaining the benefits intact. However, we have seen that this remedy can be costly for both closed and open economies. Thus, the alternatives may be going to an FF-DC system or modifying the benefits of the PAYG-DB system to make the latter viable over time.

In this appendix, we derive equation (2). Let the production function be of the CES type

$$F(K+I, L) = [\alpha_1(K+1)^\beta + \alpha_2 L^{\frac{1}{\beta}}]^\beta, \quad (\text{A1})$$

where  $L$  is labor input and  $\sigma=1/(1-\beta)$  is the constant elasticity of substitution ( $\alpha_1$  and  $\alpha_2$  are two positive coefficient). Then, the marginal products of capital and of labor yield the competitive market returns to capital and labor,  $r$  and  $w$ , respectively:

$$r = \alpha_1[\alpha_1(K+I)^\beta + \alpha_2 L^{\frac{1}{\beta}}]^\beta (K+I)^{\beta-1}, \quad (\text{A2})$$

and

$$w = \alpha_2[\alpha_1(K+I)^\beta + \alpha_2 L^{\frac{1}{\beta}}]^\beta L^{\beta-1}. \quad (\text{A3})$$

Hence,

$$\frac{dr}{dI} = \alpha_1^2(1-\beta)[\alpha_1(K+I)^\beta + \alpha_2 L^{\frac{1}{\beta}}]^\beta (K+I)^{2\beta-2} - \alpha_1(1-\beta)[\alpha_1(K+I)^\beta + \alpha_2 L^{\frac{1}{\beta}}]^\beta (K+I)^{\beta-2}. \quad (\text{A4})$$

Evaluating the latter derivative at  $I=0$  and employing (A2) we conclude that:

$$\begin{aligned} \frac{dr}{dI} &= (1-\beta) r [\alpha_1(\alpha_1 K^\beta + \alpha_2 L^\beta)^{-1} K^{\beta-1} - K^{-1}] = -\frac{(1-\beta)r\alpha_2 L^\beta}{K(\alpha_1 K^\beta + \alpha_2 L^\beta)} \\ &= -\frac{(1-\beta)r}{K[(\alpha_1 K^\beta / \alpha_2 L^\beta) + 1]}. \end{aligned} \quad (\text{A5})$$

Employ (A2) and (A3) to get

$$\frac{rK}{wL} = \frac{\alpha_1 K^\beta}{\alpha_2 L^\beta}. \quad (\text{A6})$$

Denoting the share of labor in national income by  $S_L = wL/(wL+rK)$ , it follows from (A6) that

$$\frac{1}{S_L} = \frac{wL+rK}{wL} = 1 + \frac{rK}{wL} = 1 + \frac{\alpha_1 K^\beta}{\alpha_2 L^\beta} . \quad (\text{A7})$$

Substituting (A7) into (A5) yields

$$\frac{dr}{dI} = - (1-\beta)rK^{-1}(1-S_K) , \quad (\text{A8})$$

where  $S_K=1-S_L$  is the share of capital in national income. Differentiate (1) with respect to  $I$ , evaluate the derivative at  $I=0$ , and employ (A8) to get:

$$\frac{dA}{dI} = 1+r + pK\frac{dr}{dI} \quad (\text{A9})$$

$$= 1 + r - p(1-\beta)r(1 - S_K) .$$

Upon some rewriting of (A9), we get

$$\frac{dA}{dI} = 1 + r - \frac{pr(1-S_K)}{\sigma} , \quad (\text{A10})$$

which completes the derivation of equation (2).

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