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Currency Crises and Foreign Reserves: A Simple Model

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Abstract

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This paper addresses the important question of how far a government will run down its stock of foreign reserves in a defense of a fixed exchange rate. An optimizing model of currency crisis is presented in which the decision of whether or not to borrow in a defense of a peg is explicitly analyzed. The threshold level of reserves is then determined endogenously and shown to be a function of fundamental economic variables. The analysis also demonstrates how an increase in the level of reserves, a credit-rating upgrade, or the imposition of capital controls can remove the multiplicity of equilibria.

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I. INTRODUCTION

How far will a government run down its stock of foreign reserves in a defense of a fixed exchange rate? Existing literature on currency crises has paid very little attention to this important question of how the threshold level of reserves is determined. The traditional literature, as expounded by Krugman (1979), arbitrarily assumes the threshold to be zero without offering any justification of why this should be so.² More recent 'second generation' models rightly purport the view that in these modern times, countries have access to world capital markets so that reserve adequacy *per se* is far less of a concern than in the 1970's when Krugman's model was constructed.³ These models however, go to the extreme by avoiding altogether an explicit consideration of reserves so that a threshold simply does not exist. The implicit assumption of course, is that foreign reserves can be borrowed freely and without limit from the world capital market. There has also been no analysis of how a government's decision to borrow reserves may affect the equilibrium determination and thus sustainability of a peg. As things stand, the meaning and relevance of a reserve threshold in a world where solvent governments can borrow from international capital markets, other central banks, or multilateral institutions is unclear.

This paper attempts to fill this gap by analyzing, within a simplified framework of a second generation model, the government's decision of whether reserves should be borrowed to mount a currency defense. In particular, the government's payoff function is extended to allow for the possibility of borrowing foreign reserves at a fixed cost, should the need and desire arise. Borrowing is made possible through the introduction of risk-neutral foreign lenders who have access to a safe return asset and stand ready to lend to the central bank at some rate of interest. The model can be used to determine endogenously, for a given stock of reserves, the largest amount of intervention that the central bank will engage in to defend the currency. At this critical level, the costs to the economy of maintaining the peg, plus the cost of any external finance, is equal to the benefits of the fixed exchange rate. The threshold level of reserves obtained will in general depend on the cost of borrowing, the state of the economy, and the current stock of reserves. In addition, the model sheds light on the issue of multiple equilibria by explaining how indeterminacy can occur for some levels of state variables (reserves for example) and not at others.

Obstfeld (1986) investigated the issue of borrowing abroad during balance of payments crises and concluded that with the possibility of lump-sum taxation, foreign reserves can become infinitely negative without violating the government's intertemporal budget constraint. There is no limit on the level of external debt, only on its rate of growth. Negative reserve positions are therefore feasible and hence a continually declining stock of foreign reserves need not, in itself, force the abandonment of a fixed exchange rate. A similar conclusion was reached by Buiter (1986) who further showed that within the confines of a Krugman-type model, borrowing abroad to replenish the central bank's stock of foreign reserves can delay the collapse of a peg.

²The large literature inspired by this model is extensively surveyed by Agenor, Bhandari, and Flood (1992).

³Examples include Obstfeld (1994), Andersen (1994), and Davies and Vines (1995). See also Flood and Marion (1998) for a review of first and second generation model.

There has been several episodes of governments choosing to borrow in defense of their currencies. For example, an emergency international loan was arranged for Argentina in 1981 and 1995 that helped to prevent a devaluation, while Sweden borrowed reserves for a similar purpose in the second week of September 1992. In practice, major central banks can generally draw on swap-line agreements with other central banks, allowing them to quickly borrow the foreign currency needed to mount a defense.⁴ Figures 1 and 2 show the evolution of reserves/M2 and exchange rates of the Asian countries that experienced a run on their currency in 1997. Note that for all the countries except Singapore, the ratio of reserves to M2 was well below one and generally declining. As highlighted by Obstfeld and Rogoff (1995), in situations where the central bank acts as a lender of last resort and the currency crisis is accompanied by a banking crisis, the central bank may potentially have to buy up all of M2 to defend the peg. These countries, therefore, would have had to resort to foreign borrowing if the situation got bad enough and they were determined to defend their currencies.

The outline for the rest of the paper is as follows. Section II sets out the model. Section III extends the analysis to incorporate endogenous expectations and discusses equilibrium determination. Section IV concludes and some technical details are presented in an appendix.

II. THE MODEL

A. The Domestic Economy

The basic structure of the model has been deliberately kept as simple as possible to focus the analysis on reserves and their role in equilibrium determination. Nonetheless, the underlying intuition will go through in more sophisticated models. Consider a standard small open economy model with perfect capital mobility. Domestic agents choose an optimal portfolio allocation of their wealth between money, domestic bonds, and foreign bonds in the traditional Tobin way. Arbitrage by agents between domestic and foreign bonds yields the familiar uncovered interest parity condition

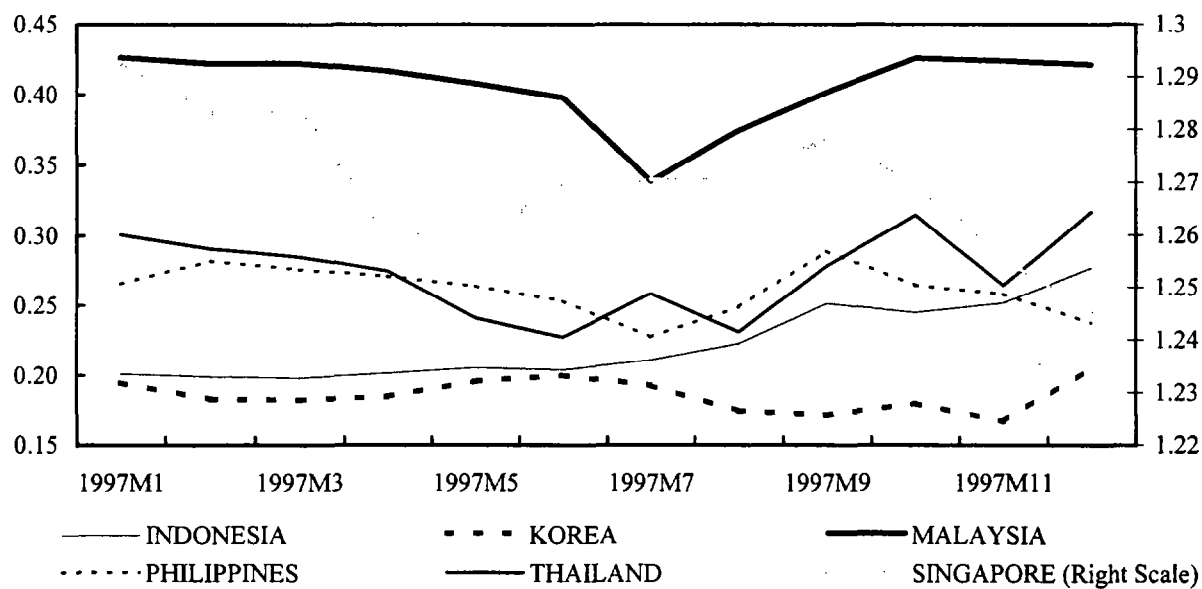
$$i^d = i^f + \pi^e, \quad (1)$$

where i^d and i^f are the domestic and foreign interest rate respectively. The expected rate of depreciation of the domestic currency is $\pi^e = \left(\frac{e - \bar{e}}{\bar{e}} \right)$, where e is the domestic currency price of foreign exchange and \bar{e} its initial fixed level. An increase in π^e will be taken to represent a loss of confidence in the peg that will be associated with an attack on the currency.

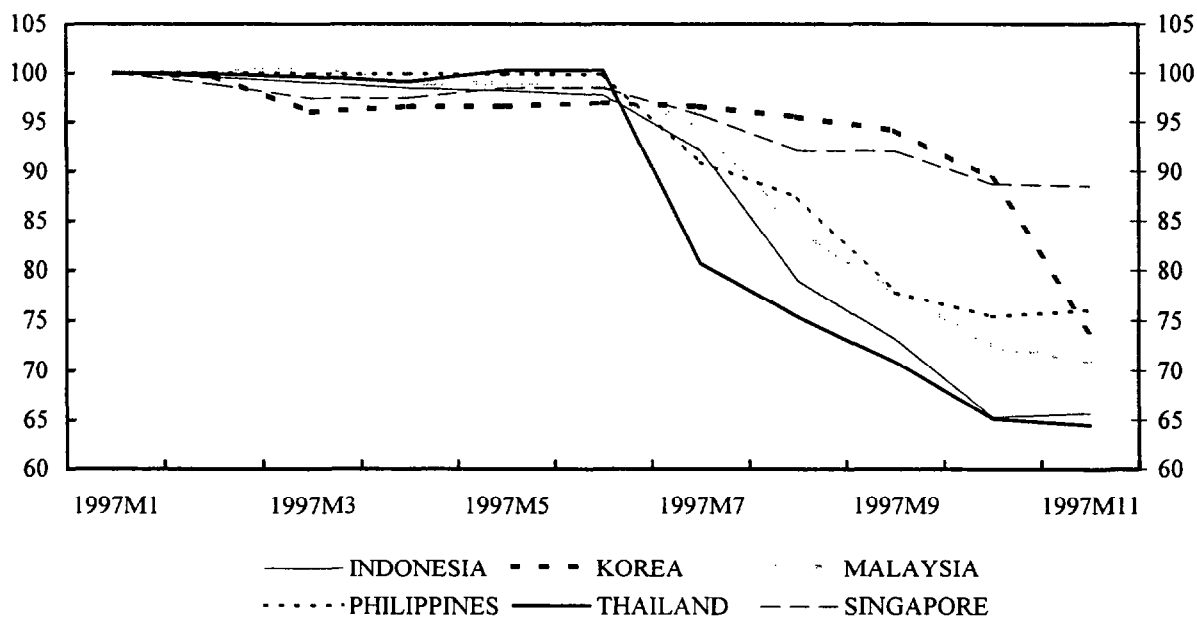
Perfect capital mobility and a fixed exchange rate implies that money supply has to adjust endogenously to clear the money market. In general, an increase in the expected rate of

⁴For example, the central banks of both the United Kingdom and Italy were entitled under ERM rules to credit lines from Germany and thus they were able to engage in direct foreign exchange intervention on a very large scale. More recently, a multilateral currency swap arrangement was set up among the East Asian countries under which countries can draw on the resources of other central banks to fend off an attack.

Figure 1. Reserves/M2



**Figure 2. Exchange Rates - \$US/National Currency
(1997M1 = 100)**



Source: IFS, author's calculations.

depreciation will result in portfolio reallocation away from domestic money towards foreign assets until domestic interest rates rise by enough to restore (1). The size of the reallocation, in terms of the domestic currency, will be proportional to the revision of depreciation expectations

$$\bar{e}G = \phi\pi^e,$$

where $\phi > 0$ and the initial expected rate of depreciation is taken, without loss of generality, to be zero.⁵ Thus $\bar{e}G$ represents the intensity of attack on the domestic currency which the central bank must meet out of its reserves in order to preserve the peg. Note that in contrast to traditional models, a run on the currency here is not modelled as one-off events characterized by a single massive all-out attack but rather as gradual pressure on the currency which becomes more intense as expected depreciation rises. It will be convenient to express the intensity of attack in terms of the foreign currency as

$$G = \alpha\pi^e, \quad (2)$$

where $\alpha \equiv \frac{\phi}{\bar{e}}$.⁶

Equations (1) and (2) capture the idea that defending a fixed exchange rate will entail an increase in the interest rate in proportion to the strength of the attack. In general, these high interest rates will, in turn, impose macroeconomic costs on the economy which make governments less willing to hang on to the peg.⁷ With this in mind, consider the familiar open-economy IS curve specification where output, y , is taken to depend negatively on the domestic interest rate and positively on the exchange rate. The latter can be due to the existence of some nominal rigidity which implies that output is demand determined so that nominal exchange rate changes have temporary real effects. Thus

$$y = b - \psi i^d + \eta e - u \quad b > 0, \psi > 0, \eta > 0, \quad (3)$$

where u is a stochastic output shock. The distribution of u is assumed to be uniform over the interval $[0, m]$. Note that under this specification, a positive value of u represents a *negative* output shock.

B. External Borrowing

The domestic central bank may borrow foreign reserves from international lenders such as other central banks, multilateral organizations, and private creditors. International lenders are risk-neutral and also have access to the risk-free foreign bond which returns i^f . Lending to the

⁵For example, a typical money market equilibrium can be characterized by (for simplicity, ignore the transaction role of money) $\bar{e}R + D = M^s = M^d = m_0 - m_1 i^d$ where the R and D are foreign reserves and domestic credit, respectively. Using (1) and holding domestic credit constant implies $\bar{e}\Delta R = -m_1 \Delta \pi^e$.

⁶The appendix outlines an alternative way of obtaining the attack schedule (2) through the introduction of risk-averse speculators.

⁷See for example Bensaïd and Jeanne (1997), Davies and Vines (1995), Ozkan and Sutherland (1994), and Obstfeld (1994).

domestic government is not risk-free, however, since it involves the possibility of default. If this occurs, lenders get back nothing, while if the sovereign decides to repay, they will receive the interest rate charged, i . Given that all lending is denominated in the foreign currency, risk neutral lenders will lend at rates which equalize their expected returns

$$(1 + i^f) = p(1 + i),$$

where p represents their assessment of the probability of repayment.

In practice, governments will not have access to unlimited international borrowing. Accordingly, assume the existence of a debt ceiling, C , such that the borrowing country will always default if the required repayments exceed C and honor its obligations with positive probability so long as repayments are no greater than this level. C can be viewed as the solvency constraint of the country which reflects the maximum level of debt that it is capable of repaying out of its own resources. The cost of borrowing will then be

$$(1 + i) = \begin{cases} \frac{(1 + i^f)}{p} & \text{for } C \geq (G - R)(1 + i) \quad [p > 0] \\ \infty & \text{for } C < (G - R)(1 + i) \quad [p = 0], \end{cases} \quad (4)$$

where R is the stock of foreign reserves measured in terms of the foreign currency and $(G - R)(1 + i)$ represents the repayment obligations of the country.⁸

C. The Government

Assume that the government wants to avoid deviations of output from its target rate and that maintaining the currency parity bestows some benefits upon policymakers captured by the variable z . The desirability of a pegged exchange rate can possibly be due to the reduced volatility in the terms of trade which promotes trade and the efficient allocation of international investment. Alternatively, z could embody the benefits of anti-inflation credibility, voter approval, and sound reputation.

Keeping in mind that the associated borrowing costs have to be taken into consideration under circumstances in which the government chooses to seek additional external finances, the loss function which the authorities minimize can be written as

$$L = (y^n - y)^2 + \delta (z - \beta \{\max [0, (G - R) i]\})^2 k, \quad (5)$$

where y^n is the target level of output, β denotes the discount factor, and δ is a dummy variable which takes on a value of 1 if the government decides to devalue and 0 otherwise. External borrowing is sought once reserves falls to zero and incurs an interest rate of i .⁹ Interest payments

⁸The debt ceiling can be allowed to depend negatively on the intensity of attack, π^e , so that it is determined endogenously without affecting the main results of the paper.

⁹Note that the model is general enough to accommodate the case where the threshold for borrowing is non-zero. It is also possible to take into account the opportunity cost of running down reserves without altering any of the results.

are discounted since they are transacted at the end of the period. Importantly, the only thing that matters is not the loss of fixing *per se*, but the loss of fixing *relative* to devaluing. Thus borrowing costs can be represented as *lowering* the loss of devaluing *relative* to that of fixing, which is the specification adopted in (5). Finally, to deal with the quadratic nature of the loss function, k is basically a dummy which takes on a value of either 1 or -1 so as to guarantee that greater borrowing will always decrease the loss of devaluing relative to fixing.¹⁰ Substituting in for y from (1) and (3) yields

$$L = [y^n - b + \psi (i^f + \pi^e) - \eta e + u]^2 + \delta (z - \beta \{\max [0, (G - R) i]\})^2 k. \quad (6)$$

If the government decided to devalue, it would attempt to minimize (6) by choosing a new exchange rate of

$$e^* = \frac{y^n - b + \psi i^f + u}{\eta}, \quad (7)$$

where the assumption is made, as in Davies and Vines (1995), that once the government devalues, no further devaluation is expected. If the authorities opt to maintain the fixed exchange rate, the loss will be

$$L^f = (v + \gamma \pi^e + u)^2,$$

where $v \equiv y^n - b + \psi i^f - \eta \bar{e}$ and $\gamma \equiv \psi$.

To simplify the exposition, assume that the initial level of the exchange rate, \bar{e} , is set such that $y = y^n$ in the absence of any output shock. This implies that $v = 0$. The respective losses under fixing and devaluing are then given by

$$L^f = (\gamma \pi^e + u)^2, \quad (8)$$

$$L^d = \{z - \beta [\max (0, (G - R) i)]\}^2 k. \quad (9)$$

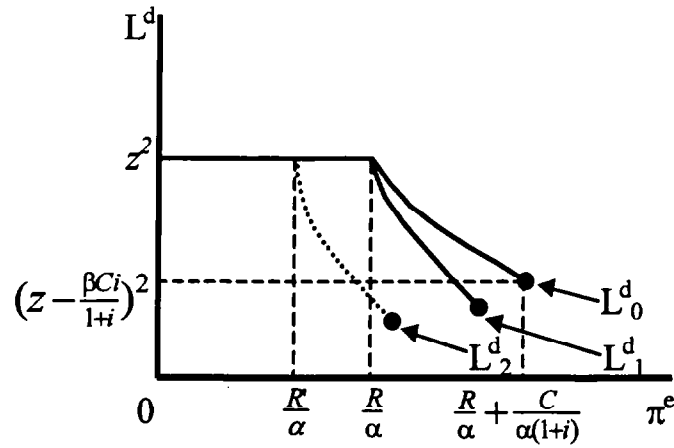
Finally, with the normalization made so far, the rate of depreciation that would take place if the peg is abandoned will be

$$\frac{e^* - \bar{e}}{\bar{e}} = \frac{u}{\eta \bar{e}}. \quad (10)$$

The government is thus faced with a dilemma. Committing to the fixed exchange rate allows it to forgo the costs incorporated in z but prevents reaction to shocks which causes output to deviate from its desired level and entails borrowing costs if reserves are insufficient. The government's decision will therefore depend on which of the alternatives seems more attractive given the prevailing circumstance. Following Obstfeld (1994), Davies and Vines (1995), and Bensaid and Jeanne (1997), the discussion will be further simplified by considering only the possibility of depreciation. While the model could easily be modified to include the possibility of

¹⁰A possible specification for k is $k = \frac{a}{\max(a, -a)}$ where $a \equiv z - \beta[(G - R) i]$.

Figure 3. The Loss of Devaluing.



appreciation, doing so does not add much insight and also removes some clarity from the analysis.

D. The Threshold Level of Reserves

The timing of the model is as follows. At the beginning of the period, private expectations are formed and the government decides whether to defend the peg or not given these expectations and after observing the output shock u . All claims are then settled at the end of the period. The mechanics and implications of the model are most easily elicited through a diagrammatic exposition. Figure 3 graphs the loss associated with devaluing, L_0^d , as a function of the expected rate of depreciation which is treated as given for now, but will be determined endogenously in the next section. Once $G (= \alpha\pi^e) > R$, sustaining the exchange rate requires foreign borrowing which, under specification (5), makes devaluing relatively more attractive than fixing thus lowering L^d . Note that the presence of a debt ceiling implies that the cost of borrowing becomes infinite at some point. The borrowing limit is defined by

$$C = (\alpha\pi^e - R)(1 + i), \quad (11)$$

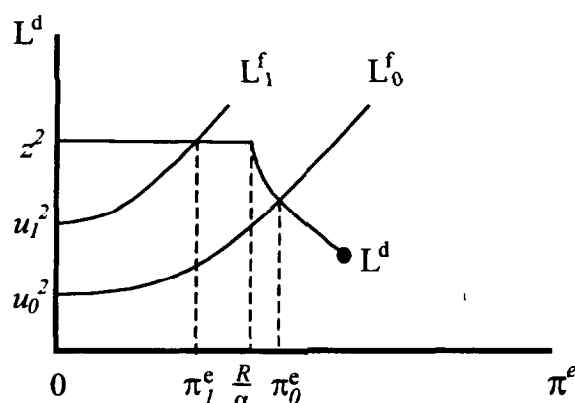
and for any π^e that violates the upper limit of this constraint, it will always be optimal to devalue since the loss relative to fixing is infinitely lower. Specifically,

$$L^d = \begin{cases} z^2 & \text{for } \pi^e \leq \frac{R}{\alpha} \\ [z - \beta(\alpha\pi^e - R)i]^2 & \text{for } \frac{R}{\alpha} + \frac{C}{\alpha(1+i)} \geq \pi^e > \frac{R}{\alpha} \quad (k = 1) \\ -\infty & \text{for } \pi^e > \frac{R}{\alpha} + \frac{C}{\alpha(1+i)} \quad (k = -1). \end{cases}$$

Figure 3 also shows how a rise in the interest rate charged on borrowing will *lower* the

loss of devaluing to L_1^d and make fixing a relatively less attractive proposition for the government. Such a worsening of the terms of borrowing can be interpreted more broadly as the result of a down-grading of the country's credit worthiness, or to reflect other costs associated with external borrowing, including conditions imposed by the lending party. A slip in credit worthiness in the eyes of the international capital market makes it harder to secure funds and so defending the peg becomes more difficult. Finally, Figure 3 shows that a fall in the stock of reserves from R to R' will shift the loss of devaluing inwards to L_2^d , and also reduce the attractiveness of fixing. The country will have to start borrowing abroad, and incur the associated costs, at a lower level of selling pressure because the extent with which it can defend the peg out of its own reserves is reduced.

Figure 4. The Government's Trade-Off.



To elicit the nature of the trade-offs which the government has to weigh, the loss functions under each possibility are brought together in Figure 4. Note that the loss associated with fixing is uniquely drawn for each possible value of the output shock. A more adverse output shock, an increase in u , will shift L_0^f to L_1^f and diminish the government's incentive to defend the parity. It will be optimal to remain in the fixed exchange rate regime so long as $L^d \geq L^f$. The intersection of the two curves will determine the *critical expected rate of depreciation*, denoted by π_T^e , beyond which the government will opt to devalue. The *critical size of attack* which speculators have to mount in order to collapse the peg is thus given by $\alpha\pi_T^e$. The *threshold level of reserves*, T , can be calculated generally as

$$T = R - \alpha\pi_T^e.$$

Traditional models of currency crises imposed $T = 0$ without offering any discussion of the underlying forces why this should be so. Under the present setup, the threshold is allowed to vary endogenously and the factors that affect its level can be clearly elicited.

In Figure 4, suppose that the initial state of the economy is reflected by u_0 so that the loss to fixing is L_0^f and $\pi_T^e = \pi_0^e$. In this case the government is willing to borrow abroad in order to defend the peg and $T < 0$. A worsening of the state of the economy to u_1 will lower the critical

expected rate of depreciation to π_1^e and the threshold level of reserves is now positive. Here the macroeconomic costs of high interest rates necessary to defend the peg will be so large that it will be abandoned before the government actually exhausts its stock of reserves. Formally, the critical expected rate of depreciation for the two states will be given as

$$\pi_0^e = \frac{z - u_0 + \beta i R}{\gamma + \beta \alpha i}$$

and

$$\pi_1^e = \frac{z - u_1}{\gamma}. \quad (12)$$

In the case where $T < 0$, a comparative statics exercise reveals that a deterioration in the terms of borrowing will reduce π_T^e , lower the critical size of attack, and increase the threshold level of reserves (T less negative):

$$\frac{\partial \pi_0^e}{\partial i} = \frac{\gamma \beta R - \beta \alpha (z - u_0)}{(\gamma + \beta \alpha i)^2} < 0$$

since

$$u < z - \frac{\gamma R}{\alpha} \quad \text{for} \quad \frac{C}{\alpha(1+i)} > \pi^e > \frac{R}{\alpha}.$$

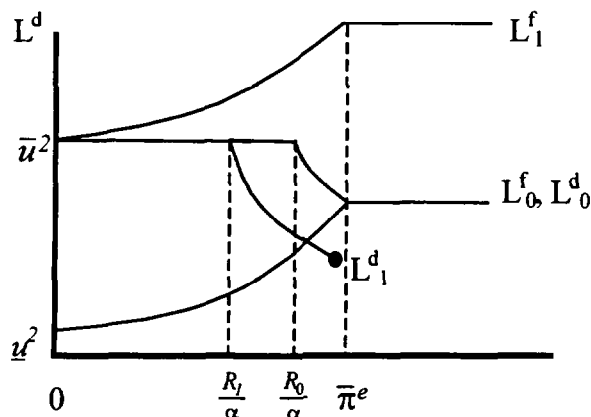
In addition, a fall in the stock of reserves will shift L^d to the left, making it harder for a government to defend a currency parity and lowering the critical size of attack. It can be verified that

$$\frac{\partial T}{\partial R} = \begin{cases} 1 > 0 & \text{for } T > 0 \\ 1 - \frac{\alpha \beta i}{\gamma + \alpha \beta i} > 0 & \text{for } T < 0. \end{cases}$$

The threshold level of reserves is therefore an increasing function of u , i , and R . A country will tend to abandon its peg with more reserves to spare (or with less borrowed reserves) the more unfavorable is the state of the economy, the more costly it is to borrow externally, and the higher is the initial stock of reserves.

In order to be successful, the speculative attacks in this model need not be so large as to exhaust reserves, but only large enough to push reserves beyond the threshold level which maybe positive or negative depending on the state of the economy and current stock of reserves. The model is therefore able to capture situations where the government chooses to give in before reserves are completely exhausted. Traditional models are not able to explain this while existing second-generation ones are consistent with such behavior simply because they completely disregard the role of reserves. Here such a situation can logically arise as a result of optimal decision making by the government. Importantly, the viability of a fixed exchange rate depends not only on the government's perceived desire not to devalue, but also vitally on the government's *ability* not to devalue. By running down foreign reserves, the likelihood of involuntary collapse due to credit constraints increases. Thus even within the confines of a second-generation model, a currency crisis can occur as a result of insufficient reserves.

Figure 5. Limited Speculative Resources.



E. Limited Speculative Resources

In practice, as highlighted by Obstfeld and Rogoff (1995), agents are constrained by their access to the domestic currency when mounting an attack. This is also true for short-sellers since a determined defense will eventually raise interest rates to a level that makes it unprofitable to speculate on the currency. To capture this, an upper bound, $\bar{\pi}^e$, will be imposed on the expected rate of depreciation. This implies an upper bound on the intensity of attack of $\bar{G} \equiv \alpha \bar{\pi}^e$ which, when multiplied by \bar{e} , represents the maximum amount of domestic currency that the central bank must absorb and exchange for foreign currency in order to defend the peg.¹¹ Figure 5 illustrates the implications of an upper bound on π^e . In particular, it is now possible to break up the possible values of the output shock into three regions:

i. The *stable* region is defined by $u \leq \underline{u}$. Over this range, agents are not powerful enough to raise the cost of intervention above the benefits of fixing even if they expected the highest possible rate of depreciation.

ii. The *unstable* region, given by $u > \bar{u}$, where the loss to fixing exceeds that of devaluing irrespective of agents' actions. The peg will collapse even if there is no attack on the currency.

iii. The *ripe for attack* region occurs when $\underline{u} < u \leq \bar{u}$. Here the size of the attack determines whether the government will choose to intervene or allow a devaluation. If agents expect the worse then a depreciation will ensue, but if they don't expect a depreciation and hold on to their balances, the peg will be maintained.

¹¹The upper bound on the intensity of attack can also be justified by assuming that agents need to hold a minimum amount of domestic currency for transaction purposes so that $M^d(i^d = \infty) > 0$, and the money demand schedule becomes vertical past a certain level of interest rate. More obviously, the expected rate of depreciation cannot exceed 100 percent. Finally, the appendix also illustrates how speculators' resource constraints can be related to $\bar{\pi}^e$.

So long as reserves exceed total possible claims, \bar{G} , it is not possible that a peg collapses because the government runs out of resources and ability to defend. It can collapse only if the macroeconomic costs of defending are severe enough. An important point shown in Figure 5 is that a decline in the stock of reserves from R_0 to R_1 lowers the loss of devaluing to L_1^d , and eliminates the stable region. Now the peg will always collapse if enough people attack. Given an upper limit to the possible capital outflow then, self-fulfilling crises can occur only when the economy is in a sufficiently bad state given the stock of reserves, or reserves are low enough given the state of the economy.

III. MULTIPLE EQUILIBRIA

In this section, the expected rate of depreciation is endogenized so that strategic behavior of both the government and speculators are explicitly considered and a more realistic equilibrium concept can be analyzed. Such an extension will make the cost of resisting an attack dependent on an endogenous variable thereby creating the possibility of 'sun spot' equilibria.

A. Government Indifference Boundary

Define the *government indifference boundary* to be the combinations of output shock and expected rate of depreciation for which the government will be indifferent between defending or devaluing. For a given π^e , let u^* be the output shock corresponding to the intersection of L^d and L^f . Over the range which reserves are sufficient to absorb the attack, the indifference boundary will be given by¹²

$$\pi^e = \frac{z - u^*}{\gamma} \quad \text{for} \quad \pi^e \leq \frac{R}{\alpha}. \quad (13)$$

Once borrowing costs have to be taken into consideration, $L^d = L^f$ dictates that π^e and u^* be related by

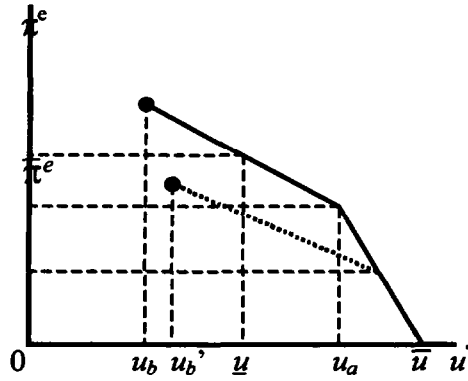
$$\pi^e = \frac{z - u^* + \beta Ri}{\gamma + \beta \alpha i} \quad \text{for} \quad \frac{R}{\alpha} + \frac{C}{\alpha(1+i)} \geq \pi^e > \frac{R}{\alpha}. \quad (14)$$

Here the country is borrowing to sustain the fixed exchange rate but never wishes to borrow more than international lenders are willing to lend. For $\pi^e > \frac{R}{\alpha} + \frac{C}{\alpha(1+i)}$, the government will always devalue and the trigger point u^* does not exist. Once the debt ceiling binds, there is no intersection of L^d with L^f and the government indifference boundary is not defined over that range.

The solid line in Figure 6 depicts the government indifference boundary. It is the inner envelope of schedules (13) and (14). The negative relation between u and π^e follows because the higher is the output shock, the larger is the deviation of output from target and hence it takes a smaller expected rate of depreciation to trigger the decision to quit. The area under the

¹²If $R > \frac{\alpha z}{\gamma}$ then the boundary is always defined by (13) and borrowing never occurs.

Figure 6. The Government Indifference Boundary.



government indifference boundary represents combinations of u and π^e where the loss under a devaluation exceeds that of maintaining the peg. It represents the *region of fixing*. Compared to the case of unconstrained borrowing, the introduction of costly borrowing convexifies the relationship between u^* and π^e and shrinks the region over which the peg is sustainable.

Given the level of reserves, borrowing starts when

$$u^* < u_a \equiv z - \frac{\gamma R}{\alpha}, \quad (15)$$

and the borrowing constraint will bind once the output shock falls below

$$u_b \equiv z - \frac{\gamma R}{\alpha} - \frac{C}{1+i} \left(i\beta + \frac{\gamma}{\alpha} \right). \quad (16)$$

This is simply the shock which makes L^d intersect L^f at $\pi^e = \frac{R}{\alpha} + \frac{C}{\alpha(1+i)}$.¹³

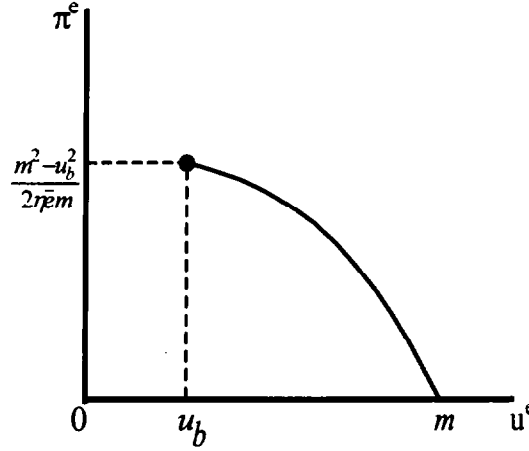
Figure 6 also illustrates the regions discussed previously in Section IIE. Given reserves of R_0 , the peg will be stable for any value of output shock $u \leq \underline{u}$, unstable for $u > \bar{u}$, and ripe for attack when $\underline{u} < u \leq \bar{u}$. These regions are given formally as

$$\bar{u} = z$$

$$\underline{u} = \begin{cases} z - \frac{\gamma \bar{G}}{\alpha} & \text{for } R \geq \bar{G} \\ z - \frac{\gamma S}{\alpha} - \beta i (\bar{G} - R) & \text{for } R + \frac{C}{1+i} \geq \bar{G} > R. \end{cases}$$

¹³Note that allowing for the possibility of being rationed in the international capital market implies the parametric restriction $u_b > 0 \iff C < (1+i) \left(z - \frac{\gamma R}{\alpha} \right) \left(\frac{\alpha}{\alpha i \beta + \gamma} \right)$.

Figure 7. The Private Decision Boundary.



Note that \underline{u} is undefined for $\bar{G} > R + \frac{C}{1+i}$ (since in this case speculative resources outweigh the government's even once all possible external sources of funds have been exhausted and no region of stability exists). The effects of a fall in reserves to R_1 is shown as a dotted line in Figure 6. The rise in u_b to u'_b reflects the fact that countries with lower reserves are more likely to be credit constrained. Crucially, the region of fixing shrinks and it is possible that the region of stability disappears because \underline{u} is no longer defined.

B. Private Decision Boundary

Assume that private agents understand the trade-off facing the government and know that a devaluation will occur when the output shock hitting the economy is bad enough. Specifically, they believe that there exists a threshold level of output shock, u^e , such that the government will opt to devalue when the actual shock is greater than this value. Hence

$$\pi = \begin{cases} 0 & \text{if } u \leq u^e \\ \frac{u}{\eta \bar{e}} & \text{if } u > u^e. \end{cases}$$

Since the public knows that there will always be a devaluation once the debt ceiling is reached, they will only form conjectures over the range $u^e \geq u_b$. The expected rate of depreciation can be calculated as

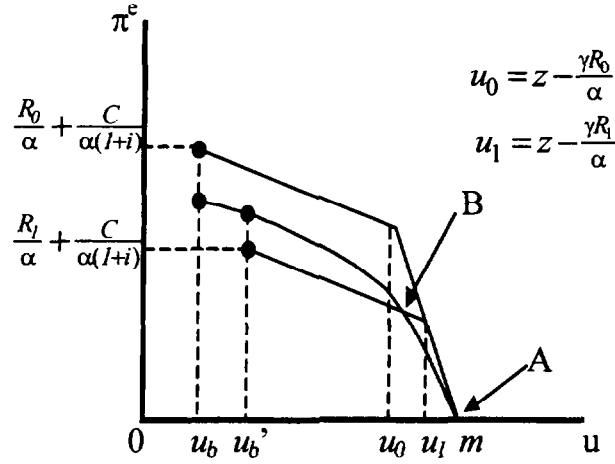
$$\pi^e = 0 \cdot \Pr(u \leq u^e) + E(\pi | u > u^e) \cdot \Pr(u > u^e).$$

Using the fact that u is uniformly distributed over $[0, m]$, one obtains

$$\pi^e = \frac{m^2 - u^{e2}}{2\eta \bar{e} m}, \quad u^e \geq u_b. \quad (17)$$

The locus (17), referred to as the *private decision boundary*, is shown in Figure 7.

Figure 8. Equilibrium Determination and the Level of Reserves.



C. Equilibrium Determination

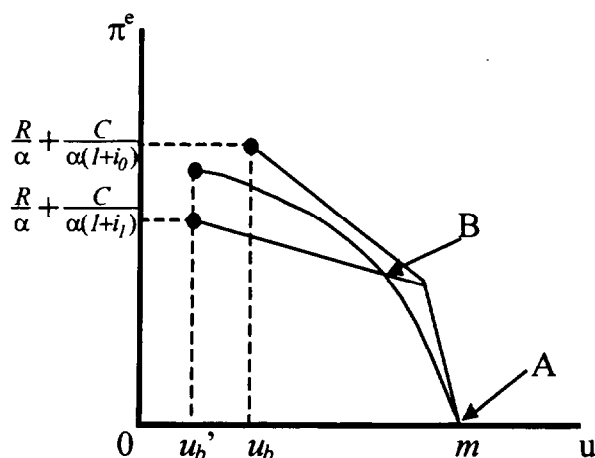
The equilibrium concept employed here is a form of Nash equilibrium commonly used in the literature.¹⁴ Equilibrium occurs when private sector expectations of the threshold output shock is correct in the sense that it corresponds exactly to the highest value of u at which the government is still willing to defend the parity. More precisely, equilibrium is defined by a level of π^e such that $u^e = u^* = u$. Both government and private sector decisions are correct given their expectations of the other player's behavior. Diagrammatically, equilibrium occurs at the intersection of the government indifference boundary with the private decision boundary.

Focussing on the most interesting case when the economy is in the ripe for attack region and $\bar{G} > R + \frac{C}{1+i}$, the exposition will be further simplified by assuming that $m = z$. Consider, then, an initial situation with reserves at R_0 so that there exists the unique equilibrium A as depicted in Figure 8. At equilibrium A , the shock which would be necessary for a regime switch is outside the range of possible shock values so a depreciation never occurs and the private sector accordingly does not expect any depreciation of the currency. Point A can be interpreted as the *full credibility equilibrium*.

Now take a country with a lower stock of reserves R_1 , the region of fixing is smaller and the system exhibits multiple equilibria. At equilibrium B , high expected depreciation creates considerable selling pressure which has to be soaked up by central bank intervention. This, in turn, drives up the domestic interest rate and inflicts pain on the government so that a devaluation will occur unless the output shock hitting the economy is quite favorable. A shift from equilibrium A with zero depreciation expectations to equilibrium B which has positive depreciation expectations, and therefore features an attack, is an example of a self-fulfilling currency crisis. The public's perception that a depreciation is possible gets validated by a lower government optimal trigger

¹⁴See, for example, Obstfeld (1994), Ozkan and Sutherland (1995), Davies and Vines (1995).

Figure 9. A Credit Rating Down-Grade.



value for u , increasing the likelihood that a shock of sufficient size to collapse the peg will actually be realized. Such an event can capture the abruptness with which speculative attacks occur in practice. Multiplicity is due to the lack of pre-commitment technology and a circularity in expectations where the choice of regime depends on private sector expectations that in turn depend on the government's perceived incentive to devalue. A shift to a more adverse equilibrium will in general be associated with market reallocation away from the domestic currency, higher domestic interest rates, and losses of foreign reserves.

This finding supports the argument by Sachs, Tornell, and Velasco (1996) that in the 1994 Mexican crises, self-fulfilling expectations became decisive in generating a panic only after the government ran down foreign reserves to precarious levels. Ghosh et. al. (1999) using probit regressions also found that lower reserve levels tend to be associated with a higher probability of crisis. It should be kept in mind, however, that since a fixed exchange rate implies that reserves are endogenously determined, their decline are often a symptom, not the cause, of a currency crisis. Rather, they serve as an indicator of the viability of a peg during times of crisis and determine how susceptible the country is to a shift in confidence. Importantly, given that a country is ripe for attack, policy measures which serve to boost the current level of reserves can remove the multiplicity problem, and thus the possibility of a self-fulfilling attack.

Figure 9 illustrates that a down-grading of a country's credit rating which raises the probability of default, $(1 - p)$, and thereby increasing the interest rate charged on external borrowing, can also create a situation of multiple equilibria. Starting with i_0 , the only equilibrium that exists is the full credibility equilibrium A . An increase in the interest rate charged by lenders to i_1 gives rise to indeterminacy as equilibrium B now becomes a distinct possibility. Sudden changes in expectations could lead to an equilibrium switch from one in which no realization of u can force the government off the peg to one in which even a relatively small u does so. An important implication, then, is that down-grading the credit worthiness of a country at a time when its currency might be vulnerable to an attack can make matters worse by actually increasing

the likelihood of such an attack. Consequently, the decision by Moody's and Standard & Poor's to reduce the credit ratings of major Asian countries periodically during the height of the Asian currency crises in 1997 could only have made things worse.

Finally, note that although speculative attacks driven by 'sunspots' contain a self-fulfilling element and are somewhat arbitrary in timing, they are not, however, completely unrelated to fundamentals. As discussed above, countries with larger stocks of reserves and better credit ratings are less susceptible to sudden speculative attacks associated with a shift in equilibria.

D. Capital Controls

This section briefly analyses the effects of capital controls which are often imposed by countries in their efforts to defend a peg. Both Malaysia and Thailand, for example, imposed exchange and capital controls during the Asian crisis of 1997. Capital controls drive a wedge in the uncovered interest parity relation and (1) will now be given as

$$i^d = (1 - t)(i^f + \pi^e), \quad (18)$$

where t is a measure of the extent of capital controls which takes on a value of 0 when there are no restrictions, and 1 in the case of complete control. Each value of π^e is now associated with a lower domestic interest rate and thus a smaller loss to fixing. The intensity of attack becomes

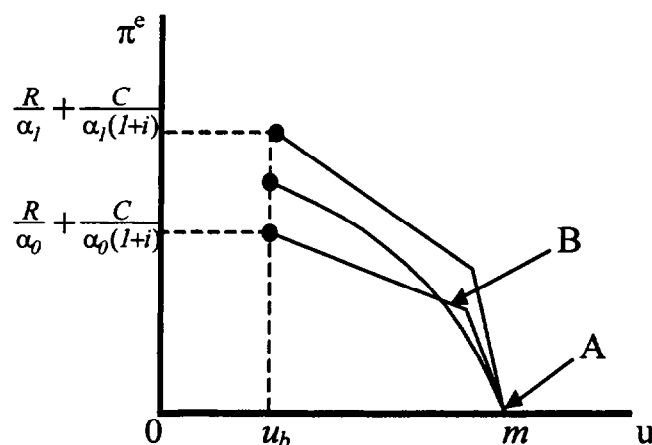
$$G = (1 - t)\alpha\pi^e. \quad (19)$$

For any given π^e , the implied intensity of attack is now lower and the existing stock of reserves can sustain a higher rate of expected depreciation before external borrowing is needed.

Importantly, the implementation of capital controls can remove multiple equilibria as illustrated in Figure 10. The initial situation with $\alpha_0 \equiv \alpha$ exhibits indeterminacy since either equilibrium A or B is possible. Increasing t reduces the impact of depreciation expectations on domestic interest rates and the system is now characterized by $\alpha_1 \equiv \alpha(1 - t)$. For a given level of expected depreciation, the associated trigger value of the output shock is larger and the government indifference boundary expands outward yielding the unique full credibility equilibrium at A . As modeled, capital controls help to preserve the stock of reserves and increase the region where fixing is viable.

It has been argued that certain capital control measures such as the Tobin tax and margin requirements are not difficult to implement and represent an effective means of mitigating speculative attacks. Eichengreen, Rose, and Wyplosz (1995), for example, put forward the view that capital controls are required for the maintenance of a fixed exchange rate when there is perfect capital mobility. These arguments are given some theoretical support by the analysis presented above. Finally, Bensaid and Jeanne (1997) suggested in their conclusion that further research be conducted to determine whether capital controls can eliminate multiple equilibria. Evidently, this is the case under the framework adopted in this paper.

Figure 10. Capital Controls and Equilibrium Determination.



IV. CONCLUSION

This paper has presented a simple stylized model in which the viability of a currency peg hinges on both political willingness and technical feasibility. By considering the possibility of limited borrowing at a cost as well as the macroeconomic costs of a defense, the model helps to elucidate the factors that influence the threshold level of reserves. In general, a higher stock of reserves, a healthier state of the economy, and a lower cost of external finance translates into a greater strength of commitment so that the size of the attack required to crack the government's resolve is larger. Furthermore, it is now possible for a currency to collapse due to *inadequate reserves* even within the context of a second generation model. This represents an important extension to the literature because it brings the essential elements of the two separate approaches to modeling currency crises closer together.

A useful feature of the model is that it highlights the conditions under which multiple equilibria may or may not exist. In particular, the depletion of foreign reserves or a deterioration of borrowing conditions can place the economy in a region of multiple equilibria and make an otherwise fully credible peg vulnerable to an attack. These results are consistent with empirical studies which find that foreign reserves are a good predictor of currency crises and also alludes to a possible explanation of why banking and currency crises tend to occur around the same time. Specifically, a banking crisis which is associated with a bailout can magnify the potential claims on the central bank, effectively reducing the size of its reserves and thereby increasing the probability of a currency crisis occurring. Interestingly, the multiplicity of equilibria can be removed and the viability of the peg enhanced through the introduction of capital controls.

APPENDIX I

This appendix outlines an alternative way of obtaining a speculative attack schedule similar in form to (2). Consider a continuum of risk averse short-term speculators distributed over the interval $[0, 1]$ active in the market. Speculators are strategic sellers of the domestic currency with the objective of making profits on the subsequent decline in currency value. In what follows, everything will be measured in terms of the domestic currency. Speculator i 's behavior depends on the profits generated from trading activity which is given by

$$\omega = S^i \cdot \left(\frac{e - \bar{e}}{\bar{e}} \right) = S^i \pi,$$

where S^i is the size of the bet, e the domestic currency price of foreign exchange, \bar{e} its initial fixed level, and π the actual rate of depreciation.

Following standard models of portfolio maximization, assume that speculators behave to maximize the generic mean-variance objective function

$$W = E(\omega) - \frac{\rho}{2} Var(\omega),$$

where $Var(\cdot)$ denotes the conditional variance given current information, $E(\cdot)$ the expectations operator, and ρ the coefficient of absolute risk aversion. Maximizing this yields the speculation schedule

$$S^i = \phi E\left(\frac{e - \bar{e}}{\bar{e}}\right) = \phi \pi^e,$$

where

$$\phi = \frac{1}{\rho Var(\pi)} = \frac{1}{\rho \sigma^2}.$$

Here π^e is the expected rate of depreciation and σ^2 its variance as perceived by speculators. Thus risk-averse speculators will gradually commit more resources as expected depreciation rises. Their bets will be proportional to the expected rate of depreciation and inversely related to their degree of risk aversion and perceived variance of returns.

Assuming that each agent has finite resources of size \bar{S} , this will represent the largest attack possible by an individual and will be associated with an expected rate of depreciation of

$$\bar{\pi}^e = \frac{\bar{S}}{\phi}.$$

Any π^e above this level will not result in greater speculative pressure on the system. Since speculators are homogenous and hold identical expectations, aggregate sales by the private sector will be

$$S = \int_0^1 S^i di = \phi \pi^e.$$

Finally, dividing through by \bar{e} yields (2).¹⁵ Note that if speculators suddenly become less risk-averse or believe that there is less risk in betting against the currency, then ϕ will rise and so will the size of their bets.

¹⁵This speculation schedule has been adopted by many authors including Eaton and Turnovsky (1984), and Calvo and Mendoza (1996).

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