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International Debt and the Price of Domestic Assets

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Abstract

This paper examines the behavior of indebtedness, consumption, and asset prices in a small open economy in which the foreign real interest rate depends not only on an exogenous world interest rate and on indebtedness, but also on the value of the capital stock, viewed as an implicit "collateral," and hence on the price of capital. The paper finds that the collateral effect magnifies the intensity of shocks to the economy and the duration of their impact. The collateral effect also generates additional distortions that could lead to overborrowing. The paper discusses the policy responses to these distortions.

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I. INTRODUCTION

The role of the price of assets in explaining economic fluctuations has been receiving increasing attention as of late, in particular in the specialized press and in policy discussions in various fora. To a large extent, this is due to the recent crises in the economies of the Far East, but also to the current stock market boom in the United States.[2] Although the link among the price of domestic assets, shocks, and fluctuations in borrowing conditions is not new and was already discussed by Irving Fisher (1933) in his debt deflation hypothesis, the issue has received scant explicit and rigorous analysis in the literature. One important exception is the recent work by Kiyotaki and Moore (1997), who, in the context of a closed economy, consider a situation in which firms' borrowing possibilities are hindered by the value of their collateral and productivity shocks can generate credit cycles. Another exception is the work by Bernanke and Gertler (1999) and Bernanke, Gertler, and Gilchrist (2000).[3]

Several well-known papers have incorporated the idea of total debt as a determinant of the interest rate faced by domestic borrowers in the discussion of capital market imperfections.[4] This literature focuses on cases in which foreign lenders do not verify the repayment capacity of individual borrowers (no verification). This implies that all residents of the country are charged the same rate, which is determined by the country's aggregate indebtedness.[5] An externality arises as a result, as individuals disregard the effect of their own indebtedness on the interest rate charged to all of them, and overborrow. Indeed, this externality is sometimes considered as an argument for the imposition of capital controls—in the form, for example, of a tax on indebtedness that would equate the private and the social costs of indebtedness.

This paper takes this analysis further by examining the behavior of indebtedness, consumption, and asset prices in a small open economy in which the foreign real interest rate depends not only on an exogenous world interest rate and indebtedness, but also on the value of the capital stock, viewed as an implicit collateral, and hence on the price of capital. In this

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[2] Consider, for example, the statement by the Chairman of the Fed, Alan Greenspan, in his address to the Federal Reserve Bank of Kansas City symposium in Jackson Hole, Wyo., Aug. 27, 1999: "...our analytic tools are going to have to increasingly focus on changes in asset values and resulting balance sheet variations if we are to understand these important economic forces. Central bankers, in particular, are going to have to be able to ascertain how changes in the balance sheets of economic actors influence real economic activity and, hence, affect appropriate macroeconomic policies".

[3] See also the recent working papers by Caballero and Krishnamurthy (1999) and Elul (1999).

[4] See, for example, Bardham (1967), Aizenman (1989), and Agénor (1997a).

[5] Agénor (1997a) takes up the case of full verification at the individual level.

respect, the paper is in the spirit of Kiyotaki and Moore (1997). In this paper, the term collateral should be interpreted more as a general indicator of repayment capacity rather than as collateral in the strict legal sense. The inclusion of the value of capital in the determination of real interest rates is a realistic feature that allows us to capture the observed association of asset-price fluctuations with booms and busts in economic activity.

The paper reaches three basic conclusions. First, the presence of collateral both magnifies the impact effect and increases the persistence of shocks, regardless of verification possibilities of debt and capital. A temporary rise in the world basic interest rate, for example, induces agents to decrease their level of indebtedness by means of selling their physical capital, which produces an immediate fall in the price of capital, and in the value of the collateral. As a result, the interest rate immediately increases (because of the fall in the price of capital) and remains higher than in the case in which the collateral does not matter. A similar magnification occurs with the level of consumption and the current account. These results are in agreement with those obtained for the closed economy case in Bernanke, Gertler, and Gilchrist (2000).

Second, when foreign lenders cannot verify the repayment capacity of individual borrowers (lack of verification), the presence of the collateral as a determinant of the interest rate leads to a disparity between the private and social benefits of holding physical capital (the variable that individuals can control). Symmetrically to what happens in the case of indebtedness, the market solution is one in which individuals hold too little capital and a Pareto optimum calls for a subsidy on the holding of capital. We will refer to this outcome as the *collateral distortion*, as opposed to the well-known *indebtedness distortion*. We show how a combination of a tax (on indebtedness) and a subsidy (on collateral) can replicate the full verification outcome and hence eliminate both distortions, and how such a scheme under fairly general conditions is self-financed.

Third, we find that even with perfect verification, and hence in the absence of indebtedness and collateral distortions, the market solution involves still another externality. The interest rate depends on the *value* of capital (units of capital times their price). Since individuals cannot affect the price of capital, they will not consider the effect of their desire to hold physical capital on the interest rate. They thus will tend to hold too low a collateral. Note that this distortion (which we call "asset price" distortion) is different from the collateral distortion referred to in the previous paragraph. We prove that the full verification solution can be improved upon via a subsidy on the holding of capital. As elaborated later, this asset-price distortion issue is remarkably akin to the question in the discussion of the optimum quantity of money.

The organization of the paper is as follows. Section 2 presents a simple general equilibrium model and discusses its steady-state and adjustment properties. Section 3 discusses the adjustment patterns following either a permanent or a transitory change in the world interest rate. Section 4 discusses the problem arising from the lack of individual verification and the policy solution to the problems, and Section 5 focuses on the asset-price distortion that arises even if verification is perfect. The results are summarized in Section 6.

II. The Basic Model

A. The Environment

The economy is assumed to be populated by an infinite number of identical agents, each endowed with an equal and fixed amount of labor. There is perfect foresight and time is continuous. The technology is given by a constant returns to scale production function, with inputs labor and capital,

$$f(k_i) \qquad f_k > 0,\ f_{kk} < 0$$

where k_i is the ratio of capital to labor for agent "i". Aggregate labor is normalized to unity, so that, in the aggregate, k is the stock of capital, which is assumed to be fixed (like non-depreciating, nonreproducible trees).[6] We will also assume that only residents can own capital.[7]

Individuals can borrow in international markets, so that the budget constraint for typical individual "i" can be written as

$$f(k_i) + \frac{db_i}{dt} = c_i + p\frac{dk_i}{dt} + r_i b_i \qquad (1)$$

where r_i is the interest rate faced by the individual, b_i is the individual's level of indebtedness, c_i is the individual's consumption, and p is the price of capital in terms of the consumption good (the price of trees in terms of fruits, for example) so that pk_i is the value of individual's existing capital.

The individual's wealth constraint, is given by,

$$a_i = pk_i - b_i \qquad (2)$$

where a_i is the individual's wealth.

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[6] Alternatively, we could assume a linear technology with no labor, and capital as the only input, with no change in the results.

[7] See below, last part of Section 2, for an elaboration of the rationale and the significance of this assumption.

At each point in time the typical individual maximizes the utility functional

$$V_i = \int_0^{\infty} u[c_i(t)]\, e^{-\rho t}\, dt \qquad (3)$$

where ρ is the rate of time preference, and $u(\,)$ is the instantaneous utility function satisfying the usual Inada conditions.

We now need to specify the mechanism governing the determination of the interest rate. We assume that the interest rate faced by the country's residents depends (positively) on some risk-free world rate, r^w (which we will assume to be lower than the rate of time preference[8]), the level of debt, b, and (negatively) on the value of capital, pk, or "collateral," which for convenience we will call v. More specifically, we will assume that the interest rate will depend on the ratio of indebtedness to the value of capital (i.e., that the corresponding function is homogeneous of degree zero in both b and v), so that

$$r = \begin{cases} r\left(r^w, \dfrac{b}{v}\right) & for \quad b \geq 0 \quad r_1 > 0, \quad r_2 > 0, \quad r_{12} = 0, \; r_{22} > 0 \\ r^w & for \quad b < 0 \end{cases} \qquad (4)$$

Note that the restrictions placed in expression (4) imply that it is a convex function in the ratio of debt to collateral, and that domestic borrowers face an upward-sloping supply curve for credit. This r is the rate that makes foreign lenders indifferent about whether to lend to domestic residents or elsewhere at the basic interest rate r^w. For a justification of an expression such as (4), but with only the level of indebtedness as an argument, see, for example, Agénor (1997a) and references therein. Although such a formulation implies uncertainty on the parts of foreign lenders, perfect foresight is assumed for domestic residents.[9]

The following analysis will distinguish between two possible extreme cases. In the first case, we assume that full verification on the individual's levels of debt and physical capital (and hence the value of capital) exists. In this case, individuals will take into account

[8] As will become clear later, given the specification of the interest rate function, a rate of time preference equal to or lower than the world basic interest rate would generate a steady state with zero or negative indebtedness.

[9] This uncertainty of the foreign lenders is what allows our model to show a steady-state solution in which those lenders can permanently receive an interest rate higher than the risk-free rate r^w.

the consequences of their indebtedness and capital acquisition decisions on their own interest rate: the private costs and benefits of holding debt and capital will be equal to the social costs and benefits. In the second case, we assume that individual verification is impossible: thus individuals will not internalize those consequences and a distortion will arise both for the level of indebtedness and the level of the value of capital. When verification is absent, the introduction of collateral as an argument in the interest rate function will yield a result symmetric to the well-known result of overborrowing.

Note a few additional features and the justification of the simple model. First, the stock of physical capital is assumed to be constant in order to capture some features of the short and medium run, for which the assumption is not only convenient but realistic. There is perfect competition in the market for capital, and individuals take the price of capital as given, while the level of their physical capital is a choice variable. At the aggregate level, considering that the aggregate stock of capital is constant, the price of capital will always adjust to clear the market. Second, even if total indebtedness is lower than the value of the capital stock, the interest rate faced by the country's residents will still be higher than the world basic rate, r^w. Third, since consumption and debt accumulation patterns satisfy overall transversality conditions, Ponzi games are ruled out. Also, we assume that individual contracts are enforced and the possibility of bankruptcies is ruled out. Finally, the introduction of competitive financial intermediaries into our framework does not provide more insights, because with no asymmetries of information and no imperfect competition in the domestic economy, the presence of financial intermediaries becomes redundant.

B. Solving the Model

Maximization of (3) subject to (1) yields the individuals' optimality Euler equations

$$\frac{dc_i}{dt} = \left\{\frac{u_{c_i}(c)}{-u_{cc_i}(c)}\right\}\left\{\frac{f_{k_i}}{p} + \left(\frac{1}{p}\frac{dp}{dt}\right) - h\, b_i \frac{\partial r_i}{\partial v_i} - \rho\right\} \qquad (5)$$

$$\frac{dc_i}{dt} = \left\{\frac{u_{c_i}(c)}{-u_{cc_i}(c)}\right\}\left\{r\left(r^w, b/pk\right) + h\, b_i \frac{\partial r_i}{\partial b_i} - \rho\right\} \qquad (5')$$

where $v_i = k_i p$. [10] In the derivation of (5) and (5') we have introduced the parameter h in order to specify either full verification (in which case $h = 1$) or complete lack of verification (in which case $h = 0$). In the first case, the interest rates charged to individual agents will

[10] In what follows, in order to minimize clutter we omit the arguments in the derivatives $\partial r / \partial b$ and $\partial r / \partial v$, which obviously depend on world interest rate, indebtedness, and the price of capital.

depend on the agents' ratio of debt to collateral; the individual would thus take into account the effect of their debt-to-collateral ratio on their own interest rate. In the second case, foreign lenders observe only the aggregate ratio, and individuals know that their own ratio has no effect on the interest rate they face.

From (5) and (5') we obtain the asset price arbitrage condition

$$\left(\frac{1}{p}\frac{dp}{dt}\right) = r\left(r^{w}, b/pk\right) - \frac{f_{k_i}}{p} + h\, b_i\left[\frac{\partial r_i}{\partial b_i} + \frac{\partial r_i}{\partial v_i}\right] \qquad (6)$$

Aggregation of (5'), (6), and (1) over all individuals yield the system of three differential equations which, using the restrictions imposed in the interest rate function (4) and after some simple substitutions, can be expressed as

$$\frac{dc}{dt} = \left\{\frac{u_c(c)}{-u_{cc}(c)}\right\}\left\{r\left(r^{w}, b/pk\right) + h(b/pk)\,\frac{\partial r}{\partial (b/pk)} - \rho\right\} \qquad (7)$$

$$\left(\frac{1}{p}\frac{dp}{dt}\right) = r\left(r^{w}, b, /pk\right) - \frac{f_k(k)}{p} + h\,(b/pk)\,\frac{\partial r}{\partial (b/pk)}\left[1 - (b/pk)\right] \qquad (8)$$

$$\frac{db}{dt} = c\ + r\left(r^{w}, b/pk\right)b - f(k) \qquad (9)$$

Note that the term $\partial r / \partial (b/pk)$ depends on the ratio of debt to the value of capital, *b/pk*.

Expressions (7), (8), and (9) describe the behavior of the economy. Observe that in this aggregate system the stock of debt, *b*, is the state variable that cannot endogenously jump (it is, as it is sometimes said, a "crawler" rather than a "jumper").[11]

[11] At the aggregate level, at any point in time *t*, total net wealth is $a_t = pk_t - b_t$. At each point in time there is a well-defined price of capital; indeed, a discontinuity in the price taking place at any time *t* is the result of a market clearing condition, so that transactions can take place at the new market clearing price at t^+, p_{t^+}. At such a well-defined price, it will be

(continued...)

Equating (7), (8), and (9) to zero yields the steady-state equilibrium conditions which, after some simple substitutions, can be expressed as

$$r\left(r^{w}, b^{*}/p^{*}k\right) + h(b^{*}/p^{*}k)\frac{\partial r}{\partial(b^{*}/p^{*}k)} = \rho \qquad (7')$$

$$p^{*} = \frac{f_{k}(k)}{\rho - h(b^{*}/p^{*}k)^{2}\dfrac{\partial r}{\partial(b^{*}/p^{*}k)}}. \qquad (8')$$

$$c^{*} = f(k) - r\left(r^{w}, b^{*}/p^{*}k\right)b^{*} \qquad (9')$$

These expressions are familiar. The first shows that in the steady-state equilibrium at which consumption is not changing, the marginal cost of borrowing is equal to the rate of time preference: the equilibrium interest rate is lower when verification exists, and therefore individuals internalize the effects of their borrowing on the interest rate they face. The second says that in the steady state, the equilibrium price of capital that would induce individuals to hold the existing stock of physical capital is equal to its marginal return (the marginal product of capital) divided by its net marginal cost—the rate of time preference, minus, if internalized, the marginal collateral gain of holding an additional unit of physical capital. The equilibrium price of capital will be higher with verification than without it. Finally, the last expression is simply the economy's overall resource constraint. Note that the steady-state levels of indebtedness and the price of capital can be solved from the first two equations. The first of these equations is sufficient to determine the equilibrium ratio of debt to the value of capital—hence the equilibrium interest rate—and in the second equation this ratio determines the equilibrium price of capital, thereby the level of indebtedness.

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true that in the aggregate $\Delta a_{t^{+}} = -\Delta b_{t^{+}}$, because, in the aggregate, $\Delta k_{t} = 0$. But note that this expression is incompatible with the sum of individual wealth constraints (2): individuals can choose the composition of their net wealth by exchanging debt for capital (or vice versa), but cannot alter its level. Notice, however, that aggregate debt—hence aggregate net wealth—could change, for example, as the result of an exogenous event, such as a condonation of part of the debt—in the same way, for example, as a state variable, such as the stock of capital, can be changed exogenously by an earthquake. On the other hand, a jump in the level of prices (from $p_{t^{-}}$ to $p_{t^{+}}$) will result in a change of aggregate wealth by the magnitude $\Delta a_{t} = k_{t}\Delta p_{t}$.

Obviously, the system (7')–(9') simplifies a great deal in the case of no verification ($h = 0$). In particular, the steady-state price of capital becomes the ratio of the marginal product of capital and the rate of time preference. When verification takes place, although the algebra is more tedious, it is possible under reasonable assumptions (as shown in the appendix) to specify the signs of the partial derivatives of the steady-state level of all variables with respect to parameters, as follows:

$$b = b\left[\underset{-}{r^w},\ \underset{+}{\rho},\ \underset{+}{k}\right]$$

$$p = p\left[\underset{-}{r^w},\ \underset{-}{\rho},\ \underset{?}{k}\right] \qquad (10)$$

$$c = c\left[\underset{+}{r^w},\ \underset{?}{\rho},\ \underset{+}{k}\right]$$

In the appendix it is also shown that the steady-state equilibrium is unique and that the system has a unique saddle path.

Note that if the sale of titles of capital's ownership to foreigners were allowed, the price of capital to foreigners would be equal to $f_k(k)/r^w$ (since the riskless return to capital is $f_k(k)$). For any positive level of indebtedness, such price in steady state would be higher than the domestic price. Residents would then find it advantageous to sell their capital to foreigners and, by decreasing their debt, increase their consumption. This is due to the fact that, the residents' interest rate being higher than the interest rate in the rest of the world, the price of capital will be consistently lower than the price that foreigners would be willing to pay. Consider, for simplicity, the case of no verification. In the steady-state, an individual's consumption is

$$c_i = f(k_i) - r_i b_i$$

Since individuals will take the interest rate as given, the change in steady-state consumption resulting from a sale of capital to foreigners, at the price $f_k(k)/r^w$, will be

$$dc_i = f_k(k_i)\, dk_i - r\ db_i = f_k(k_i)\,[1-(r/r^w)]\, dk_i$$

Since $r > r^w$ as long as $b > 0$, individuals will find that consumption can be increased by selling capital at the international price and repaying debt, as long as debt is positive. The result, then, would be the elimination of debt. This is certainly not a desirable feature in a model that is studying an economy that is indeed holding debt.[12] For this reason, the model in this paper assumes that only residents can own capital.

III. Response to Shocks: The Case of the World Basic Interest Rate [13]

In order to gain some understanding of the implications of the dependence of the interest rate on collateral, we now study the effects of an exogenous shock in the form of a permanent or transitory unanticipated increase in the world basic interest rate, r^w. More specifically, we compare the effects of the shock when the value of the collateral is a factor determining the interest rate, and when it is not. In the latter case the general system (7)–(9) reduces to the two equations (7) and (9) in the variables c and b. Once the paths of these variables are obtained, equation (8) determines the path of prices.

Because of the complexity of the system with three differential equations for the case in which the value of the collateral is a factor determining the interest rate, we study these responses by performing some simulations.[14] To this effect (also for the analysis in the following sections), we assume the production function to be of the Cobb-Douglas type, that is

$$f(k) = y_o \, k^{\alpha} \qquad (11)$$

and the utility function to be of the logarithmic type

$$u = ln \; c \qquad (12)$$

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[12] What is essentially going on here is that trade in financial assets (borrowing and lending) is a substitute for trade in real capital (the possibility of selling property titles to foreigners). Since in our model foreigners perceive risk when lending, and no risk while holding domestic capital, if both types of exchange are allowed then the second (trade in capital) will dominate and eliminate the first.

[13] The characterization of a shock in the context of this model is different than in the case of stochastic discrete time models. Here, a shock is defined as an exogenous change in a parameter that lasts either forever (when the shock is permanent) or for a well-defined length of time (when it is transitory).

[14] For each simulation, the eigenvectors were calculated using a Maple program; once those values were obtained, the paths of the variables were simulated in a simple worksheet.

The interest rate function (4), in turn, is assumed to take the particular form

$$r = r^w + \mu\left[\frac{b}{v}\right]^\gamma \quad \gamma > 1, \quad \mu > 0 \tag{13}$$

Notice that $\gamma > 1$ means that the function is convex in the ratio of debt to collateral. In the simulations we use the parameter values

$$y_o = 1,\ k = 1.000,\ \alpha = .4,\ r = .04,\ r^w = .03,\ \mu = .01,\ \gamma = 1.2$$

Note that such values, and the simulations themselves, are not intended as a calibration exercise, but as a way to obtain information on the qualitative behavior of the variables. For the case where collateral does not matter we use a particular case of (4):

$$r = r^w + \mu\, b^\gamma \tag{13'}$$

In these simulations we take the case in which there is no verification ($h = 0$). Alternative simulations show that for this experiment the qualitative results are the same for both verification and without verification.

Note that for the two cases, we are comparing the steady-state values of the interest rate and the price of capital that will be the same, but not the steady-state levels of indebtedness and consumption. In order to facilitate the comparison, we will portray these last two variables as percentages of their initial steady-state values. An alternative formulation would have been to specify a form $r = r^w + \bar{\mu}\, b^\gamma$, with $\bar{\mu} = \mu / (p^* k)^\gamma$, where p^* is the steady state price of capital. Under this alternative formulation the initial steady-state levels of indebtedness and consumption would be the same whether collateral matters or not. In either case, results of simulations performed under this alternative assumption are similar to those reported in this section.

A. A Permanent, Unanticipated Shock

Consider, first, the case of an initial steady-state situation in which at $t = 0$ a permanent unanticipated increase in the basic world interest rate from $r^w = .03$ to $r^w = .035$ takes place.

The response to the shock is presented in Figure 1. The graphs show the responses of indebtedness and consumption (as a proportion of their original levels), the price of capital, the current account, and the interest rate both for when collateral matters and when it does not. Here, the results are very much in line with what one would intuitively expect. Debt

starts to fall and consumption initially drops before starting to increase to its higher new steady-state value.[15] The current account (which is simply the negative of the slope of indebtedness) becomes positive and remains positive throughout the adjustment, reflecting the asymptotic decrease in indebtedness.

The important thing to observe, however, is the much stronger effect of the change in the world interest rate when collateral matters, both in terms of the initial impact and the path of adjustment on the control variables (consumption, the price of capital, the current account, and the interest rate). At the same time, notice the much slower response, when collateral matters, of the state variable—the level of indebtedness. In fact, these two features are part of the same phenomenon: when collateral matters, the adjustment of indebtedness is much slower, and as a result the control variables vary with higher amplitude. In a sense, it could be said that the presence of the collateral decreases the ability of debt to act as a shock absorber. The interest rate initially rises by more than the increase in the world basic rate, because the fall in the price of capital magnifies the exogenous change. The price of capital falls, because as individuals face a rise in the cost of keeping debt, they tend to readjust their portfolio—at the individual level, decreasing their level of debt via sale of real capital. Since the stock of capital is fixed, its price falls. Not only does the interest rate initially rise by more, but it remains considerably higher for longer than in the noncollateral case.[16] In turn, when collateral matters, indebtedness remains at higher levels, and the price of capital at lower levels. Note also the much higher (proportional) changes in steady-state levels of both indebtedness and consumption when collateral matters.[17]

B. A Transitory Shock

Consider now the case of a transitory shock of the same magnitude taking place under the same initial conditions. In particular, we assume that it is to last for five periods. Results of the simulations are reported in Figure 2, again for cases in which collateral matters and those in which it does not.

For the case of a transitory shock, essentially similar conclusions apply to the behavior of the domestic interest rate and the level of consumption. The presence of

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[15] The result that consumption rises in the long run should not be misinterpreted as an indication that the economy is better off because of the rise of the world interest rate—notice the lower level of consumption during most of the adjustment period.

[16] Observe that when the collateral does not matter, the initial rise reflects only the rise in the world's basic rate, since the level of indebtedness is a state variable.

[17] In both cases the new steady-state level of consumption is higher, although the increase is too small when collateral does not matter for the graph in Figure 1 to reflect it clearly, due to the scale of the vertical axis.

collateral magnifies the initial impact and the effect of the shock while the shock lasts and diminishes the fall of the interest rate when the shock ends, so that the interest rate is always higher than when collateral does not matter. The level of consumption, in turn, initially falls by a much higher proportion and is (proportionally) lower at all times. As in the case of a permanent change in the world interest rate, the level of debt remains at higher (proportional) levels during the whole episode, and the price of capital remains lower than when collateral does not matter. More importantly, for the particular transitory shock that we have simulated, the level of indebtedness starts to increase and remains higher than the steady- state level throughout the adjustment to the shock. This is clearly reflected in the graph depicting the current account, which becomes negative and remains so during the duration of the shock.

A transitory increase in interest rates brings about both a wealth and a substitution effect. The former calls for higher debt in order to smooth consumption; the latter calls for lower debt in response to its higher cost. In the case of a permanent change a substitution effect clearly dominates; if the shock is of short duration the wealth effect dominates. In fact, a simulation with the same parameters but lasting 10 periods, rather than 5, would have also shown an initial fall in indebtedness, rather than a rise. In this respect the simulations suggest that the wealth effect is enhanced when collateral matters.[18]

IV. Correcting for the Lack of Verification Problem

It is well known that when the level of indebtedness influences the interest rate an externality arises—as in the classic "road congestion problem"—to the extent that verification at the individual level is incomplete or nonexistent: individuals will not internalize the effect of their indebtedness on the interest rate paid by all, and a gap will exist between the private and the social cost of holding debt. This case is presented in the literature as a justification for a tax on foreign debt to close such a gap.[19] When collateral matters, a similar problem arises in the absence of individual verification: since the interest rate depends on the value of aggregate collateral and individuals cannot capture the full benefits of holding a higher level of physical capital (the variable that they see as the one they can privately control), they will hold collateral at less than the aggregate optimal level. If this is the case, then the policy called for is the granting of a subsidy on the value of collateral.

When there is no individual verification, the appropriate level of tax (on debt)-cum-subsidy (on collateral) that replicates perfect verification can be easily calculated for the steady state.[20] Let ℓ be percentage tax on the holding of debt and s percentage subsidy on the

[18] These results are similar to the findings in Bernanke, Gertler, and Gilchrist (2000) analysis of the "financial accelerator."

[19] See, for example, Bardhan (1969), Aizenman (1989), and Calvo (1988).

[20] We should stress that these are comparative statics results valid for the steady state and that, as usual, they need to be adjusted for events taking place during the adjustment process.

value of collateral. By introducing these values into the individual's budget constraint, it is easy to show that, for the general case in which the coefficient h takes the values of zero or one for the cases of no verification and perfect verification, the system becomes

$$\frac{dc}{dt} = \left\{\frac{u_c(c)}{-u_{cc}(c)}\right\}\left\{r\left(r^w, b/pk\right) + \ell + hb\frac{\partial r}{\partial b} - \rho\right\} \qquad (14)$$

$$\left(\frac{1}{p}\frac{dp}{dt}\right) = r\left(r^w, b/pk\right) + \ell - s - \frac{f_k}{p} + h\,b\left[\frac{\partial r}{\partial b} + \frac{\partial r}{\partial v}\right] \qquad (15)$$

$$\frac{db}{dt} = c + r\left(r^w, b/pk\right)b - f(k)^{21} \qquad (9)$$

In the steady state the following two general conditions will obtain

$$\frac{f_k}{p} + s - h\,b\,\frac{\partial r}{\partial v} = \rho \qquad (15')$$

$$r\left(r^w, b/pk\right) + \ell + hb\frac{\partial r}{\partial b} = \rho \qquad (14')$$

where we have used $v = k\,p$. With full verification ($h = 1$) and no taxes or subsidies, the steady state values satisfy

$$\frac{f_k}{p} - b\,\frac{\partial r}{\partial v} = \rho \qquad (15'')$$

[21] Equation (9) implicitly assumes that government will always balance its budget by imposing (granting) neutral head taxes (subsidies) so as to offset differences between expenditures generated by the subsidy on collateral and revenues generated by the imposition of the tax on debt.

$$r\left(r^{w}, b/pk\right)+b\frac{\partial r}{\partial b}=\rho \qquad (14'')$$

The values of p and b satisfying (14'') and (15'') are those we wish to replicate through the use of the available taxes and subsidies for the case without verification. We call these values p^* and b^*. Without verification ($h = 0$), the subsidy rates on the value of collateral and tax on indebtedness to replicate the values p^* and b^* are given by

$$s^{*}=\rho-\frac{f_{k}(k)}{p^{*}} \qquad (16)$$

$$\ell^{*}=\rho-r(r^{w}, b^{*}/p^{*}k) \ ^{22} \qquad (17)$$

Note that from the previous expressions (14') to (17), after performing a few simple substitutions, the values of s and ℓ required for the replication of the full verification solution can also be expressed as levels for which

$$s^{*}=-b^{*}\frac{\partial r}{\partial v} \qquad (16')$$

$$\ell^{*}=b^{*}\frac{\partial r}{\partial b} \qquad (17')$$

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[22] For the case of the interest rate function (13), the levels of subsidy on collateral and the tax on indebtedness become

$$\ell^{*}=\frac{\left[\rho-r^{w}\right]\gamma}{1+\gamma}$$

and $$s^{*}=\ell^{*}(b^{*}/v^{*})=\gamma\mu\left[\frac{(\rho-r^{w})}{\mu(1+\gamma)}\right]^{\frac{1+\gamma}{\gamma}}.$$

Although we can always assume the government can balance its budget by imposing (granting) additional neutral head taxes (subsidies), we are interested in analyzing the budgetary impact of the subsidy-cum-tax policy that replicates the full verification solution. For that purpose, let us define the difference between the government's payment of subsidy and tax receipts as

$$D = s^* v^* - \ell^* b^* \qquad (18)$$

Replacing (16') and (17') into (18) yields

$$D = -b\frac{\partial r}{\partial v}v - b\frac{\partial r}{\partial b}b = -b\left(\frac{\partial r}{\partial v}v + \frac{\partial r}{\partial b}b\right) \qquad (18')$$

If the function $r\,(r^w,\ b,\ v)$ is homogeneous of degree zero in b and v, as we have assumed, then

$$\left(\frac{\partial r}{\partial v}v + \frac{\partial r}{\partial b}b\right) = 0$$

so that $D = 0$. The tax-cum-subsidy strategy that replicates the verification equilibrium is self-financed; that is, it does not require the imposition or the granting of additional head taxes or subsidies.

V. The Asset-Price Externality and Some of its Implications

Suppose that perfect verification exists for both debt and collateral (or, alternatively, that full verification is perfectly replicated via the appropriate tax-cum-subsidy). It is possible to show that the presence of collateral as an argument of the interest rate faced by residents generates another externality, which is totally independent of verification.

When the value of collateral matters for the interest rate, both the quantity of physical capital and the price of capital will influence the interest rate. In our model, the aggregate level of capital is fixed, but individuals see their own physical capital as the variable they can control. Without verification, individuals seek to hold physical capital taking account only of capital's productive capacity (fruit from a tree, in our analogy). With perfect verification, they will, in addition, take into account the effect of their holding of physical capital on their

collateral and hence on the interest rate they face. But they still *will not take into account the effect of their desire to hold physical capital on the price of capital*, since they cannot individually capture such an effect.

There is a clear analogy between collateral in our model and the "optimal quantity of money" arguments. Take the case in which utility depends on the level of the real money stock. If every individual decides to hold additional nominal money (in violation of their private marginal optimality conditions) the result would be a rise in the price of money (i.e., a fall in the price level), an increase in the real money stock, and higher utility for all individuals. But this would not be a sustainable Nash equilibrium, because individuals will have an incentive to satisfy their optimality conditions and depart from it. The social optimum needs to be generated by a social planner via a subsidy on the holdings of money, either by payment of interest on money or by deflation. In our case, suppose that all individuals simultaneously decide to hold more collateral by holding additional physical capital (again, in violation of their private optimality conditions). The result will be a rise in the price of capital and the value of collateral, hence a fall in the interest rate.[23] As in the case of money, however, this is not a Nash equilibrium, and it would need to be induced by a policy subsidizing [the holding of] collateral. However, as it is shown below, whereas in the case of money the optimal subsidy is the one for which the level of satiation is reached (i.e., when the nominal interest rate is driven to zero), in the context of our simple model a corresponding well-defined social optimum does not correspond to a finite subsidy.[24]

Consider some of the operational implications of the price externality we just described. First, it is possible to show that, starting from an initial steady-state equilibrium with perfect verification (i.e., in the absence of the indebtedness and the collateral externalities), there is a policy that will result in higher price of capital, and the same level of indebtedness, that will increase welfare. If a subsidy s on collateral is imposed, the result will be a rise in the price of capital, but it will also have the side effect of increasing the level of indebtedness. Consider, instead, a policy that grants the subsidy to collateral, but at the

[23] Note how remarkably appropriate the analogy is. The real money stock is the product of nominal money (which is seen as the control variable by individuals, but is fixed in the aggregate) times the price of money (the inverse of the price level, which is taken as given by individuals, but is the adjusting variable in the aggregate). The real value of the collateral is the product of physical capital (which is seen as the control variable by individuals, but is fixed in the aggregate) times the price of capital (which is taken as given by individuals, but is the adjusting variable in the aggregate). In both cases, in the language of optimal control theory, what is the control variable for individuals (nominal money in one case, physical capital in the other) is the state variable in the aggregate, and what is taken as the state variable by individuals (the price of money in one case, the price of capital in the other) is the control variable in the aggregate.

[24] But see below, last paragraph of this section.

same time introduces a tax on debt up to the point at which debt returns to the initial level. Such a combination would result in a new steady state at which indebtedness would be the same as without intervention and the interest rate would be lower, so that the level of consumption would be higher. Moreover, the new equilibrium would be attained at once, so that the reaction to such policy would be an immediate adjustment of consumption to its new higher steady-state level, allowing for a simple unambiguous conclusion about welfare.

We need to show, however, that such combination of subsidy-cum-tax exists. The procedure is similar to the one used in the previous section. Start from an initial equilibrium with full verification, with the values b_0, p_0, and c_0, and consider again expressions (14') and (15'), with $h = 1$. From (14'), the equilibrium debt-to-collateral ratio (b/pk) depends (inversely, as it can be easily shown) only on the tax on debt, and not on the subsidy to the value of collateral. On the other hand, from (15'), the price of capital depends on the subsidy on collateral and the debt-to-collateral ratio, and therefore also on the tax on debt. Assume we wish to preserve the initial no intervention level of indebtedness, b_o, and generate a rise in the price of capital (which implies a fall in the ratio of debt to collateral, and hence a fall in the interest rate). Suppose we introduce, first, a small tax on the holding of debt. As indicated by (14'), this will generate a different (smaller) ratio of debt to the price of capital. Calculate now the price that would generate this ratio for the original level of debt, b_o. Replacing the value of this price in (15'), as well as the value of the tax, we can then solve for the value of the required subsidy. The immediate result of such policy will be a rise in the price of capital, with the same level of debt, and hence an instantaneous adjustment to a higher steady-state level of consumption—a welfare improvement.

This finding proves that it is always possible to improve over the nonintervention equilibrium. It *does not* follow that such policy achieves an *optimum optimorum*. Indeed, a policy of subsidy-cum-tax that leaves the level of indebtedness constant leads to the unrealistic conclusion that higher and higher levels of appropriately combined taxes and subsidies would entail higher and higher levels of welfare. It is obvious from (13) that for a given level of indebtedness, as the price of capital increases, the interest rate decreases approaching asymptotically the world interest rate. This happens because in our simple model the price of capital matters only for determining the value of collateral and the interest rate.[25] In a more complete model with capital accumulation, the price of capital would matter for other variables and not only for determining the interest rate. We conjecture that in such a case, a well-defined optimum level of subsidy on the holding of capital would exist.

[25] Continuing the analogy with the case of money, note that the optimal subsidy on the holdings of money (interest on money, or deflation) is bounded, when real money is an argument in the utility function, only when there is a satiation point. When this is not the case, the same problem as in our model arises.

Within the context of our model, an alternative would be to define a threshold level κ of the debt-to-collateral ratio below which $r = r^w$, so that, for any given level of debt, a subsidy higher than the one generating such a ratio would be unnecessary.[26]

VI. Conclusions

We have analyzed the effects of the value of physical capital (implicit collateral) as one of the determinants of the borrowing world interest rate faced by residents of a small open economy, in addition to indebtedness. The main findings can be summarized as follows.

(i) The *collateral effect* tends to magnify the consequences of a permanent or a transitory shock, as an increase in the world basic interest rate, on variables such as the interest rate, consumption, the price of capital, and the current account. This magnification (the counterpart of which is a more delayed response in the level of indebtedness) takes the form of a higher impact effect of the shock and longer persistence of its consequences. As mentioned before, these results are similar to those found for the closed economy case reported in Bernanke, Gertler (1999) and Bernanke, Gertler and Gilchrist (2000).

(ii) In the absence of verification of the value of collateral at the individual level, agents will take the interest rate as given, without internalizing the effect that their decisions on how much physical capital to hold will have on the rate. The presence of this "collateral effect" introduces the same well-known distortion that occurs when debt influences the interest rate and individual verification is nonexistent. The solution to these twin distortions is the simultaneous imposition of a tax on debt and a subsidy on collateral. We show how such a policy can be self-financed, in the sense that government's revenues from the appropriate tax on indebtedness exactly matches government's expenditures on the appropriate subsidy on the value of capital.

(iii) We have also discussed the presence of yet another externality when collateral matters, which occurs even in the presence of full verification. Since even with full verification individuals take the price of capital as given, they will not internalize the effects of their decisions regarding holding physical capital on the price of capital, and hence on the interest rate. We established an analogy between this case and the distortion discussed in the literature on the optimum quantity of money, and proved that a subsidy on the value of collateral with a tax on indebtedness can improve on the market outcome even when verification exists.

[26] We are particularly indebted to Peter Montiel for discussions concerning this point.

Figure 1
A Permanent Rise in the Basic World Interest Rate

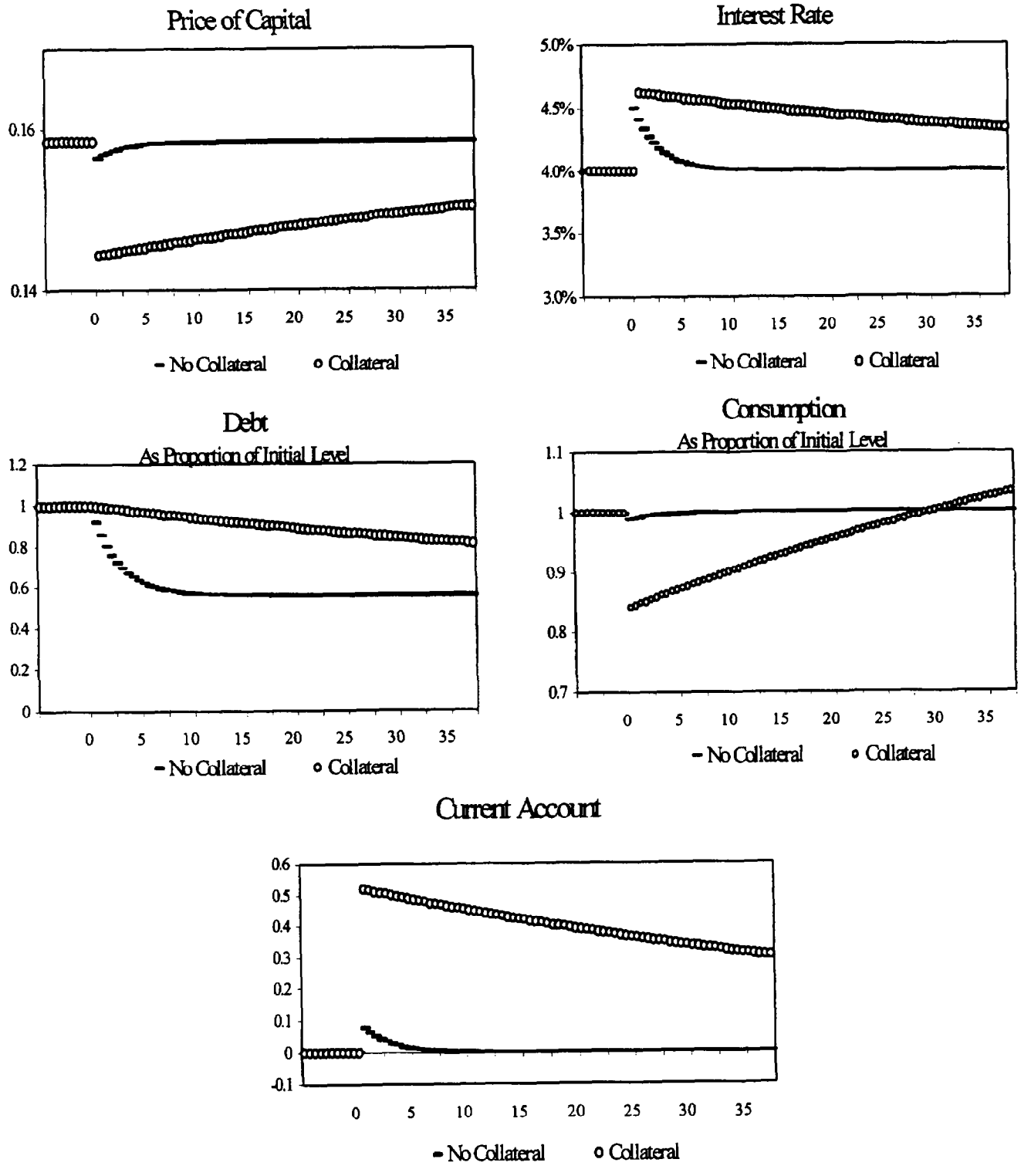
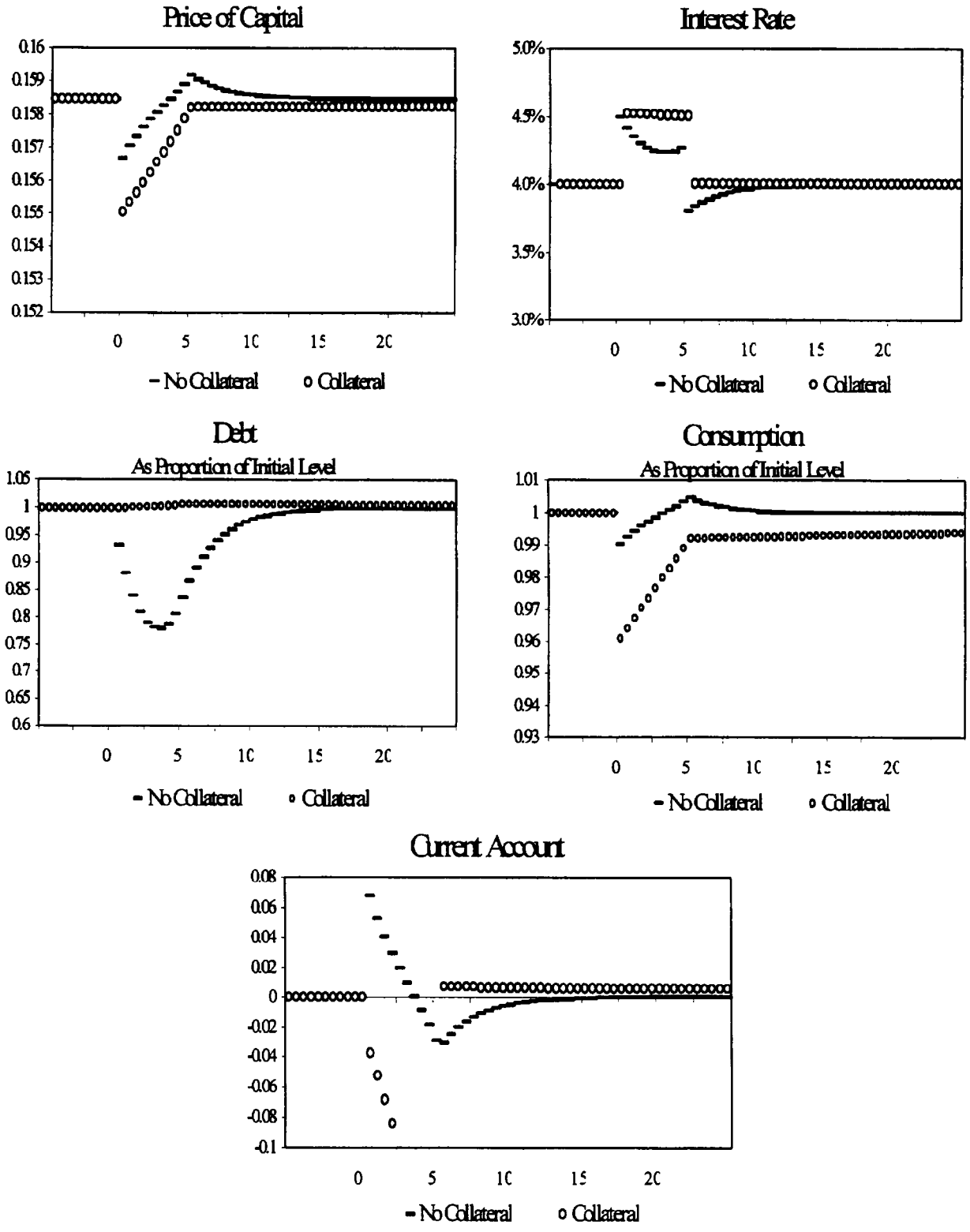


Figure 2
A Transitory Rise in the Basic World Interest Rate



Appendix

a) Dynamic properties

The assumptions on strict concavity of the utility and production functions presented in Section 2, guarantee the existence of a unique interior solution with positive levels of debt, consumption, and asset prices. In addition, we need to prove that the system described by equations (7), (8), and (9) exhibits saddle point stability. The proof proceeds in several steps:

First, we compute the linear approximation of the system (7)–(9) in the vicinity of the steady state.

$$\dot{x} \approx J\left(x - x_o\right) \tag{A.1}$$

Where x is the vector of endogenous variables (b, c, p), J is the Jacobian, and x_o (b_o, c_o, p_o) is the vector of steady-state values. In what follows we will denote partial derivatives with subscripts under the variable (e.g., $r_x \equiv \partial r / \partial x$).

The specific form of the matrix J is given by

$$J = \begin{bmatrix} r^* + b^* r_b & 1 & b^* k r_v \\ \dfrac{u_c(c^*)}{-u_{cc}(c^*)}\left(r_b + h r_b + h b^* r_{bb}\right) & 0 & \dfrac{u_c(c^*)}{-u_{cc}(c^*)}\left(k r_v + h b^* k r_{bv}\right) \\ p^*\left(r_b + h r_b + h b^* r_{bb} + h b^* r_{vb}\right) & 0 & v^* r_v + h b^* v^* r_{bv} + h b^* v^* r_{vv} + \dfrac{f_k(k)}{p^*} \end{bmatrix}$$

In order to prove the saddle point property we need to show that the sign of the determinant of matrix J is negative. For the case of full verification ($h=1$) the determinant is equal to

$$|J| = \frac{u_c(c^*)}{-u_{cc}(c^*)}\left[2 v^* b^*\left(r_v r_{vb} - r_b r_{vv}\right) - v^* b^{*2}\left(r_{bb} r_{vv} - r_{bv}^2\right) - \frac{f_k(k)}{p^*}\left(2 r_b + b^* r_{bb}\right) - v^* r_v^2\right] \tag{A.2}$$

For the case of no verification ($h=0$) the determinant is equal to

$$|J| = -\left[\frac{u_c(c^*)}{-u_{cc}(c^*)}\right]\frac{f_k(k)r_b}{p^*} < 0 \qquad (A.3)$$

Clearly, for the case of no verification, the determinant of J is negative and the system (7)–(9) exhibits saddle-point stability. However, for the case of full verification the direct computation of the sign of the determinant of J is cumbersome. We get around this problem by computing the derivative of p with respect to r^w, using expression (7') from the text,

$$\frac{\partial p^*}{\partial r^w} = -\frac{\left(\frac{b^*}{v^*}\right)\frac{\partial r^*}{\partial r^w}}{\rho - \left(\frac{b^*}{v^*}\right)^2 \frac{\partial r}{\partial (b^*/v^*)}} < 0 \qquad (A.4)$$

which can also be obtained by applying Cramer's rule to the linearized version of the system (7)–(9). In fact, by means of this procedure we obtain

$$\frac{\partial p^*}{\partial r^w} = -\frac{\left(\frac{\partial r}{\partial r^w}\frac{\partial r}{\partial v} + b^*\frac{\partial r}{\partial r^w}\frac{\partial^2 r}{\partial v \partial b}\right)}{|J|} \qquad (A.5)$$

Notice that the denominator of (A.5) is the same determinant of matrix J shown in (A.1).

The right hand side of expressions (A.4) and (A.5) are of the same sign. From the assumptions on the interest rate function, we know that the numerator of (A.5) is positive. This means that in order for (A.4) and (A.5) to be equivalent, the denominator in the right-hand side of (A.5) is negative.

b) Comparative statics

$$\frac{\partial b^*}{\partial r^w} = \frac{\left[\frac{u_c(c^*)}{-u_{cc}(c^*)} r_{r^w}\right]\left(h v^* b^* r_{vv} + \frac{f_k(k)}{p^*}\right)}{|J|} < 0 \qquad (A.6)$$

$$\frac{\partial b^*}{\partial \rho} = \frac{\left(v^* r_v + h v^* b^* r_{vb} + h v^* b^* r_{vv} + \frac{f_k(k)}{p^*}\right)}{|J|}; \qquad (A.7)$$

$$\frac{\partial b^*}{\partial \rho} > 0, \text{ for } \left(v^* r_v + h v^* b^* r_{vb}\right) < \left(h v^* b^* r_{vv} + \frac{f_k(k)}{p^*}\right)$$

$$\frac{\partial b^*}{\partial k} = \left\{\frac{\frac{u_c(c^*)}{-u_{cc}(c^*)}}{|J|}\right\}\left(r_v + h b^* r_{vb}\right)\left(2v^* p^* r_v + h2v^* p^* b^* r_{vb} + h2v^* p^* b^* r_{vv} + f_k(k) - k f_{kk}(k) p^{*2}\right) \qquad (A.8)$$

$$\frac{\partial b^*}{\partial k} > 0, \text{ for } \left(2v^* p^* r_v + h2v^* p^* b^* r_{vb}\right) < \left(h2v^* p^* b^* r_{vv} + f_k(k) - k f_{kk}(k) p^{*2}\right)$$

$$\frac{\partial p^*}{\partial r^w} = -\frac{\left(\frac{b^*}{v^*}\right)\frac{\partial r^*}{\partial r^w}}{\rho - \left(\frac{b^*}{v^*}\right)^2 \frac{\partial r}{\partial (b^*/v^*)}} < 0 \qquad (A.9)$$

$$\frac{\partial p^*}{\partial \rho} = \frac{\left[p^* r_b + h p^*\left(r_b + b^* r_{bb} + r_v + b^* r_{vb}\right)\right]}{|J|}; \qquad (A.10)$$

$$\frac{\partial p^*}{\partial \rho} < 0, \text{ for } \left(r_b + b^* r_{bb}\right) > \left(r_v + b^* r_{vb}\right)$$

$$sign \quad of \quad \frac{\partial p^*}{\partial k} = sign \quad of \quad \left\{ \frac{\frac{u_c(c^*)}{-u_{cc}(c^*)}}{|J|} \right\} \varphi \; = ? \qquad (A.11)$$

Where

$$\varphi = -f_{kk}(k)\left(r_b + hr_b + hb^* r_{bb}\right) + hp^{*2} b^* r_{vv}\left(2r_b + b^* r_{bb}\right) - (1+h)p^{*2} b^* r_v r_{vb} - p^{*2} r_v \left(hr_v + b^* r_{vb}\right)$$

$$\frac{\partial c^*}{\partial r^w} = -r^* \frac{\partial b^*}{\partial r^w} - b^* \frac{\partial r^*}{\partial r^w} > 0 \quad for \quad -r^* \frac{\partial b^*}{\partial r^w} > b^* \frac{\partial r^*}{\partial r^w} \qquad (A.12)$$

(A.13)

$$\frac{\partial c^*}{\partial \rho} = -r^* \frac{\partial b^*}{\partial \rho} - b^* \frac{\partial r^*}{\partial \rho} = ?$$

$$\frac{\partial c^*}{\partial k} > 0 \quad for \quad f_k(k) > r^* \frac{\partial b^*}{\partial k} \qquad (A.14)$$

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