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Real Effective Exchange Rate and the  
Constant Elasticity of Substitution  
Assumption

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**IMF Working Paper**

Research Department

**Real Effective Exchange Rate and the Constant Elasticity  
of Substitution Assumption**

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**Abstract**

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

The real effective exchange rate is an aggregation of several bilateral real exchange rates with respect to other countries. The aggregation is usually done under the assumption of constant elasticity of substitution (CES) between products from different countries. We investigate the validity of this assumption by estimating manufacturing export equations for 56 countries over 26 years. We find that the hypothesis of CES is rejected and that the export equations that contain two real effective exchange rates (one in relation to OECD countries and one in relation to non-OECD countries) perform on average considerably better than the traditional ones.

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## I. INTRODUCTION

A commonly used measure of competitiveness is the real effective exchange rate (REER), often calculated as a weighted average of the relative prices of a country with its main trade competitors. The trade shares of a country in each industry can be used to determine the weight each country should have in these calculations. This methodology can take into account both domestic and third-market effects. The Information Notice System (INS) of the IMF provides such weights for almost all countries, which are used extensively to calculate REERs. These weights are supposed to reflect the major competitors of each economy.

The current methodology for determining the weights in the REER estimations assumes that the elasticity of substitution is constant for products of different countries. In particular, the level of development of the country of origin is not relevant (the elasticity of substitution between U.S. manufactures and Japanese manufactures, is equal to the elasticity of substitution between U.S. manufactures and any other country's manufactures, either developed or developing).

If this assumption does not hold, then the current weights in the REER calculation are not accurate. A country with a high trade share for a product will be assigned a large weight, even when the elasticity of substitution is relatively small for the same product exported by other countries. In contrast, a country with a relatively small trade share for a product will be assigned a small weight, even when the elasticity of substitution is large.

The implications for both economic analysis and economic policy are obvious. If the elasticity of substitution assumption does not hold, each country will have the wrong idea of who its competitors are by using the current weights in the REER calculation. This implies that the exchange rate may be characterized as overvalued when in fact it is not, and the other way around.

One of the many examples that could be used to illustrate this point is the debate on a possible devaluation in China during the recent East Asian financial crisis. There was concern that if the Chinese authorities devalued their currency, they would fuel a new wave of competitive devaluation in the region. Was this concern justified? According to the INS weights, the answer is negative. China's weight is less than 3 percent for all East Asian economies, except for Hong Kong SAR. If we consider the list of the ten major competitors for each East Asian country, defined as the ten economies with the largest weights, China only makes it to the list for Hong Kong (Hong Kong in turn only makes it to the list for Korea and Singapore, but with very small weights, at 2.5 percent and 3 percent respectively). However, most economists and politicians involved in the East Asian crisis would disagree that a devaluation in China during this period would have had no impact. If they are right, then the current REER weights are inappropriate. One reason may be that if the exports of China have a relatively high elasticity of substitution for the exports of the other East Asian countries, then China should be assigned a higher weight than is currently assigned by its trading shares in the standard REER calculation.

This paper tests the validity of the hypothesis that the elasticity of substitution is constant between products coming from different countries. The test determines if the assumption holds, and if it matters when it does not, by estimating manufacturing export equations for 56 countries, for a period of 26 years. The equations include either the standard aggregated REER or two REERs disaggregated into two components: one REER with respect to Organization for Economic Cooperation and Development (OECD) countries and one with respect to non-OECD countries. If the assumption of the constant elasticity of substitution holds, any disaggregation of the REER should result in a reduction of the adjusted  $R^2$ . In addition, the estimated coefficients for any disaggregated REERs (two in our case) should be equal.

The results show that the hypothesis of constant elasticity of substitution is rejected and the export equations which contain two REERs perform on average considerably better than traditional ones. The estimates of the two REERs are significantly different, and export equations with two REERs perform better than the export equation with the currently used REER. These results are robust to panel and time series regressions, as well as to regressions for levels and changes and to several estimation techniques.

The disaggregation of the standard REER in the OECD REER and a non-OECD REER is arbitrary, but justified by the idea that products from similar countries should have a similar elasticity of substitution. We assess the plausibility of this idea by performing a simulation exercise in which we show that it is possible to increase the adjusted  $R^2$  in most cases by simply using two REERs with respect to randomly chosen groups of countries. However, choosing a grouping of OECD and non-OECD countries performs better than most other random country groupings.

Other studies that have disaggregated the REER in a similar way have also found that the explanatory power of trade equations improves significantly. Giorgianni and Milesi-Ferretti (1997) split the exchange rate for industrialized versus nonindustrialized countries to explain export demand for Korea. Faini, Clavijo and Senhadji-Semlali (1992) used such disaggregation to determine the benefits of devaluation for developing economies following an export-led strategy, when other developing economies were following similar policies. These studies in part motivate the chosen disaggregation in our paper.

This paper focuses only on the REER relevant for international trade. There are many different methodologies for calculating the REER, some of them more appropriate than others, depending on its end use. Maciejewski (1983) provides a review of the different real exchange rates used in the economic literature. The calculation based on trading weights is more relevant for trade purposes and this is why it is the measure often used in export equations.

Finally, a clarification is needed on why this paper focuses only on manufacturing exports. Weights for other sectors such as agriculture and tourism are available from INS. However, it is difficult to decide how to disaggregate the REER for these sectors and draw any clear conclusions for the elasticity of substitution. In this case, it is not clear that an OECD versus a non-OECD REER will be justified. Furthermore, the constant elasticity

assumption may not be as strong for nonmanufacturing sectors. As an example, it is easier to argue that the elasticity of substitution is constant (or at least doesn't differ much) for coffee coming from different countries, than for textiles.

The rest of the paper is organized in three main sections and two appendices. Section II is a brief description of the theoretical foundations behind the REER estimation. Section III is the empirical section, and presents the methodology, the empirical results and robustness tests. Section IV concludes the paper. Appendix I has the data sources and description, and Appendix II the detailed algebraic calculations for Section II.

## II. THEORETICAL FOUNDATIONS

This section briefly describes a demand system that is used as the theoretical foundation of the way the REER is estimated.<sup>2</sup> As will be clear from what follows, the assumption of constant elasticity of substitution between products coming from the same industry in different countries is central in this methodology. The weights assigned to each country in the REER calculations broadly used today are derived from the first-order conditions of a utility maximization problem, based on some restrictive assumptions. The constant elasticity of substitution assumption is one assumption that can be easily tested as this section shows.

Theories of demand for tradable goods often assume that a good supplied by one country is a perfect substitute for the same good supplied by other countries (the elasticity of substitution is infinite). This assumption is not realistic, since quality differences are significant among different products within the same category. This is one of the reasons that the law of one price does not hold and that countries engage in intra-industry trade. This assumption can be relaxed by assuming that products within an industry but coming from different countries are not perfect substitutes (imperfect substitutes model). This feature results in a demand system in which products are differentiated by their kind and by their country of origin. This is a key feature of the theoretical foundation for the derivation of the country weights broadly used in REER estimations.

Assume there are  $m$  countries, and symbolize the set of countries with  $C=(C_1, C_2, \dots, C_m)$ .  $C$  is also the vector of the sources of demand. Assume there are  $n$  goods, and symbolize the set of goods with  $X=(X_1, X_2, \dots, X_n)$ . The set of products that belong in good  $i$  is  $X_i = (X_{i1}, X_{i2}, \dots, X_{im})$ , where each product  $X_{ij}$  is produced by a different country. Therefore, the product vector is  $X = (X_{11}, X_{12}, \dots, X_{1m}, X_{21}, X_{22}, \dots, X_{2m}, \dots, X_{n1}, X_{n2}, \dots, X_{nm})$ , and there are  $mn$  products.

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<sup>2</sup> The analysis in this section follows Armington (1969). A similar presentation can be found in Dixit and Stiglitz (1977) and in Helpman and Krugman (1985), in their analysis of consumer preferences for variety.

Each good is differentiated according to the country of origin. Therefore,  $X_{ij}$  is assumed to be an imperfect substitute for  $X_{ip}$ , for  $j \neq p$ , which means that the law of one price does not hold. Therefore, each country has a demand function for each product  $X_{ij}$ . Since there are  $nm$  different products, there are  $m^2n$  product demands, of which  $nm$  are domestic demands.

Since products belonging to the same good category are not perfect substitutes, there are  $nm$  prices. Symbolize the price vector with  $P = P_{11}, P_{12}, \dots, P_{1m}, P_{21}, P_{22}, \dots, P_{2m}, \dots, P_{n1}, P_{n2}, \dots, P_{nm}$ .

Utility  $U$  will be a function of all  $nm$  products:  $U = U(X) = U(X_{11}, X_{12}, \dots, X_{1m}, X_{21}, X_{22}, \dots, X_{2m}, \dots, X_{n1}, X_{n2}, \dots, X_{nm})$ . If we assume that relative product evaluation at the margin in any market is not affected by purchases in the other markets, the utility function can be written in a "two-level form" as:

$$U = U(X_{11}, X_{12}, \dots, X_{1m}, X_{21}, X_{22}, \dots, X_{2m}, \dots, X_{n1}, X_{n2}, \dots, X_{nm}) = U'(X_1, X_2, \dots, X_n), \quad (1)$$

$$\text{where } X_i = \phi_i(X_{i1}, X_{i2}, \dots, X_{im}) \quad (2)$$

This two-level utility function is the same as in Helpman and Krugman (1985, p. 115). In their model, each industry produces a variety of products, and each variety enters the utility function because consumers like variety. To use their example, consumers like food, but they don't like to eat the same food every day. In the present context, consumers like variety, but variety is defined in terms of products within an industry coming from different countries. In other words, each country produces only one variety of each good. Otherwise, the two presentations are similar.

The maximization problem has two steps. First, the consumer maximizes  $\phi_i$  for given allocation of spending across all goods. Second, the consumer chooses the expenditure allocation to maximize  $U(\cdot)$ , subject to the overall budget constraint. This approach results in demand functions that depend on money income ( $D$ ), on the price of each product and the price of that product relative to the prices of the other products in each market  $\psi$  ( $\psi = 1, \dots, m$ ), and in all markets. Therefore, the general form of the demand functions will be:

$$X_i = X_i(D, P_1, P_2, \dots, P_n) \quad (3)$$

and

$$X_{ij} = X_{ij}(D, P_{11}, P_{12}, \dots, P_{1m}, P_{21}, P_{22}, \dots, P_{2m}, \dots, P_{n1}, P_{n2}, \dots, P_{nm}), \text{ for all } i \text{ and all } j. \quad (4)$$

The existing model assumes a constant elasticity of substitution (CES) function for utility:

$$X_i = \left( \sum_j b_{ij} X_{ij}^{-\rho_i} \right)^{\frac{1}{\rho_i}} \quad (5)$$

where  $\rho_i = \frac{1}{\sigma_i} - 1$ , and where  $\sigma_i$  is the elasticity of substitution between any pair of products in the  $i$  industry.

According to this utility function, the elasticity of substitution between any pair of products in industry  $i$  is the same. The CES assumption is convenient because it significantly simplifies the calculations and there is little information on the elasticities of substitution between pairs of varieties within an industry. However, CES may not hold for many industries.

**Proposition 1:** Assuming that  $\sigma_{i/lj}^\psi = \sigma_i^\psi, \sigma_i^\psi = \sigma^\psi = \sigma, l \neq j$  and that there is only one good (for example manufactures), the following export equation for country  $j$  can be derived from the first-order conditions:

$$\ln D_j = \sum_{\psi=1}^m \varepsilon^\psi \ln D^\psi - \sum_{\psi=1}^m T_j^\psi (1 - S_j^\psi) (\sigma - 1) \ln P_j + (\sigma - 1) \sum_{\omega \neq j} TW_{j\omega} \ln P_\omega \quad (6)$$

where  $D_j = P_j X_j$  is the demand for the product of country  $j$ ,  $D^\psi$  is the total demand in market  $\psi$ ,  $\varepsilon^\psi$  is the elasticity of demand in market  $\psi$ ,  $TW_{j\omega} = \sum_{\psi=1}^m T_j^\psi S_\omega^\psi$  is the weight of

country  $j$  on country  $\omega$ ,  $S_j^\psi = \frac{P_j X_j^\psi}{\sum_{j=1}^m P_j X_j^\psi}$ , is the share of exports of country  $j$  in country  $\psi$

over the total external demand in country  $\psi$ , and  $T_j^\psi = \frac{P_j X_j^\psi}{\sum_{\psi=1}^m P_j X_j^\psi}$  is the share of country's  $j$  exports in country  $\psi$  over the total exports of country  $j$ .

**Proof:** See Appendix II.

According to equation (6), the demand for country's  $j$  exports is a function of its export price index and a weighted average of the prices of the other countries' exports. The weights are independent from the elasticities of substitution of the products, because it has been assumed that:  $\sigma_{i/lj}^\psi = \sigma_i^\psi, \sigma_i^\psi = \sigma^\psi = \sigma, l \neq j$ . This makes the estimation of equation (6) easier because it is difficult to find data for each  $\sigma_{i/lj}^\psi$ .

However, there are many reasons to believe that the assumption  $\sigma_{ii'}^y = \sigma_i^y$  may not hold. For example, a product produced and exported by a developed economy is often very different from a product produced and exported by a developing economy, even when the two products belong in the same category. Even when the two products are in the same industry, they often target different consumer categories and do not necessarily compete. Products coming from developed countries are often more advanced technologically and are relatively expensive compared to the simpler and cheaper products coming from developing economies. The assumption that the elasticities of substitution among products is the same regardless of their origin does not seem realistic, but it is a key assumption for the calculation of the REER weights.

Equation (6) is an export equation and can be estimated either in changes, or in levels. The hypothesis that the elasticities of substitution are the same can be tested by splitting the sample in two or more groups. If the assumption holds, the estimated coefficient of the second term in equation (6) should be equal to  $(\sigma - 1)$  regardless of how the sample is split.

### III. EMPIRICAL ANALYSIS

The methodology of REER estimation presented in the theoretical section assumes that the elasticity of substitution between products and goods coming from different countries is constant. The purpose of this section is to test the validity of the CES assumption.

A natural experiment to assess the validity of CES can be provided by estimating an export demand equation. If the CES assumption is valid, splitting the real exchange rate into two or more components should not increase its predictive power in an export demand equation. If it does, then the CES assumption does not hold and the real exchange rate as usually computed is not correct. This empirical experiment is conducted by estimating export equations for a set of 56 countries during a period of 26 years—1970 through 1995.

Ideally, we should estimate a system comprising export demand and export supply for each country (see Goldstein and Khan, 1985). However, the uncertainty about the supply curves and the lack of proper supply data for many countries make this task impossible. There is a trade-off between estimating the demand equation with potential simultaneity bias or estimating a system of two equations with potential misspecification of the supply curve. The second problem becomes more relevant when working with panel data with a relative large number of countries because of the uncertainty about the supply equations for many countries.<sup>3</sup> Moreover, estimating only a demand equation does not give rise to simultaneity

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<sup>3</sup> This is probably the reason why many authors focus only on the demand side when working with relatively large panel data (see Reinhart, 1995, or Bayoumi, 1996).

bias if the supply curve is perfectly elastic to price. In support of this, Muscatelli, et al. (1995) find that supply is very elastic to price. Giorgianni and Milesi-Ferretti (1997) find that the supply curve for Korean exports is flat, even with quarterly data, and hence they focus only on demand.<sup>4</sup>

The ideal specification of an export demand equation should split the real exchange rate into bilateral components—potentially using all the bilateral real exchange rates. Obviously, lack of degrees of freedom makes it impossible to use all the bilateral exchange rates in one regression. An intermediate solution, as explained in the theoretical section, is to use two exchange rates: one with respect to OECD countries and the other with respect to non-OECD countries. Other splits could be used, as discussed in more detail below, but the one here seems to be the most obvious to test the validity of the CES assumption. This split assumes that the elasticity of substitution is the same within the groups of the OECD and non-OECD economies, but is different between the two groups.

The demand equation to estimate is:

$$\ln(D_{jt}) = \alpha_j + \beta_j * \ln\left(\frac{P_{t1t}}{P_{jt}}\right) + \gamma_j * \ln\left(\frac{P_{t2t}}{P_{jt}}\right) + \theta_j * \ln(WGDP_t) + \zeta_j + \varepsilon_{jt} \quad (7)$$

where  $\beta_j$  is the coefficient on the relative prices with respect to OECD countries and  $\gamma_j$  the coefficient on the relative prices with respect to non-OECD countries (these price ratios are weighted by the existing INS weights for manufacturing exports), WGDP is the world real GDP, and all the export prices are in U.S. dollar terms.<sup>5</sup> Under the null hypothesis that the CES assumption holds, we should find that:  $\beta_j = \gamma_j$  and that the fit of the export equation increases if the two exchange rates are collapsed into one.

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<sup>4</sup> Note that the debate is not settled. Riedel (1989), in a response to Muscatelli (1995), argues that specification of export demand and supply can drive the results on elasticity. In particular, he argues that export supply is more upward sloping than what is found in other studies.

<sup>5</sup> There are several possible alternative scale variables. Bayoumi (1995) uses a weighted average of trading partners' GDP; Giorgianni and Milesi-Ferretti show that investment is better than GDP in the case of Korea. We avoid scaling GDP by the existing weights since the purpose of the exercise is to test if these weights are calculated correctly. The price index in the regression is the export price index, since the dependent variable is exports. The consumer price index includes import and nontraded domestic good prices and so it may not be as relevant. Unit labor costs are more appropriate for REER calculations, but not many developing economies have available data.

The existing REER weights assume equal weights for competition among all domestic and foreign producers in a third market (domestic producers have a weight of 0.5, while all foreign producers combined have a weight of 0.5). This assumption is strong and arbitrary, but there are no easy alternatives since there are no comprehensive input-output tables for most countries.<sup>6</sup> Therefore, this assumption is taken as given, and is left to future research to test its validity and importance.

There are two remaining major econometric issues involved in the estimation of equation (7). First, it is necessary to establish the dynamic properties of the data in order to use the proper estimation techniques; second, a formal test should determine whether to pool the data and estimate the system as a panel, or to estimate separate equations for each country. The section proceeds by addressing these two issues.

### A. Dynamic Properties of the Data

At a theoretical level it is not clear whether an export demand equation should be estimated in differences or in levels. Therefore, the issue should be resolved at the empirical level.<sup>7</sup> The variables in equation (7) include (log) of real manufacturing, (log) of aggregated real exchange rate, (log) of disaggregated real exchange rates, and (log) of real world GDP.

A first pass is to test the data for the presence of unit roots by applying standard single equation methodologies—augmented Dickey Fuller (ADF) and Phillips-Perron. Looking at the ADF, the presence of a unit root is rejected at the 5 percent significance level only in two countries for manufacturing exports, in five countries for the real exchange rate, and is not rejected even at the 1 percent significance level for world GDP; the evidence is similar for Phillips-Perron tests.

Unsurprisingly, given the low power of these tests, especially in small samples, it is difficult to reject the null hypothesis of unit roots.<sup>8</sup> Therefore, we investigate the issue of the

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<sup>6</sup> For a more detailed analysis of this issue see Wickham (1987).

<sup>7</sup> Senhadji and Montenegro (1998) derive an export equation from an intertemporal maximization model and show that estimation of both in levels and in differences are theoretically justified depending on the nature of the income innovations.

<sup>8</sup> For instance, through a Monte Carlo simulation, Nelson Mark finds that the percentage of rejection of a unit root at 5 percent confidence interval is below 6 percent when the alternative of a  $\rho=0.96$  is true.

stationarity in the data by using two tests for unit root data in the context of panel data.<sup>9</sup>

Im, Pesaran, and Shin (1995) propose a test based on separate Dickey Fuller unit root tests for each  $N$  cross-section unit. They show that the average of the  $t$ -statistics associated with the unit roots tests is distributed according to a normal variable with mean  $\mu$  and variance  $\sigma^2$  that depends on  $N$  and  $T$ .<sup>10</sup> The same study also suggests that this test can be extended to the case of panels with unobserved common time-specific components by taking out cross-sectional averages; however, they alert us that this procedure is not robust to misspecification of time trends if the effect of the common component varies across groups. Table 1 presents the results for panel unit root tests with and without the cross sectional average.

Maddala and Wu (1998) proposed an alternative test based on the  $p$ -values of the separate Dickey Fuller unit roots tests for each  $N$  cross-section units. Under the assumption that the tests are independent, the sum of the (log) significance levels  $p_i$  is distributed as a  $\chi^2$  with  $2N$  degrees of freedom. Unfortunately, the  $p$ -values associated to the augmented Dickey Fuller test are not tabulated. We constructed our  $p$ -values with a bootstrapping of 10000 draws. The results of both tests are reported in Table 1. Both tests are more general than the Levin Lin (1993) test, because in the alternative hypothesis each panel group is allowed to have a different process.

Table 1. Panel Unit Root Test

	Im-Pesaran-Shin	Im-Pesaran-Shin (Controlling for timespecific effects)	Maddala-Wu
REER	-4.43 (0.00)	-3.70 (0.00)	169.01 (0.00)
REER vs OECD Countries	-3.98 (0.00)	-3.78 (0.00)	159.83 (0.00)
REER vs Non-OECD countries	-4.28 (0.00)	-4.28 (0.00)	162.37 (0.02)
Log (Manufacturing)	0.15 (0.56)	-1.97 (0.02)	115.58 (0.39)

Note: The  $p$ -value are in parentheses. The IPS statistic is distributed according to a Normal(0,1) and the Maddala-Wu statistics is distributed according a  $\chi^2$  with  $2N$  degrees of freedom.

<sup>9</sup> Levin and Lin (1993) propose such a test, in which the null hypothesis is the presence of unit root ( $\rho_i=1$  for all  $i$ ), while the alternative is that none of the panel has a unit root and all follow the same dynamic process ( $\rho_i=\rho<1$  for all  $i$ ). We do not use such a test here because the alternative hypothesis is too restrictive.

<sup>10</sup> Ito, Isard, Symansky and Bayouni (1996) provide tables with the tabulated values of mean  $\mu$  variance  $\sigma^2$ .

The results of both tests show that the hypothesis of a unit root is rejected for the real exchange rates at the 1 percent significance level while it cannot be rejected for (log) manufacturing.<sup>11</sup> Given these results, it is appropriate to test whether there is a cointegrating relationship between (log) manufacturing and (log) world GDP.

We perform a cointegration test for each country taken singularly and in a panel framework. By applying the standard residual-based test as suggested by Engle and Yoo (1987) on each equation the null hypothesis of no-cointegration is rejected in 25 cases at the 10 percent significance level.<sup>12</sup> However, the power of tests based on single groups is limited, and therefore, panel cointegration tests proposed by Pedroni (1998) may be more appropriate. Under the null hypothesis of no cointegration all the statistics are distributed as a normal (0,1). Under the alternative hypothesis, the first statistic diverges to positive infinity while the others six diverge to negative infinity.

Table 2. Panel Cointegration Tests Among the Variables:

$$\ln(D_{jt}), \ln\left(\frac{P_{11t}}{P_{jt}}\right), \ln\left(\frac{P_{12t}}{P_{jt}}\right), \ln(WGDP_t)$$

Panel	v-stat	2.25
Panel	rho-stat	0.31
Panel	pp-stat	-0.80
Panel	adf-stat	-0.89
Group	rho-stat	2.26
Group	pp-stat	-0.13
Group	adf-stat	-0.59

See text for explanations.<sup>13</sup>

<sup>11</sup> In recent years there has been a considerable literature on the existence of unit roots for real exchange rates. O'Connell (1998) finds that panel data evidence in favor of stationarity of the REER disappears if the resting procedure controls for cross-sectional dependence among the error terms; however, Higgins and Zakrajšek (1999) argue that, using SUR techniques and correcting for upward bias, there is clear evidence for REER stationarity. Our results are not directly comparable given that we considered an exchange rate based only on manufacturing products.

<sup>12</sup> The critical level for the cointegration tests are based on an augmented Dickey-Fuller for the residuals from Phillips and Ouliaris (1990).

<sup>13</sup> The statistics shown in Table 2 are calculated using a program kindly provided by Pedroni.

Table 2 reports the results for the panel cointegration tests. The null of no-cointegration is not rejected in six out of seven tests. The v-statistic, which rejects the null at the 3 percent significance level is often subject to higher small sample size distortion.<sup>14</sup>

Given these results, equation (7) is rewritten by taking the first differences of the (log) manufacturing export and (log) world GDP to obtain:

$$\Delta \ln(D_{jt}) = \alpha_j + \beta_j * \ln\left(\frac{P_{11t}}{P_{jt}}\right) + \gamma_j * \ln\left(\frac{P_{12t}}{P_{jt}}\right) + \theta_j * \Delta \ln(WGDP_t) + \zeta_j + \varepsilon_{jt} \quad (8)$$

Finally, note that the conclusions on the presence of unit roots and on the lack of cointegration do not depend on the hypotheses that coefficients on the parameters are the same, given that the analysis is based on the residuals of regressions which involve only one country at a time.

### B. Should the Data Be Pooled Together?

A Chow test on the joint restrictions ( $\beta_j = \beta \cup \gamma_j = \gamma \cup \theta_j = \theta$ ) in equations (7) and (8) can determine whether the data can be pooled together in a panel. This test is applied separately to OECD and non-OECD countries to see if countries within each group behave differently.

Table 3: Test for Poolability. F-tests for the Restrictions ( $\beta_j = \beta \cup \gamma_j = \gamma \cup \theta_j = \theta$ )

	Levels		Differences	
	F-test <sup>0</sup>	p-values	F-test	p-values
All Countries	34.20	0.00	1.69	0.00
OECD	18.74	0.00	0.92	0.65
Non-OECD	33.84	0.00	1.75	0.00

<sup>14</sup> We thank Pedroni for useful clarification on the subject.

The results, which are reported in Table 3, show that the null hypothesis is always rejected except in the case of differences in OECD countries. This is not surprising given that non-OECD countries are a more heterogeneous group.

Rejecting the hypothesis that the coefficients are the same across the panel does not imply that estimating the pooled data is less efficient. If the final aim is to obtain more precise estimates of the parameters one may prefer a biased estimator but with low variance to an unbiased estimator with high variance (see Baltagi, 1995). For this reason, what follows presents both panel estimates, which are biased, and estimates based on single equations, which are not biased but may be less efficient.

### C. Results

The previous sections have shown that the export equations should be estimated using the first differences of manufacturing export and world GDP, and without pooling the data. Table 4 shows the average adjusted R<sup>2</sup> and coefficients obtained by estimating equations (9a) and (9b) for each country:

$$\Delta \ln(D_{jt}) = \alpha_j + \phi_j * \ln\left(\frac{P_{11t}}{P_{jt}}\right) + \theta_j * \Delta \ln(WGDP_t) + \zeta_j + \varepsilon_{jt} \quad (9a)$$

$$\Delta \ln(D_{jt}) = \alpha_j + \beta_j * \ln\left(\frac{P_{11t}}{P_{jt}}\right) + \gamma_j * \ln\left(\frac{P_{12t}}{P_{jt}}\right) + \theta_j * \Delta \ln(WGDP_t) + \zeta_j + \varepsilon_{jt} \quad (9b)$$

We allow up to two lags in the exchange rates in each equation if they are significant at the 10 percent level in order to eliminate any serial correlation in the error terms.

Table 4. Average Coefficients (equations (9a) and (9b))

Variable	Observations	Mean	Standard Deviation	Minimum	Maximum
Adjusted R <sup>2</sup> (9a)	56	0.26	0.19	-0.08	0.78
Adjusted R <sup>2</sup> (9b)	56	0.36	0.19	-0.08	0.79
β <sub>j</sub> (9b)	56	0.34	0.82	-1.54	2.85
t-stat (β <sub>j</sub> ) (9b)	56	0.82	1.75	-2.77	4.67
γ <sub>j</sub> (9b)	56	3.24	5.56	-10.29	19.92
t-stat (γ <sub>j</sub> ) (9b)	56	1.38	1.63	-1.24	4.78
World GDP (9b)	56	0.41	0.81	-1.71	2.72
test (γ <sub>j</sub> = β <sub>j</sub> ) (9b)	56	3.73	4.55	0.00	21.01
p-value (9b)	56	0.27	0.27	0.00	1.00

A comparison of the adjusted  $R^2$  and the coefficients on the real exchange rate can show if equation (9b) is an improvement with respect to equation (9a). According to the results in Table 4, the average adjusted  $R^2$  is 0.26 in equation (9a), and 0.36 in equation (9b). Therefore, it is clear that splitting the real exchange rate improves considerably the fitness of the model.

The improvement is not limited to a small number of countries. Figure 1 shows both  $R^2$  for equations (9a) and (9b) and the 45-degree line. The vast majority of the countries lie above the 45-degree line, indicating that (9b) fits the data better than (9a). In addition, while there are many cases where the improvement is significant, there is no case of significant worsening of the fit.

The second criterion is to compare the coefficients on the real exchange rates. If the existing weights are correct, the two coefficients on the exchange rate in equation (9b) should be the same. Table 4 shows that the average  $\gamma_j$  is equal to 3.24, while the average  $\beta_j$  is equal to 0.34. Unfortunately, these coefficients are estimated quite imprecisely, and this makes it difficult to test the hypothesis that  $\gamma_j = \beta_j$  for all  $j$ . However, the null hypothesis that  $\gamma_j = \beta_j$  can be rejected at the 10 percent significance level based on a t-test to each equation in 26 out of 56 countries. For the remaining countries we fail to reject the hypothesis that  $\gamma_j = \beta_j$ , largely because the parameters are estimated with so much imprecision that the power of the t-test is very low in the single equation setting.

In order to investigate further whether  $\gamma_j = \beta_j$  and to avoid the problem of lack of power in the single equation approach, two approaches are followed. The first applies the procedure suggested by Fisher (1932) to test the significance of the results from  $N$  independent tests of hypothesis as explained in Maddala and Wu (1998): "...If the test statistics are continuous, the significance levels  $p_i$  ( $i=1,2,\dots,N$ ) are independent uniform (0,1) variables, and  $-2 \ln(p_i)$  has a  $\chi^2$  distribution with 2 degrees of freedom. Using the additive property of  $\chi^2$  variables, we get  $\lambda = -2 \sum \ln(p_i)$  has a  $\chi^2$  distribution with  $2N$  degrees of freedom...". In our case,  $\lambda$  is 258.36 and the corresponding p-value is 0.000 so that we can reject the hypothesis that  $\gamma_j = \beta_j$  for every  $j$ .

The second approach is to estimate a panel. While this will lead to biased estimates, given that the equations have different coefficients, it could still improve the estimates and gain more power for the test.

Figure 1. Comparison between Adjusted  $R^2$

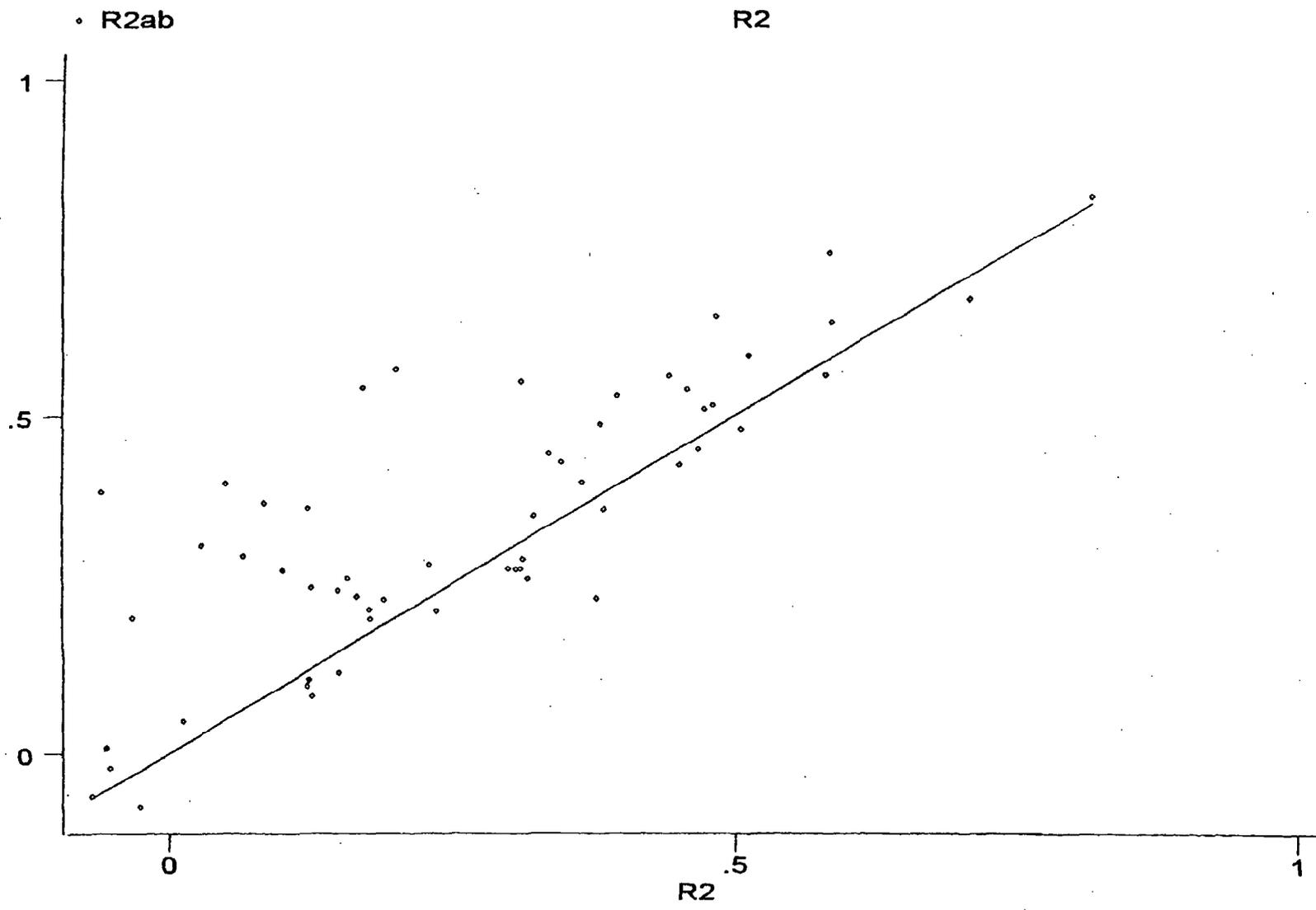


Table 5: Equations (9a) and (9b) Estimated as a Panel with Fixed Effects  
Dependent Variable  $\Delta \ln(\text{manufacturing})$

Equations	All Countries		OECD		Non-OECD	
	10a	10b	10a	10b	10a	10b
REER	0.22 (7.1) 2/		0.21 (4.7) 2/		0.21 (5.5) 2/	
REER with OECD		0.13 (2.9) 2/		0.15 (2.6) 1/		0.13 (2.1) 1/
REER with non-OECD		0.62 (3.4) 2/		0.71 (2.3) 1/		0.61 (2.6) 2/
$\Delta \ln(\text{wgdp}t-1)$	0.40 (4.5) 2/	0.41 (4.6) 2/	0.17 (2.7) 2/	0.17 (2.8) 2/	0.56 (4.0) 2/	0.56 (4.0) 2/
Observations	1400	1400	550	550	850	850
Number of Countries	56	56	22	22	34	34
Adjusted R <sup>2</sup>	0.05	0.06	0.05	0.06	0.06	0.06

Absolute value of t-statistics in parentheses

1/ Significant at 5 percent level.

2/ Significant at 1 percent level.

The first two columns of Table 5 report the results for the panel estimation with fixed effects of equations (9a) and (9b). The coefficients have the expected signs and are all significant, but the adjusted R<sup>2</sup> are low; the two coefficients  $\gamma_j$  and  $\beta_j$  have quite different values and we can formally reject that they are equal at the 3 percent significance level. Finally, the coefficient  $\gamma_j$  estimated in a panel setting is considerably smaller than the average single equation estimate of  $\gamma_j$ ; this is probably due to the presence of big outliers in the equation-by-equation approach.

Between the two extreme approaches of estimating equation by equation and estimating a panel, there is an intermediate approach that consists of grouping countries into broad groups with similar characteristics. This can be done by splitting the countries in the sample into 22 OECD countries and 34 non-OECD countries.<sup>15</sup> The coefficients on the real exchange rate show the same pattern as in the regression with all the countries. However, the coefficients on the world GDP growth are quite different, much higher in non-OECD countries than in OECD countries, which is in line with the usual findings in the literature.

<sup>15</sup> For a list of the countries see Appendix I.

In all regressions we find that the elasticity of exports with respect to the OECD REER is less than the one with respect to the non-OECD REER. This is consistent with the idea that exports from developing countries are more labor intensive and more sensitive to price competition, while exports from developed countries are less sensitive to price.

#### D. Robustness Tests

The estimations have used levels of REER, but so far have not imposed cointegration. In this section, the robustness of the results is tested to the way in which the export equations are specified.

In the previous section, the estimations include differences for the series which had a unit roots, namely (log) of world GDP and (log) of manufacturing exports. This is justified by the fact that there is no evidence of cointegration in the data. However, other studies find and estimate a cointegrating relationship in the export equation (for instance, Reinhart, 1995 and Bayoumi, 1997). In this section, the tests assume that there is a cointegrating relationship between (log) manufacturing exports and (log) world GDP. Accordingly, equations (10a) and (10b) below are the long run equations, and equation (10c) is the error correction of equation (10b):

$$\ln(D_{jt}) = \alpha_j + \phi_j * \ln\left(\frac{P_{11t}}{P_{jt}}\right) + \theta_j * \ln(WGDP_t) + \zeta_j + \varepsilon_{jt} \quad (10a)$$

$$\ln(D_{jt}) = \alpha_j + \beta_j * \ln\left(\frac{P_{11t}}{P_{jt}}\right) + \gamma_j * \ln\left(\frac{P_{12t}}{P_{jt}}\right) + \theta_j * \ln(WGDP_t) + \zeta_j + \varepsilon_{jt} \quad (10b)$$

$$\Delta \ln(D_{jt}) = \alpha_j + \phi_j * \ln\left(\frac{P_{11t-1}}{P_{jt-1}}\right) + \tau_j * \ln\left(\frac{P_{12t-1}}{P_{jt-1}}\right) + \theta_j * \Delta \ln(WGDP_{t-1}) + \mu_j * EC_{jt} + \varepsilon_{jt} \quad (10c)$$

The variable  $EC_{jt}$  is the error term from equation (10a). Table 6 reports summary statistics for the average adjusted  $R^2$  and coefficients for equations (10a)-(10c). Consistently with the results in the previous section, the specification with two REERs performs better than the specification with only one REER. Moreover, the null hypothesis that  $\gamma_j = \beta_j$  for all  $j$  is rejected.

While the majority of the 56 equations perform reasonably, some equations do not behave very well (for instance the maximum value for  $\gamma_j$  is due to an outlier). Our results do not depend on the inclusion of specific countries and are robust to the exclusion of these outliers.

Table 6. Summary Statistics for Equations (10a), (10b) and (10c)

Variable	Observations	Mean	Standard Deviation	Minimum	Maximum
R <sup>2</sup> (10a)	56	0.84	0.25	-0.04	0.99
R <sup>2</sup> (10b)	56	0.87	0.19	0.10	0.99
β <sub>j</sub> (10b)	56	0.43	2.04	-6.75	5.31
t-stat (β <sub>j</sub> ) (10b)	56	0.75	2.40	-8.33	5.05
γ <sub>j</sub> (10b)	56	4.88	13.43	-23.81	68.44
t-stat (γ <sub>j</sub> ) (10b)	56	7.58	18.57	-3.09	84.89
World GDP (10b)	56	0.97	0.70	-0.55	3.17
test (γ <sub>j</sub> = β <sub>j</sub> ) (10b)	56	5.22	13.13	0.00	95.82
Pval (γ <sub>j</sub> = β <sub>j</sub> ) (10b)	56	0.31	0.31	0.00	0.95
Pval (τ <sub>j</sub> = φ <sub>j</sub> ) (10c)	56	0.41	0.33	0.00	1.00

An additional robustness test is to estimate a regression using differences of the REER instead of levels. The debate on the time series properties of the REER is not yet settled-see O'Connell (1998), and Higgins and Zakrajšek (1999)-but as suggested by the tests in the previous section, the level of the REER may be more appropriate than the differences. However, in order to check if the results are driven by the chosen specification above, the same exercise is performed using differences of the REER. The specification is given by the following equations:

$$\Delta \ln(D_{jt}) = \alpha_j + \phi_j * \Delta \ln\left(\frac{P_{1jt}}{P_{jt}}\right) + \theta_j * \Delta \ln(WGDP_t) + \zeta_j + \varepsilon_{jt} \quad (11a)$$

$$\Delta \ln(D_{jt}) = \alpha_j + \beta_j * \Delta \ln\left(\frac{P_{1jt}}{P_{jt}}\right) + \gamma_j * \Delta \ln\left(\frac{P_{2jt}}{P_{jt}}\right) + \theta_j * \Delta \ln(WGDP_t) + \zeta_j + \varepsilon_{jt} \quad (11b)$$

Table 7 reports the summary statistics of equations (11a) and (11b). As before, the adjusted R<sup>2</sup> of the equations with two REERs is on average much higher than the R<sup>2</sup> of the equation with one REER; the other results are also very similar.

Table 7. Summary Statistics for Equations (11a) and (11b)

Variable	Observations	Mean	Standard Deviation	Minimum	Maximum
R <sup>2</sup> (11a)	56	0.27	0.21	-0.07	0.82
R <sup>2</sup> (11b)	56	0.35	0.20	-0.08	0.83
β <sub>j</sub> (11b)	56	0.12	0.80	-1.57	2.40
t-stat (β <sub>j</sub> ) (11b)	56	0.32	1.58	-3.14	4.32
γ <sub>j</sub> (11b)	56	4.82	7.51	-13.35	38.04
t-stat (γ <sub>j</sub> ) (11b)	56	1.65	1.43	-1.58	6.24
World GDP (11b)	56	0.36	0.92	-3.42	2.74
test (γ <sub>j</sub> = β <sub>j</sub> ) (11b)	56	0.75	5.82	0.01	28.75
pval (γ <sub>j</sub> = β <sub>j</sub> ) (11b)	56	0.31	0.29	0.00	0.92

Finally, equations (9a) and (9b) were estimated using random-coefficients regression. The results, which are not reported here, confirmed the other findings.

#### E. Is OECD versus Non-OECD Countries Grouping Efficient?

In the previous sections, we have compared the standard REER with two REERs, one with respect to the OECD and one with respect to the non-OECD countries. This distinction is arbitrary but is justified by the idea that products from similar countries should have a similar elasticity of substitution.<sup>16</sup> In order to see whether this idea is correct, we perform the following exercise. First, we form two groups of countries randomly selected;<sup>17</sup> second, we calculate the REERs vis-à-vis these two groups; third, we run the export equations described in equations (10a) and (10b) for each of 56 countries in the sample; fourth, we compute the proportions of countries for which the adjusted R<sup>2</sup> is higher when we use the two exchange rates instead of only one.

We have repeated this exercise 1,240 times and the results are reported in Table 8. Two things are quite striking: first, it is possible to increase the adjusted R<sup>2</sup> by simply using two REERs with respect to two randomly chosen groups of countries in 75 percent of the cases; second, grouping the countries into OECD and non-OECD, increases the explanatory power in 69 percent of the cases, well above of the improvement with random grouping. Moreover, while random grouping decreases the adjusted R<sup>2</sup> only 25 percent of the times, the average random grouping increases the adjusted R<sup>2</sup> only 55 percent of times.

<sup>16</sup> Giorgianni and Milesi-Ferretti (1997) split the exchange rate using the similar criterion of industrialized versus nonindustrialized countries.

<sup>17</sup> One group contains 22 countries, the other one the remaining 34 countries in order to have the same size as the OECD versus non-OECD classification.

Table 8: Percentage of Cases Improved by Using Two REERs

Percentage	Frequency	Percent	Cumulative
0.37	3	0.24	0.24
0.39	2	0.16	0.40
0.41	14	1.13	1.53
0.43	15	1.21	2.74
0.45	31	2.50	5.24
0.46	62	5.00	10.24
0.48	79	6.37	16.61
0.50	101	8.15	24.76
0.52	137	11.05	35.81
0.54	124	10.00	45.81
0.55	136	10.97	56.77
0.57	113	9.11	65.89
0.59	121	9.76	75.65
0.61	94	7.58	83.23
0.62	67	5.40	88.63
0.64	51	4.11	92.74
0.66	32	2.58	95.32
0.68	21	1.69	97.02
0.70 1/	15 1/	1.21 1/	98.23 1/
0.71	11	0.89	99.11
0.73	8	0.65	99.76
0.75	2	0.16	99.92
0.77	1	0.08	100.00

1/ Value based on OECD versus non-OECD country groupings.

This simulation exercise provides two conclusions. First, from a theoretical point of view, it shows that the elasticities of substitution are generally asymmetric across countries and, as a first approximation, the biggest asymmetry is between the OECD and non-OECD countries. From a practical point of view, we show that there is little to be gained by using two exchange rates randomly and that the partition of the OECD versus non-OECD country groupings can improve considerably the fit of export demand equation for many countries.

#### IV. CONCLUSIONS

The real effective exchange rate of a country is an aggregation of several bilateral real exchange rates with respect to other countries. The aggregation is usually done under the assumption of constant elasticity of substitution between goods coming from different countries. We investigate the validity of this assumption, by estimating manufacturing export equations for 56 countries over 26 years.

The results show that the specifications that do not impose the restriction of constant elasticity of substitution perform better than the specifications that do impose such restrictions. The hypothesis of constant elasticity of substitution is rejected and the export equations that contain two REERs perform on average considerably better than the traditional ones. These results are robust to several estimation techniques. These results suggest that it may be useful to use disaggregated REERs, or even bilateral REERs with respect to the countries believed to be the main competitors of a country, when trying to reach conclusions on competitiveness.

In particular, we find that the elasticity of export to the REER with respect to the OECD countries is less than that with respect to non-OECD countries. This is consistent with the idea that exports from developing countries are more labor intensive and more sensitive to price competition, while exports from developed countries are less sensitive to price competition. This qualifies the policy implications of Faini et al. (1991), who found that competitive devaluations do not expand aggregate exports from developing countries. We confirm their results for developing countries, but show that this result is not the case for developed countries.

Our results are not only important for the empirical literature, but suggest important implications for the theoretical trade literature. Several models study the relationship between trade and growth in the context of the product variety model with constant elasticity of substitution (Grossman and Helpman, 1991). However, we have shown that data reject this hypothesis and that there is a pattern for the elasticity of substitution between OECD and non-OECD countries. We do not know whether the main results of models based on product variety hold when taking into account our results, and think an examination of this question should be undertaken.

### Data Sources and Description

The weights in the REER calculations are export weights for manufacturing derived in Zanello and Desruelle (1997). The methodology is explained in their paper, and is the same methodology used for the IMF-INS trade weights.

All other data are from the World Development Indicators of the World Bank (WDI). WDI provides data for manufacturing exports. The export price index is used as the price index for all REER calculations. World real GDP is taken by simply adding the GDP of all countries with consistently available data during the period being considered.

The 22 OECD countries are: Australia, Austria, Belgium-Luxembourg, Canada, Denmark, Finland, France, Germany, Federal Republic, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. The 34 non-OECD economies in the sample are: Algeria, Argentina, Brazil, Chile, China, Colombia, Costa Rica, Ecuador, Egypt, El Salvador, Guatemala, Honduras, Hong Kong, Hungary, India, Indonesia, Israel, Jamaica, South Korea, Malaysia, Mauritius, Mexico, Morocco, Nicaragua, Pakistan, Paraguay, Peru, Philippines, Singapore, Thailand, Trinidad and Tobago, Tunisia, Turkey, Uruguay.

#### The Summary Statistics are: All Countries

Variable	Observations	Mean	Standard deviation	Minimum	Maximum
Ln(manufacturing exports)	1456	21.76	2.47	15.68	26.69
REER	1456	-0.08	0.22	-1.18	1.15
REER-OECD	1456	-0.09	0.18	-1.04	0.77
REER-non-OECD	1456	0.01	0.05	-0.17	0.38
Ln (world GDP)	1456	9.35	0.62	8.14	10.27

#### OECD Countries

Variable	Observations	Mean	Standard deviation	Minimum	Maximum
Ln(manufacturing exports)	572	23.59	1.93	16.64	26.69
REER	572	-0.01	0.11	-0.41	0.38
REER-OECD	572	-0.02	0.10	-0.37	0.32
REER-non-OECD	572	0.01	0.02	-0.04	0.09
Ln (wgdp)	572	9.35	0.62	8.14	10.27

#### Non-OECD Countries

Variable	Observations	Mean	Standard deviation	Minimum	Maximum
Ln(manufacturing exports)	884	20.57	2.01	15.68	25.57
REER	884	-0.12	0.26	-1.18	1.15
REER-OECD	884	-0.13	0.21	-1.04	0.77
REER-non-OECD	884	0.01	0.06	-0.17	0.38
Ln (wgdp)	884	9.35	0.62	8.14	10.27

### Algebraic Calculations for Section II

According to the first-order conditions in the maximization problem of Section II, the marginal rate of substitution between a product  $j$  and a product  $k$  of industry  $i$  in a market  $\psi$  should be equal to their price ratio:

$$\frac{\frac{\partial \phi_i^\psi}{\partial X_j^\psi}}{\frac{\partial \phi_i^\psi}{\partial X_{ik}^\psi}} = \frac{b_j}{b_{ik}} \left( \frac{X_{ik}^\psi}{X_j^\psi} \right)^{\frac{1}{\sigma}} = \frac{P_j}{P_{ik}} \quad (A1)$$

The prices should be such that the demand of  $X_i$  is consistent with the optimum selection of products in the  $i$  market. This is satisfied if:<sup>18</sup>

$$P_i = \frac{P_j}{\frac{\partial \phi_i}{\partial X_j}}, \text{ for } i = 1, \dots, n \text{ and } j = 1, \dots, m. \quad (A2)$$

It can be easily shown that the first-order conditions yield the following demand function for each product  $X_{ij}$  in value terms, in market  $\psi$ :<sup>19</sup>

$$P_j X_{ij}^\psi = b_j^{\sigma} P_i X_i^\psi \left( \frac{P_j}{P_i} \right)^{1-\sigma} \quad (A3)$$

It can also be shown that  $P_i$  is equal to the following:<sup>20</sup>

<sup>18</sup> See Armington (1969).

<sup>19</sup> See Armington (1969, Appendix I).

<sup>20</sup> First, notice that:  $\sum_{j=1}^m \frac{\partial \phi_i}{\partial X_j} X_j = X_i$ . However,  $\frac{\partial \phi_i}{\partial X_j} = \frac{P_j}{P_i}$ . Therefore:  $X_i = \sum_{j=1}^m \frac{P_j}{P_i} X_j$ .

Solving for  $P_i$ :  $P_i = \sum_{j=1}^m \frac{X_j}{X_i} P_j$ .

$$P_i = \sum_{j=1}^m \frac{X_{ij}^\psi}{X_i^\psi} P_{ij} \quad (\text{A4})$$

Taking the total differential of equation (A3):

$$\frac{dP_{ij} X_{ij}^\psi}{P_{ij} X_{ij}^\psi} = \varepsilon_i^\psi \frac{dD^\psi}{D^\psi} - (\eta_i^\psi - 1) \frac{dP_i}{P_i} + \sum_{l \neq i}^n \eta_{il}^\psi \frac{dP_l}{P_l} - (\sigma_i^\psi - 1) \left( \frac{dP_{ij}}{P_{ij}} - \frac{dP_i}{P_i} \right) \quad (\text{A5})$$

where  $\varepsilon_i^\psi$  is the elasticity of demand with respect to good  $X_i$  in market  $\psi$ , and  $\eta_i/l^\psi$  is the elasticity of demand of good  $i$ , with respect to the price of good  $l$  in market  $\psi$ .

For simplicity, assume that total demand and the prices of goods other than  $i$  do not change:  $dD^\psi=0$  and  $dP_l=0$  for  $l \neq i$ . It is also convenient to assume that the elasticity of a good  $i$  with respect to its price is one:  $\eta_i^\psi = 1$ . Finally, we assume that there is only one good (for example manufacturing):  $n=1$ . Taking the total differential of equation (A4), making use of equation (A2) and substituting into equation (A5) we get:

$$\frac{dP_j X_j^\psi}{P_j X_j^\psi} = -(1 - s_j^\psi)(\sigma^\psi - 1) \frac{dP_j}{P_j} + \sum_{\omega \neq j}^m S_\omega^\psi \frac{dP_\omega}{P_\omega} (\sigma^\psi - 1) \quad (\text{A6})$$

where  $S_j^\psi = \frac{P_j X_j^\psi}{\sum_{j=1}^m P_j X_j^\psi}$ , is the share of exports of country  $j$  in country  $\psi$ , over the total demand in country  $\psi$ .

Introducing some more notations, define as the share of country's  $j$  exports in country  $\psi$ , over the total exports of country  $j$ :  $T_j^\psi = \frac{P_j X_j^\psi}{\sum_{\psi=1}^m P_j X_j^\psi}$

To aggregate over all markets, we need to take the weighted sum of equation (A5) over all markets  $\psi$ :

$$\sum_{\psi=1}^m T_j^\psi \frac{dP_j X_j^\psi}{P_j X_j^\psi} = - \sum_{\psi=1}^m T_j^\psi (1 - S_j^\psi) (\sigma^\psi - 1) \frac{dP_j}{P_j} + \sum_{\omega \neq j}^m \sum_{\psi=1}^m T_j^\psi S_\omega^\psi \frac{dP_\omega}{P_\omega} (\sigma^\psi - 1) \quad (\text{A7})$$

Which can be rewritten as follows:

$$\frac{dD_j}{D_j} = -\sum_{\psi=1}^m T_j^\psi (1 - S_j^\psi) (\sigma - 1) \frac{dP_j}{P_j} + (\sigma - 1) \sum_{\omega \neq j}^m TW_{j\omega} \frac{dP_\omega}{P_\omega} \quad (\text{A8})$$

where  $D_j = P_j X_j$ ,  $TW_{j\omega} = \sum_{\psi=1}^m T_j^\psi S_\omega^\psi$  is the weight of country  $j$  on country  $\omega$ , and we have assumed that  $\sigma$  is the same in all markets (the elasticity of substitution between a pair of products is the same in all markets). Equation (A8) can be also written in levels as in equation (6) in the text.

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