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Life-Cycles, Dynasties, Saving: Implications for Closed and Small, Open Economies

Hamid Faruquee and Douglas Laxton

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**Life-Cycles, Dynasties, Saving:
Implications for Closed and Small, Open Economies**

Prepared by Hamid Faruquee and Douglas Laxton¹

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Abstract

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This paper examines the macroeconomic implications of life-cycle and dynastic saving behavior for closed and small, open economies. Using an extended version of Blanchard's overlapping agents model, the analytical framework nests these two competing views, treating agents as either dynastic households or disconnected generations. Calibrating the life-cycle variant using empirical age-earnings profiles, the analysis compares the long-run effects of fiscal policy shocks under both perspectives. The results quantify the implications of life-cycle considerations for the effects of deficit finance on real interest rates and the capital stock or net foreign assets.

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Author's E-Mail Address: hfaruquee@imf.org; dlaxton@imf.org

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Contents	Page
I. Introduction.....	4
II. The Basic Model.....	6
A. Consumption.....	6
B. Dynastic Assumption.....	7
C. Life-Cycle Income.....	8
III. Small Open Economy Case.....	9
A. Investment.....	10
B. Government.....	10
C. External Sector.....	10
IV. Closed Economy Case.....	11
V. Income Profiles: Theory and Calibration.....	13
A. Specification Issues.....	13
B. Data and Estimation.....	15
VI. Steady-State Profiles and Simulations.....	16
A. Steady-State Effects of Government Debt.....	18
B. Closed Economy Results.....	18
C. Small Open Economy Results.....	20
VII. Concluding Remarks.....	21
Appendices	
1. The Basic Model.....	31
2. Further Extensions: Liquidity Constraints.....	33
3. Population and Productivity Growth.....	35
References.....	38
Tables	
1. Small Open Economy Model with Dynastic Household.....	22
2. Closed Economy Model of Saving and Investment.....	23
3. Relative Income Profiles—Non-Linear Least Squares Estimates.....	24
A1. Extended Model with Liquidity Constraints and Population & Productivity Growth.....	37

Figures

1. Age-Earnings Distribution: United States, 1980-199	25
2a. Steady-State Income and Consumption Profiles with Dynastic Households	26
2b. Steady-State Wealth Profiles with Dynastic Households	26
3a. Steady-State Income and Consumption Profiles with Life-Cycle Income	26
3b. Steady-State Wealth Profiles with Life-Cycle Income	26
4a. Steady-State Effect of a Fiscal Debt Shock Closed Economy with Dynamic Households	27
4b. Steady-State Effect of a Fiscal Debt Shock Closed Economy with Life-Cycle Income	27
4c. Steady-State Effect of a Fiscal Debt Shock Closed Economy with Dynamic Households	28
4d. Steady-State Effect of a Fiscal Debt Shock Closed Economy with Life-Cycle Income	28
5a. Steady-State Effect of a Fiscal Debt Shock Closed Economy with Life-Cycle Income	29
5b. Steady-State Effect of a Fiscal Debt Shock Closed Economy with Life-Cycle Income	29
6a. Steady-State Effect of a Fiscal Debt Shock Small Open Economy with Dynastic Households	30
6b. Steady-State Effect of a Fiscal Debt Shock Small Open Economy with Life-Cycle Income	30

I. INTRODUCTION

Over the postwar period, there has been an appreciable increase in government debt among most OECD countries. And while this peacetime expansion in debt has been substantial in many cases, the reasons underlying this trend are not fully known.² More recently, fiscal consolidation has slowed the debt build-up and in some cases reversed it. An interesting question remains, however, as to the likely economic consequences of these persistent budgetary deficits (and their reversals)—an issue that has been the subject of a somewhat strident and ongoing debate, in which no clear consensus has emerged.

Proponents of Ricardian equivalence downplay the economic consequences of public debt on the real economy.³ After all, according to the government's own (intertemporal) budget constraint, deficit financing merely represents a change in the timing of taxes, while the present value of taxes remains unchanged given public expenditure. Thus, higher taxes in the future should offset the benefits of any current reduction in taxes, leaving current consumption unaffected. In the case of government spending, higher public consumption implies an increase in the present value of tax liabilities which would act to lower private consumption, changing the composition (but not the level) of aggregate demand. In either case, so long as agents internalize the future consequences of government debt, an increase in public borrowing should be met with an increase in private saving, offsetting the effects on national saving.

In contrast, from a life-cycle perspective,⁴ increases in public debt may have real consequences as agents perceive that the prospective tax burden partly shifts to future generations of taxpayers. In a primarily closed economy, a decline in public saving would be reflected in lower national saving; the resultant upward pressure on interest rates would tend to crowd out investment and retard the rate of capital accumulation. In a small open economy, the decline in domestic saving would crowd out net exports and lead to a greater reliance on foreign borrowing. In either case, the increase in current consumption (public or private) would take place at the expense of lower living standards in the future, either through a lower level of the capital stock or higher foreign claims on national output.

²See Alesina and Perotti (1995) for a discussion on this issue.

³Interest in the hypothesis of Ricardian equivalence was revived by Barro (1974). See Barro (1989) for a review. However, it should be noted that Ricardo himself did not believe in Ricardian equivalence and indeed was very concerned that there could be deleterious crowding-out effects associated with high levels of government debt.

⁴The seminal paper is by Diamond (1965). See Bernheim (1989) for a more recent review.

As is well known, the disparate economic implications of public debt between the Ricardian and life-cycle approaches stem from their underlying conceptual differences with respect to the behavior of economic agents. When agents are viewed as dynastic lineages, linked to all future generations through operative bequests to their descendants, the level of national saving is invariant to the choice between deficit or tax finance, and the economy obeys Ricardian equivalence.⁵ In contrast, where agents represent individuals disconnected from each other and influenced by life-cycle considerations, the real effects of government deficits may be large.

This paper revisits the contrasting economic and policy implications of the Ricardian and life-cycle approaches, examining how large quantitatively these differences can be.⁶ Extending Blanchard's (1985) overlapping agents model, we consider a framework that nests these two competing views: treating agents either as dynastic households or as individuals with life-cycle characteristics. The "life-cycle" version of the Blanchard model is developed by incorporating age-earning profiles, calibrated from empirical income distributions in the United States. We then examine the quantitative importance of the distinction between dynastic and life-cycle saving behavior in terms of their comparative implications for the effects of government debt in closed and small open economies. The analysis also identifies the key economic parameters affecting the long-run comparative statics of the models.

It should be noted that the life-cycle version of this model differs somewhat from more traditional models in one important aspect. In the presence of life-cycle income profiles and lifetime uncertainty, agents are motivated to hold "precautionary wealth" in this context. Unlike standard life-cycle analyses which posit negative saving among retirees, the present model suggests that agents accumulate wealth (albeit to a lesser degree) until their eventual death. The current analysis thus avoids the common criticism levied against the life-cycle paradigm that dissaving among the elderly is not strongly supported by the empirical evidence. In this context, individual wealth accumulation over time will be a feature of both the dynastic and life-cycle versions of the model.

The paper is organized as follows: section II describes the basic analytical framework incorporating both dynastic and life-cycle saving behavior—extensions to productivity and population growth, and liquidity constraints are developed in the appendix; sections III and IV then consider the respective cases of the closed and small open economy; section V provides a calibration of age-earnings profiles in the life-cycle version of the model; section

⁵Evans (1991) further shows that in the simple Blanchard (1985) model—i.e., without any life-cycle features—the departures from Ricardian Equivalence are quite small. In other words, though debt neutrality does not hold exactly, it is nevertheless a good approximation.

⁶Using a similar framework, Romer (1988) examines the effects of "excessive" deficits, without incorporating life-cycles features to the model. Consequently, the economic effects (e.g., on interest rates) of fiscal deficits are second-order; nevertheless, he argues that the *normative* consequences may still be first-order on (intergenerational) welfare.

VI discusses the steady-state implications of changes in government debt. Finally, section VII offers some concluding remarks.

II. THE BASIC MODEL

This section presents the basic model which incorporates the behavior of either dynastic households or individuals with "life-cycle characteristics." Following Blanchard (1985), we consider an economy with overlapping agents who have finite planning horizons (i.e., positive probability of death). In the case of dynasties, the probability of death represents the likelihood that the family line will end, while with overlapping generations of individuals, the probability of death is related to life expectancy.

Specifically, consider an economy populated by finitely lived agents, each facing a constant probability p of dying at each moment in time and a planning horizon—or the expected time until death given by $1/p$.⁷ Also at each point in time, a new generation (or dynasty) is born of relative size normalized to p . Consequently, the number of survivors from a cohort born at time s remaining at time t is equal to $p e^{-p(t-s)}$, leaving the number of total agents—aggregating over all existing cohorts (indexed by s)—constant and normalized to unity.⁸

A. Consumption

Agents are assumed to maximize expected utility over their "lifetimes" subject to a budget constraint. Specifically, the evolution of wealth $w(s, t)$ for an individual or household is determined by their saving, defined as the difference between income and consumption: $\dot{w}(s, t) = [r(t) + p] w(s, t) + y(s, t) - \tau(s, t) - c(s, t)$; where r is the interest rate, $y - \tau$ is disposable income, and c is consumption, all expressed in real terms (units of consumption).⁹ In the small open economy case, the real interest rate is also assumed to be exogenous and fixed at the world real rate of interest. Explicitly solving the consumer's problem under the assumption of no capital market imperfections individual consumption behavior (under log

⁷See Blanchard (1995). This well-known assumption of a constant death or hazard rate allows for analytical tractability and implies that all agents have been the same expected length of life remaining. This inherent feature of the model is often equated with the absence of a life-cycle dimension; however as well as we shall see, it is only one part of what constitutes "life-cycle behavior," and perhaps not the most relevant aspect, depending on the issue at hand.

⁸The case of population (and productivity) growth is addressed in the appendix.

⁹The term $p w(s, t)$ in the dynamic budget constraint reflects the efficient operation of life insurance of annuities market. See Yaari (1965) or Blanchard (1985). The budget constraint would also incorporate the depreciation of capital assets, omitted here for simplicity.

utility) is given by: $c(s, t) = (\theta + p) [w(s, t) + h(s, t)]$ where $h(s, t)$ is a measure of an agent's human wealth—equal to the present value of future labor income.¹⁰

Aggregating over all agents, total consumption as a function of (financial and human) wealth is given by (dropping the time index).¹¹

$$C = (\theta + p) [W + H], \quad (1)$$

where uppercase letters denote economy-wide aggregates. Total financial wealth W consists of domestic equity and bond holdings and, in the open-economy case, holdings of net foreign assets ($W \equiv K + B + F$). As for aggregate human wealth H , its definition depends on the treatment of agents as dynasties or individuals with life-cycle characteristics.

B. Dynastic Assumption

In the case of dynasties, the agent or household's planning horizon may far exceed the lifetime of any individual member if they care as much about the welfare and circumstances of their descendants as they do about their own. Correspondingly, human wealth is expressed in terms of the disposable income stream available to the dynastic household. As dynasties themselves do not possess any life-cycle dimension, different households regardless of age can be treated identically with respect to labor earnings. Consequently, income and taxes (and thus human wealth) are not generation-specific (i.e., $y(s, t) = Y(t)$, $\tau(s, t) = T(t)$, $h(s, t) = H(t)$). Hence, the dynamics for aggregate human wealth under a dynastic interpretation can be written as:

$$\dot{H} = [r + p]H - [Y - T]. \quad (2)$$

Under this dynastic assumption, note that an agent's labor income—which is independent of age—can grow monotonically over time with productivity. In this case the future income stream of the dynasty might be interpreted as the income stream of a family business where p would represent the probability that the dynasty would end in any period (and $1/p$ represents the effective planning horizon of the dynasty). Indeed, in the special case where p equals zero it is well known that the model becomes one with infinitely lived agents where exact Ricardian equivalence holds.¹² It is important to note, however, that this result assumes that there is no increase in the number of infinitely-lived dynasties in the future; otherwise, the

¹⁰For a given (world) real interest rate, individual human wealth can be written as:

$$h(s, t) \equiv \int_t^{\infty} [y(s, v) - \tau(s, v)] e^{-(r+p)(v-t)} dv.$$

¹¹In terms of notation, time arguments have been dropped in the text except where potential ambiguities may arise. The time index is reintroduced in the tables.

¹²See Blanchard and Fischer (1989).

timing of taxes would matter.¹³ This representative agent assumption represents a crucial difference between the case of dynastic and life-cycle behavior as will be come apparent below.

C. Life-Cycle Income

In the case where agents represent overlapping generations (rather than dynasties), the planning horizon reflects an individual's expected life span, during which time life-cycle considerations are relevant. To incorporate life-cycle features, the basic framework can be modified to the case where the time profile of labor income has a life-cycle dimension:¹⁴ rising with age and experience when young, before eventually declining with retirement when old.

To introduce a concave earnings profile over an individual's lifetime, we assume that the income $y(s, t)$ accruing to an individual from generation s at time t can be expressed in terms of age-dependent weights on aggregate labor income $Y(t)$, to allow for aggregation, equal to the sum of two exponential functions:¹⁵

$$y(s, t) = \left[a_1 e^{-\alpha_1(t-s)} + a_2 e^{-\alpha_2(t-s)} \right] Y(t); \quad a_1 > 0, a_2 < 0, \alpha_2, \alpha_1 > 0. \quad (3)$$

The first exponential can be interpreted as the gradually declining endowment of labor (i.e., gradual retirement) which is inelastically supplied. The second exponential can be interpreted as the relative productivity and wage gains from experience with increasing age.

In the case of age-earnings profiles, the dynamics governing aggregate human wealth are modified accordingly:

¹³Weil (1989) shows clearly that what matters for Ricardian equivalence to break down is not that $p=0$ but that agents alive today are disconnected from some agents in the future. This would be the case for example if new dynasties were being created in the future as a result of immigration, or if some members of existing dynasties severed their relationships and formed new strands. Buiter (1998) confirms this result, showing that the death rate is neither necessary nor sufficient for Ricardian Equivalence to fail; instead, it is the birth rate that matters.

¹⁴Blanchard (1985) examines the case of individually declining income profiles. The more realistic case of non-monotonic (concave) earnings profiles is mentioned only in passing (footnote 8).

¹⁵As discussed in section III, the parameters in (3) are chosen such that the weighting function is assumed to be non-negative and initially increasing; by an adding up constraint, we also require that $\frac{a_1 p}{\alpha_1 + p} + \frac{a_2 p}{\alpha_2 + p} = 1$.

$$H = \beta H_1 + (1 - \beta) H_2 \quad (4)$$

$$\dot{H}_1 = [r + p + \alpha_1] H_1 - [Y - T] \quad (5)$$

$$\dot{H}_2 = [r + p + \alpha_2] H_2 - [Y - T] \quad (6)$$

Human wealth, which measures the present value of future disposable labor income, is now expressed as the sum of two components, reflecting the concave (or hump-shaped) dimension, of each individual's income over the life cycle.¹⁶

In terms of their economic implications, life-cycle income profiles tend to augment the real effects of government debt on the real economy, as will be shown explicitly later. The basic intuition can be seen by comparing equation (2) with equations (3)-(5). In the life-cycle case, age-earnings profiles further increase the wedge between public and private discount rates, seen by the terms above, beyond the effects of a positive birth rate ($p > 0$).¹⁷ In other words, the policy choice between tax financing versus deficit financing (i.e., the timing of taxes) will have larger consequences for national consumption and saving if agents perceive that the prospective tax burden falls partly on future generations who have higher taxable income.

This basic framework with dynastic or life-cycle agents can be further generalized to incorporate population growth (n), long-run productivity growth (μ), and a broader range of intertemporal substitution elasticities in consumption (σ^{-1}). These extensions are taken up in the appendix. In the text, we turn our attention to the small open economy and closed economy variants of the model.

III. SMALL OPEN ECONOMY CASE

The small open economy is assumed to be a price taker in the world market for goods and capital; hence, consumption is expressed in terms of a single internationally-traded good (numeraire) whose price is taken as given. The open economy version of the model is closed by specifying simple models for the external and government sectors as well as by including a standard dynamic model for investment behavior.

¹⁶Integrating up equation (9) yields the definition of the human wealth component H_1 :

$H_1(t) \equiv \int_t^\infty [Y(v) - T(v)] e^{-(r+p+\alpha_1)(v-t)} dv$, where the following boundary is assumed to be satisfied: $\lim_{t \rightarrow \infty} H_1(t) e^{-(r+p+\alpha_1)t} = 0$; H_2 is derived equivalently.

¹⁷Because there is no population growth at this point in the analysis the birth rate, b , is assumed to be equal to the probability of death p .

A. Investment

In deriving investment behavior, it is assumed that domestic firms can freely borrow at the (exogenous) world real interest rate in making their investment decisions [see appendix]. With (convex) installation costs of capital, the investment decision can be derived as:

$$I = [q - 1 + \delta] K \quad (7)$$

Where I is gross investment excluding installation costs, K is the domestic capital stock, δ is the rate of depreciation of capital, and q is the value of an additional unit of capital (related to Tobin's q). Total investment expenditure \tilde{I} is given by the sum of gross investment plus adjustment costs $I + A$. Note that in equation 7, domestic investment is independent of domestic saving and consumption behavior. In other words, with the ability to borrow and lend freely at a given world real rate of interest, a small open economy will choose an investment rule that is separable from its consumption behavior (Fisherian separability).

As for capital accumulation, net investment—defined as gross investment net of depreciation—determines the incremental change in the domestic capital stock:

$$\dot{K} = I - \delta K, \quad (8)$$

where again δ is the (constant) rate of depreciation or obsolescence for capital.

B. Government

As for the public sector, it is assumed that government expenditures G are financed either through (lump-sum) taxation T or the issuance of government debt. Debt accumulation and the government's dynamic budget constraint is given by:

$$\dot{B} = rB + G - T, \quad (9)$$

where B is the stock of public debt. In equation (9), the primary deficit plus interest payments on the existing stock determines the government's bond-financing requirements and the corresponding rate of debt issue.

C. External Sector

Using national accounting identities, the current account can be expressed in terms of income, saving and absorption. First, domestic production or GDP is given by $f(K)$, which is a concave, twice-differentiable aggregate production function (labor L normalized to 1),¹⁸

¹⁸ Assuming that $F(K, L)$ is homogenous-of-degree-one in its arguments, we can write the production function as $LF(K/L, 1) = f(K) [= F(K, 1)]$ at $L = 1$. Also, the following

(continued...)

and national income or GNP is defined by GDP plus net interest income (factor payments) from abroad: $GNP = f(K) + rF$. In turn, national saving S equals national income less consumption (public and private): $S = GNP - C - G$.

In terms of external balance, the difference between domestic production and domestic absorption equals the trade balance (i.e., net exports): $f(K) - C - G - \tilde{I} = NX$, and the difference between income and absorption or between saving and investment is given by the current account: $CA = NX + rF = S - \tilde{I}$.¹⁹ In terms of dynamics, since the gap between income and expenditure must be met by international lending or borrowing, the current account also reflects changes in the stock of net foreign assets:

$$\dot{F} = CA. \quad (10)$$

Table 1 summarizes the basic equations and laws of motion in the dynastic model in the case of the small open economy. The version of the model that is based on life-cycle income can be obtained by replacing the definition of human wealth in the table with equations 5 and 6 above.

IV. CLOSED ECONOMY CASE

The discussion thus far has considered the case of fixed world real interest rate facing a price-taking small open economy. However, in the face of (say) global shocks (e.g., changes in public debt across countries), one might expect the world real interest rate to be affected and change over time. This example can also be examined in the same basic framework by noting that the world as a whole is a closed economy and introducing an endogenous real interest rate to be determined by tastes and technology.

In a closed economy, domestic saving must equal investment in the absence of international capital flows —e.g., the world current account is zero. Hence, the rate of capital accumulation will depend on preferences or the willingness of households to forgo current consumption (save) as well as on the return to investment as determined by technology. To ensure that the level of saving equals investment, the domestic real interest rate $r(t)$ must adjust to equate the supply and demand for these funds. Under profit maximization by firms, the real interest rate must also equal the net marginal product of capital:

$$r(t) = f'(K(t)) - \delta \quad (11)$$

conditions are assumed to apply to guarantee the existence of an interior steady-state solution: $0 \leq \lim_{K \rightarrow \infty} f'(K) \leq r + \delta \leq \lim_{K \rightarrow 0} f'(K) \leq \infty$. Strict concavity of $f(K)$ —an increasing function—guarantees uniqueness.

¹⁹In this model we abstract from sticky prices and terms-of-trade effects. For a discussion about how these could be included into the model see Macklem (1993).

As before, net investment—defined as gross investment net of depreciation—determines the incremental change in the capital stock, but now domestic investment is also equal to domestic (net) saving:

$$\dot{K} = f(K) - C - G - A - \delta K$$

With installation costs, the capital accumulation equation also includes a term reflecting these costs of adjustment, so that the incremental increase in the capital stock (net of depreciation) is equal to saving less installation costs A .²⁰

The equations characterizing the closed economy under the dynastic interpretation are summarized in Table 2. Note that in the case of a closed economy, net foreign assets and the current account are identically zero: $F, \dot{F} = 0$. Note also that the real interest rate now carries a time argument in Table 2.²¹ The extension to the case of life-cycle income in the closed economy case follows exactly as in the small open economy example.

In the simulations that follow, we examine the comparative effects of government debt that emerge in both closed and small open economies under dynastic and life-cycle saving behavior. The simulations are based on an extended version of the basic model with liquidity constraints and also allows for both population and productivity growth [see the appendix for the derivation of the optimality conditions].

In the case of liquidity constraints, it is assumed that younger generations are initially denied access to borrowing²² and that their consumption is constrained to equal current income [see appendix]. The parameter λ measures the proportion of agents faced with constrained consumption. Overall, aggregate consumption, reflecting the behavior of both

²⁰In the numerical simulations that follow, we assume that a (fixed) equity premium exists in calibrating the baseline levels; this is necessary to obtain both a sensible equilibrium capital-output ratio and real interest rate. Otherwise, the real interest rate would be unrealistically high—see for example Romer (1988).

²¹With a time-varying rate of interest, the present value of (dynastic) labor income which comprises of human wealth is given by:

$$H(t) \equiv \int_t^{\infty} [Y(v) - T(v)] e^{-\int_t^v (r(x) + p) dx} dv$$

Differentiating this expression with respect to time yields the dynamic equation for human wealth shown in Table 2.

²² Hayashi (1985), Zeldes (1989) and Jappelli (1990) each find empirical evidence suggesting liquidity constraints are more likely for younger families with lower wealth and income.

permanent- and current-income consumers will display excess sensitivity to current income.²³

From the perspective of fiscal policy, the presence of liquidity constraints allows changes in taxes to affect private consumption directly through affecting the level of disposable income available to constrained households. For example, a reduction in taxes given fiscal spending, would free-up current resources for individuals to spend while the government borrows against future resources to meet its current expenditures. The government in effect borrows on behalf of individuals who otherwise would not have access to their future (taxable) income. Consequently, the choice of financing and changes in public debt can have real economic consequences.

V. INCOME PROFILES: THEORY AND CALIBRATION

In order to simulate the effects of life-cycle savings in the model, we must first calibrate the shape of age-earnings profiles. This section presents an empirical methodology and investigation of this issue, beginning with specification issues before turning to data and estimation.

A. Specification Issues

To characterize the time profile of earnings, individual incomes can be represented as a time-varying, generation-specific weight $\omega(s, t)$ on income per capita for the economy as whole. Specifically, labor income $y(s, t)$ for a member of generation s at time $t (\geq s)$ as a proportion of average income per capita can be written as:

$$y(s, t) = [a_1 e^{-a_1(t-s)} + a_2 e^{-a_2(t-s)}] \frac{Y(t)}{N(t)} \quad (13)$$

where Y is aggregate labor income and N is the size of the population. From a theoretical perspective, several characteristics of the earnings profiles and parameters restrictions with respect to equation (13) are worth noting:

-
- *Non-monotonicity.* To guarantee that income profiles do not rise or fall monotonically, we require that a_1 and a_2 are of opposite sign. Without loss of generality, we further

²³In the dynastic case—where labor income is identical across agents— λ also reflects the degree of excess sensitivity in the aggregate consumption to disposable income; in the life-cycle case, the coefficient of excess sensitivity reflects the amount of labor income (and hence consumption) associated with the proportion λ of the population who face borrowing constraints [see appendix]. Departures from the predictions of the strict permanent income hypothesis have been characterized in terms of excess sensitivity of consumption to *anticipated* changes or excess smoothness to *unanticipated* innovations in income. See Campbell and Deaton (1989). These issues can be viewed as aspects of the same phenomenon, generated by liquidity constraints; see Flavin (1993).

specify $\alpha_1 > 0$ and $\alpha_2 < 0$. To ensure concavity, two additional restrictions are needed: *Initially increasing*. For incomes to rise initially, the time derivative of $\omega(s, t)$ at $s=t$ must be strictly positive, requiring: $\alpha_1 \alpha_1 < -\alpha_2 \alpha_2$.

- *Eventually declining*. To also assure that labor earnings eventually fall off with retirement, a sufficient condition has $\alpha_1 \alpha_2 > 0$ which in combination with the previous assumptions is sufficient to generate a hump-shaped time profile for labor income.²⁴
- *Non-negativity*. For individual incomes to always remain positive given aggregate income (i.e. for $\omega(s, t) \geq 0, \forall t$), a necessary condition has $\alpha_1 \geq -\alpha_2$, which is also sufficient provided that we also have $\alpha_2 > \alpha_1$.²⁵
- *Adding-up*. Integrating over all generations, individual labor incomes must add up to aggregate labor income, requiring that:

$$\frac{\alpha_1 b}{\alpha_1 + b} + \frac{\alpha_2 b}{\alpha_2 + b} = 1 \text{ where } b \text{ is the birth rate.}^{26}$$

The simple two-exponential specification can also be generalized to allow for a broader range of time profiles for labor income. Specifically, we can expand (13) as follows:

$$y(s, t) = \left[\sum_{i=1}^k \alpha_i e^{-\alpha_i(t-s)} \right] \frac{Y(t)}{N(t)} \quad (14)$$

for some integer k .²⁷ This more general specification is used later in the estimation along with the corresponding parameter restrictions on the α_i and α_i terms to ensure adding-up and concavity.

²⁴A common explanation for the presence of borrowing constraints involves agency problems—e.g., moral hazard and adverse selection—in credit markets stemming from collateral issues or asymmetric information. See Buiter (1994) or Stiglitz and Weiss (1981).

²⁵Together, the conditions for non-negative and initially-increasing income profiles imply:
 $\alpha_2 \alpha_1 > -\alpha_2 \alpha_2 > \alpha_1 \alpha_1 > -\alpha_1 \alpha_2$

²⁶This is the more general adding-up condition that allows for population growth (see appendix). The text describes the special restriction with zero population growth ($b=p$).

²⁷Increasing the number of exponential terms increases the number of inflection points.

B. Data and Estimation

Using data on labor income and employment by age for the United States for 1980 to 1995, a data set was constructed containing the cross-sectional distribution of the real labor income across age groups for each intervening year. The ages used were 20, 30, 40, 50, 60, and 75 years, essentially representing the mid-point (or median) ages for each of six cohort ranges.²⁸ Hence, for each point in time (16 years from 1980-95), we have a cross-sectional distribution characterizing labor income across 6 age groups for a total of 96 observations.

To characterize the time profile of labor earnings, we assume that a typical individual's earnings over his or her lifetime follows the same time pattern suggested by the average income profile seen in the cross-sectional distribution. To account for productivity growth (i.e., cohort effects), we focus on relative income rather than income in absolute terms. In particular, we express individual labor income for a particular generation or cohort as a proportion of income per capita for the aggregate economy: $ry(s, t) \equiv y(s, t)N(t)/Y(t)$. Working with relative income distributions has the advantage that the shapes of these profiles are likely to be more stable (and thus comparable) over time, given time-variation in labor productivity.²⁹ More to the point, relative income profiles are more likely to directly reflect the structural or institutional aspects of labor markets (e.g., seniority wages, age of retirement, etc.) that affect relative earnings, summarized by the parameters entering the age-dependent weighting function.

To estimate the shape of the earnings profile, we apply non-linear least squares estimation to equation (14) using our data on relative income distributions. Note that the specification with two or more exponential terms has a multiplicity of possible parameterizations (i.e., local maxima). However, the key parameters of economic interest here are the exponents α_i . Consequently, we turn to *conditional* estimates of these parameters, based on a given birth rate b and/or set of coefficients a_i , which narrows the parameter search considerably and provides more robust estimates to alternative starting values.³⁰ Conditional NLLS estimates of equation (14) are shown in Table 3 for one case with $k=2$ and two cases with $k=3$.

²⁸The cohort ranges are: 18-24, 25-34, 35-44, 45-54, 55-64, 65+. Earnings, employment, and population data are from the U.S. Bureau of Labor Statistics and Census Bureau.

²⁹The presumption here is that productivity growth does not affect the *relative* income distribution very much, i.e., is not biased toward any particular age group of workers. See appendix for more on productivity growth.

³⁰Imposed parameters reported in the table are obtained through grid search. Over the sample period, the average "birth rate"—defined as the relative sized of the newest cohort—of the U.S. Adult population was around $2\frac{1}{2}$ percent.

The estimates in Table 3 do reasonably well in fitting the cross-sectional income distributions for the United States, and are generally sensible. The plots of the fitted income profiles are shown in Figure 1. The specifications with an added exponential term ($k=3$) have somewhat better fits, although the income specification with the highest R^2 i.e., in column (2)) yields an implausibly high birth rate (6 percent) and eventually turns negative [see Figure 1]. For these reasons, the preferred estimates are given in column (3) of the table.

VI. STEADY-STATE PROFILES AND SIMULATIONS

Based on the consumption behavior in both the closed- and open-economy versions of the model, one can derive the implied steady-state paths for consumption, saving and wealth. By first solving for the real interest rate that obtains in general equilibrium (in the closed economy case),³¹ we present these steady-state profiles in figures 2 and 3 under dynastic and life-cycle behavior. In the dynastic case, labor income at the aggregate *and* household level is simply a constant (normalized to unity).³² Hence, human wealth or the present value of labor income is also a constant, shown in figure 2b. An agent's consumption, meanwhile, can be shown to be rising over time, as dynastic households maintain a constant rate of saving (as a share of total income),³³ allowing financial wealth and asset income of the household to rise over time.³⁴

³¹Dynamic and steady-state versions of the model are simulated numerically in TROLL to solve for the equilibrium real interest rate and the capital stock in the closed economy and net foreign asset position for the small open economy, assuming no public debt as an initial condition. This numerical work has been made considerably easier by the development of state-of-the-art newton-based methods that are considerably more robust and efficient than first-order iterative techniques. See Juillard et al. (1988) for a discussion of the algorithm and a comparison with other techniques.

³²We assume no population or productivity growth here. The parameters used to derive the steady-state profiles shown include: $\theta = 0.05$, $\sigma = 1.0$, $b = p = 0.021$. In the life-cycle income case, the α 's are taken from Table 3, column (3). An assumption that the probability of death is the same across the dynastic and life-cycle cases is used so that we can isolate the implications of life-cycle income. Allowing p to differ significantly across the two models would further strengthen the economic differences between the two models.

³³Because in the dynastic case agents do not choose to borrow—i.e., no intergenerational lending, borrowing constraints placed on some agents (considered later) is not really relevant here. Hence, current-income consumers *must* represent naive or rule-of-thumb consumers a la Campbell and Mankiw, rather than liquidity-constrained agents.

³⁴Note that individual wealth, which is rising, also includes life insurance or annuity income. This transfer, from agents dying each period to surviving agents, does *not* add to aggregate financial wealth, which is constant even though individual wealth profiles are rising.

With life-cycle saving behavior, the rate of saving may vary significantly over time and across individuals, depending on where agents are within their respective life cycles. Younger agents expecting a rising earnings profile choose to borrow (if able) and consume against their permanent income, which exceeds current income initially as shown in figure 3a.³⁵ At middle-age, agents enjoying a relatively higher level of earnings choose to accumulate assets and save for their eventual retirement.³⁶ But unlike traditional life-cycle models where individuals reaching retirement would tend to run down their assets (dissave), in the present setting agents continue to save, albeit to a lesser degree, well into their old age (see charts 3a and 3b).

The intuition for this result is as follows. The presence of age-earnings profiles and life-time uncertainty induces individuals to continue to accumulate wealth as they get older. Not knowing exactly when they might die, agents must "replace" the decline in labor income that accompanies retirement by building up asset income (including annuities) to maintain their consumption levels.³⁷ Because planning horizons are constant and independent of age, individuals who live a very long time would eventually reach and maintain a given (target) level of financial wealth. Hence, this model possesses a "precautionary wealth" motive, to guard against the possibility of remaining alive without labor income.³⁸ Consequently, wealth holdings tend to be higher among older agents, as in the dynastic case—although in that instance, wealth accumulation is spurred through an operative bequest motive.

³⁵The demand for loans—i.e., intergenerational lending—with lifecycle income allows the possibility for binding borrowing constraints, if they appear early in the life cycle. Jappelli and Pagano (1994) introduce a similar implication in a 3-period OLG model, where liquidity constraints appear in the first period, but income is earned only in the middle period.

³⁶The "saving for retirement" motive (absent in the dynastic case) tends to lead to greater wealth accumulation; in the closed economy case, this leads to a *lower* steady-state real interest rate r (or other things equal). For example, whereas $\bar{r} > \theta$ in the dynastic case, the interest rate can be below the rate of time preference in the life-cycle case, opening up the possibility of dynamic inefficiency. See Blanchard (1985).

³⁷Because life insurance or annuity dividends are paid only in fixed proportions to the level of financial wealth (i.e., zero profit condition), agents must build up their financial estates—which are turned over to the insurance company at the time of death—in order to receive higher annuity income while alive.

³⁸The role of "buffer-stock" or precautionary saving is another well-known channel through which individuals seek to maintain a target level of wealth. However, this type of saving behavior is spurred by income (rather than lifetime) uncertainty and should be more prevalent in the earlier stages of the life-cycle. When labor income is earned. See Carroll and Samwick (1997) for a recent and the references cited therein.

A. Steady-State Effects of Government Debt

Simulations of the model in both the dynastic and life-cycle cases are conducted to examine the comparative steady-state implications of a 10 percentage point increase (from zero) in the public debt ratio as a share of GDP. The simulations are conducted over a range of parameter values for the intertemporal elasticity of substitution in consumption σ^{-1} and the extent of liquidity constraints λ

B. Closed Economy Results

In a closed economy, we examine the effects of a change in the government debt on the steady-state real interest rate and capital-output ratio, measured as deviations from their baseline levels.³⁹ The interest rate change is measured in basis points and the change in the capital stock is measured in percent of GDP.⁴⁰ If Ricardian equivalence holds exactly or approximately, these effects should be zero or near zero.

With dynastic households, the long-run effects of government debt are indeed small. Regardless of the values of σ and λ , the effects of a 10 percentage point change in the debt ratio are less than 10 basis points on the real interest rate (figure 4a) and less than one percent of GDP on the capital-output ratio (figure 4c).⁴¹ Adding productivity growth or population growth (not shown) does not overturn this finding of approximate Ricardian equivalence. In fact, adding productivity growth further *reduces* the effects of government debt.⁴² In effect, with dynastic saving behavior, agents essentially internalize the future tax implications of public debt and tend to offset the effects of deficit finance on the real economy.

³⁹The closed economy model is calibrated so that the baseline real interest rate is always the same across different parameterizations, by fixing the rate of time preference—given other taste parameters—at the level required to obtain an initial long-run real interest rate of 4 percent.

⁴⁰The relation between long-run changes in interest rates and the capital stock is as follows: with Cobb-Douglas production and capital share around a third, a 10 basis-point change in the real interest rate translates to around $1\frac{1}{2}$ percentage point change in capital-output ratio. In turn the relationship between the steady-state change in private consumption and the capital stock (given public consumption) is given by $d\bar{C} = rd\bar{K}$.

⁴¹These results for the dynastic case essentially replicate the findings in Evans (1991), who shows that changes in p do not alter the finding of approximate Ricardian equivalence in this class of models.

⁴²With long-run productivity growth, dynastic agents have income profiles that increase monotonically over time, representing an increasing tax base and, thus, greater sensitivity to the future tax implications of public debt.

However, in the life-cycle case, the effects of government debt can be substantially larger. Figures 4b,d show that life-cycle income considerations tend to *augment* the real effects of deficit financing. The differential effects are even larger in the case of long-run productivity and population growth (figures 5a,b). Intuitively, with eventually declining earnings and retirement, existing agents further discount the impact of future tax liabilities from an increase in public debt since the prospective tax base increasingly shifts to future generations *with* higher taxable income. This greater wedge between private and public discount rates underlies the larger non-neutral effects of government debt. Without an altruistic link between generations, an increase in the fiscal deficit will not be fully offset by an increase in saving of present agents, as a share of the debt burden falls on future generations, whose marginal propensity to save presently is zero.

Also evident from figures 4-6 is that life-cycle income is a necessary but *not* sufficient condition to generate large debt non-neutralities. A small intertemporal elasticity of substitution in consumption (large σ) is also needed. The intuition for this result is as follows. A smaller substitution elasticity implies that consumption is *less* sensitive to changes in the interest rate.⁴³ Consequently, in the life-cycle case where changes in government debt matter, a *larger* interest rate adjustment is required to accommodate changes in consumption patterns.⁴⁴

Overall, the role of liquidity constraints on affecting the degree of departure from Ricardian equivalence is comparatively much *smaller*. The reason can be found in the aggregate consumption function. Liquidity-constraint effects operate through the consumption of current-income consumers—who comprise a much smaller share of total consumption. Moreover, the channel through which liquidity constraints matter involves the (somewhat smaller) effects of changes in the stock of government debt on the *flow* of disposable labor income; life-cycle considerations, on the other hand, operate through their implications for the *stock* of human wealth—i.e., the expected accumulation of disposable income, thus allowing the effects of government debt on consumption and the real economy to be substantially greater.

⁴³Interest rates affect saving and consumption through 3 channels: discount rate effects, income effects, and substitution effects. With a low substitution elasticity, the substitution effect is attenuated, allowing the (positive) income effect to more greatly offset the (negative) discount rate—i.e., human wealth revaluation—effect resulting from higher interest rates.

⁴⁴Because the substitution elasticity effects consumption (of permanent-income consumers) directly through the marginal propensity to consume and indirectly through influencing the amount of interest rate adjustments, changes in σ can have non-linear implications for the effects of government debt as shown in figure 5.

C. Small Open Economy Results

In the case of a small open economy, things are qualitatively different in several important dimensions. First, the real effects (if any) of government debt will be reflected in terms of the impact on the economy's net foreign asset position and not the capital stock. Because the economy can borrow and lend freely at the (fixed) world real interest rate—i.e., assuming perfect capital mobility, the capital stock is determined completely by technology or the supply side;⁴⁵ meanwhile, domestic consumption and saving (private and public) have no role in determining the level of investment. Hence, changes in government debt will not affect the capital stock or interest rate.

Second, because the (world) interest rate is invariant to changes in public debt, the substitution elasticity σ^{-1} (i.e., interest sensitivity) in consumption *no* longer appreciably affects the degree of debt non-neutrality in the life-cycle model.⁴⁶ Instead, the degree of "crowding out" of the net external assets from an increase in government indebtedness is more sensitive to parameters like the rate of time preference θ .

Finally, the effects of public debt significant under *both* dynastic and life-cycle saving in the case of a small open economy. Figures 6a and 6b show the steady-state effects on net foreign assets (as a share of GDP) of a 10 percentage point increase in the government debt ratio in a small open economy, across different parameter values for λ and θ .⁴⁷ Changes in the net foreign assets—and, thus, external debt servicing—imply, in turn, changes in long-run consumption.⁴⁸ Once again, the effects of public debt are larger in the case with life-cycle income than with dynastic households, but the effects are now non-trivial under *both* types of saving behavior.

The reason why the real effects of public debt (on net foreign assets) in a small open economy are much larger with dynastic saving than (on the capital stock) in a closed economy with dynastic saving stems from our assumption of capital mobility. In a closed economy with no capital mobility, higher government borrowing competes with private borrowing, placing upward pressure on domestic interest rates. The prospect of higher interest rates now and in the future in turn helps boost private saving to largely offset the

⁴⁵This result could easily be overturned with imperfect capital mobility—e.g., imperfect asset substitutability, if (say) the domestic interest rate was determined by the world rate plus a risk premium, sensitive to the degree of fiscal indebtedness.

⁴⁶Simulations (not shown) confirm that the impact of different substitution elasticities is negligible in a small open economy in both the dynastic and life-cycle cases.

⁴⁷As a stability condition in the dynastic case, we require that $\theta > \bar{r} - p$, placing a lower bound on the rate of time preference. See Blanchard (1985).

⁴⁸The relationship between the long-run change in private consumption and net foreign assets is: $d\bar{C} = rd\bar{F}$; the (fixed) world interest rate r is 4 percent in the simulations.

decline in public saving. In a small open economy, the fall in government saving is largely absorbed by foreigners willing to lend freely at a fixed world real interest rate. Consequently, the presence of elastic foreign saving magnifies the effects of deficit finance on national saving.

VII. CONCLUDING REMARKS

The distinction between the dynastic and life-cycle paradigm is a quantitatively important one, in terms of its implications for fiscal policy. In the simple Blanchard model, overlapping generations of otherwise identical (or representative) agents characterize a world inhabited by dynastic households—a world where the interest rate effects of government debt are close to zero. Adding a life-cycle dimension to the analysis through age-earnings profiles is a necessary (but not sufficient) condition for generating significant real effects of public debt. Coupled with a low elasticity of intertemporal substitution, life-cycle considerations can generate economically meaningful debt non-neutralities in a closed economy. The fiscal implications are even more disparate in the presence of economic growth.

In a small open economy that can borrow or lend freely at a fixed world rate of interest, the effects of government debt on net external assets are sizable with either dynastic or life-cycle saving. With the perfectly elastic supply of foreign saving, the effects of deficit finance on national saving are magnified in both instances, though the economic impact is still larger in the case of life-cycle saving. These results highlight the important role that behavioral and parametric assumptions play in the conduct of policy analysis.

**Table 1 – Small Open Economy Model with Dynastic Households:
Behavioral Equations and Laws of Motions**

$$C(t) = (\theta + p)[W(t) + H(t)]; W = K + B + F \quad (15)$$

$$I(t) = [q(t) - 1 + \delta] K(t) \quad (16)$$

$$\dot{K}(t) = I(t) - \delta K(t) \quad (17)$$

$$\dot{B}(t) = rB(t) + G(t) - T(t) \quad (18)$$

$$\dot{F}(t) = rF(t) + f(K(t)) - C(t) - G(t) - [I(t) + A(t)] \quad (19)$$

$$\dot{H}(t) = [r(t) + p]H(t) - [Y(t) - T(t)] \quad (20)$$

$$\dot{q}(t) = (r + \delta) q(t) - f'(K(t)) - \frac{I(t)}{K(t)} [q(t) - 1] + \frac{1}{2} [q(t) - 1]^2 \quad (21)$$

Table 2 - Closed Economy Model of Saving and Investment:
Behavioral Equations and Laws of Motions

$$C(t) = (\theta + p)[W(t) + H(t)]; W = K + B \quad (22)$$

$$I(t) = [q(t) - 1 + \delta]K(t) \quad (23)$$

$$\dot{K}(t) = f(K(t)) - C(t) - G(t) - A(t) - \delta K(t) \quad (24)$$

$$\dot{B}(t) = rB(t) + G(t) - T(t) \quad (25)$$

$$\dot{H}(t) = [r(t) + p]H(t) - [Y(t) - T(t)] \quad (26)$$

$$\dot{q}(t) = \left[r(t) + \delta - \frac{I(t)}{K(t)} \right] [q(t) - 1] + \frac{1}{2} [q(t) - 1]^2 \quad (27)$$

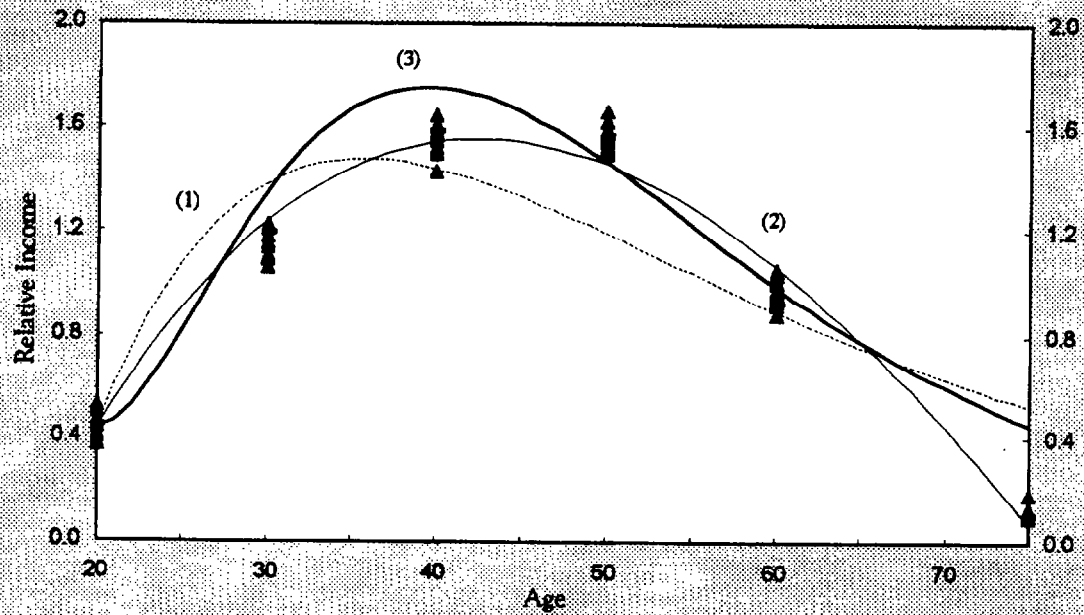
$$r(t) = f'(K(t)) - \delta \quad (28)$$

Table 3. Relative Income Profiles—Non-linear Least Squares Estimates

<i>model:</i>	$ry_t = \sum_{i=1}^k a_i e^{-\alpha_i(t-20)}$		
<i>restrictions:</i>	$\sum_{i=1}^k \frac{a_i b}{\alpha_i + b} = 1; \sum_{i=1}^k a_i = r\bar{y}_{20}$		
	k = 2	k = 3	k = 3
<i>parameters:</i>	(1)	(2)	(3)
α_1	0.051**	0.010**	0.080**
α_2	0.061	0.019**	0.107**
α_3	...	-0.004	0.098
a_1	20.00	40.00	100.00
a_2	-19.56	-30.00	200.00
a_3	...	-9.56	-299.56
b	0.031**	0.057**	0.030
\bar{R}^2	0.78	0.98	0.88
<i>D.W.</i>	1.95	2.49	2.00

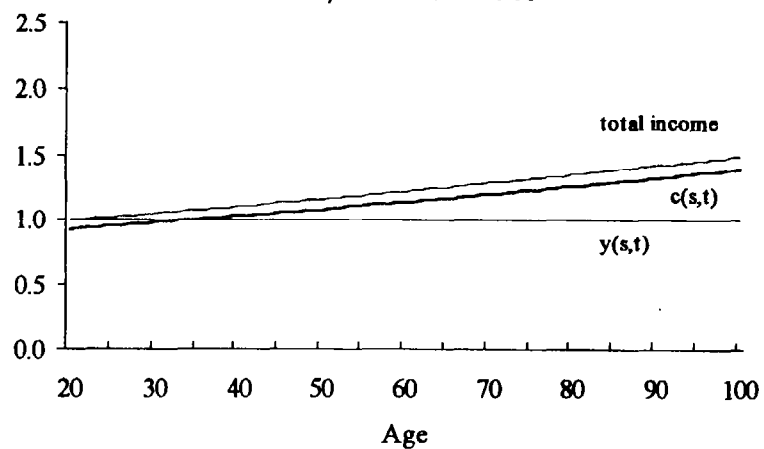
A * (**) indicates significance at the 5 (1) percent level; entries in *italics* denote parameter values that were imposed (or redundant) in the conditional estimates. For example, the first $k-1$ exponents (α 's) are estimated directly, but the k th term is determined implicitly from the adding-up restriction.

Figure 1. Age-Earnings Distributions
United States, 1980-1995



Data source: U.S. Bureau of Labor Statistics and U.S. Census Bureau

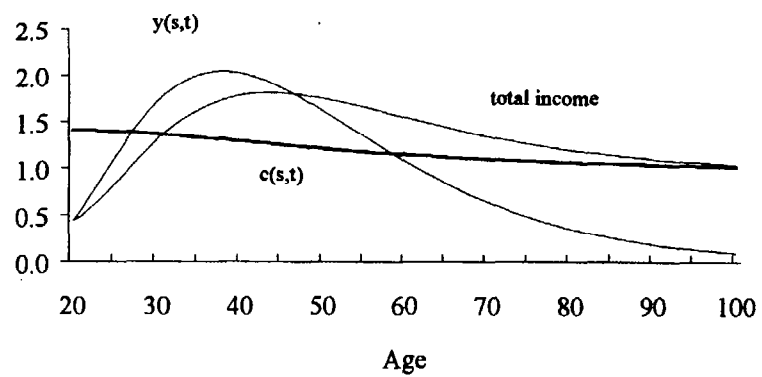
**Figure 2a. Steady-State Income and Consumption Profiles
with Dynastic Households**



**Figure 2b. Steady-State Wealth Profiles
with Dynastic Households**



**Figure 3a. Steady-State Income and Consumption Profiles
with Life-Cycle Income**



**Figure 3b. Steady-State Wealth Profiles
with Life-Cycle Income**

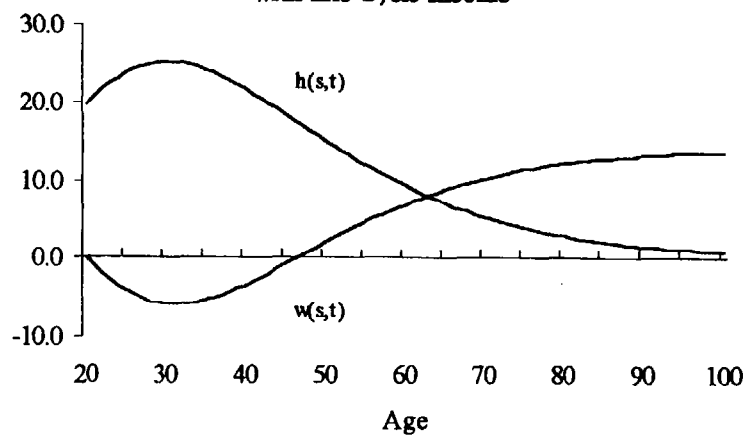


Figure 4a. Steady-State Effects of a Fiscal Debt Shock
Closed Economy with Dynastic Households

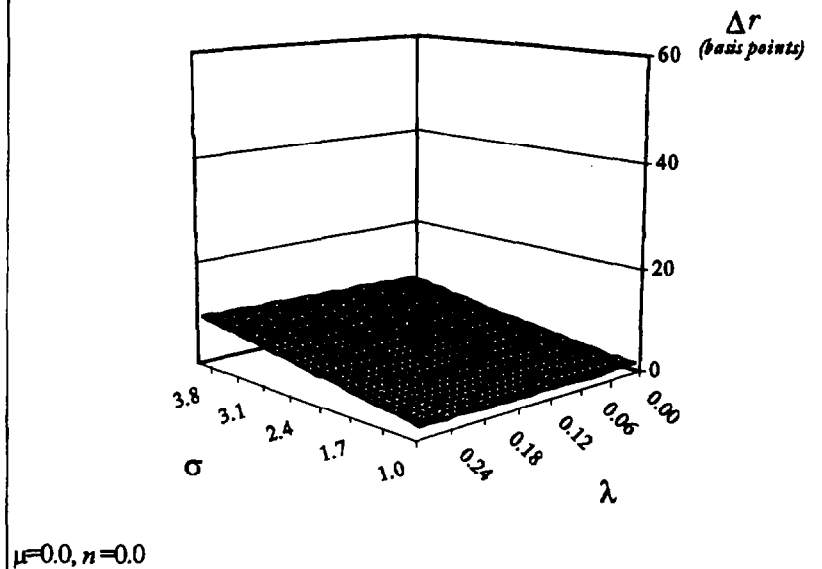


Figure 4b. Steady-State Effect of a Fiscal Debt Shock
Closed Economy with Life-Cycle Income

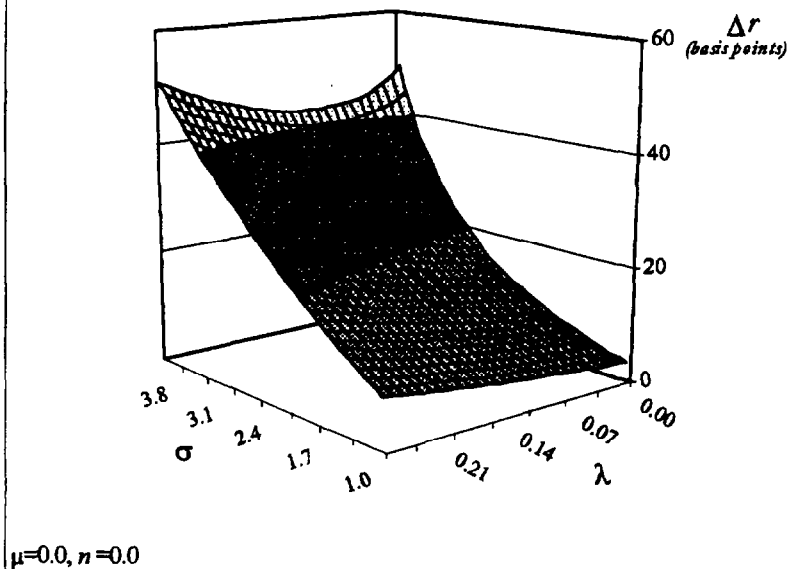


Figure 4c. Steady-State Effect of a Fiscal Debt Shock
Closed Economy with Dynastic Households

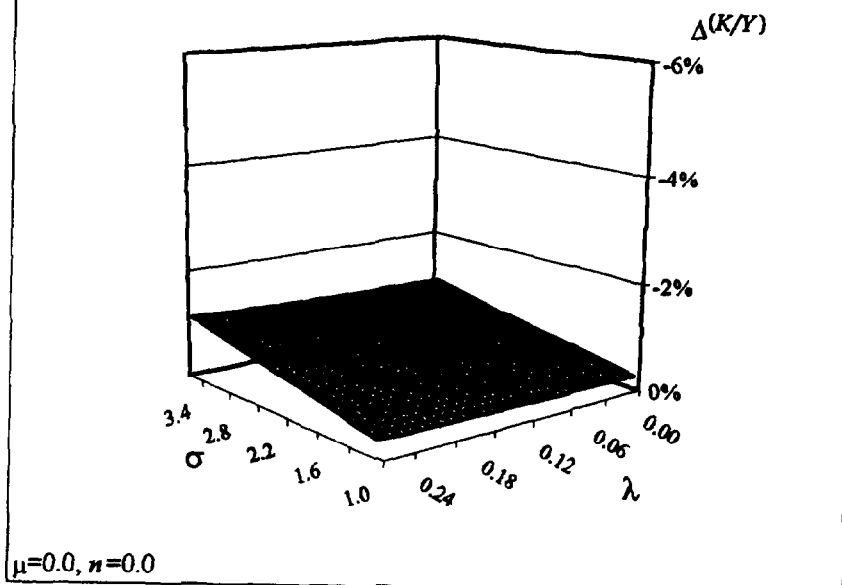


Figure 4d. Steady-State Effect of a Fiscal Debt Shock
Closed Economy with Life-Cycle Income

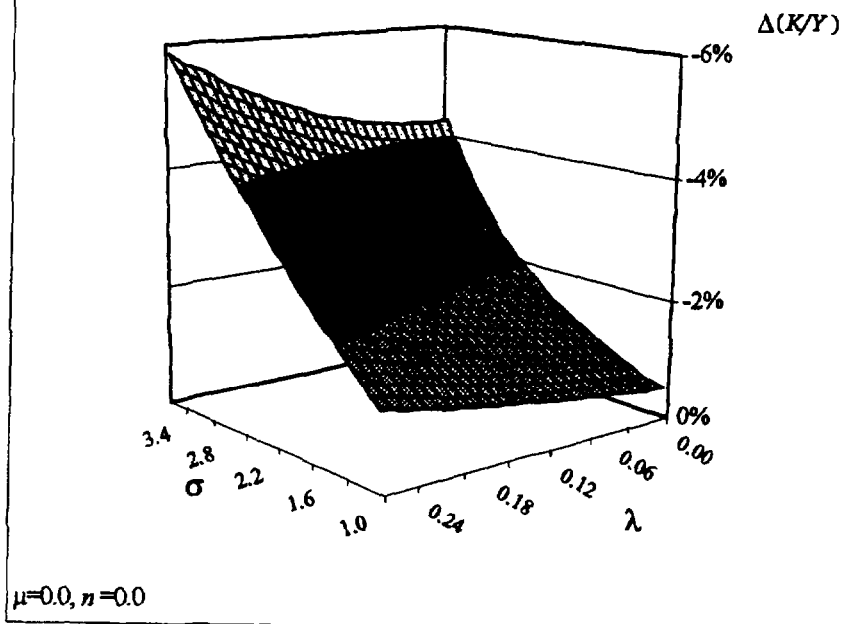


Figure 5a. Steady-State Effect of a Fiscal Debt Shock
Closed Economy with Life-Cycle Income

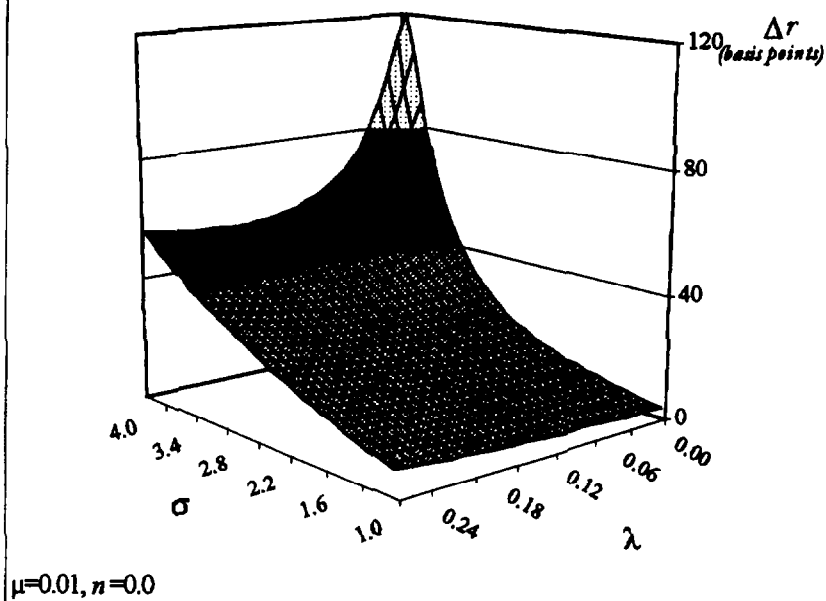


Figure 5b. Steady-State Effect of a Fiscal Debt Shock
Closed Economy with Life-Cycle Income

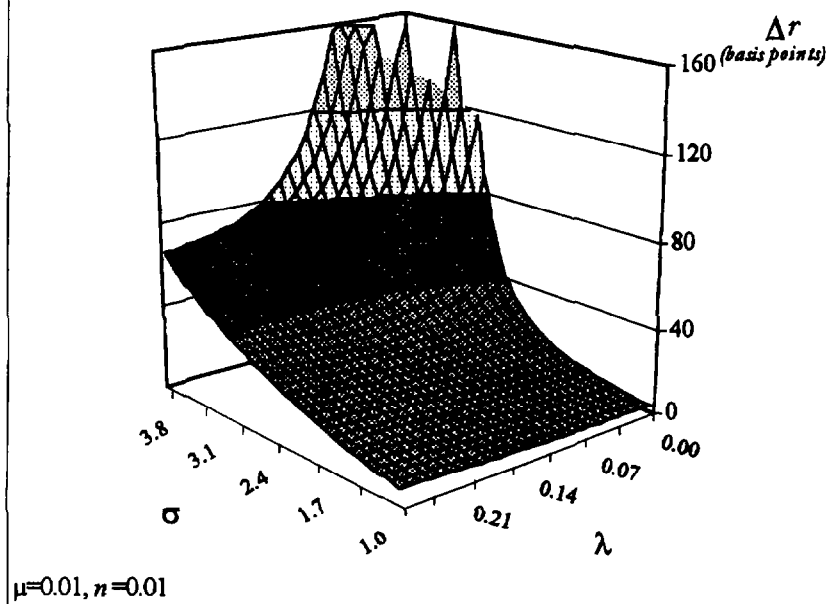


Figure 6a. Steady-State Effect of a Fiscal Debt Shock
Small Open Economy with Dynastic Households

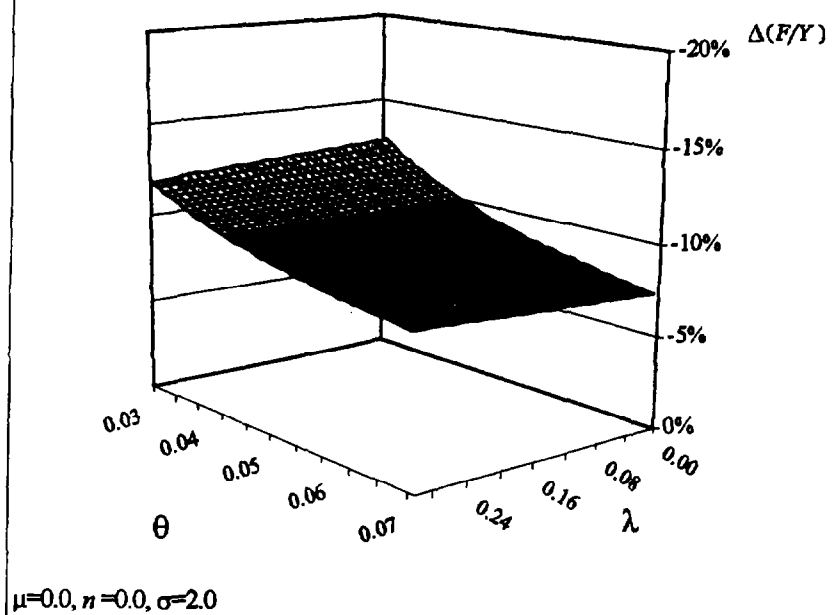
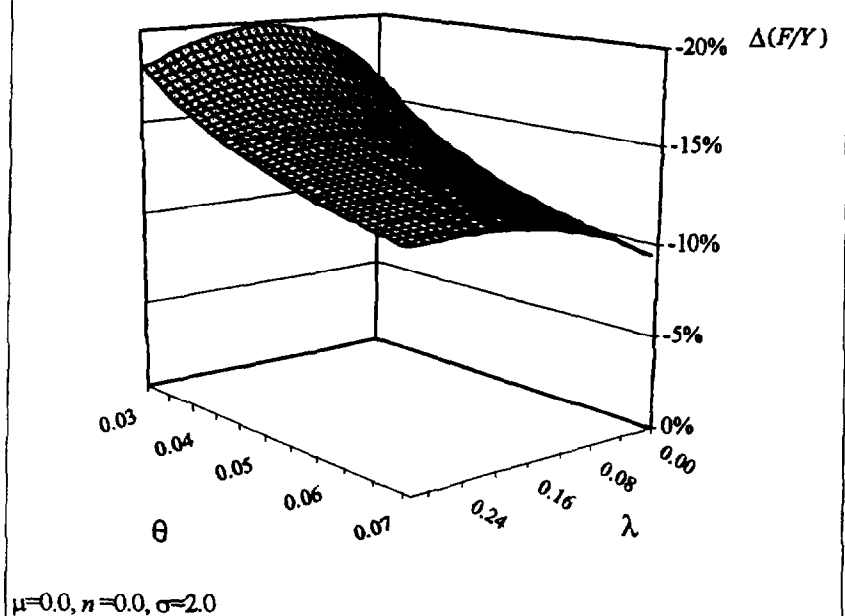


Figure 6a. Steady-State Effect of a Fiscal Debt Shock
Small Open Economy with Life-Cycle Income



The Basic Model

Consumer's Problem

To guard against the uncertainty of lifetimes or lineage, agents contract with insurance companies to receive (or make) payments contingent upon their death. To hedge against dying or dying without heirs unexpectedly, households agree to have all of their net wealth—which may be positive or negative—turned over to the insurance firm at the time of their death, in return for flow transfers at rate p while alive. Specifically, survivors with net wealth $w(s, t)$ receive $pw(s, t)$ from the insurance company.⁴⁹

Hence, the dynamic budget constraint facing each consumer or household can be written as: $\dot{w}(s, t) = [r(t) + p]w(s, t) + y(s, t) - \tau(s, t) - c(s, t)$.⁵⁰ Subject to this constraint, permanent-income consumers maximize expected utility over their lifetimes:

$$\max E_t \int_t^\infty \log[c(s, z)] e^{-\theta(z-t)} dz = \int_t^\infty \log[c(s, z)] e^{-(\theta+p)(z-t)} dz, s < i(t), \{c(s, z)\} \quad (A1)$$

where $E_t[\cdot]$ represents (rational) expectations conditional on the information set at time t , and θ is the rate of time preference. Note that the uncertainty of lifetimes, and hence uncertainty about future consumption, raises the effective discount rate. Utility is assumed to be logarithmic for convenience.⁵¹ Solving this dynamic optimization problem facing permanent-income consumers (with log utility), we derive the consumption function for these individuals given in the text.⁵²

⁴⁹This is the zero profit condition for the perfectly-competitive insurance industry. Insurance firms pay $pw(s, t)$ to surviving member of cohort s and inherit estates worth $w(s, t)$ from the proportion p of that cohort who die at time t . See Yaari (1965).

⁵⁰Leaving aside the depreciation of capital assets.

⁵¹Under the general class of CRRA utility the marginal propensity to consume (mpc) out of wealth depends on the interest rate: $c(s, t) = \Delta^{-1}[w(s, t) + h(s, t)]$, where the inverse of the mpc Δ evolves according to: $\dot{\Delta} = [(1 - 1/\sigma)r(t) + p + \theta/\sigma]\Delta - 1$ and where σ^{-1} is the intertemporal elasticity of substitution. With log utility, $\sigma = 1$ and $\dot{\Delta} = 0$.

⁵²In deriving consumption, a transversality (no-Ponzi game) condition on wealth is imposed, preventing agents from accumulating debt indefinitely at a rate higher than the effective rate of interest: $\lim_{z \rightarrow \infty} w(s, z) e^{-(r+p)z} = 0$.

Investment and the Firm's Problem

We consider the behavior of the firm. Assuming price-taking (infinitely-lived) firms maximize the present value of the cash flows and face (convex) installation costs of investment, the representative firm employs labor and chooses investment in order to maximize:

$$\max \int_t^{\infty} [F(K(z), L(z)) - w(z)L(z) - I(z) - A(I(z))]e^{-r(z-t)} dz \quad (A2)$$

$$\{L(z), I(z)\}$$

Note that cash flows for the firm derive from revenues less labor costs, gross investment I , and adjustment or installation costs of capital $A(\cdot)$ —the last two terms comprising total investment expenditure \tilde{I} . With costly adjustment of capital, greater investment expenditure $(I + A)$ is required for a given incremental increase in the capital stock (net of depreciation): $\dot{K} = I - \delta K$. In what follows, it is assumed that installation costs are of the form:⁵³

$$A(I, K) = \frac{1}{2} \left(\frac{I}{K} - \delta \right)^2 K$$

Solving the optimal control problem defined in (A1) given the transition equation for capital (state variable), one can express the optimal investment decision in terms of the shadow price of capital q (co-state variable) as shown in Table 1, where q measures the value of an additional unit of installed capital (i.e., "marginal q " given the price of capital goods), and evolves according the dynamic equation (7) in Table 1.⁵⁴ As for the employment decision, with no population growth, L is normalized to unity. Hence, the first order (optimality) condition for employment equates the marginal product of labor with the real wage or labor income Y (given inelastic labor supply): $f'(K) = Y$.

⁵³See Lucas (1967) or Treadway (1969)

⁵⁴See Abel (1990) or Hayashi (1982) for a discussion of neoclassical investment theory and the q -theory of investment from Tobin (1969).

Further Extensions: Liquidity Constraints

The overlapping generations framework can be further extended to consider the case where capital market imperfections preclude some agents from borrowing against their future income. In particular, it is assumed that younger generations who have little or no financial wealth are initially denied access to credit markets and, hence, are left to consume out of current resources.⁵⁵ Specifically, we define $i(t)$ as the index denoting the oldest generation still credit-rationed at time t . Assuming that a generation graduates out of the pool just as another is born into it, the fixed proportion λ of liquidity-constrained individuals in the economy is given by:

$$\int_{i(t)}^t p e^{-p(t-s)} ds + 1 - e^{-p(t-i(t))} = \lambda; di(t)/dt = 1. \quad (A3)$$

For these current-income consumers belonging to generations $s > i(t)$, their consumption is constrained by current disposable income: $c(s, t) = y(s, t) - \tau(s, t)$, where $y(s, t)$, $c(s, t)$ and $\tau(s, t)$ are labor income, consumption, and taxes for generation s at time t .

Consequently, overall consumption is characterized by the behavior of *both* permanent- and current-income consumers. Leaving aside life-cycle income momentarily, aggregate consumption can be written compactly as:

$$\begin{aligned} C &= (\theta + p)[W + (1 - \lambda)H] + \lambda[Y - T], \\ &= C^p + C^c \end{aligned} \quad (A4)$$

Agents with the ability to borrow choose their consumption (C^p) based on permanent income as before, which consists of financial wealth W and human wealth H . Meanwhile, agents who face borrowing constraints have their consumption (C^c) constrained by current disposable income, where Y is labor income and T is lump-sum taxes. The parameter represents the proportion of households in the latter category, and total consumption is simply the sum of consumption by permanent-income and current-income consumers.

In equation (A4), λ can be interpreted as the degree of *excess sensitivity* of consumption to current disposable income compared to the case where every agent behaves according to the permanent income hypothesis. Note from the equation that permanent-income consumers who make up $1 - \lambda$ of the population, own $1 - \lambda$ of the human wealth but hold *all* of the financial wealth in the economy. This follows since every individual is born without assets and newer generations do not save initially while they are credit-rationed.

⁵⁵ A common explanation for the presence of borrowing constraints involves agency problems (moral hazard and adverse selection) in credit markets stemming from collateral issues or asymmetric information. See Buiter (1994) or Stiglitz and Weiss (1991)

Finally, in the presence of age-dependent income (and taxes) as seen in equation (2), aggregate consumption can be summarized as follows:

$$C = (\theta + p)[W + \beta(1 - \lambda_1)H_1 + (1 - \beta(1 - \lambda_2))H_2] + [\beta\lambda_1 + (1 - \beta)\lambda_2](Y - T) \quad (A5)$$

With generation-specific income, the excess sensitivity of consumption to income now depends on the relative share of aggregate disposable income held by current-income consumers—seen by the coefficient $\beta\lambda_1 + (1 - \beta)\lambda_2$ —rather than the proportion of liquidity-constrained consumers in the population.⁵⁶

⁵⁶By adding up, we have $\beta \equiv a_1 p / (p + \alpha_1)$, $\lambda_1 \equiv 1 - e^{-(\alpha_1 + p)(t-i(t))}$, $\lambda_2 \equiv 1 - e^{-(\alpha_2 + p)(t-i(t))}$. Note that $\lambda_2 > \lambda_1$ by definition, and both parameters can be shown to be $> \lambda$ for plausible income profiles.

Population and Productivity Growth

The basic model can be further extended to the case of population and productivity growth.⁵⁷ In the case of a growing population, the rate of population growth n is equal to the difference between the birth and death rates: $n=b-p$. The size of the total "population" at each moment in time is given by $N(t) = e^{nt}$, where $N(0)$ is normalized to unity. In the case of dynasties, N would represent the number of dynastic families; if the number of members within these family were constant then the total population would be proportional to the number of dynasties. In terms of specific cohorts, the number of individuals (or dynasties) born as part of cohort s is a proportion of the contemporaneous population given by $N(s,s)=bN(s)$, and the number of these individuals surviving at time $t \geq s$ is given by $N(s,t) = bN(s)e^{-p(t-s)}$. Hence, the population at time t can also be expressed as the sum of surviving individuals from all generations: $N(t) = \int_{-\infty}^t bN(s)e^{-p(t-s)} ds = e^{nt}$.⁵⁸

Similarly, we can introduce long-run growth in productivity. Assuming (Harrod-neutral) labor-augmenting technical change, labor productivity is assumed to grow at a constant rate μ . In other words, labor input L , measured in efficiency units, depends on both the number of workers and the efficiency of each worker: $L(t) = N(t)e^{\mu t}$. In the case where N represents the number of dynasties (rather than individuals), the labor force would be proportional to the number of these households. As with the population, the level of productivity at $t=0$ is set equal to unity.

For n or $\mu > 0$, we normalize aggregate variables (denoted by lowercase)⁵⁹ in terms of labor, measured in efficiency units: $x(t) \equiv X(t) / L(t) = X(t)e^{-(n+\mu)t}$. Based on this renormalization, the revised equations for the closed economy model under our dynastic assumption with population and productivity growth and liquidity constraints is summarized in Table A1. Note that since the denominator implicit in the time derivatives is growing at rate $n+\mu$, this growth factor must be accounted for on the right hand side of the dynamic equations. So for example, if government debt were to increase in terms of efficiency units of

⁵⁷See also Buiter (1988)

⁵⁸More generally, with time-varying birth and death rates (demographic shocks), the population level would reflect the past accumulation of these shocks to population growth

$n(t): N(t) = e^{\int_0^t (b(v)-p(v)) dv}$ where the population at time zero is again normalized to unity. In what follows, we assume that birth and death rates are *constant* but unequal.

⁵⁹Lowercase variables with a time and generation index refer to individual measures whereas lowercase variables with only a time argument reflect per capita measures (in units of labor efficiency).

labor, it must more than keep pace with the growth of productivity and the population (i.e., scale effect).

In the case of human wealth, the dynamic equation in the table reflects further modifications which are needed in the case of population growth. Specifically, the aggregate dynamics of absolute human wealth (without life-cycle income) with population growth must be revised:

$$\dot{H} = [r + n + p]H - [Y - T] \quad (A6)$$

The instantaneous change in human wealth now also includes the rate of population growth (previously, $n=0$). This reflects the fact that new additions of agents to the overall population contribute to the present value of aggregate labor income. Expressing this transition equation in labor efficiency units then yields the dynamic equation in Table A1.

In the case of overlapping generations with life-cycle income, further substantive modifications are also needed in the case of population growth. Specifically, to ensure adding up, individual labor income are now expressed as function of aggregate labor income *per capita*:

$$y(s, t) = [a_1 e^{-\alpha_1(t-s)} + a_2 e^{-\alpha_2(t-s)}] Y(t) e^{-nt}; \quad \frac{a_1 b}{\alpha_1 + b} + \frac{a_2 b}{\alpha_2 p} = 1 \quad A(7)$$

The second part of this expression reflects the adding-up restriction on the parameters in terms of the *birth* rate (rather than death rate) so that individual incomes sum to aggregate income; the earlier example in the text showed the simpler case where $b=p$ (i.e., stationary population). The dynamic equations for human wealth (in labor efficiency units) under life-cycle income are derived analogously as before:⁶⁰

$$h = \beta h_1 + (1 - \beta) h_2 \quad A(8)$$

$$\dot{h}_1 = [r + p + \alpha_1 - \mu] h_1 - [y - \tau] \quad A(9)$$

$$\dot{h}_2 = [r + p + \alpha_2 - \mu] h_2 - [y - \tau] \quad A(10)$$

These equations would replace the dynamics for human wealth shown in Table A1 in the life cycle case. Finally, (normalized) consumption with life-cycle income and liquidity constraints would also be modified accordingly as follows:

$$c = (\theta + p) [w + \beta(1 - \lambda_1) h_1 + (1 - \beta)(1 - \lambda_2) h_2] + (\beta \lambda_1 + (1 - \beta) \lambda) \quad A(11)$$

⁶⁰We now have: $\beta \equiv a_1 b / (b + \alpha_1)$, $\lambda_1 \equiv 1 - e^{-(\alpha_1 + b)(t-i(t))}$, $\lambda_2 \equiv 1 - e^{-(\alpha_2 + b)(t-i(t))}$.

Table A1 - Extended Model with Liquidity Constraints and
Population & Productivity Growth

$$c(t) = (\theta + p)[w(t) + h(t)] + \lambda[y(t) - \tau(t)] \quad (12)$$

$$i(t) = [q(t) - 1 + \delta + n + \mu]k(t) \quad (13)$$

$$\dot{k}(t) = f(k(t)) - c(t) - g(t) - a(t) - [\delta + n + \mu]k(t) \quad (14)$$

$$\dot{b}(t) = [r(t) - n - \mu]b(t) + g(t) - \tau(t) \quad (15)$$

$$\dot{h}(t) = [r(t) + p - \mu]h(t) - [y(t) - \tau(t)] \quad (16)$$

$$\dot{q}(t) = \left[r(t) + \delta - \frac{i(t)}{k(t)} \right] (q(t) - 1) + \frac{1}{2} [q(t) - 1]^2 \quad (17)$$

$$r(t) = f'(k(t)) - \delta \quad (18)$$

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