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Determinants of Dollarization: The Banking Side

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Abstract

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Dollarization in financial intermediation has exhibited a widely diverse pattern across countries. Empirical work relating it to macroeconomic variables has had only limited success in explaining the phenomenon. This paper presents a two-currency banking model to show that deposit and loan dollarization are determined by a broader set of factors. These include interest rates and exchange rate risk, as well as structural factors related to costly banking, credit market imperfections, and availability of tradable collateral. The direction in which dollarization tends to move with macroeconomic shocks is shown to depend on those factors as well as on initial dollarization levels.

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I. INTRODUCTION

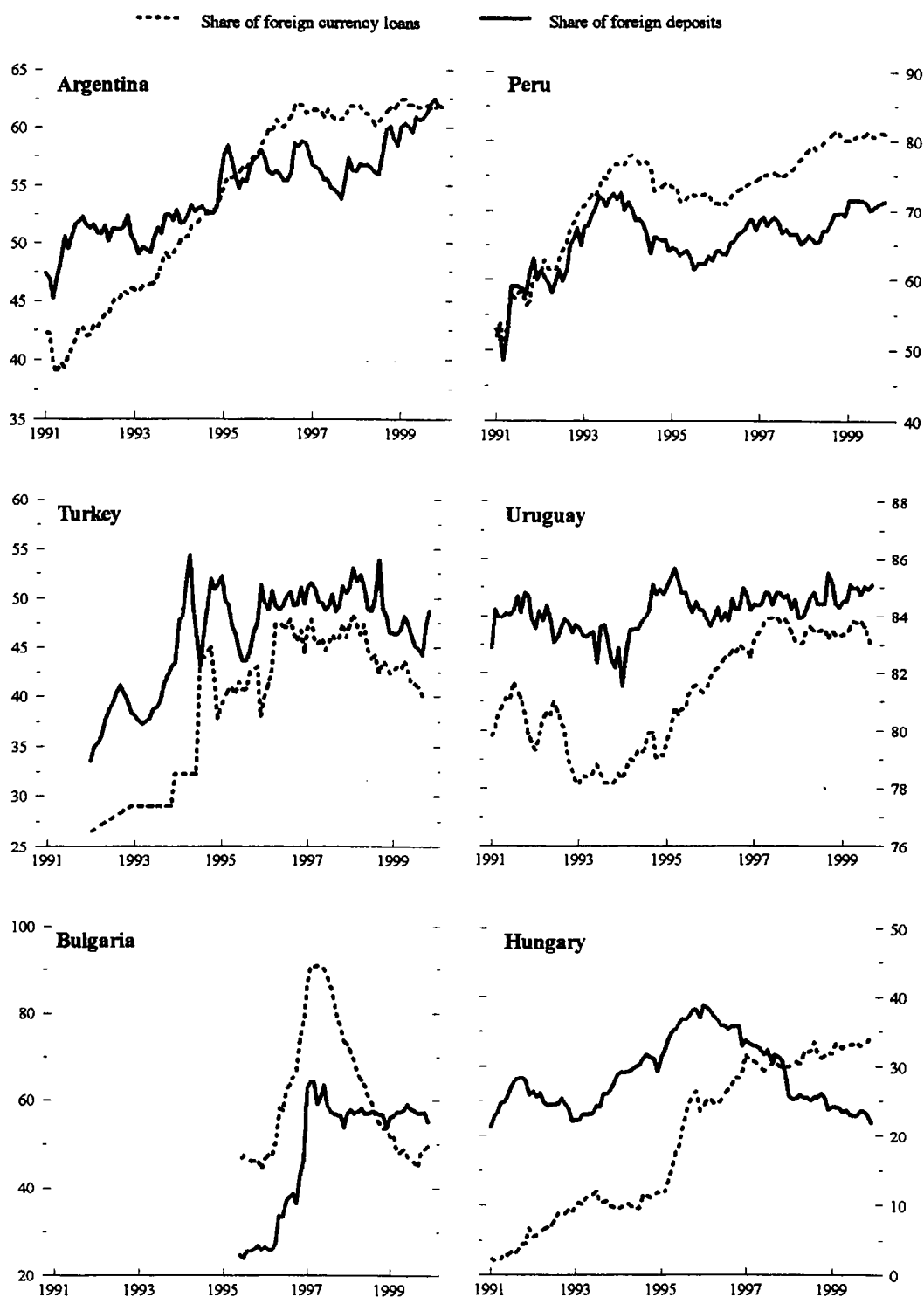
In an effort to end long histories of monetary disarray and move toward a well-functioning market system, several developing countries and transition economies embarked upon far-reaching financial liberalization and economic reform over the past decade. During this period, tighter links between their respective national currencies and a strong international reserve currency such as the U.S. dollar or the Deutsche Mark were established,² often through hard exchange rate pegs which underpinned stabilization programs in several of these countries. Financial integration with the United States or the European Union took a step further wherever capital account restrictions were eliminated and domestic financial intermediation was allowed to be conducted in both domestic and foreign currencies. With residents being able to denominate and settle domestic contracts in either currency, as well as arbitrage freely between onshore and offshore accounts, competition between the two monies intensified. In countries where such a dual currency arrangement was introduced, the share of foreign currency denominated assets and liabilities in the banking system rose very rapidly. By the end of the 1990s some of these economies became so highly "dollarized" that full dollarization has arisen as a serious policy option.³

Yet, the increase in dollarization has not been monotonic across countries. As shown in Figure 1, some countries have experienced very rapid increases in the share of foreign currency denominated deposits, sometimes followed by partial reversals or, more often, by a leveling-off. Moreover, the increased share of foreign currency denominated deposits in total deposits ("deposit dollarization") has not been always matched by a similar increase in the share of foreign currency loans in total bank lending ("loan dollarization"); as a result, some national banking systems have operated with some degree of currency mismatch between asset and liabilities, and the size of such mismatches also have varied over time. But perhaps even more strikingly is the fact that in none of the countries where the domestic and the foreign currency competed freely in loan and deposit markets, has full dollarization arisen as

² In the case of transition economies in Eastern Europe, links with the Deutsche Mark (and more recently with the euro) have been stronger, whereas in Latin America links with the U.S. dollar have been overwhelming. In this paper, we use the term "dollarization" as a generic for greater foreign currency participation in domestic financial intermediation, regardless of whether it refers to the U.S. dollar or the euro.

³ Highly dollarized economies include Argentina, Bolivia, Peru, and Uruguay, all of which display ratios of foreign currency deposits to broad money in excess of 50 percent. To date, only one country in Latin America (Panama) has fully replaced the domestic currency by the US dollar and another (Ecuador) is completing its dollarization process. Indicators of the extent of dollarization in a large sample of countries are provided in Baliño et al. (1999).

Figure 1. Dollarization in Selected Emerging Markets
(Percent)



Sources: national authorities; and IMF staff calculations.

a natural outcome of agents' preferences and optimal portfolio allocation by financial intermediaries.

This diversity of dollarization patterns, both across countries and over time, calls for an explanation. Most of the literature has focused on the dollarization of currency transactions ("currency substitution") and of bank deposits.⁴ The basic analytical framework has been the consumer's portfolio selection model, where dollarization is determined by the relative rates of returns of domestic currency and foreign currency denominated assets (Thomas, 1985). While there is evidence that rate of return differentials help explain swings in deposit dollarization in a few countries—notably in Eastern Europe (Sahay and Végh, 1996; Baliño et al., 1999), the same model has proved much less successful in explaining dollarization trends in others, notably (but not solely) in Latin America. As Figure 1 illustrates, dollarization in some Latin American countries has remained persistently high, even after real rates of return in domestic currency assets have risen relative to their foreign currency counterparts and single-digit inflation drastically reduced the opportunity cost of holding domestic currency demand deposits. This has given rise to "hysteresis" models which explain the persistence of dollarization by switching costs between currencies (Guidotti and Rodriguez, 1992; Uribe, 1997; Sturzenegger, 1997). However, the switching costs emphasized in this strand of the literature refer to the transactions component of broad money (currency and demand deposits), while the bulk of dollarization in recent years has occurred in the savings component (time deposits) which is relatively costless to switch. This suggests that other variable(s) not contemplated in those models may be at play.

But perhaps the most significant gap in the literature is the lack of work on the dollarization of bank loans. The focus on currency substitution and deposit dollarization overlooks the fact that bank credit has not only accounted for a substantial bulk of financial sector dollarization, but also displayed a distinct pattern from deposit dollarization in a number of instances. One notable exception is Ize and Levy-Yeyati (1998), who examine the determinants of deposit *and* credit dollarization in a minimum variance portfolio model where dollarization is driven by the volatility of inflation and the real exchange rate. Their model yields the interesting result that dollarization increases with inflation volatility and decreases with the volatility of the real exchange rate, but is only able to explain half of the

⁴ Useful overviews are provided in Calvo and Végh (1992) and Savastano (1996). As noted by those authors, the use of the term dollarization in the literature is sometimes confusing: in some studies it refers to currency substitution (or the dollarization of *narrow* money), whereas in others it also encompasses the dollarization of less liquid assets (such as time deposits). The problem with this is that currency substitution and the dollarization of time deposits tend to be determined by different sets of variables. Yet, because of deficient data on foreign currency holdings, several empirical studies use broad money indicators to test hypotheses regarding currency substitution.

changes in dollarization in their panel data sample of countries. Thus, one is again left with the impression that other significant variables have been omitted.

This paper tries to fill some of this gap. It shows that dollarization in financial intermediation is determined by a broader set of variables, once we incorporate into the analysis the roles of costly banking and credit market imperfections typically found in emerging markets. We consider an economy where a domestic and a foreign currency compete in deposit and loan markets, there is exchange rate risk, and banking activity is costly but plays a central and irreplaceable role in financial intermediation.⁵ In addition, banks in this economy are subject to loan default and face two types of credit market “imperfections”. One is the segmentation between tradable and non-tradable borrowers. This segmentation stems from the fact that tradable producers can usually pledge their loans against foreign currency denominated revenues or a tradable collateral, and have some access to international capital markets, as discussed in Caballero and Krishnamurthy (1998) and documented in Krueger and Tornell (1999); this makes the tradable sector more willing and apt to take foreign currency loans when borrowing from domestic banks. In contrast, non-tradable borrowers typically comprise the majority of households and smaller firms producing non-tradable goods, which generate revenues in domestic currency, have no direct access to foreign borrowing, and are unable to pledge their loans against a tradable collateral; this makes them less suited to borrow in foreign currency from local banks. The other type of credit market imperfection prevalent in emerging markets is limited bank competition, affecting in particular the latter class of borrowers. To the extent that banks do exert some monopoly power over clients who can only borrow locally—as it appears to be often the case (Cañonero, 1997; Vicens, 1997; Barajas et al. 1999), interest rate margins in this segment of the credit market will tend to be higher, making domestic currency lending more attractive in some circumstances.

In this set-up, banks will lend in domestic or foreign currency depending on the net balance of distinct forces. On the one hand, foreign currency loans will be more attractive the higher the devaluation risk, the greater their collateralization (itself a function of the availability of tradable collateral), and the lower the operating and funding costs of those loans. On the other hand, domestic currency loans will be more attractive the higher banks’ monopoly power over non-tradable borrowers, the lower the probability of devaluation and their impact on loan default, and the smaller the cost of raising deposits and managing loans in domestic currency. Thus, loan and deposit dollarization will be driven by the interplay between credit market structure, banking costs and macroeconomic shocks, rather than by macroeconomic variables or their variance alone, as in previous studies.

⁵ Costly banking and its central role in financial intermediation have featured prominently in the recent literature on the monetary transmission mechanism in emerging markets (e.g., Mishkin, 1997; Edwards and Végh, 1997). In this literature, banks take deposits and lend in domestic currency only. Here we extend these features to a two-currency framework.

The remainder of the paper is divided into three sections as follows. Section II lays out the basic model and derives its main propositions regarding the impact of shocks to external interest rates and exchange rate risk on the volume and currency composition of bank assets and liabilities. In this "stripped-down" version of the model, the incidence of loan default is assumed to be exogenous. This assumption is instrumental in facilitating understanding of the mechanics of the model and yields crispier results that may be relevant in some contexts. However, in light of evidence that interest rate and exchange rate shocks tend to have a significant impact on the quality of banks' loan portfolio, section III extends the model to the more complex case where loan default is endogenous, being *inter alia* a function of those two variables. We then discuss the roles of unhedged currency positions by the non-financial private sector, costly credit recovery, and loan collateralization in affecting the quality of banks' portfolio, showing how these impact on dollarization. Section IV concludes.

II. THE BASIC MODEL

Let the economy consist of n banking firms, indexed $i = 1, 2, \dots, n$, which are free to intermediate resources in domestic and foreign currencies. At any given moment of time, a bank i holds as assets loans in domestic (L_i) and foreign currency (L_i^*), and reserves in domestic (R_i) and foreign currency (R_i^*), and finances itself with deposits in domestic (D_i) and foreign currency (D_i^*), and net foreign borrowing (B_i^*).⁶ Bank i operates in a small open economy with free capital mobility and competitive deposit markets, so that it faces given interest rates on deposits in the domestic currency (r_D) and the foreign currency (r_D^*).⁷

On the asset side, credit markets are assumed to be imperfectly competitive and segmented between domestic and foreign currency borrowers. In particular, bank i faces downward sloping demand curves for loans in domestic and foreign currencies:

⁶ This approach to modeling the banking firm follows a long tradition starting with Klein (1971). A useful review and references are provided in Freixas and Rochet (1997). In this paper we extend this basic framework to the two currency case building on earlier work by Terrones (1994).

⁷ The assumption of perfectly competitive deposit markets is not unreasonable for some emerging markets and has been adopted in other studies (see Barajas et al. 1999, and references therein). One main reason is that long histories of high inflation and unexpected devaluations have encouraged depositors to arbitrage more effectively between domestic banks as well as between onshore and offshore accounts. In several countries, competition in local deposit markets appears to have been further intensified in recent years by financial liberalization and foreign entry (Claessens et al., 1998).

$$r_L = r_L(L) \text{ where } \frac{\partial r_L}{\partial L} < 0 \quad (1.a)$$

$$r_L^* = r_L^*(L^*) \text{ where } \frac{\partial r_L^*}{\partial L^*} < 0, \quad (1.b)$$

where r_L and r_L^* stand for the respective lending rates in domestic and foreign currency (call it "dollar"), and the inverse demand functions rule out substitutability between domestic and foreign currency loans. While such an assumption is a convenient simplification to make the model more tractable, it is also firmly grounded on evidence from other studies, as discussed in section I. Imperfect substitutability between loans in the two currencies is also corroborated by the wide interest rate differential between domestic and foreign currency loans in several dollarized economies, even after correcting for measures of implicit devaluation risk, such as the interest rate differential between domestic and foreign currency denominated deposits (Figure 2 and 3).

Bank i is risk-neutral and seeks to maximize profits in dollars.⁸ Abstracting from bankruptcy constraints, it will take three factors into account. One is the expected change in the exchange rate between the domestic currency (call it "peso") and the dollar during the interval since it disburses the loan and when it is due to be repaid. The other is the percentage share of non-performing loans in the peso and the dollar segments of the credit market, i.e., how much of its total loans the bank expects to recover fully. Third, banks will consider the structure of minimum reserve requirements on their liabilities set by the regulatory authorities, as these can change the relative costs of raising peso and foreign currency deposits or that between deposits and foreign borrowing.

In light of the above, bank's i optimization problem can be formalized as:

$$\begin{aligned} \text{Max } E(\Pi) = & \alpha[r_L(L) + E(\tilde{\delta})]L_i + \alpha^* r_L^*(L^*) L_i^* + [r_E + E(\tilde{\delta})]R_i + r_E^* R_i^* - \\ & [r_D(D) + E(\tilde{\delta})]D_i - r_D^*(D^*) D_i^* - r^* B_i^* - C(D_i, D_i^*, L_i, L_i^*; 1 - \alpha, 1 - \alpha^*) \end{aligned} \quad (2)$$

subject to

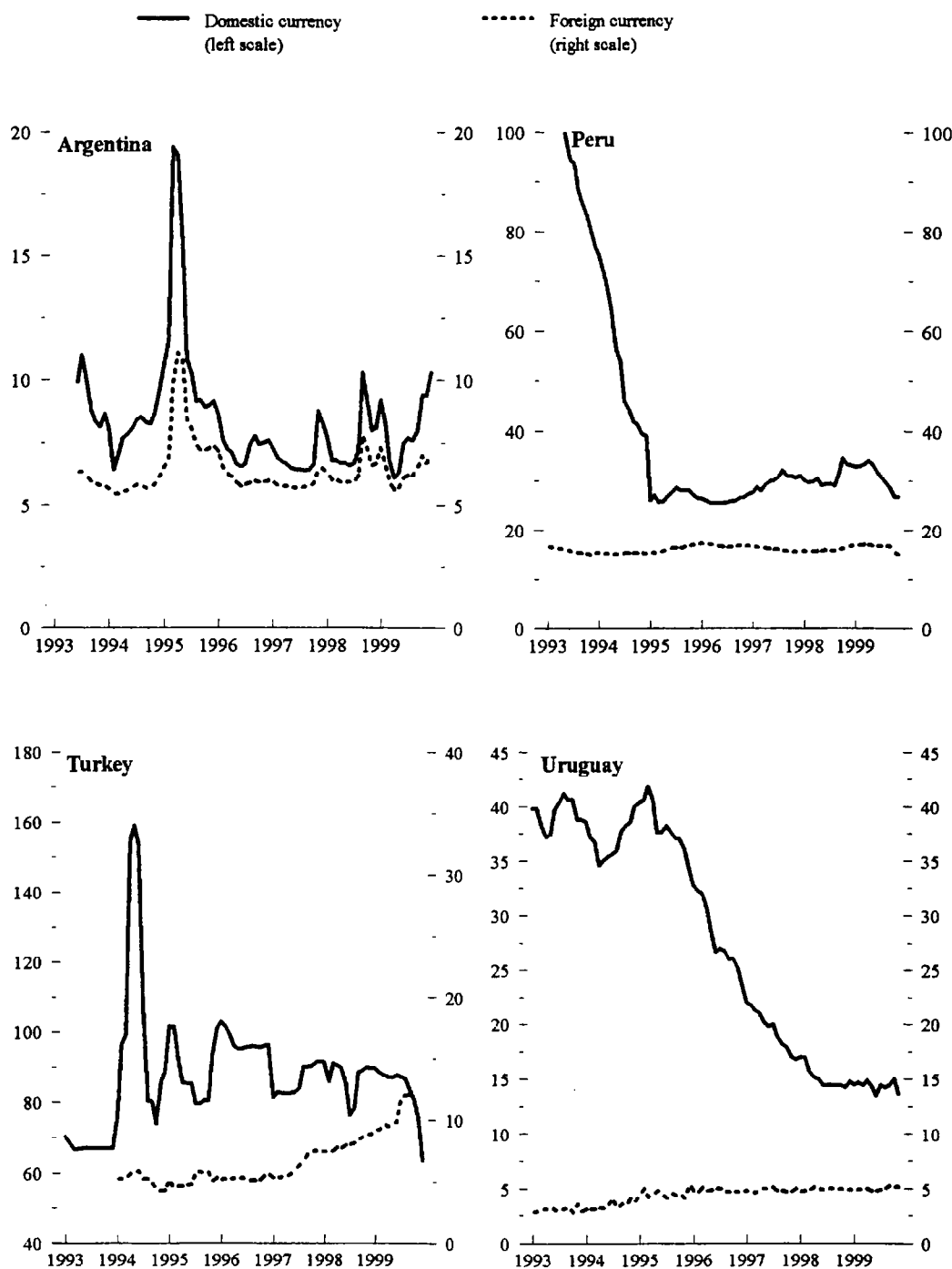
$$L_i + L_i^* + R_i + R_i^* = D_i + D_i^* + B_i^* \quad (3)$$

$$\varepsilon D_i \leq R_i \quad (4)$$

$$\varepsilon^* D_i^* \leq R_i^* \quad (5)$$

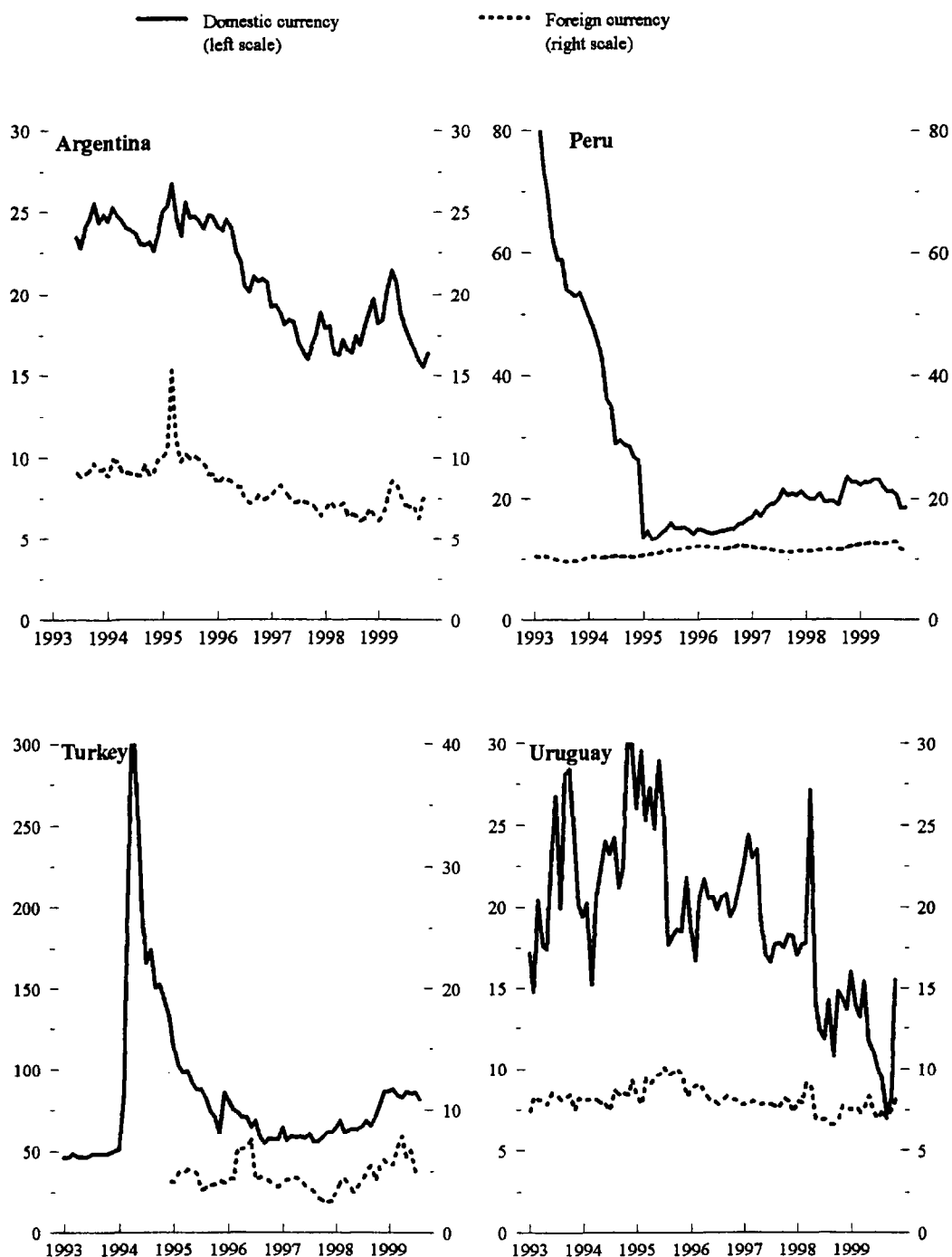
⁸ The assumption that banks maximize profits in dollars rather than in domestic currency seems very appropriate in an economy where the dollar has taken an increasing role as unit of account and store of value. The greater participation of foreign banks in emerging markets in recent years adds further realism to this assumption, since these institutions seek to maximize profits denominated in a foreign reserve currency which is often the U.S. dollar.

Figure 2. Deposit Interest Rates in Selected Emerging Markets
(Percent a year)



Sources: national authorities; and IMF staff calculations.

Figure 3. Loan-Deposit Interest Spreads in Selected Emerging Markets
(Percent a year)



Sources: national authorities; and IMF staff calculations.

where E is the expectation operator and π stands for the bank's profits; $\varepsilon, \varepsilon^*$ are the reserve requirements remunerated at interest rates r_E and r_E^* ; $\tilde{\delta}$ is the rate of appreciation of the peso relative to the dollar, where the base exchange rate has been normalized to one;⁹ α, α^* are the percentage share of performing loans in total lending in each currency which for now we shall assume as exogenously given;¹⁰ r^* is the parametric world interest rate denominated in U.S. dollars; and $C(L_i, L_i^*, D_i, D_i^*; 1-\alpha, 1-\alpha^*)$ is an operating cost function which depends on the banks' intermediation activities and the quality of the bank's loan portfolio.¹¹

Equation (3) is the balance sheet identity constraint. Equations (4) and (5) spell out the minimum reserve requirements on deposits set by the regulatory authority for the general case where these requirements may differ according to the currency denomination of deposits, and may be remunerated or not.¹²

Solution to the optimization problem laid out in equations (2) to (5) requires specifying the probability distribution function of $\tilde{\delta}$ as well as how the lending decisions by bank i relate to lending decisions by the rest of the banking sector under imperfect competition. In line with the experience of several emerging economies that have adopted fixed exchange rate regimes in recent years, assume that the probability distribution function of $\tilde{\delta}$ is as follows:

$$\tilde{\delta} = \begin{cases} 0, & 1-p \\ -\Delta, & p \end{cases} \quad (6)$$

⁹ To simplify the algebra and make the presentation crispier, we assume that $r_L \delta \approx 0$, $r_D \delta \approx 0$, and $r_E \delta \approx 0$. This will not affect, however, the thrust of our results.

¹⁰ We relax this assumption later in the paper and discuss how this affects our main results.

¹¹ The inclusion of the shares of non-performing portfolio ($1-\alpha, 1-\alpha^*$) in the operating cost functions can be rationalized in two ways. One is that it represents the cost of setting aside a certain amount of loan provisions required by the outside regulator. Alternatively, the term could be thought of as capturing the costs of collecting the loan principal.

¹² Consistent with regulations in various (but not all) countries we simply assume that net foreign borrowing B^* is not subject to reserve requirements or any quantitative restriction. However, this assumption can be readily modified. Assuming, for instance, an infinitely elastic supply of international capital, the imposition of reserve requirements on banks' foreign borrowing is tantamount to an increase in the external interest rate.

where the peg has a $1-p$ probability of being maintained and a probability p of being abandoned with a rate of devaluation Δ . This implies that $E(\tilde{\delta}) = -p\Delta$.

A simple way of modeling imperfect competition in credit markets and canonically express the interdependence between bank's i lending decisions and those of the rest of the industry is through Dixit's (1986) concept of "conjectural variations".¹³ Let aggregate bank loans be denoted by $L = \sum L_i$ and $L^* = \sum L_i^*$, so that we can define total lending by other banks as

$$\Lambda_i = L - L_i = \sum_{j \neq i} L_j \quad (7.a)$$

$$\Lambda_i^* = L^* - L_i^* = \sum_{j \neq i} L_j^* \quad (7.b)$$

The "conjectural variation" coefficients that relate individual banking lending with that of the rest of the banking system are thus defined as:

$$\frac{\partial \Lambda_i}{\partial L_i} = \psi(L_i, \Lambda_i), \quad (8.a)$$

$$\frac{\partial \Lambda_i^*}{\partial L_i^*} = \psi^*(L_i^*, \Lambda_i^*), \quad (8.b)$$

It can be shown that under **perfect competition** $\psi(L_i, \Lambda_i) = \psi^*(L_i^*, \Lambda_i^*) = -1$, under **competitive collusion** $\psi(L_i, \Lambda_i) = \Lambda_i / L_i$ and $\psi^*(L_i^*, \Lambda_i^*) = \Lambda_i^* / L_i^*$ —which in the particular case in which the banking firms are identical become constant and equal to $n-1$,¹⁴ and under **competitive oligopoly** of the Cournot type, $\psi(L_i, \Lambda_i) = \psi^*(L_i^*, \Lambda_i^*) = 0$.

The Lagrangian associated with bank i 's optimization problem is obtained by substituting equations (3) to (8) into (2). As $L_i, L_i^*, R_i, R_i^*, D_i, D_i^*, B_i^* > 0$, and provided that legal reserve requirements are binding, the model can be solved for the optimal levels of these variables. In the Appendix we show that, under standard assumptions about the bank's

¹³ For a discussion of advantages and shortcomings of the conjectural variation approach, see Quirmback (1988). Its use here has been motivated by the intuitive and simple way in which it allows us to represent different market structures within a unified framework.

¹⁴ If all banks have an equal share of the loan market, $L = nL_i$. Substituting in (8.a), we obtain that $\Lambda_i = nL_i - L_i = L_i(n-1)$. Hence, $\Lambda_i / L_i = n-1$. The same relations apply to dollar lending.

cost functions, this optimal is a maximum. On this basis, we define loan dollarization (l) and deposit dollarization (d) as:

$$l = \frac{L^*}{(L + L^*)} \quad (9.a); \quad \text{and} \quad d = \frac{D^*}{(D + D^*)} \quad (9.b)$$

which in equilibrium are function of the model's parameters. In what follows, we will examine how l and d will move with shocks to r^* and $\tilde{\delta}$, and examine the role of market structure and loan default in this connection.¹⁵ For the sake of generality, we impose only the minimally necessary amount restrictions on the model's parameters, leaving further specialization for future empirical work. These restrictions amount to standard assumptions about the convexity of banks' cost function (see the Appendix) and the following simplifying assumption about aggregation. Variables l and d are defined for the banking system as a whole, whereas the optimization problem above was set up in terms of the individual bank. Without having to specialize the results for the perfect competition case, and maintaining instead the conjectural variation framework laid in equations (7) and (8), an usefully simple approach to aggregation is to assume that the economy has identical banks.¹⁶ We follow this approach through the remainder of the paper.

II.1. Impact of External Interest Rate Shocks

Proposition 1. A decline (increase) in the external interest rate r^* will lead to an expansion (contraction) in loans financed by higher (lower) net external borrowing, as deposits in both currencies will drop (rise). However, whether loan dollarization will increase or decrease depends on the pre-shock level of loan dollarization, credit market structures, the share of performing loans in the bank's domestic and foreign currency portfolio, and the marginal cost of intermediation in each currency. Likewise, whether deposit dollarization will fall or rise depends on all these factors plus the reserve requirements on deposits.

Proof: The first part of the proposition follows from applying the implicit function theorem to the first order conditions and using Cramer's rule. Under standard assumptions about the bank cost function discussed in the Appendix, the following results hold:

¹⁵ Due to space constraints, we shall not report here the effects of changes in reserve requirements. The results of this exercise can be obtained from the authors upon request.

¹⁶ This rules out the existence of complex kinks in aggregate cost curves which would complicate considerably the algebra possibly without adding any further significant insight.

$$\frac{\partial L_i}{\partial r^*} = -\frac{w^*}{|H|} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} < 0, \quad \frac{\partial L_i^*}{\partial r^*} = -\frac{w}{|H|} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} < 0 \quad (10.a)$$

$$\frac{\partial D_i}{\partial r^*} = -\frac{(1-\varepsilon)}{|H|} \frac{\partial^2 C}{\partial D_i^{*2}} w^* w > 0, \quad \frac{\partial D_i^*}{\partial r^*} = -\frac{(1-\varepsilon^*)}{|H|} \frac{\partial^2 C}{\partial D_i^2} w^* w > 0 \quad (10.b)$$

$$\frac{\partial B_i^*}{\partial r^*} = -\frac{(w+w^*)}{|H|} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} + \frac{w^* w}{|H|} \left[\frac{\partial^2 C}{\partial D_i^2} (1-\varepsilon^*)^2 + \frac{\partial^2 C}{\partial D_i^{*2}} (1-\varepsilon)^2 \right] < 0 \quad (10.c)$$

where $|H|$ is the determinant of the Hessian associated with bank's i optimization problem which is shown in the Appendix to be strictly negative, and

$$w_i = \alpha a_i - \frac{\partial^2 C}{\partial L_i^2}, \text{ where } a_i = 2 \frac{\partial r_L}{\partial L_i} (1 + \Psi^i) + \frac{\partial r_L}{\partial L_i} L_i \left(\frac{\partial \Psi^i}{\partial L_i} + \frac{\partial \Psi^i}{\partial \Lambda_i} \Psi^i \right) + \frac{\partial^2 r_L}{\partial L_i^2} L_i (1 + \Psi^i)^2 \quad (11.a)$$

$$w_i^* = \alpha^* a_i^* - \frac{\partial^2 C}{\partial L_i^{*2}}, \text{ where } a_i^* = 2 \frac{\partial r_L^*}{\partial L_i^*} (1 + \Psi^{*i}) + \frac{\partial r_L^*}{\partial L_i^*} L_i \left(\frac{\partial \Psi^{*i}}{\partial L_i^*} + \frac{\partial \Psi^{*i}}{\partial \Lambda_i^*} \Psi^{*i} \right) + \frac{\partial^2 r_L^*}{\partial L_i^{*2}} L_i (1 + \Psi^{*i})^2 \quad (11.b)$$

with a and a^* being coefficients that capture market structure effects and shown to be zero when the loan market is perfectly competitive, while w and w^* are shown to be negative (see Appendix).

Using (9.a) and (9.b), loan and deposit dollarization will thus be said to increase when $\partial l / \partial r^* > 0$ and $\partial d / \partial r^* > 0$, respectively. Deriving (9.a) and (9.b) in relation to r^* and using equations (10) and (11), it can be shown that:

$$\frac{\partial l_i}{\partial r^*} = \begin{cases} \geq 0, & \text{if } \frac{l_i}{1-l_i} \leq \frac{w_i}{w_i^*} \\ < 0, & \text{if } \frac{l_i}{1-l_i} > \frac{w_i}{w_i^*} \end{cases} \quad (12.a); \text{ and } \frac{\partial d_i}{\partial r^*} = \begin{cases} \geq 0, & \text{if } \frac{d_i}{1-d_i} \leq \frac{(1-\varepsilon^*)}{(1-\varepsilon)} \frac{\partial^2 C / \partial D_i^2}{\partial^2 C / \partial D_i^{*2}} \\ < 0, & \text{if } \frac{d_i}{1-d_i} > \frac{(1-\varepsilon^*)}{(1-\varepsilon)} \frac{\partial^2 C / \partial D_i^2}{\partial^2 C / \partial D_i^{*2}} \end{cases} \quad (12.b)$$

thus indicating that loan and deposit dollarization will increase or decrease depending on the initial or pre-shock levels of dollarization and the various cost, market structure and regulatory parameters discussed. ■

The intuition behind the first sentence of proposition 1 is straightforward. When the funding cost of raising funds abroad drops, bank i will borrow and lend more in both currencies. At the same time, it will also take advantage of lower borrowing costs abroad and substitute net external borrowing for deposits on the liability side. The results regarding the direction in which dollarization will take following an external interest rate shock are, however, ambiguous. For the sake of generality we presented those findings without specifying likely ranges for the parameters involved, but more specificity is needed to help understand actual dollarization episodes.

Since emerging economies are often categorized in terms of their currency arrangements as low (or un-) dollarized, semi-dollarized, and highly dollarized by some analysts, a simple formalization of these categories can help us to specialize the results of proposition 1. Let the three polar cases be: that of an economy in its initial stages of dollarization (i.e. $l \rightarrow 0$ and $d \rightarrow 0$), that of a nearly fully dollarized economy (i.e. $l \rightarrow 1$ and $d \rightarrow 1$) and the case of a semi-dollarized economy (i.e. $l \rightarrow 1/2$ and $d \rightarrow 1/2$). In the semi-dollarization case, we can specify things further, drawing on the results of the literature referred to above which suggests that domestic currency loan markets are substantially less competitive than foreign currency counterpart. Once we consider these specific cases, unambiguous predictions about the impact of external interest rate shocks on the direction of dollarization arise, as the following corollaries demonstrate.¹⁷

Corollary 1a. A decline in the external interest rate unambiguously increases loan dollarization if the economy is very little dollarized on the loan side (i.e. $l_i \rightarrow 0$), but reduce loan dollarization if the economy is nearly fully dollarized (i.e. $l_i \rightarrow 1$). If the economy is semi-dollarized (i.e. $l_i \rightarrow 1/2$), dollarization can increase, decrease or remain unchanged depending on the relative convexities of the marginal cost function for each currency and the degree of competition in the loan market in foreign currency relative to the one in domestic currency.

¹⁷ For the sake of conciseness we do not provide here the expressions for deposit dollarization. Needless to say, when loan dollarization increase, liability dollarization will also increase. However, the composition of liabilities between deposits and net foreign borrowing will change, as shown by equations (10.a) and (10.b). The derivations of the expressions for deposit dollarization are available from the authors upon request.

Proof. It can be readily seen from (13) that

$$l_i \rightarrow 0 \therefore \frac{\partial l_i}{\partial r^*} \rightarrow -\frac{w_i}{|H|(L_i + L_i^*)} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} < 0 \quad (13.a)$$

$$l_i \rightarrow 1 \therefore \frac{\partial l_i}{\partial r^*} \rightarrow \frac{w_i^*}{|H|(L_i + L_i^*)} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} > 0 \quad (13.b)$$

$$l_i \rightarrow 1/2 \therefore \frac{\partial l_i}{\partial r^*} \rightarrow -\frac{1/2}{|H|(L_i + L_i^*)} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} [w_i - w_i^*], \quad (13.c)$$

where in the latter case: $\frac{\partial l_i}{\partial r^*} = \begin{cases} < 0, & \text{if } w_i^* > w_i \\ \geq 0, & \text{otherwise} \end{cases}$. ■

Corollary 1b. Consider the latter case, where the loan portfolio is semi-dollarized (i.e. $l_i \rightarrow 1/2$), and assume that the bank's marginal cost function have a similar convexity with regard to domestic and foreign currency loans (i.e. $\partial^2 C / \partial L_i^2 = \partial^2 C / \partial L_i^{*2}$ for any $L_i \approx L_i^*$). Loan dollarization will increase (decrease) with a fall (rise) in the external interest rate if the foreign currency segment of the loan market is perfectly competitive and the market for domestic currency loans is imperfectly competitive.

Proof. A glance at equation (11) shows that, for $a_i < 0$ (imperfect competition in the peso loan market) and $a_i^* = 0$ (perfect competition in the dollar loan market), $|w_i| > |w_i^*|$. Since $w_i < 0$, the term $[w_i^* - w_i]$ in (13.c) will be strictly positive. Since $|H| < 0$, it is clear that $\partial l_i / \partial r^* < 0$. ■

The upshot is that, for low or semi-dollarized economies, dollarization will tend to increase as the external interest rates falls and decrease as it rises. The basic intuition is that lower external interest rates will induce banks to fund their loans from abroad, and since all external borrowing is in foreign currency, banks will be also more inclined to lend in US dollars domestically relative to lending in pesos, as they seek to hedge against devaluation risk. Thus, asset dollarization will increase as external financing conditions ease, and the converse happens when the external financing conditions tighten. There is some evidence of such an inverse relationship between the external interest rate on the one hand, and bank lending growth and dollarization on the other in many emerging markets during the 1990s. In the first half of the 1990s, for instance, when interest rates in advanced economies were low and overall market sentiment toward emerging economies was upbeat, bank lending in several of these countries experienced a rapid expansion while the share of dollar-

denominated assets and liabilities increased. Conversely, as external financial conditions got tightened during the 1995 “Tequila” crisis in Latin America as well as toward the end of the decade following the 1997/98 financial crises in Asia, the pace of both overall financial intermediation and dollarization slowed considerably.

II.2. Impact of Shocks to Devaluation Expectations

Proposition 2. An increase in devaluation risk lowers domestic currency lending, while increasing domestic currency deposits. The counterpart of the increase in peso deposits is the drop in net foreign borrowing. Thus, loan dollarization will increase but deposit dollarization will decrease.

Proof. The first part of the proposition follows from applying the implicit function theorem to the first order conditions and using Cramer’s rule (see Appendix). Under standard assumptions about the bank cost function discussed in the Appendix, the following results hold:

$$\frac{\partial L_i}{\partial(p\Delta)} = - \frac{\alpha w_i^*}{|H|} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D^{*2}} < 0, \quad \frac{\partial L^*}{\partial(p\Delta)} = 0 \quad (14.a)$$

$$\frac{\partial D_i}{\partial(p\Delta)} = - \frac{(1-\varepsilon)}{|H|} \frac{\partial^2 C}{\partial D^{*2}} w_i w_i^* > 0, \quad \frac{\partial D_i^*}{\partial(p\Delta)} = 0, \quad (14.b)$$

$$\frac{\partial B_i^*}{\partial(p\Delta)} = \frac{w_i^*}{|H|} \frac{\partial^2 C}{\partial D^{*2}} \left[(1-\varepsilon)^2 w_i - \alpha \frac{\partial^2 C}{\partial D_i^2} \right] < 0 \quad (14.c)$$

The second part of the proposition follows from taking the derivatives of (9) in relation to $p\Delta$ and substituting in (14.a) and (14.b), which yields:

$$\frac{\partial l_i}{\partial(p\Delta)} = \frac{l_i}{(L_i + L_i^*)} \frac{\alpha w_i^*}{|H|} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D^{*2}} > 0 \quad (15.a)$$

$$\frac{\partial d_i}{\partial(p\Delta)} = \frac{d_i}{(D_i + D_i^*)} \frac{(1-\varepsilon) w_i w_i^*}{|H|} \frac{\partial^2 C}{\partial D^{*2}} < 0. \quad \blacksquare \quad (15.b)$$

The intuition behind these results is straightforward. As banks seek to maximize their profits in US dollars, and the interest rate on of peso loans is fixed, if a devaluation occurs after the loan is disbursed its return in US dollars will decline, reducing the bank’s profits. So, if devaluation expectations are high, banks will tend to minimize the peso component of their loan portfolio. By the same token, banks will try to take on more peso deposits as devaluation

expectations rise, shifting the burden of possible losses to fixed-term depositors. As total loans will decrease with devaluation risk, the counterpart of an increase in peso liabilities will be a reduction in dollar deposits as well as in net foreign borrowing.

As documented in the empirical literature, periods of exchange rate pressure and heightened devaluation risk in countries with hard exchange rate pegs have been generally associated with a shift away from peso lending. For instance, in Argentina lending in domestic currency was more sharply curtailed than lending in US dollars as pressure on the foreign exchange market mounted during the 1995 “Tequila” crisis, with a similar phenomenon being observed in other Latin America countries (see Figure 1). A similar phenomenon took place in several emerging countries in the wake of the financial turmoil triggered by Russia’s default in late 1998. Also as predicted by the model, evidence from the Argentine and Mexican experiences in 1995, as well as from that of other countries between August 1998 and early 1999, indicates that the share of net foreign borrowing in banks’ total liabilities dropped dramatically during those periods of financial turmoil and heightened devaluation expectations.¹⁸

III. Augmented Model with Endogenous Default and Loan Collateralization

The results above were derived on the assumption that the shares of non-performing loans in the bank’s portfolio ($1 - \alpha$; $1 - \alpha^*$) are exogenous to all other variables in the model, including its two key macro variables – the external interest rate and exchange rate risk. This assumption is consistent with the view that the loan default is negatively related to the pace of economic activity,¹⁹ where the latter is mainly determined by forces stemming from the real side of the economy, such as world income growth, productivity gains, and external terms-of-trade. While this assumption may well be realistic in a number of instances, there is abundant empirical evidence that the quality of banks’ loan portfolios in emerging markets deteriorates dramatically (i.e. α and α^* fall) during periods of credit crunch; and, as

¹⁸ In the case of Asia, banks’ net foreign liabilities continued to contract through the first half of 2000, nearly two years after the worst of the crisis was over (see IMF, 2000, Chapter 2).

¹⁹ Catão (1997) and Keeton (1999) provide evidence that as real GDP grows faster (slower), so does borrowers’ repayment capacity, leading to lower (higher) delinquency ratios. Looking at the issue from the supply-side, Rajan (1994) cautions, however, that the positive correlation between demand growth and the quality of the loan portfolio can be weakened if, yielding to reputational pressures, bankers loosen credit standards to maximize short-term profits at the expense of long-term solvency. In this case, while the short-run effect of an exogenous demand pull on loan performance tends to be positive, it may end up being small or even perverse in the longer-run if the banking system is poorly regulated and moral hazard fosters reckless lending.

witnessed by the several financial crises of the 1990s, such a sharp deterioration in bank's loan portfolio have been more closely associated with monetary shocks—notably, with rising external interest rates and heightened devaluation risk—rather than with shocks stemming from the “real” side of the economy (Domaç and Ferri, 1999; Krueger and Tornell, 1999). Thus, it seems desirable to augment the basic model laid out in the previous section so that the shares of non-performing loans in bank i 's portfolio (i.e. $1-\alpha$ and $1-\alpha^*$) are endogenized, to become *inter alia* a function of the external interest rate r^* and the devaluation risk ($p\Delta$).

There are several mechanisms through which r^* and $p\Delta$ can affect α and α^* . Tighter international liquidity (e.g. resulting from monetary policy decisions in advanced countries) and unfavorable market sentiment toward emerging market debt tend to increase r^* and dampen capital inflows to individual emerging economies (Calvo, Leiderman, and Reinhart, 1996). These impact negatively on output growth and hence on borrowers' repayment capacity, leading to lower α and α^* . There is also a well-documented connection between capital inflows and exchange rate risk in emerging markets. Sharp slowdowns or sudden stops in capital flows usually make it extremely difficult for a typical emerging market to prevent a sharp currency depreciations or avoid unwelcome devaluations (Calvo and Reinhart, 1999). Large devaluations can greatly increase default risk in countries where foreign currency denominated liabilities of the non-financial private sector exceed its foreign currency assets. As witnessed by the experience of various Asian countries in the run-up to the 1997 financial crisis, such currency mismatches can be rather sizeable once residents take for granted the maintenance of a fixed exchange rate regime or assume that they will be bailed-out by the government should a devaluation occur. In this context, loan default will clearly be a function of the exchange rate, and its incidence will tend to be higher among foreign currency borrowers with larger currency mismatches between their assets and liabilities.²⁰

In light of these considerations, we follow a simple approach to modeling loan default. Regarding banks' domestic currency portfolio, we assume the expected share of non-performing loans (or its converse, α) to be a function of the international interest rate (r^*) and the degree of loan collateralization and its enforceability (γ). The expected default ratio on dollar loans will in turn depend on these two variables plus the devaluation risk multiplied by a measure of borrowers' net short position in foreign currency. Default risk in the dollar lending will thus increase with the size of the borrower's net liability position in dollars, which the bank is assumed to know about prior to its lending decision. Algebraically:

²⁰ This is because peso borrowers will either benefit from the devaluation (if part of their net revenues is US dollar denominated) or, at worst, will be neutral (if all their liabilities and assets are all peso-denominated).

$$\alpha = \alpha(\gamma r^*), \quad \text{where} \quad \frac{\partial \alpha}{\partial r^*} < 0, \quad (16.a)$$

$$\alpha^* = \alpha^*(\gamma^* r^*, \gamma^* \mu p \Delta), \quad \text{where} \quad \frac{\partial \alpha^*}{\partial r^*} < 0, \quad \text{and} \quad \frac{\partial \alpha^*}{\partial (p \Delta)} < 0 \quad (16.b)$$

where:

$$\gamma = 1 - c \quad \text{and} \quad \gamma^* = 1 - c^*$$

with c and c^* being the collateralization shares of the peso and dollar loans, respectively, implying that γ and γ^* range from zero (full collateralization) to one (no loan collateralization); μ stands for the ex-ante net liability position of the borrower in dollars as a percentage of its net worth.

Substituting (16) back into equation (2), and re-working the comparative static exercises for shocks to r^* and $p\Delta$, yields the following results.

III. 1. Impact of external interest rate shocks

Proposition 3. *With endogenous loan default, the decline (increase) in the external interest rate r^* will lead to higher (lower) lending financed by higher (lower) net external borrowing, as deposits in both currencies will drop (rise), as in proposition 1. However, the magnitude of these effects will now be stronger. Whether loan and deposit dollarization increases or decreases will depend on the factors already highlighted in proposition 1 plus the level of r^* , the degree of collateralization of loans in each currency, the elasticity of marginal costs with respect to α and α^* , as well as the elasticity of these two parameters with respect to r^* .*

Proof. Applying the implicit function theorem to the first order conditions of the optimization problem in equation (2) to (5), and using Cramer's rule yields:

$$\frac{\partial L_i}{\partial r^*} = -\frac{w_i^*}{|H|} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^2} \left[1 - \gamma \frac{\partial \alpha}{\partial r^*} \left(\Omega_i r_L(L) - p\Delta - \frac{\partial C / \partial L_i}{\partial \alpha} \right) \right] < 0 \quad (17.a)$$

$$\frac{\partial L_i^*}{\partial r} = -\frac{w_i}{|H|} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} \left[1 - \gamma^* \frac{\partial \alpha^*}{\partial r} \left(\Omega_i^* r_L^*(L^*) - \frac{\partial C / \partial L_i^*}{\partial \alpha^*} \right) \right] < 0 \quad (17.b)$$

$$\frac{\partial D_i}{\partial r^*} = \frac{w_i w_i^*}{|H|} \frac{\partial^2 C}{\partial D_i^{*2}} \left[-(1 - \varepsilon) + \gamma \frac{\partial C / \partial D_i}{\partial \alpha} \frac{\partial \alpha}{\partial r^*} \right] > 0 \quad (18.a)$$

$$\frac{\partial D_i^*}{\partial r^*} = \frac{w_i w_i^*}{|H|} \frac{\partial^2 C}{\partial D_i^2} \left[-(1 - \varepsilon^*) + \gamma^* \frac{\partial C / \partial D_i^*}{\partial \alpha^*} \frac{\partial \alpha^*}{\partial r} \right] > 0 \quad (18.b)$$

$$\frac{\partial B_i^*}{\partial r^*} < 0^{21} \quad (18.c)$$

where Ω_i and Ω_i^* capture market structure effects and are both > 0 (see Appendix equations A.7 and A.8). If loans in neither currency are fully collateralized (i.e. $0 < \gamma, \gamma^* < 1$), it follows that the terms in brackets in (17.a) and (17.b) will be greater than 1. Thus, the right hand side of each of these expressions will be greater than the right-hand sides of the corresponding equations in (10.a), implying that with endogenous loan default, the impact of external interest rate shocks on bank lending will be greater. A similar conclusion follows from comparing (18.a) and (18.b) with the corresponding expressions in (10.b).

The remainder of the proposition can be proved by deriving (9.a) and (9.b) with respect to r^* , using (17) and (18) and the arbitrage relationship between lending rates in the two currencies derived from the first order conditions,²² to show that:

²¹ Since this equation is very large and unwieldy, we preferred to omit it here, but is available from the authors upon request.

²² That is, $\Omega_i r_L(L) - p\Delta = \Omega_i^* r_L^*(L^*) + \left(\frac{1}{\alpha} - \frac{1}{\alpha^*}\right) r^* + \frac{1}{\alpha} \frac{\partial C}{\partial L_i} - \frac{1}{\alpha^*} \frac{\partial C}{\partial L_i^*}$

$$\frac{\partial l_i}{\partial r^*} = \begin{cases} \geq 0, & \text{if } \frac{l_i}{1-l_i} \geq \frac{w_i \left[\gamma^* \frac{\partial \alpha^*}{\partial r^*} \left(\Omega_i^* r_L^* - \frac{\partial C}{\partial \alpha^*} \right) - 1 \right]}{w_i \left[\gamma^* \frac{\partial \alpha^*}{\partial r^*} \left(\Omega_i^* r_L^* (L) + \left(\frac{1}{\alpha} - \frac{1}{\alpha^*} \right) r^* + \frac{1}{\alpha} \frac{\partial C}{\partial L_i} - \frac{1}{\alpha^*} \frac{\partial C}{\partial L_i^*} - \frac{\partial C}{\partial \alpha^*} \right) - 1 \right]} \\ < 0, & \text{otherwise} \end{cases} \quad (19.a)$$

$$\frac{\partial d_i}{\partial r^*} = \begin{cases} \geq 0, & \text{if } \frac{d_i}{1-d_i} \leq \frac{\left[(1-\varepsilon^*) - \gamma^* \frac{\partial \alpha^*}{\partial r^*} \frac{\partial C}{\partial \alpha^*} \right] \frac{\partial^2 C}{\partial D_i^2}}{\left[(1-\varepsilon) - \gamma \frac{\partial \alpha}{\partial r^*} \frac{\partial C}{\partial \alpha} \right] \frac{\partial^2 C}{\partial D_i^{*2}}} \\ < 0, & \text{otherwise} \end{cases} \quad \blacksquare \quad (19.b)$$

As with proposition 1, these results can be specialized to cases of interest. These are presented in the following corollary.²³

Corollary 3a. *A decline (increase) in r^* unambiguously increases (decrease) loan dollarization if the economy is very little dollarized (i.e. $l \rightarrow 0$), and the lending interest rate in dollars is higher than the elasticity of the marginal cost of dollar lending to the share of dollar-denominated performing loans (i.e. $\Omega_i^* r_L^* (L^*) > \frac{\partial C}{\partial \alpha^*}$). However, a decline (increase) in r^* unambiguously reduce (increase) loan dollarization if the economy is nearly fully dollarized (i.e. $l \rightarrow 1$) and the lending interest rate in pesos adjusted for expected devaluation is higher than the elasticity of the marginal cost of peso lending to the share of peso-denominated performing loans (i.e. $\Omega_i r_L (L) - p\Delta > \frac{\partial C}{\partial \alpha}$). If the economy is semi-dollarized on the loan side (i.e. $l \rightarrow 1/2$), dollarization can increase, decrease or remain unchanged. Unlike in the exogenous loan default case (see corollary 1.a) this will no longer only depend on the relative convexities of the marginal cost function for each currency and the degree of competition in the peso and dollar loan markets. Depending on the relative degree of collateralization of peso and dollar loans, α and α^* , and the level of r^* , and the marginal cost increase due to problem loans in each currency, that result may be overturned or reinforced.*

²³ As before, we shall not discuss here the special cases for deposit dollarization. Those results are available from the authors upon request.

Proof. It can be readily seen from (19.a) that once these conditions are met, we have

$$l_i \rightarrow 0 \therefore \frac{\partial l_i}{\partial r^*} \rightarrow -\frac{w}{|H|(L_i + L_i^*)} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} \left[1 - \gamma^* \frac{\partial \alpha^*}{\partial r^*} \left(\Omega_i^* r_L^*(L^*) - \frac{\partial C / \partial L_i^*}{\partial \alpha^*} \right) \right] < 0 \quad (20.a)$$

$$l_i \rightarrow 1 \therefore \frac{\partial l_i}{\partial r^*} \rightarrow \frac{w^*}{|H|(L_i + L_i^*)} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} \left[1 - \gamma \frac{\partial \alpha}{\partial r^*} \left(\Omega_i r_L(L) - p\Delta - \frac{\partial C / \partial L_i}{\partial \alpha} \right) \right] > 0 \quad (20.b)$$

$l_i \rightarrow 1/2 \therefore$

$$\frac{\partial l_i}{\partial r^*} \rightarrow \frac{1/2}{|H|(L_i + L_i^*)} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} \left[w_i \left(1 - \gamma^* \frac{\partial \alpha^*}{\partial r^*} \left(\Omega_i^* r_L^*(L^*) - \frac{\partial C / \partial L_i^*}{\partial \alpha^*} \right) \right) - w_i^* \left(1 - \gamma \frac{\partial \alpha}{\partial r^*} \left(\Omega_i r_L(L) - p\Delta - \frac{\partial C / \partial L_i}{\partial \alpha} \right) \right) \right] \quad (20.c)$$

where in the latter case, the sign of $\partial l_i / \partial r^*$ will depend on the sign of the term in brackets. Comparing (20.c) with (13.c), one can see that dollarization can increase or decrease in response to an external interest rate shock. Corollary 1.a established that dollarization would increase (decrease) with a negative (positive) shock to r^* if $|w_i| > |w_i^*|$, with these two parameters being a function of the degree of competitiveness of peso and dollar loan markets and the convexity of the bank's marginal cost function with respect to lending in each currency. However, equation (20.c) indicates that, with endogenous loan default, that condition is no longer sufficient. In particular, making using of the arbitrage condition between $r_L(L)$ and $r_L^*(L^*)$ it follows that:

$$\frac{\partial l_i}{\partial r^*} < 0, \text{ if } \frac{w_i}{w_i^*} > \frac{1 - \gamma \frac{\partial \alpha}{\partial r^*} \left(\Omega_i^* r_L^*(L^*) + \left(\frac{1}{\alpha} - \frac{1}{\alpha^*} \right) r^* + \frac{1}{\alpha} \frac{\partial C}{\partial L_i} - \frac{1}{\alpha^*} \frac{\partial C}{\partial L_i^*} \frac{\partial C / \partial L_i^*}{\partial \alpha} \right)}{1 - \gamma^* \frac{\partial \alpha^*}{\partial r^*} \left(\Omega_i^* r_L^*(L^*) - \frac{\partial C / \partial L_i^*}{\partial \alpha^*} \right)}, \text{ and } \frac{\partial l_i}{\partial r^*} \geq 0 \text{ otherwise.} \blacksquare$$

The results above therefore show that in the case of semi-dollarized economies dollarization can increase, decrease, or remain constant following a shock to r^* . The direction as well as the magnitude of the effect would depend on a broader array of variables than the case discussed in section 2, when we assume the incidence of loan default not to be a function of r^* . With endogenous loan default, whether dollarization increases or decreases with changes in r^* will depending the degree of competitiveness in the domestic and foreign currency loan markets, the level of r^* (provided that $\alpha \neq \alpha^*$), the degree of collateralization of loans in the two currencies, and a number of banking cost parameters. In particular if $w_i \leq w_i^*$ and $\alpha > \alpha^*$, and collateralization of foreign currency lending is sufficiently higher than of domestic currency lending (i.e. $\gamma > \gamma^*$) to offset other differences in the remaining

cost parameters, then $\frac{\partial L_i}{\partial r^*} \geq 0$. Thus, dollarization and external interest rate changes will tend to be positively correlated, rather than negatively correlated as in the special case presented in corollary 2.a. Moreover, it can clearly be seen from the equation above that, once $\alpha > \alpha^*$, such a positive correlation will rise with the level of r^* .

III.2. Impact of Shocks to Devaluation Expectations

Proposition 4. With endogenous loan default, an increase (decrease) in devaluation risk lowers (rises) domestic currency lending, and also foreign currency lending if at least part of the foreign currency lending is uncollateralized (i.e. $\gamma^* > 0$) and the average dollar borrower is net short in foreign currency (i.e. $\mu > 0$). Peso deposits will increase (decrease), while both dollar deposits and net foreign borrowing will decrease (increase). Deposit dollarization will unambiguously decrease (increase) with higher (lower) devaluation risk, while loan dollarization will increase or decrease, depending on market structure, loan collateralization, and the various cost function parameters.

Proof. The first part of the proposition proceeds along similar as proposition 2, yielding the following expressions:

$$\frac{\partial L_i}{\partial (p\Delta)} = -\frac{\alpha w_i^*}{|H|} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^*} < 0 \quad (21.a)$$

$$\frac{\partial L_i^*}{\partial (p\Delta)} = \frac{w_i}{|H|} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} \mu \gamma^* \frac{\partial \alpha^*}{\partial (p\Delta)} \left[\Omega_i^* r_L^*(L^*) - \frac{\partial C / \partial L_i^*}{\partial \alpha^*} \right] < 0 \quad (21.b)$$

$$\frac{\partial D_i}{\partial (p\Delta)} = -\frac{w_i^* w_i}{|H|} \frac{\partial^2 C}{\partial D_i^{*2}} (1 - \varepsilon) > 0 \quad (22.a)$$

$$\frac{\partial D_i^*}{\partial (p\Delta)} = \frac{w_i w_i^*}{|H|} \frac{\partial^2 C}{\partial D_i^2} \gamma^* \mu \frac{\partial C / \partial D_i^*}{\partial \alpha^*} \frac{\partial \alpha^*}{\partial r} < 0 \quad (22.b)$$

$$\frac{\partial B_i}{\partial (p\Delta)} < 0 \quad (22.c)$$

To prove the second part of the proposition, take the derivatives of (9.a) and (9.b) with respect to $p\Delta$ and substitute those into equations (21) and (22), to yield:

$$\frac{\partial l_i}{\partial(p\Delta)} = \begin{cases} > 0 & \text{if } \frac{l_i}{(1-l_i)} > -\frac{w_i}{w_i^*} \left(\frac{\gamma^* \mu}{\alpha} \frac{\partial \alpha^*}{\partial p\Delta} (\Omega_i^* r_L^*(L^*) - (\partial C / \partial L_i) / \partial \alpha^*) \right) \\ \leq 0, & \text{otherwise} \end{cases} \quad (23)$$

$$\frac{\partial d_i}{\partial(p\Delta)} = \frac{w_i w_i^*}{|H|(D_i + D_i^*)} \left[(1-d_i) \gamma^* \mu \frac{\partial \alpha^*}{\partial(p\Delta)} \frac{\partial^2 C}{\partial \alpha^* \partial D_i^2} + d_i (1-\varepsilon) \frac{\partial^2 C}{\partial D_i^{*2}} \right] < 0. \quad \blacksquare \quad (24)$$

From the perspective of depositors, the results of equation (24) seem counter-intuitive. For when expectations of a devaluation increase, households will tend to convert peso deposits into foreign currency, leading to an incipient increase in dollarization of bank deposits. The model shows, however, that this runs against the bank's intentions. Whether in practice peso deposits will end up increasing or decreasing depends on the whether such increase in the supply of peso deposits will be countervailed by a demand contraction for such deposits on the side of households. Here, as our analysis focuses on the banking side, and banks are assumed to face an infinitely elasticity supply of time deposits at given interest rates r_D and r_D^* , domestic currency deposits end up increasing and dollarization decreasing with higher devaluation risk.²⁴

On the loan side, the results in proposition 4 qualify those for the exogenous loan default case (see proposition 2). That is, loan dollarization will **not** necessarily increase with devaluation risk insofar as banks endogeneize the behavior of the non-financial private sector, taking account of its vulnerability to a devaluation resulting from a net liability position in foreign currency. In this case, it may just be optimal for domestic banks to rebalance their portfolio towards peso lending as peso borrowers will be less vulnerable to a devaluation risk. Whether this will actually be the outcome of a rise in $p\Delta$ depends, however, on other factors. Two key factors are the initial level of dollarization of financial intermediation, and banks' vulnerability to insufficient currency hedging on the part of domestic borrowers, as the following corollary shows.

Corollary 4. *Assume that loan default is a function of devaluation risk. An increase (decrease) in $p\Delta$ unambiguously lowers (rises) loan dollarization if the economy is very*

²⁴ Episodes in which domestic currency deposits increase relative to foreign currency denominated deposits in the run-up to devaluation are not unheard of. Rogers (1992), for instance, has found evidence of a negative and statistically significant relation between devaluation expectations and the ratio of dollar-denominated to peso-denominated deposits in Mexico for the period 1973–1985.

little dollarized (i.e. $l_i^ \rightarrow 0$), but increases (decreases) loan dollarization if the economy is nearly fully dollarized (i.e. $l_i \rightarrow 1$). If the economy is semi-dollarized on the loan side (i.e. $l_i \rightarrow 1/2$), dollarization can increase, decrease or remain unchanged depending on: the degree of competition between peso and dollar loan markets (captured in w and w^*), banks' vulnerability to borrowers' lack of currency hedging (captured by μ and γ^*), and banks' marginal cost parameters.*

Proof. Taking the derivative of (9.a) and (9.b) with respect to $p\Delta$ and using (21) it follows that

$$l_i \rightarrow 0 \therefore \frac{\partial l_i}{\partial(p\Delta)} \rightarrow \frac{w_i}{|H|(L_i + L_i^*)} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} \gamma^* \mu \frac{\partial \alpha^*}{\partial(p\Delta)} \left(\Omega_i^* r_L^*(L^*) - \frac{\partial C / \partial L_i^*}{\partial \alpha^*} \right) < 0 \quad (25.a)$$

$$l_i \rightarrow 1 \therefore \frac{\partial l_i}{\partial(p\Delta)} \rightarrow \frac{w_i^*}{|H|(L_i + L_i^*)} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} > 0 \quad (25.b)$$

$$l_i \rightarrow 1/2 \therefore \frac{\partial l_i}{\partial(p\Delta)} \rightarrow -\frac{1/2}{|H|(L_i + L_i^*)} \frac{\partial^2 C}{\partial D_i^2} \frac{\partial^2 C}{\partial D_i^{*2}} \left[w_i \gamma^* \mu \frac{\partial \alpha^*}{\partial(p\Delta)} \left(\Omega_i^* r_L^*(L^*) - \frac{\partial C / \partial L_i^*}{\partial \alpha^*} \right) + \alpha w_i^* \right] \quad (25.c)$$

where it follows from the latter equation that:

$$\frac{\partial l_i}{\partial(p\Delta)} = \begin{cases} > 0, & \text{if } w_i < -\frac{\alpha w_i^*}{\gamma^* \mu \frac{\partial \alpha^*}{\partial(p\Delta)} \left(\Omega_i^* r_L^*(L^*) - \frac{\partial C / \partial L_i^*}{\partial \alpha^*} \right)} \\ \leq 0, & \text{otherwise} \end{cases} \quad \blacksquare \quad (25.d)$$

A salient implication of equations (25.c) and (25.d) is that the higher the degree of collateralization of dollar loans (i.e. the lower γ^*), the more dollarization will tend to increase with devaluation risk. In the limit, when collateralization of dollar loans is nearly complete (i.e. $\gamma^* \rightarrow 0$), even if the private sector has a large net short position in dollars (i.e. $\mu \gg 0$) and is thus very vulnerable to a devaluation, it follows from (25.d) that dollarization will increase and the more so the closer γ^* is to zero. While, as discussed below, there are practical limits about the extent to which banks can be immune to widespread private sector insolvency brought about by a devaluation, the results above underscore the direct links between the availability of tradable collateral in the economy and equilibrium dollarization levels. Yet, three important assumptions have to be made for the results of equation (25) to hold, namely: i) that the collateral is appropriately priced to reflect the full value of the loan; ii) that banks can costlessly (or near costlessly) recover the collateral; and iii) that collateral

values in dollars are not negatively affected by a devaluation. Conditions (ii) and (iii), in particular, are not easily met in practice. Legal arrangements in emerging market economies often make it costly—and sometimes even impossible—for banks to fully recover the collateral pledged against loans. Thus, even if in the loan contract bank lending is fully backed by a fairly priced collateral, in net terms the bank may simply be able to recover only some percentage of it.²⁵ Moreover, collateral values—even if denominated in foreign currency—may not be insensitive to an abrupt devaluation if the latter is accompanied by a deep recession, as has been often the case with “twin crisis” episodes (Kaminsky and Reinhart, 1999). Once again, this implies that banks are not guaranteed to recover the full value of the loan even if the latter is fully collateralized. Hence, the positive relationship between dollarization and devaluation risk in partially dollarized economies will tend to be weaker wherever the availability of tradable collateral is relatively small (as in closed economies), existing legal arrangements make it more costly to recover, and collateral values may be adversely affected by a devaluation.

IV. Conclusions

This paper has been motivated by the shortcomings of existing dollarization models in explaining the diverse pattern of deposit and loan dollarization across countries and over time. Instead of modeling dollarization as consumer’s portfolio selection problem, we look at the issue from the point of view of a key player in the dollarization process in emerging markets—the domestic banking system. A simple partial equilibrium banking model was developed, incorporating key structural characteristics of financial intermediation that have featured prominently in the recent literature on the monetary transmission in emerging markets, but which had been overlooked in previous studies on dollarization.

We found that equilibrium dollarization varies with the external interest rate and devaluation risk but the direction and amplitude of such variations will depend on: i) initial conditions (i.e. whether the economy is low-, high- or semi-dollarized to begin with); ii) the degree of bank competition in the domestic currency relative to the foreign currency segments of the loan market; iii) banks’ cost structure; and iv) the availability of tradable collateral in the economy and how costly loan enforcement is. Beyond these general findings, we were able to establish some specific propositions by imposing some restrictions on the signs and relative magnitude of certain parameters. First, dollarization should be expected to increase as external interest rates fall if initial dollarization levels are low; if, instead, the economy is semi-dollarized but the domestic currency segment of the loan market is less

²⁵ In empirical implementations of the model, this can be readily accommodated by either redefining γ^* net of loan recovery costs or incorporating recovering costs into the marginal cost term $\frac{\partial C / \partial L_i^*}{\partial(p\Delta)}$.

competitive than its foreign currency counterpart and banks' dollar loan portfolio is substantially more collateralized than the peso portfolio, this result will also tend to hold. However, when the economy is close to full dollarization, such a relationship between external interest rates and dollarization turns positive, i.e., dollarization will tend to increase as r^* rises and vice-versa.

Second, the model shows that dollarization will unambiguously increase with devaluation risk if loan default is solely a function of idiosyncratic and real macro shocks; if, however, the incidence of problem loans is endogenous and becomes a function of external interest rate and exchange rate risk, banks can no longer insulate themselves from the devaluation risk by reallocating their loan portfolio into foreign currency, as the incidence of default in dollar loans will also be a function of exchange rate risk. So, whether dollarization will increase or decrease with devaluation risk will depend on the extent of collateralization of dollar loans, the degree of currency hedging by the non-financial private sector, and how dollarized is the economy: if dollarization is very high, it will tend to increase further with greater devaluation risk, but if it is low, it will tend to decrease further. Once again, these results clearly highlight the importance of initial conditions or "hysteresis" in determining the direction of dollarization following a macroeconomic shock.

Third and more generally, the model indicates that equilibrium dollarization levels will be higher the more competitive the domestic credit market and the higher the availability of tradable collateral. Greater availability of the latter, coupled with its ownership being largely the monopoly of dollar borrowers, enables banks to collateralize further their dollar portfolio and thus reduce its exposure to interest rate and exchange rate shocks.²⁶ To the extent that the dollar loan portfolio becomes less exposed to such shocks than the peso portfolio, dollarization is thereby fostered. Finally, we show that deposit dollarization will not always move in tandem with loan dollarization. In particular, when the external interest rate rise and some other ancillary conditions contribute to lower loan dollarization, liability dollarization will tend to fall, as banks seek to cut down net external borrowing, substituting part of it by domestic currency deposits. Conversely, as devaluation risk rises, banks will increase their demand for peso-denominated deposits, thus tending to reduce deposit dollarization.

²⁶ Caballero and Krishnamurthy (1998) developed a model where there is a domestic market for tradable collateral and the latter is transacted between tradable and non-tradable sectors. This allows non-tradable sectors to collateralize their loans, thus being able to borrow in dollar and at lower lending rates. However, the underdevelopment of such a market in most emerging economies to date has prevented large scale ownership of tradable collateral by non-tradable firms. This is at the root of the segmentation hypothesis discussed earlier and built into the model.

An obvious implication of these results is that, to the extent that banking cost, credit structure parameters, and the availability of tradable collateral vary from country to country as well as over time, so will equilibrium dollarization levels. This seems consistent with the diversity of dollarization patterns in practice and the incapacity of models using solely macroeconomic variables to explain this diversity. Yet, dollarization is a macroeconomic phenomenon, so caution must be exercised to avoid stretching the power of a partial equilibrium banking model to explain it all. This points to two main avenues for future research. One is to build a general equilibrium model of dollarization where the banking system plays a key role, but firms' and government behavior, together with households' portfolio allocation decisions, also play a part; our aim in developing a banking model along the lines above was to make a first stride in that direction. The other is to estimate empirically the magnitudes of the banking sector parameters involved, so as to permit a realistic calibration of such a model. Given the various channels through which macroeconomic and banking policies can affect dollarization in financial intermediation, realistic model calibration in this case can have useful policy applications.

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Appendix: Optimality Conditions and Comparative Statics

i) First-order Conditions

The first-order conditions for bank's i (where $i = 1, \dots, n$) optimization problem are:

$$\alpha \left(-\frac{S_i (1 + \psi^i)}{\eta} + 1 \right) r_L(L) - \alpha p \Delta - \frac{\partial C}{\partial L} - \lambda \leq 0 \quad (\text{A.1})$$

(=0, if $L_i > 0$)

$$\alpha^* \left(-\frac{S_i^* (1 + \psi^{i*})}{\eta^*} + 1 \right) r_L^*(L^*) - \frac{\partial C}{\partial L^*} - \lambda \leq 0 \quad (\text{A.2})$$

(=0, if $L_i^* > 0$)

$$-r_D + \varepsilon r_E + (1 - \varepsilon) p \Delta - \frac{\partial C}{\partial D_i} + \lambda (1 - \varepsilon) \leq 0 \quad (\text{A.3})$$

(=0, if $D_i > 0$)

$$-r_D^* + \varepsilon^* r_E^* - \frac{\partial C}{\partial D_i^*} + \lambda (1 - \varepsilon^*) \leq 0 \quad (\text{A.4})$$

(=0, if $D_i^* > 0$)

$$\lambda - r^* \leq 0 \quad (\text{A.5})$$

(=0, if $B_i^* > 0$)

$$B_i^* + (1 - \varepsilon) D_i + (1 - \varepsilon^*) D_i^* - L_i - L_i^* \leq 0 \quad (\text{A.6})$$

(=0, if $\lambda > 0$)

where λ is the Lagrangian multiplier, ψ^i and ψ^{i*} are the conjectural variations coefficients defined in equations (8.a) and (8.b); S_i and S_i^* are the shares of bank i 's loans in each currency in banking system lending in domestic and foreign currencies respectively, and η and η^* are the price elasticities of demand for loans in each currency.²⁷

²⁷ That is, $\eta = -\frac{\partial L / L}{\partial r_L / r_L}$ and $\eta^* = -\frac{\partial L^* / L^*}{\partial r_L^* / r_L^*}$.

In the above expressions, the market structure terms can be compactly redefined as:

$$\Omega_i = 1 - \frac{S_i(1+\psi^i)}{\eta} \quad (\text{A.7})$$

$$\Omega_i^* = 1 - \frac{S_i^*(1+\psi^{i*})}{\eta^*} \quad (\text{A.8})$$

where $\Omega_i > 0$, $\Omega_i^* > 0$, as necessary for the first order conditions to hold. In the particular case of perfect competition, $\Psi = -1$ and $\Psi^* = -1$; and $\Omega = 1$ and $\Omega^* = 1$ for all banks.

ii) Second Order Conditions and Comparative Statics

The bordered Hessian associated with Bank's i optimization problem is:

$$H = \begin{pmatrix} 0 & -1 & -1 & (1-\varepsilon) & (1-\varepsilon^*) & 1 \\ -1 & w_i & -\frac{\partial^2 C}{\partial L_i \partial L_i^*} & -\frac{\partial^2 C}{\partial L_i \partial D_i} & -\frac{\partial^2 C}{\partial L_i \partial D_i^*} & 0 \\ -1 & -\frac{\partial^2 C}{\partial L_i^* \partial L_i} & w_i^* & -\frac{\partial^2 C}{\partial L_i^* \partial D_i} & -\frac{\partial^2 C}{\partial L_i^* \partial D_i^*} & 0 \\ (1-\varepsilon) & -\frac{\partial^2 C}{\partial D_i \partial L_i} & -\frac{\partial^2 C}{\partial D_i \partial L_i^*} & -\frac{\partial^2 C}{\partial D_i^2} & -\frac{\partial^2 C}{\partial D_i \partial D_i^*} & 0 \\ (1-\varepsilon^*) & -\frac{\partial^2 C}{\partial D_i \partial L_i^*} & -\frac{\partial^2 C}{\partial D_i^* \partial L_i} & -\frac{\partial^2 C}{\partial D_i \partial D_i^*} & -\frac{\partial^2 C}{\partial D_i^{*2}} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (\text{A.9})$$

where w_i and w_i^* are defined in equations (10.a) and (10.b) and can be shown to be strictly negative under fairly general conditions (Dixit, 1986).

In line with the mainstream literature, we assume that the cost function is convex and has decreasing returns to scale in lending and borrowing; we also assume that there may exist economies of scope between deposits and loans in the same currency but these are relatively

small, while those between the domestic and foreign currencies are zero.²⁸ Algebraically, this implies:

$$\begin{aligned} \frac{\partial C}{\partial L_i} > 0, \quad \frac{\partial^2 C}{\partial L_i^2} > 0, \quad \frac{\partial C}{\partial L_i^*} > 0, \quad \frac{\partial^2 C}{\partial L_i^{*2}} > 0, \quad \frac{\partial C}{\partial D_i} > 0, \quad \frac{\partial^2 C}{\partial D_i^2} > 0, \quad \frac{\partial C}{\partial D_i^*} > 0, \quad \frac{\partial^2 C}{\partial D_i^{*2}} > 0, \\ \frac{\partial^2 C}{\partial L_i \partial L_i^*} \approx 0, \quad \frac{\partial^2 C}{\partial D_i \partial D_i^*} \approx 0, \quad \frac{\partial^2 C}{\partial L_i \partial D_i} \leq 0, \quad \frac{\partial^2 C}{\partial L_i \partial D_i^*} \approx 0, \quad \frac{\partial^2 C}{\partial L_i^* \partial D_i} \approx 0, \quad \frac{\partial^2 C}{\partial L_i^* \partial D_i^*} \leq 0 \quad ; \end{aligned}$$

$$\text{and } \frac{\partial^2 C}{\partial L_i^2} \frac{\partial^2 C}{\partial D_i^2} > \left(\frac{\partial^2 C}{\partial L_i \partial D_i} \right)^2 ; \quad \frac{\partial^2 C}{\partial L_i^{*2}} \frac{\partial^2 C}{\partial D_i^{*2}} > \left(\frac{\partial^2 C}{\partial L_i^* \partial D_i^*} \right)^2 \quad (\text{A.10})$$

Under these conditions, minors of (A.9) can be shown to alternate in sign, implying that matrix [H] is negative definite— a sufficient condition for the optimal to be a maximum.

Applying the implicit function theorem to equations (A.1) to (A.6) and using the Cramer rule, we can calculate the impact of an interest rate shock on $B_i^*, D_i, D_i^*, L_i, L_i^*$ from the following system:

$$\begin{pmatrix} 0 & -1 & -1 & (1-\varepsilon) & (1-\varepsilon^*) & 1 \\ -1 & w_i & -\frac{\partial^2 C}{\partial L_i \partial L_i^*} & -\frac{\partial^2 C}{\partial L_i \partial D_i} & -\frac{\partial^2 C}{\partial L_i \partial D_i^*} & 0 \\ -1 & -\frac{\partial^2 C}{\partial L_i^* \partial L_i} & w_i^* & -\frac{\partial^2 C}{\partial L_i^* \partial D_i} & -\frac{\partial^2 C}{\partial L_i^* \partial D_i^*} & 0 \\ (1-\varepsilon) & -\frac{\partial^2 C}{\partial D_i \partial L_i} & -\frac{\partial^2 C}{\partial D_i \partial L_i^*} & -\frac{\partial^2 C}{\partial D_i^2} & -\frac{\partial^2 C}{\partial D_i \partial D_i^*} & 0 \\ (1-\varepsilon^*) & -\frac{\partial^2 C}{\partial D_i^* \partial L_i} & -\frac{\partial^2 C}{\partial D_i^* \partial L_i^*} & -\frac{\partial^2 C}{\partial D_i^* \partial D_i} & -\frac{\partial^2 C}{\partial D_i^{*2}} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial \lambda}{\partial r^*} \\ \frac{\partial L_i}{\partial r^*} \\ \frac{\partial L_i^*}{\partial r^*} \\ \frac{\partial D_i}{\partial r^*} \\ \frac{\partial D_i^*}{\partial r^*} \\ \frac{\partial B_i^*}{\partial r^*} \end{pmatrix} = \begin{pmatrix} 0 \\ -\left[\Omega_i r_L(L) - p\Delta - \frac{\partial C / \partial L_i}{\partial \alpha} \right] \gamma \left(\frac{\partial \alpha}{\partial r^*} \right) \\ -\left[\Omega_i^* r_L^*(L^*) - \frac{\partial C / \partial L_i^*}{\partial \alpha^*} \right] \gamma^* \left(\frac{\partial \alpha^*}{\partial r^*} \right) \\ \gamma \frac{\partial C / \partial D_i}{\partial \alpha} \frac{\partial \alpha}{\partial r^*} \\ \gamma^* \frac{\partial C / \partial D_i^*}{\partial \alpha^*} \frac{\partial \alpha^*}{\partial r^*} \\ 1 \end{pmatrix} \quad (\text{A.11})$$

²⁸ The rationale behind the existence of economies of scope between loans and deposits (which helps justify universal banks) is discussed in Fama (1985). The assumption that they are negligible across currencies, however, is consistent with hedging strategies generally adopted by financial intermediaries and also accords well with the segmentation hypothesis discussed earlier in the paper, which sets apart the types of clients that usually operate in domestic and foreign currency markets.

where the restrictions on the cost functions discussed above apply to the left-hand side matrix.²⁹ The basic model with exogenous loan default is simply a particular case of the above, once $\frac{\partial \alpha}{\partial r^*} = \frac{\partial \alpha^*}{\partial r^*} = 0$.

Likewise, one can proceed in a similar fashion to calculate the impact of exogenous changes in devaluation risk. The respective matrix representation is:

$$\begin{pmatrix} 0 & -1 & -1 & (1-\varepsilon) & (1-\varepsilon^*) & 1 \\ -1 & w_i & \frac{\partial^2 C}{\partial L_i \partial L_i^*} & \frac{\partial^2 C}{\partial L_i \partial D_i} & \frac{\partial^2 C}{\partial L_i \partial D_i^*} & 0 \\ -1 & -\frac{\partial^2 C}{\partial L_i^* \partial L_i} & w_i^* & \frac{\partial^2 C}{\partial L_i^* \partial D_i} & \frac{\partial^2 C}{\partial L_i^* \partial D_i^*} & 0 \\ (1-\varepsilon) & \frac{\partial^2 C}{\partial D_i \partial L_i} & \frac{\partial^2 C}{\partial D_i \partial L_i^*} & \frac{\partial^2 C}{\partial D_i^2} & \frac{\partial^2 C}{\partial D_i \partial D_i^*} & 0 \\ (1-\varepsilon^*) & \frac{\partial^2 C}{\partial D_i^* \partial L_i} & \frac{\partial^2 C}{\partial D_i^* \partial L_i^*} & \frac{\partial^2 C}{\partial D_i^* \partial D_i} & \frac{\partial^2 C}{\partial D_i^* \partial D_i^*} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial \lambda}{\partial(p\Delta)} \\ \frac{\partial L_i}{\partial(p\Delta)} \\ \frac{\partial L_i^*}{\partial(p\Delta)} \\ \frac{\partial D_i}{\partial(p\Delta)} \\ \frac{\partial D_i^*}{\partial(p\Delta)} \\ \frac{\partial B_i^*}{\partial(p\Delta)} \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha \\ -\left[\Omega_i^* r_L^*(L) - \frac{\partial C / \partial L_i^*}{\partial \alpha^*} \right] \gamma^* \mu \left(\frac{\partial \alpha^*}{\partial(p\Delta)} \right) \\ -(1-\varepsilon) \\ \gamma^* \mu \frac{\partial C / \partial D_i^*}{\partial \alpha^*} \frac{\partial \alpha^*}{\partial(p\Delta)} \\ 0 \end{pmatrix} \quad (A.12)$$

with the basic model with exogenous loan default being a particular case of the above, once $\frac{\partial \alpha}{\partial(p\Delta)} = \frac{\partial \alpha^*}{\partial(p\Delta)} = 0$.

²⁹In the expressions provided in the main text, we assumed away the existence of economies of scope. Provided that conditions (A.10) hold, this does not affect the postulated direction of changes while allowing us to work with more manageable mathematical expressions.