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The Terms of Trade Effect in Monetary Models of Exchange Rate Determination

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Summary

This paper discusses the relationship between the terms of trade and the exchange rate in a flexible-price monetary model. In such a model, a change in the terms of trade is shown to affect the exchange rate only through its effect on the price level as determined by the money market equilibrium condition. The paper shows, in a simple two-country, two-good model, that this occurs only when the prices of goods are weighted differently across countries in the price deflators relevant to money demand; this result can be generalized to an n-good case under some simplifying assumptions.

The precise nature of the role of this monetary asymmetry in exchange rate determination depends crucially on the complex interaction of changes in relative prices and incomes that result from initial disturbances to which the economy is subjected. In general, a positive relationship between the terms of trade and the exchange rate, such that a depreciation is associated with a deterioration of the terms of trade, requires that the domestic price of the domestic good be given the dominant weight in the specification of the price deflator. In fact, this is the most reasonable outcome and is supported by the recent experience with flexible exchange rates.

I. Introduction

Recent experience with flexible exchange rates has shown that a nominal appreciation of the currency of a country often leads to a prolonged rise in the relative price of the goods that are produced in that country. This sustained co-movement of the exchange rate and the relative price illustrates the limitation of two types of popular theories.

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First, an equilibrium theory based on a single traded good and a nontraded good as in Dornbusch (1973b) is not capable of explaining relative price movements among traded goods produced by different countries. Second, a disequilibrium theory based on the assumption of slow adjustment in the goods market as in Dornbusch (1976) is not capable of explaining changes in the relative price that do not return to the original level after a reasonable period of time. 1/ Thus, there seems to be a need for an equilibrium model of exchange rate determination that explicitly incorporates heterogeneous traded goods; 2/ in such a model relative price movements can be analyzed as an equilibrium phenomenon.

Such an attempt was first made by Stockman (1980) who showed that the observed co-movement of the exchange rate and the terms of trade can be generated in a monetary model through a Clower-type cash-in-advance constraint, i.e. requiring that the economic agent hold money to purchase goods. More recently, Lucas (1982) and Svennson (1985) have proposed models of exchange rate determination with a similar set-up. In these monetary models, the equilibrium exchange rate consists of not only the usual relative money stock term but also the relative price term. As Kareken and Wallace (1981) pointed out in a different context, however, the solution form for the exchange rate in monetary models crucially depends on the specification of the role of money in the model. Thus it is of interest to see how different specifications of the role of money would affect the way in which the exchange rate is determined.

Thus, the paper will investigate the manner in which a particular specification of the demand function for money influences the determination of the exchange rate in a multi-commodity setting. The purpose is not so much to derive an explicit solution for the exchange rate as to examine a general structure that is required of a monetary model to generate the observed co-movement of the exchange rate and the relative price in an equilibrium setting. It will be shown that, in order for the terms of trade to be relevant in exchange rate determination, the specification of the price deflator in money demand functions must be asymmetrical across countries. Moreover, a positive relationship between the exchange rate and the terms of trade further requires that the weight of the domestic good be dominant in that specification.

1/ Unless we assume that there are continuous shocks.

2/ See Kimbrough (1983) for a further discussion on these two classes of models. He also suggested yet another explanation of PPP deviations based on differential speeds of information in the goods and the assets markets.

The paper is organized as follows. Section II presents a simple two-country two-good model of the world economy. Section III derives the solution for the determination of the exchange rate. Section IV analyzes the role of the terms of trade in exchange rate determination. Section V extends the analysis to the determination of output and the international transmission of economic disturbances. Section VI discusses the role of the specification of the money demand function in a few representative monetary models of exchange rate determination. Section VII presents some concluding remarks. Finally, the Appendix generalizes the results obtained for the case of 2 goods to that of n goods under some simplifying assumptions.

II. The Simple Model

The model presented here is a world economy consisting of two countries, two traded goods, and two monies; each country is assumed to be specialized in the production of one good. In order to highlight the role of the terms of trade, other factors that are important in exchange rate determination are assumed away; thus there are no interest-bearing assets, no currency substitution, and no systematic inflation; there are no nontraded goods and no systematic growth; and constant market clearing is assumed. Aggregate output in each country, however, is subject to both real and monetary disturbances. Moreover, the two traded goods are imperfect substitutes, such that the relative price (which is equivalent to the terms of trade) can be affected by supply and demand conditions.

1. The Real Sector

Assume that the following aggregate supply characterizes the real sector for each country:

(1-1) $y_{1t} = \alpha+\beta(p_{1t}-E_{t-1}p_t)+ u_t,$

(1-2) $y^*_{2t}= \alpha^*+ \beta^*(p^*_{2t}-E_{t-1}p^*_t) + u^*_t,$

where asterisks are placed on the foreign variables, and subscript 1 refers to good 1 (the home good), 2 to good 2 (the foreign good), and t and t-1 to discrete time periods; y is a log of output, p a log of price, E_{t-1} a mathematical expectations operator based on the set of information available at t-1, and u a random output shock (e.g., a crop failure) and α's and β's are constant parameters. Equations (1-1) and (1-2) express the log of aggregate output as a function of three terms: a "natural" level of output, an unanticipated price change, and a supply disturbance.

The first terms, α and α^*, are assumed to be determined by real factors that are independent of cyclical factors, and can thus be appropriately normalized to zero. The second term is assumed to reflect the realized real wage at t, where p and p^* are general price levels that are relevant to the setting of the real wage rate [given by (4-1) and (4-2)]; this can be thought of as a two-good analogue of the terms suggested by Friedman (1968), Sargent and Wallace (1975), and Fischer (1977). It is assumed here that the setting of the wage rate involves one-period nominal wage contracting, where the workers' objective at t-1 is to maintain the desired real wage at t. Furthermore, in setting the expected future price level (or the nominal contracted wage), the economic agent is assumed to form expectations based on the knowledge of the economic model as well as the stochastic environment facing him. 1/ The supply elasticities with respect to an unanticipated price change, β and β^*, are assumed to be identical in both countries; that is, $\beta \equiv \beta^*$. This will considerably simplify the algebra without altering any of the substantive results of the analysis. 2/ Finally, the third terms, u_t and u_t^*, are assumed to be normally distributed and serially uncorrelated with zero means. This simplification allows us to focus on the initial effect of a supply shock independently of its propagation mechanism.

The two goods are assumed to be traded without transportation costs or other impediments to trade. Then commodity arbitrage assures that the law of one price holds for each good,

(2-1) $$p_{1t} = s_t + p_{1t}^*,$$

(2-2) $$p_{2t} = s_t + p_{2t}^*,$$

where s_t denotes the log of the spot exchange rate at t, expressed as the price of the foreign currency in terms of the home currency (i.e., an increase in s_t denotes a depreciation of the home currency).

1/ The problem of asymmetric (heterogeneous) information, as discussed in Weiss (1980) and King (1982), is assumed away. Instead, it is assumed that everybody in the world has the identical information.

2/ This assumption allows the two-step procedure followed in the text, where the solution for output (and prices) is obtained independently of the solution for the spot exchange rate; otherwise, they must be determined simultaneously. This follows from the fact that, in the solution for the exchange rate, the price terms have $(\beta-\beta^*)$ as their coefficients. This assumption of identical elasticities, however, does not change the form and economic meaning of the implied solutions.

Given the law of one price and constant market clearing, global supply and demand conditions will determine the relative price of the two traded goods. It is convenient to assume that the structure of demand is such that the relative price is determined in the following manner,

$$(3) \quad p_{2t} - p_{1t} = \Phi(y_{1t} - y^*_{2t}),$$

where the prices are expressed in the home currency, and Φ is the elasticity parameter, defined as the reciprocal of the elasticity of substitution (σ), i.e., $\Phi \equiv (1/\sigma)$. This implies that the underlying demand conditions in the two countries are such that their aggregate sum has the property of a *constant elasticity of substitution (CES) utility* function; Φ is zero if the two goods are perfect substitutes; Φ is unity if the elasticity of substitution is unitary; and Φ increases as the elasticity becomes smaller. However, this does not necessarily mean that the economic agents in the two countries have identical preferences.

The price level in each country is assumed to be specified as,

$$(4\text{-}1) \quad p_t \equiv \theta\, p_{1t} + (1-\theta)p_{2t} ,$$

$$(4\text{-}2) \quad p^*_t \equiv (1-\theta)p^*_{1t} + \theta p^*_{2t},$$

where θ is the consumption share of the domestically produced good (for each country) and, in order to preserve symmetry, θ is assumed to be the same for both countries. Although, to be more general, a separate parameter (such as θ^*) can be introduced for the foreign country, the present symmetrical set-up makes the analysis simpler without altering any of the substantive results.

From the above specification of the real sector, the following expression can be obtained for the relative price expressed in the home currency,

$$(5) \quad p_{2t} - p_{1t} = (\beta\Phi/1+\beta\Phi)(s_t - E_{t-1}s_t) + (\Phi/1+\beta\Phi)(u_t - u^*_t).$$

This fully describes the real side of the economy.

2. The Monetary Sector

Turning to the specification of the money market, the supply of money is assumed to be exogenously determined by, (6-1) $m_t^s = m_{t-1}+x_t$

(6-2) $m_t^{*s} = m_{t-1}^*+x_t^*$

where x_t and x_t^* are white-noise random money supply shocks. The main focus here is on once-and-for-all money shocks. This is consistent with the specification of the aggregate supply, where a persistent effect of lagged output and systematic growth were ruled out.

In the absence of systematic inflation, currency substitution, and interest-bearing assets, the demand for money can be specified in the most general terms as,

(7-1) $m_t^d-p_t = \gamma y_{1t}+v_t$,

(7-2) $m_t^{*d}-p_t^*=\gamma^* y_{2t}^*+v_t^*$,

where γ and γ^* are income elasticities of the demand for money; v and v^* are white-noise random money demand shocks.

Further assuming for simplicity that the foreign and domestic income elasticities are identical, i.e., $\gamma \equiv \gamma^*$, and by imposing the following money market equilibrium conditions,

(8-1) $m_t^s=m_t^d=m_t$,

(8-2) $m_t^{*s}=m_t^{*d}=m_t^*$,

the following two expressions can be obtained,

(9-1) $m_{t-1}+w_t=\gamma\ y_{1t}+\theta p_{1t}+(1-\theta)p_{2t}$,

(9-2) $m_{t-1}^*+ w_t^*= \gamma\ y_{2t}^*+(1-\theta)p_{1t}^*+\theta p_{2t}^*$,

where w_t and w_t^* are unanticipated money shocks, i.e., $w_t \equiv x_t-v_t$, $w_t^* \equiv x_t^*-v_t^*$. These fully describe the monetary side of the model.

III. The Determination of the Spot Exchange Rate

Given (5), the substitution of (1-1) into (9-1) and of (1-2) and (2-2) into (9-2) gives the following intermediate solution for s_t,

$$(10)\ s_t = (\beta\Phi+1/\Delta)(m_{t-1}+w_t-m^*_{t-1}-w^*_t) - \{[\gamma+\Phi(1-2\theta)]/\Delta\}(u_t-u^*_t) + \{[\beta\Phi(1-2\Phi)+\beta\gamma]/\Delta\}E_{t-1}\, s_t,$$

where $\Delta \equiv (1+\beta\gamma)+2\beta\Phi(1-\theta) > 0$. Equation (10) is a rational expectations reduced form equation, where the spot rate is expressed as a function of exogenous variables and its own expected value. The next step is to express s_t as a function only of the exogenous variables. Since the six random disturbance terms and lagged money supplies fully describe the state of the world at t, the final solution for s_t must be a linear function of these eight exogenous varialbes; moreover, since x's and v's enter the solution identically, we can substitute w's for x's and v's. Thus, one might postulate the following solution form,

$$(11\text{-}1)\quad s_t = H_1\, m_{t-1} + H_2\, m^*_{t-1} + H_3\, w_t + H_4\, w^*_t + H_5\, u_t + H_6\, u^*_t,$$

where H's are the coefficients to be determined. Given the assumption about the stochastic processes for the random disturbances, the expectation of s_t conditional upon the information available at t-1 is given by,

$$(11\text{-}2)\quad E_{t-1}s_t = H_1\, m_{t-1} + H_2\, m^*_{t-1}.$$

The final solution can be obtained by substituting (11-1) and (11-2) into (10) and finding the values of H's, as follows,

$$(12)\ s_t = (m_{t-1}-m^*_{t-1}) + (\beta\Phi+1/\Delta)(w_t-w^*_t) + \{[\Phi(2\theta-1)-\gamma]/\Delta\}(u_t-u^*_t),$$

where the first term is the usual liquidity effect of money: it says that anticipated money alters the exchange rate one-to-one, corresponding to the neutrality of money. The second term captures the effect of unanticipated monetary shocks and the third term the effect of real shocks on the exchange rate. The interpretation of these last two terms will be the subject of the following section.

IV. The Terms of Trade Effect in Exchange Rate Determination

An unanticipated money shock $(w_t - w_t^*)$ or a real shock $(u_t - u_t^*)$ will alter the relative price of the two traded goods. This will have an effect on the exchange rate under certain conditions. The purpose of this section is to determine what conditions are needed in order for the terms of trade to become a factor in exchange rate determination.

Since both an unanticipated money shock and a real shock alter the relative price, the coefficients of both of these disturbance terms in (12) contain the terms of trade component. Although these coefficients appear radically different from each other, however, the terms of trade effect can be analyzed identically for both types of shocks. This is the case because the coefficient of money shock terms can be decomposed as,

$$(13) \quad (\beta\Phi+1)/\Delta = 1+\beta\{[\Phi(2\theta-1)-\gamma]/\Delta\},$$

where the first term captures the effect of the change in money supply (the liquidity effect)--which is always equal to unity--and the second captures the effect of the change in output (the output effect) on the exchange rate. The output effect in (13) is identical to the coefficient of real shock terms in (12), except that the former is multiplied by β, which can be interpreted as $(\partial y/\partial w)$; for real shocks, u's, $(\partial y/\partial u)$ is unity. Thus the expression $\{[\Phi(2\theta-1)-\gamma]/\Delta\}$ applies to both types of shocks.

We note that this output effect consists of the pure terms of trade effect and the income effect. The income effect, i.e., $-(\gamma/\Delta)$, is always negative; this is so because an increase in output in the home country, for instance, will depress the price level and hence appreciate the exchange rate. On the other hand, the nature of the pure terms of trade effect, i.e., $[\Phi(2\theta-1)/\Delta]$, depends on the elasticity of substitution parameter (Φ) and, more importantly, on the share of the home good in the price deflator used in specifying the demand for money (θ). The pure terms of trade effect will be positive for $\theta > 0.5$, negative for $\theta < 0.5$, and zero for $\theta = 0.5$.

In discussing the economic interpretation of this terms of trade effect in exchange rate determination, it is useful to focus on three extreme cases, i.e., $\theta = 1$, $\theta = 0$ and $\theta = 0.5$. First, consider the case where the price deflator in money demand includes only the price of the domestic good, i.e., $\theta = 1$. In this case, the effect of a positive real shock in the home country is to depreciate the exchange rate by (Φ/Δ). The depreciation of the exchange rate occurs because, given the domestic price of the domestic good that is in some sense "fixed" by the specification of the money market equilibrium condition, the only way to lower the relative price of the domestic good is for the domestic price of the foreign good to increase. In practice, the depreciation of the home currency would take place in response to the excess supply of the home goods (or the excess demand for the foreign good) that would exist at the initial exchange rate. The net output effect that includes both this terms of trade effect and the income effect, however, may not depreciate the exchange rate if the latter is sufficiently large relative to the value of Φ. A depreciation will occur for $\Phi > \gamma$; an appreciation will occur for $\Phi < \gamma$; and the exchange rate will be invariant for $\Phi = \gamma$. In the case of a positive monetary shock, there is the additional effect on the price level of an increase in money supply. Inclusive of this liquidity effect, the exchange rate will always depreciate regardless of the magnitude of the output effect, as is evident from (13).

Second, consider the (absurd?) case where the price deflator in money demand includes only the price of the foreign good, i.e., $\theta = 0$. In this case, the effect of a positive real shock in the home country is to appreciate the exchange rate by $-(\Phi/\Delta)$. The appreciation occurs because, given the foreign price of the domestic good that is in some sense "fixed" by the specification of the money market equilibrium condition, that is the only way to lower the relative price of the domestic good. Since the income effect reduces the domestic price of the foreign good initially, the total appreciation must be large enough to offset the income effect in the new equilibrium, such that the final relative price of the domestic good is indeed lower. 1/ Thus, both the pure terms of trade effect and the income effect will reinforce each other in appreciating the exchange rate. For a positive monetary shock, however this is offset by the additional liquidity effect, which is always greater the output effect; the net effect is thus always to depreciate the exchange rate.

Finally, consider the case where the prices of the domestic and foreign goods are equally weighted in the price deflator, i.e., $\theta = 0.5$. In this case, there is no pure terms of trade effect; there is only an income effect on money demand. This result obtains because, given the law of one price, the same average of the two individual prices can be maintained across countries without requiring an adjustment in the exchange rate. In fact, this result holds whenever the weighted-average deflators are identical in both countries; $\theta = 0.5$ is a special

1/ In practice, there may be no price mechanism to bring the exchange rate to the new equilibrium, as this requires that an excess demand for the foreign good be translated into a depreciation of the foreign currency. Thus the solution under the second case is an unstable one.

case of this. 1/ Thus, the presence of the terms of trade effect in exchange rate determination requires that the price deflators relevant to money demand be specified differently across countries.

To summarize, a depreciation of the exchange rate on the account of an increase in output--exclusive of the liquidity effect of money--will coincide with a worsening of the terms of trade if a greater weight is placed on the domestic price of the domestic good in the price deflator relevant to money market equilibrium, i.e., $\theta > 0.5$. 2/ In the opposite case where $\theta < 0.5$, a depreciation will coincide with an improvement in the terms of trade. Finally, in a situation where the specification of the price deflator is identical across countries, i.e., $\theta = 0.5$, there will be no terms of trade effect associated with an exchange rate change. Since economic intuition would lead us to expect the share of the price of the domestic good to dominate that of the foreign good in the price deflator, the first outcome seems to be the most likely. In fact, the co-movement of the exchange rate and the terms of trade that we observe in practice is consistent with such an outcome.

V. The Exchange Rate and the Determination of Output

How the exchange rate is determined influences the way in which output is determined. To investigate this linkage, it is first necessary to obtain the expression for p_{1t}. (Only the solution for the home country is considered in this section, as the symmetry in the model makes it possible to obtain the solution for the foreign country by analogy.) This can be done by substituting (1-1), (5) and (12) into (9-1), and obtaining,

$$
\begin{aligned}
(14)\quad p_{1t} = {} & [1/(1+\gamma\beta)]\, m_{t-1} + [\gamma\beta/(1+\gamma\beta)]\, E_{t-1}\, p_t \\
& + \{[1+\gamma\beta+\beta\Phi(1-\theta)/\Omega\}w_t - \{[\Delta\gamma+(1-\theta)\Delta]/\Omega\}u_t \\
& + [(1-\theta)\Phi/\Omega](\beta w_t^* + u_t^*),
\end{aligned}
$$

1/ To see this point, suppose we introduce a separate parameter θ^* for money market equilibrium in terms of the foreign prices can be expressed as $\theta p_{1t}^* + (1-\theta)p_{2t}^* + s_t$, and the foreign equivalent as $\theta^* p^*_{1t} + (1-\theta^*)p_{2t}$. Clearly, as long as $\theta = \theta^*$, no adjustment in s_t is needed in response to a change in the relative price.

2/ This corresponds to the condition discussed in Flood (1981) that is needed to generate exchange rate volatility in excess of price level volatility.

the foreign country in (4-2). Then, the domestic price level relevant to where $\Omega \equiv \Delta(1+\gamma\beta) > 0$. Following the same solution procedure used for s_t, we obtain the solution for p_{1t} as,

$$(15)\ p_{1t} = m_{t-1}+[1+\gamma\beta+\beta\Phi(1-\theta)/\Omega]w_t$$
$$- \{[\Delta\gamma+(1-\theta)\Phi]/\Omega\}u_t+[(1-\theta)\Phi/\Omega](\beta w_t^*+u_t^*),$$

where use was made of the fact that $E_{t-1}p_t = E_{t-1}p_{1t}$. We note that foreign disturbances will affect the domestic price of the domestic good only if $\theta < 1$. The solution for the output of the domestic good is given by substituting (15) into (1-1) as,

$$(16)\ y_{1t} = \{[1+\gamma\beta+\beta\Phi(1-\theta)]/\Omega\}(\beta w_t+u_t) + [\beta\Phi(1-\theta)/\Omega](\beta w_t^*+u_t^*).$$

First, consider the effect of foreign disturbances on domestic output. It can be seen in (16) that no foreign disturbances will affect domestic output if $\theta = 1$. This follows from the specification of the output equation (1-1) in which output responds only to (the unanticipated component of) the realized domestic price of the domestic good; given this specification, no output response is possible when $\theta = 1$ insures that the domestic price of the domestic good is determined only by domestic variables and is invariant to foreign disturbances.

However, as long as $\theta < 1$, foreign disturbances will positively affect the domestic price of the domestic good, and hence the level of output. This follows from the fact that an increase in foreign output increases the relative price of the domestic good (provided that $\Phi \neq 0$), which is at least partially reflected in an increase in the domestic nominal price of the domestic good. Thus, there is a synchronization of output movements across countries--that is to say, output expansion in one country is transmitted as output expansion in another under flexible exchange rates, regardless of whether the disturbance is real or monetary. 1/

Second, consider the effect of domestic disturbances on output. It can be seen that, for a given positive domestic shock (e.g., a bumper crop), the output response is maximized under $\Theta = 1$ at $[1/(1+\gamma\beta)]$--multiplied by β for a monetary shock. This result follows from the fact that a

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1/ In a popular class of models with capital mobility known as the Mundell-Fleming model, the differential effects of monetary vs. real disturbances have been stressed; in such models, for example, monetary disturbances that affect output will have an asymmetric affect across countries under flexible exchange rates. See Mussa (1979).

greater value of θ implies a smaller price dampening effect on p_{1t} that results from a given change in the relative price. In the limiting case of $\theta = 1$, a fall in the relative price that results from a positive shock will leave p_{1t} intact, so that the full output expansion--limited only by the income effect--is made possible. In a sense, the case of $\theta = 1$ corresponds to the assumption of "sticky" prices and the full output expansion can be compared to the full Keynesian multiplier result. Another case of full output expansion occurs when the two traded goods are perfect substitutes, i.e., $\Phi = 0$; in this case, full output expansion results regardless of the value of θ. This is the case because p_{1t} is invariant on the account of the (nonexistent) relative price effect, such that there is only a uniform income effect.

The foregoing analysis does not mean, however, that a larger value of θ leads to a larger welfare gain to a country that experiences a positive disturbance, because a larger output gain also means a larger deterioration in the terms of trade. To make such a welfare judgement, therefore, it is necessary to consider a measure of real income that explicitly takes account of the offsetting influences of changes in output and the terms of trade. To do so, first obtain the terms of trade by substituting (12) and (15) into (5) as,

$$(17)\quad R_t \equiv p_{2t} - p_{1t}$$
$$= (\Phi/\Delta)[\beta(w_t - w_t^*) + (u_t - u_t^*)].$$

Following Jones (1979), 1/ let

$$(18)\quad dI_t \equiv -(1-\theta)dR_t + dy_{1t},$$

define a change in real income, where the first term captures the negative effect of the worsening in the terms of trade and the second a direct positive effect of an increase in output. Substituting (16) and (17) into (18), we can find the condition for (dI_t/du_t) or (dI_t/dw_t) to be negative, i.e., *immiserization*, as,

$$(19)\quad \sigma < (1-\theta) - \{[1+\beta(\gamma-1)]/(1+\gamma\beta)\},$$

where $\{...\} < 1$. This condition is more likely to hold for a larger value of θ, indicating that the larger worsening of the terms of trade tends to outweigh the larger output gain.

1/ To derive (18), start with the Jones formula $dr = -C_2\, d(P_2/P_1) + dY_1$, where r is a measure of real income, C_2 consumption of imports, and all variable are expressed in levels. Dividing through by Y_1, and defining $(dr/Y_1) \equiv dI$, we have (18).

VI. The Specification of the Demand for Money

This crucial role of the specification of the money demand in the context of fixed exchange rates was well recognized by Dornbusch (1973a, p. 904) who argued that differences in money demand functions across countries would allow changes in the relative price to have a feedback effect on the equilibrium rates of hoarding. Unfortunately, this recognition of the crucial role of the specification of the money demand has not been stressed in subsequent analyses of exchange rate determination, perhaps owing to the dominance of models based on the small country assumption. 1/ It is, however, of interest to see how a particular specification of the money demand function has resulted in a specific solution for the exchange rate in a few representative models of exchange rate determination.

In the two-good, two-country models of Stockman (1980), Lucas (1982) and Svensson (1985), the cash-in-advance constraint requires that a dollar's worth of consumption be made by an equivalent amount of money. Thus, in equilibrium, a dollar's worth of production must have the backing of an equivalent amount of money demand. Moreover, the cash-in-advance constraint in such a two-good model requires that the home currency be used to purchase the home good and the foreign currency to purchase the foreign good. This type of specification thus amounts to $\theta = 1$ and $\gamma = 1$; Mussa (1979) and Flood (1981) postulated a similar specification of the money demand function in a two-commodity world. Thus, the exchange rate is given by,

$$(20)\quad s_t = (m_{t-1} - m^*_{t-1}) + [(1+\beta\Phi)/(1+\beta)](w_t - w^*_t) + [(\Phi-1)/(1+\beta)](u_t - u^*_t).$$

In this case, a positive output shock will either appreciate or depreciate the exchange rate depending on whether the elasticity parameter is less than or greater than the unit income elasticity of the demand for money.

Hamada and Sakurai (1978) assumed not only a quantity equation in each country but also a Cobb-Douglas utility function. These assumptions amount to $\theta = 1$, $\gamma = 1$ and $\Phi = 1$ in our present framework. The exchange rate is given by,

$$(21)\quad s_t = (m_{t-1} - m^*_{t-1}) + (w_t - w^*_t).$$

1/ A notable exception is Mussa (1979, p. 168) who correctly recognized that the result he obtained was sensitive to the specification of the money demand function.

In this case, a real shock does not affect the exchange rate, since a Cobb-Douglas utility function implies a constant expenditure share of each good; a change in the relative price does not require a change in the price level relevant to money demand. Hamada and Sakurai (1978) in this manner ruled out a priori the terms of trade effect in exchange rate determination.

The foregoing analysis suggests that, whenever there are more than two goods that are traded internationally, a specification of the price deflator in the money demand function is not an innocuous matter. Equally important is the implication of that specification for the international transmission of disturbances. In this context, it is interesting to note the empirical works of Fisher (1935) and Choudhri and Kochin (1980) that indicated that the regime of flexible exchange rates successfully insulated domestic output from foreign disturbances during the Great Depression. This may be indicative of the relevance of the money demand function in which the price of the domestic good is given the dominant weight. The synchronization of output movements during the recent regime of flexible exchange rates may suggest either that Θ has fallen over time as the world economy has become more integrated, or that countries are subject to the same shocks. In a model that includes a more complete set of economic variables, it may also suggest the increasing importance of capital flows and currency substitution.

VII. Concluding Remarks

This paper has presented a general structure that is required of a monetary model of exchange rate determination to generate a co-movement of the terms of trade and the exchange rate. It has been shown that the presence of the terms of trade effect in exchange rate determination requires not only that traded goods be imperfect substitutes but also that the prices of goods be weighted differently in money demand across countries. This sensitivity of the exchange rate to a specification of the money demand function is a general result that is independent of the particular set-up of the model. Moreover, a positive co-movement of the terms of trade and the exchange rate requires that the domestic price of the domestic good be given a greater weight in the price deflator, and such an outcome is certainly consistant with economic intuition (i.e., the prices of domestic goods must be more important than those of foreign goods) as well as the stability consideration and seems to be supported by recent experience.

Generalization to the Case of n Goods

The result that was obtained in the text concerning the role of monetary asymmetry in generating the terms of trade effect in exchange rate determination is not specific to a model that involves only two goods, although such an explicit result is most readily obtainable in a simple two-good model. It would be algebraically much more difficult to explicitly obtain a similar result in a model that involves more than two goods because such a model would involve a far more complex interaction of changes in relative prices and income. However, once we make the simplifying assumptions that output is exogenous and there is no income effect on money demand, the result can be easily extended to the case of n goods.

First, let the expressions for the law of one price [(2-1) and (2-2)] be replaced by,

$$(22)\quad p_{it} = s_t + p^*_{it}. \quad (i = 1,n)$$

Second, let the expression for the relative price (3) be replaced by,

$$(23)\quad p_{it} = (MU_i - MU_1)_t, \quad (i = 2,n)$$

where each of the (n-1) relative prices are expressed in terms of the log of the marginal rate of substitution between good i and good 1, with the latter as the arbitrarily chosen numeraire.

Third, let the expressions for the price level [(4-1) and (4-2)] be replaced by,

$$(24\text{-}1)\quad p_t \equiv \sum_{i=1}^{n} \theta_i p_{it}, \quad (\sum_{i=1}^{n} \theta_i = 1)$$

$$(24\text{-}2)\quad p^*_t \equiv \sum_{i=1}^{n} \theta^*_i p^*_{it}, \quad (\sum_{i=1}^{n} \theta^*_i = 1)$$

Then, the substitution of (22), (23), (24-1), (24-2) into the monetary equilibrium conditions (8-1) and (8-2) would yield,

$$(25)\quad s_t = (m_{t-1} - m^*_{t-1}) + (w_t - w^*_t) + \left\{\sum_{i=2}^{n} (\theta^*_i - \theta_i)(MU_i - MU_1)\right\}_t,$$

where the last term is the composite effect of relative prices. This last term vanishes only if the weights in the money demand are equal across countries, i.e, $\theta_i^* = \theta_i$ for all i or if all goods are perfect substitutes, i.e, $(MU_i - MU_1) = 0$ for all i.

Alternatively, (25) can be rewritten as,

$$(26)\ s_t = (m_{t-1} - m^*_{t-1}) + (w_t - w^*_t) + \{\sum_{i=2}^{n}(\theta^*_i(MU_i - MU_1) - \sum_{i=2}^{n}\theta_i(MU_i - MU_1)\}_t,$$

where the last term can be interpreted as the terms of trade between the home and foreign countries. Thus, as in the case of two goods, the terms of trade becomes a factor in exchange rate determination only with the asymmetry in the specification of the money demand functions across countries.

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