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MINIMOD: Specification and Simulation Results 1/

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Contents

Summary

This paper treats the construction and use of a small macroeconomic model of the United States and the rest of the world. The goal is to have a readily understandable and transparent model of manageable size that is suitable for policy analysis. Consequently, an eclectic theoretical model has been specified for each of the two economies, with equations for aggregate demand and supply of goods and capital accumulation, and with consistent treatment of government and private sector flows of funds. The model was specified such that it has desirable long-run properties, including the neutrality of money and the property that government debt cannot grow without limit relative to output. Values for the parameters of the model were obtained, with a few exceptions, from the properties of a larger, multi-country model; the paper describes the methodology of reducing a larger model to its core interactions using partial simulation techniques.

The model is simulated to gauge the effects of changes in monetary and fiscal policies, under two alternative assumptions concerning expectations of future rates of inflation, of long-term bond rates, and of the exchange rate: (i) expectations adapt to past movements in the variables, or (ii) expectations are consistent with the model's own predictions. The simulations imply that an increase in the money supply is likely to depreciate the exchange rate and to stimulate output in the home country, as prices are slow to adjust; in the long run, however, real magnitudes will be unaffected. Government spending increases also have a temporary stimulatory effect on output in the home country, but, for unchanged money supplies, tend to appreciate the exchange rate.

These conclusions are common to many macroeconomic models. However, MINIMOD also makes it possible to see how sensitive the results are to assumptions concerning expectations. It is shown that the paths of major macroeconomic variables may be quite different in the two cases mentioned above. In particular, in response to a money supply change, the exchange rate is likely to overshoot its equilibrium value under consistent expectations, though not under adaptive expectations, and output effects are likely to be smaller under consistent expectations. Government expenditure changes seem to have more similar effects in the model under the two expectations assumptions, though the changes induced in the exchange rate and in long-term bond rates are larger with model-consistent expectations. It is also shown that in this case, credible, pre-announced policy changes may have substantial effects before they are actually implemented: a future fiscal contraction may in fact have a stimulatory effect on output when it is announced, because of a decline in long-term interest rates and a depreciation of the currency.

I. Introduction

This paper describes MINIMOD, a small international macroeconomic model. The construction of MINIMOD was motivated by two concerns. The first was the desire to have a readily understandable and transparent model of manageable size suitable primarily for policy analysis rather than forecasting. Transparency of results is especially important because it is clearly the case that models are only a rough, and often flawed, representation of reality; it is therefore important to be able to explain why simulation results are what they are, and to ensure that they are related to important economic linkages and not to errors of specification. This is much easier to do for a small model than for a large one. In the context of a multicountry model it is also an advantage to have small, identically-specified models for all countries because they permit comparison of certain key parameters and identification of relationships that may explain differences of behavior between economies--for example, the degree of wage indexation or the interest elasticity of money demand.

The second concern was a need for a macro model small enough to allow what are often called "rational expectations" simulations--that is, simulations where expectations are made consistent with the model's predictions-to be performed inexpensively. Such simulations are perhaps better termed "consistent expectations" simulations, and we will use this latter terminology in what follows. 1/ Although advances in computer technology have meant that the cost of simulations is not usually a factor for a model without forward-looking endogenous variables, the calculation of consistent expectations solutions can be prohibitively expensive for large non-linear models because they require iteration backward and forward in time. If a model is small, these costs are reduced considerably.

MINIMOD is a small, two-country model in which the equations for the United States and an aggregate rest of the industrial world region are based on the same theoretical framework. The model's size is the minimum needed to capture the major macroeconomic relationships. The parameters of the model were obtained not by direct estimation but rather by simulation of a larger model. Endogenous variables in the larger model were aggregated together and exogenous variables which are not relevant for policy simulations were dropped, yielding a much more compact version of the model while retaining its essential properties.

The next section describes the theoretical structure of MINIMOD. Section III explains how its parameters were extracted from the larger model using simulation techniques. Section IV describes the results of

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1/ The term "consistent expectations" was proposed by Walters (1971), but it has not been widely used by economists.

several policy-simulation experiments using both consistent expectations and adaptive expectations, in which expectations are formed on the basis of past values of the variable. The final section contains our concluding observations and suggestions for further work. Appendix I presents the simulation model; Appendix II describes the methodology employed in creating MINIMOD.

II. The Theoretical Model

MINIMOD is a model of two economies with the same structure (see Table 1). The models are driven by conventional aggregate demand functions and are linked by goods and financial markets. Expectations of the future levels of inflation and of long-term bond rates in the two economies, as well as exchange rate expectations, are made explicit in the model. In the simulations discussed below, the model is simulated with both adaptive expectations and consistent expectations of these variables.

A number of characteristics of the model are worth mentioning at the outset. Outside assets, in the form of government bonds, net claims on foreigners, and the physical capital stock, are endogenous to the model; budget constraints link these asset stocks to the appropriate flow variables. 1/ The investment function is specified as a lagged adjustment process of the actual capital stock to its desired long-run level, which is derived from a Cobb-Douglas production function. In the long run, the marginal product of capital is equal to the user cost of capital, which depends on the long-term real rate of interest and on tax rates.

There are two composite final goods in the model, with each economy specialized in the production of one of the goods. However, consumption and investment in each country includes both goods, and the deflators for consumption, investment, and government spending are assumed to be the same. Correspondingly, there are four prices in the model: an output price--i.e., the GNP deflator--and an absorption deflator, for each of the two countries. The absorption deflator is a weighted sum of home output prices and of import prices, the latter being simply the trading partner's output price expressed in the home currency.

The labor market is implicit in the model; both wages and employment have been solved out. MINIMOD contains an equation for the rate of change of the GNP deflator, which depends positively on the rate of capacity utilization and on the expected rate of change of the absorption deflator (with a unit coefficient). Such an equation can be derived from an expectations-augmented Phillips curve which determines contract wages; a

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1/ Base money is also included in the model, but it is exogenous.

price equation with a constant markup over actual wages calculated as an exponentially-declining weighted average of contract wages; and an Okun's Law relationship between unemployment and output. 1/ The absorption deflator enters the equation for the GNP deflator because it affects the real consumption wage relevant to workers. The rate of capacity utilization, calculated as the ratio of actual output to capacity output obtained from the production function, captures demand pressures on both prices and wages. The model implies that there is a normal rate of capacity utilization, which corresponds to a "natural rate" of unemployment, at which changes in output prices will equal the expected rate of change of absorption prices. Therefore, at this rate of capacity utilization, if import prices grow at a fully-anticipated, constant rate, output prices will also grow at that rate.

Consumption depends on real wealth, real disposable income and real long-term interest rates. Real disposable income is defined to include only the real portion of interest payments and thus differs from the conventional national accounts definition. Taxes are a function of national income and the private sector's interest earnings. There is a parameter, λ_t, that measures the neutrality of the tax system with respect to the inflation component in interest rates; if it is unity, then taxes are levied only on real interest receipts. Real wealth is defined as the sum of outside money, government bonds, and net claims on foreigners, each divided by the absorption deflator, and the real capital stock. Government bonds are assumed to pay the short-term interest rate, and to be fixed in price; net claims on foreigners are assumed to be denominated in U.S. dollars, and their foreign-currency value thus varies with the exchange rate, e. No attempt is made to explain the market valuation of the physical capital stock, as opposed to its replacement cost.

A parameter, λ_b, specifies the degree to which government bonds are net wealth; if it is zero, then Ricardian equivalence holds, and the choice between financing government expenditure through bond issues or lump-sum taxes has no real effects. If it is unity, the private sector does not offset anticipated future taxes needed to service the debt against any part of its current holdings of bonds. Correspondingly, one minus λ_b, multiplied by the change in government bonds outstanding, is subtracted from disposable income (see Hodrick (1981)). There is also a parameter, α, that measures the extent to which government consumption is a direct substitute for private consumption. Government expenditure is assumed to fall on home and foreign goods in the same proportions as private spending; thus import and export volumes can be expressed as functions of relative prices and total domestic absorption.

1/ Such a model is a restricted version of the model described in Taylor (1979), where expected unemployment (or output) during the duration of a contract is also important.

Perfect substitutability is assumed between home and foreign short-term bonds and between short-term and long-term bonds in each economy. The first assumption--open parity--means that the short-term home and foreign interest rates differ only by the expected exchange rate change. 1/ The second assumption implies that holding-period yields on short-term and long-term bonds, allowing for expected capital gains on the latter, are equalized.

Table 1. MINIMOD: Theoretical Model

Home Country

1. Real domestic absorption

$$a \equiv c + \dot{k} + \delta \cdot k + g$$

2. Real domestic GDP

$$y = a + x - i$$

3. Real consumption

$$c = -\alpha \cdot g + c(w, yd, rl-\bar{\pi})$$

where

$$w \equiv \lambda_m \cdot m/p + \lambda_b \cdot b/p + f/p + k$$

and

$$yd \equiv y \cdot p_q/p - \delta \cdot k - t/p + (r-\pi)(b+f)/p - (1-\lambda_b)\dot{b}/p$$

4. Net investment

$$\dot{k} = \eta \cdot (\beta \cdot y/cc-k) + n \cdot k$$

where

$$cc = (rl-\bar{\pi}+\delta)/(1-\xi)$$

1/ This equation in the model, however, has a residual that can be interpreted as an exogenous risk premium. In the simulations plotted in Chart 5 below, this residual is changed in order to gauge the effect of a portfolio shift away from U.S. dollar assets.

Table 1. MINIMOD: Theoretical Model (continued)

5. Government budget constraint

$$\dot{b} + \dot{m} = p \cdot g - t + r \cdot b$$

6. Nominal tax receipts

$$t = t\left(p_q \cdot y - \delta \cdot p \cdot k + (r - \lambda_t \cdot \pi)(b+f)\right)$$

7. Capacity output

$$y^c = A \cdot e^{(1-\beta)n\tau} \cdot k^{\beta}$$

8. GNP

$$q = y + r \cdot f / p_q$$

9. Domestic absorption deflator

$$p = [p_q \cdot (y-x) + e \cdot p_q^* \cdot i]/a$$

10. Inflation rate

$$\dot{p}_q / p_q = \pi^e + \phi(y/y^c)$$

11. Demand for m_1

$$m_1/p = m(q, r)$$

where

$$m_1 = \mu \cdot m$$

12. Long-term interest rate

$$r = rl - \dot{rl}^e/rl$$

13. Exports of goods and nonfactor services

$$x = x(e \cdot p_q^*/p_q, a^*)$$

Table 1. MINIMOD: Theoretical Model (continued)

14. Imports of goods and nonfactor services

$$i = i(e \cdot p_q^*/p_q,\ a)$$

15. Open parity condition

$$r = r^* + \dot{e}^e/e$$

16. Accumulation of net claims on foreigners

$$\dot{f} = r \cdot f + p_q \cdot x - p_q^* \cdot e \cdot i$$

Foreign Country

17. Real domestic absorption

$$a^* \equiv c^* + \dot{k}^* + \delta^* \cdot k^* + g^*$$

18. Real domestic GDP

$$y^* = a^* + i - x$$

19. Real consumption

$$c^* = -\alpha^* \cdot g^* + c(w^*,\ yd^*,\ rl^*)$$

where

$$w^* \equiv \lambda_m^* \cdot m^*/p^* + \lambda_b^* \cdot b^*/p^* - f/(p^* \cdot e) + k^*$$

and

$$yd^* \equiv y^* \cdot p_q^*/p^* - \delta^* \cdot k^* - t^*/p^* + (r^* - \pi^*) \cdot b^*/p^*$$
$$- (r - \dot{e}/e - \pi^*) \cdot f/(p^* \cdot e) - (1 - \lambda_b^*) \cdot \dot{b}^*/p^*$$

20. Net investment

$$\dot{k}^* = \eta^* \cdot (\beta^* \cdot y^*/cc^* - k^*) + n^* \cdot k^*$$

Table 1. MINIMOD: Theoretical Model (continued)

20. (Continued)

where

$$cc^* = (rl^* - \bar{\pi}^* + \delta^*)/(1-\xi^*)$$

21. Government budget constraint

$$\dot{b}^* + \dot{m}^* = p^* \cdot g^* - t^* + r^* \cdot b^*$$

22. Nominal tax receipts

$$t^* = t\big(p_q^* \cdot y^* - \delta^* \cdot p^* \cdot k^* + (r^* - \lambda_t^* \cdot \pi^*) \cdot b^* - (r - \dot{e}/e - \lambda^*_t \cdot \pi) f/e\big)$$

23. Capacity output

$$y^{c*} = A^* \cdot e^{(1-\beta^*) n^* \tau} \cdot k^{*\beta^*}$$

24. GNP

$$q^* = y^* - r \cdot f/(e \cdot p_q^*)$$

25. Domestic absorption deflator

$$p^* = [p_q^* \cdot (y^* - i) + p_q \cdot x/e]/a^*$$

26. Inflation rate

$$\dot{p}_q^*/p_q^* = \pi^{*e} + \phi^*(y^*/y^{c*})$$

27. Demand for m_1

$$m_1^*/p^* = m(q^*, r^*)$$

where

$$m_1^* = \mu^* \cdot m^*$$

28. Long-term interest rate

$$r^* = rl^* - \dot{rl}^{*e}/rl^*$$

Table 1. MINIMOD: Theoretical Model (continued)

MINIMOD Variables

* Indicates foreign variables
• Indicates a time derivative

EXOGENOUS

g = real government expenditure
m = nominal base money
μ = money multiplier
ξ = marginal tax rate
τ = time
$\bar{\pi}$ = long-run expected inflation

ENDOGENOUS

Real variables

a = absorption
c = private consumption
i = imports of goods and nonfactor services
k = capital stock
q = gross national product
w = wealth
x = exports of goods and nonfactor services
y = gross domestic product
y^c = potential gross domestic product
yd = disposable income

Prices

p = absorption deflator
p_q = gross national product deflator
π = inflation rate (rate of change of p)
π^e = short-run expected inflation

Financial variables

b = nominal stock of government bonds
m_1 = nominal money supply (M1)
f = nominal stock of net claims on foreigners (denominated in home currency)
t = government tax receipts, net of transfers

Table 1. MINIMOD - Theoretical Model (concluded)

ENDOGENOUS (Continued)

Interest rates and exchange rates

cc = user cost of capital
e = exchange rate (the unit price of foreign currency in terms of home currency)
e^e = expected exchange rate
r = short-term interest rate
rl = long-term interest rate
rl^e = expected long-term interest rate

PARAMETERS

α = proportion of a change in government expenditure that is directly offset by a change in private consumption
β = the relative share of capital in output
δ = the depreciation rate
λ_m = the proportion of base money that is included in wealth
λ_b = the proportion of government bonds that is included in wealth
λ_t = the degree to which the tax system is neutral with respect to the inflation premium in interest receipts
η = the speed of adjustment of the capital stock to its desired level
n = the rate of growth of the labor force in efficiency units, i.e., the economy's steady state real growth rate (assumed the same for the two countries)

III. The Empirical Model

The parameters appearing in behavioral equations of the theoretical model described above were for the most part obtained by simulating the Federal Reserve Board's Multi-Country Model (MCM). Our decision not to try to estimate the parameters directly reflected a concern that estimation of an aggregated model may give unsatisfactory results. Given the small samples that are available, data on macroeconomic variables are often dominated by special events and by institutional changes that heavily influence the historical data but may not be relevant to future periods over which the model will be simulated. Model builders often have to include dummy variables or make ad hoc adjustments for these past events; not to do so would distort the coefficients on the variables of interest. Such events may be considered random from a longer-term perspective, and hence in principle they should not bias the structural coefficients. However, in practice it is not possible in small samples to relegate these events to the error terms of structural equations; the change in parameter estimates due to adding only a few observations could be unacceptably large. 1/ A large part of the time spent in estimating a model consists in identifying and adjusting for these special factors; it is more straightforward to do so when a model is disaggregated and contains institutional detail. A small version of the resulting model can retain the interesting interactions, while discarding the extra exogenous variables that are needed to track the historical data but that are not relevant for simulations of future policy changes.

We therefore chose to profit from the work of other modellers and used the MCM because it is a well-documented model of 5 industrial countries with a large data base. The structure of the MCM is roughly consistent with the theoretical model described above; where there were conflicts, we imposed our theoretical structure and functional forms. For instance, we identified lagged price variables in the MCM's wage equation as resulting from expectational lags, to be consistent with equations (10) and (26) in Table 1, and we scaled real interest rates in the consumption equation by GNP in order to allow consumption to grow with output in steady state. We have based the "home economy" of the MINIMOD on the MCM's U.S. model and the "foreign economy" of the MINIMOD on an aggregation of the MCM models of Canada, the Federal Republic of Germany, Japan and the United Kingdom. The model is, in principle, a closed model, as we scale up the foreign economy's exports and imports to make them consistent with U.S. imports and exports, respectively. Implicitly, then, variables for countries not included in the MCM are assumed to move proportionally with those of the countries that are included.

The idea of creating smaller models from larger models is not new; in the mid-1970s a small structural 2/ model of the Federal Reserve Board's MPS model was constructed in order to perform optimal control experiments. 3/ More recently, Malgrange and others have studied the properties of larger models by constructing "maquettes" of those models that capture the essential dynamic features but ignore "second order" linkages. 4/ Masson and others have applied these procedures to the OECD's INTERLINK model to obtain a small structural model that is in many ways a precursor to the MINIMOD. 5/

1/ Sims (1980) has argued for treating regime changes as random errors. However, the resulting estimates of model parameters would be so unstable as to remove any confidence in simulation results.

2/ The word "structural" is used here in opposition to "reduced-form." The models are not structural in the sense of identifying utility function parameters.

3/ See Battenberg, Enzler and Havenner (1975).

4/ Deleau, Malgrange and Muet (1984).

5/ Masson and Richardson (1985) and Masson and Blundell-Wignall (1985).

An alternative to creating a small structural version of the larger model would be generating its reduced form, aggregated appropriately, and doing simulation experiments with that. 1/ This has two disadvantages for our purposes. First, it does not allow us easily to modify the model by replacing some of the structural equations or by changing some of the structural parameters. This may be desirable either because of perceived inadequacies of the large model, or because of a desire to gauge the sensitivity of simulation results to certain key parameters. Second, the reduced-form approach means working with a linear model, which may not give satisfactory long-term properties. Though it is usually possible to linearize most of the model in the logarithms of appropriately-defined variables, there are essential non-linearities, for instance those resulting from budget or balance-sheet constraints, that one may want to retain. The use of partial simulations in order to generate a small structural model allows one to do this.

The construction of the MINIMOD, described in detail in Appendix II, can be summarized as follows. First, sections of the large model that correspond to single equations in the small model are isolated. Right hand side (RHS) variables of the small-model equations are exogenous to the isolated block of the large model (though not necessarily exogenous to the model itself). Each of these variables is, in turn, given a shock, and the model is simulated for a sufficient number of periods so that the endogenous variable settles down to its long-run value. The "shock-minus-control" results of the left hand side (LHS) variable are then regressed on the "shock-minus-control" values of the RHS variables in order to generate coefficient estimates.

Following Jorgenson (1966), we have used ratios of polynomials in the lag operator to capture the dynamic responses of the model. By regression of the dependent variable on lagged values of itself as well as on contemporaneous and lagged values of the independent variable, we were able adequately to model the dynamic patterns of the large model in a parsimonious way. In practice lags of more than two periods on either dependent or independent variables were seldom required. Since the equations presented below are based on regression analysis that uses simulation results as raw data, the goodness of fit statistics refer to how well the equations replicate the larger model's properties, not to the equations' fit of historical data.

The MCM, as published in 1983, was used to generate the simulation results, with the following four exceptions. First, the link between capacity utilization and wage inflation in the United States incorporates more recent information. Second, the demand for M1 in the United States is based on work done by Porter and Brayton (1984), also at the Federal

1/ Such a methodology is used by Maciejowski and Vines (1984).

Reserve Board. Third, the M1 demand function in other industrial countries is based on work done at the OECD and published in Blundell-Wignall et al. (1984). Finally, we imposed a Cobb-Douglas function with parameters that reflect the relative shares of labor and capital.

There are a number of features of our regressions that deserve mention, since they use data generated by deterministic simulations of an existing model with errors set equal to zero. The data do not conform to a classical regression model, as endogenous variables do not result from drawings from a joint probability distribution. Instead, the "time series" of partial effects of a change in an exogenous variable result solely from the autoregressive properties of the MCM. We expect the residuals in our regressions to be serially correlated, and hence we do not report Durbin-Watson statistics.

In each case the equations were estimated without a constant term; this insures that when the RHS variables are at their steady-state values, so are the LHS variables. Two types of constraints were imposed in estimation: the effect of inflationary expectations on actual inflation is constrained to have a unit coefficient in the long run and, in the net investment equations, the coefficients on the desired and actual capital stocks are constrained such that the actual capital stock is equal to the desired capital stock in the long run. It was not possible to calculate R^2's for these constrained equations.

The equations are presented in the same order as in the theoretical model, first for the United States and then for the other industrial economy; t-values are given in parentheses, though, for the reasons mentioned above, their statistical properties are unknown. Variable notation is explained in Appendix I. In what follows, each of the variables takes on values that are deviations from its baseline path, when the exogenous variable on the RHS of the equation is changed by an arbitrary amount. Thus the regressions capture the partial effects of the exogenous variable on the endogenous one--for instance, the effect of UYD on UC in equation (1) below--and its dynamic pattern. The full set of effects of other variables on UC consists of equations (1), (2), and (3).

<u>Consumption</u>

$$\text{(1)}\quad UC = \underset{(6.10)}{.252096}\ UYD + \underset{(4.77)}{.021982}\ UYD(-1) + \underset{(42.75)}{.594668}\ UC(-1)$$

OBS = 28 $\quad \bar{R}^2 = .998 \quad$ SER = .035

$$\text{(2)}\quad UC = \underset{(2700)}{.004878}\ UW(-1)$$

OBS = 28 $\quad \bar{R}^2 = 1 \quad$ SER = .000

(3) $$UC = \underset{(-91.12)}{-3.05811} \cdot URL(-2) - \underset{(-7.32)}{.456031} \cdot URL(-3) + \underset{(24.87)}{.42672}\ UC(-1)$$

OBS = 28 $\qquad \bar{R}^2 = .998 \qquad$ SER = .034

(4) $$RC = \underset{(27.28)}{.006344}\ RW + \underset{(91.28)}{.022152}\ RW(-1) + \underset{(22.66)}{.24771}\ RC(-1) + \underset{(9.56)}{.101305}\ RC(-2)$$

OBS = 25 $\qquad \bar{R}^2 = .998 \qquad$ SER = .005

(5) $$RC = \underset{(128.73)}{.201055}\ RYD + \underset{(89.09)}{.56857}\ RC(-1)$$

OBS = 25 $\qquad \bar{R}^2 = .999 \qquad$ SER = .015

Net Investment

(6) $$\Delta UK = \underset{(3.79)}{.02937} \left(\frac{UBETA}{UUCSTCAP} \cdot UGDP - UK(-1)\right) + \underset{(12.20)}{.27905}\ \Delta UK(-1)$$

OBS = 25 $\qquad \bar{R}^2 = NA \qquad$ SER = .0121

(7) $$\Delta RK = \underset{(NA)}{.00233} \left(\frac{RBETA}{RUCSTCAP} \cdot RGDP)\right) + \underset{(4.73)}{.00534} \left(\frac{RBETA}{RUCSTCAP(-1)} \cdot RGDP(-1)\right)$$

$$+ \underset{(2.20)}{.00252} \left(\frac{RBETA}{RUCSTCAP(-4)} \cdot RGDP(-4)\right) + \underset{(-7.35)}{.65450}\ RK(-1) - \underset{(7.45)}{.66217}\ RK(-2)$$

$$- \underset{(2.20)}{.00252}\ RK(-5)$$

OBS = 25 $\qquad \bar{R}^2 = NA \qquad$ SER = .033

Nominal Taxes, Net of Transfers

(8) $$UTAX = \underset{(84,483)}{.242635}\ UACT + \underset{(87,759)}{.252042}\ UACT(-1)$$

OBS = 25 $\qquad \bar{R}^2 = .999 \qquad$ SER = .000

UACT = U.S nominal net national product plus net real interest payments

(9) $$RTAX = \underset{(219.63)}{.332668}\ RACT + \underset{(37.49)}{.168227}\ RTAX(-1)$$

OBS = 28 $\bar{R}^2$ = .999 SER = .019

RACT = ROW nominal net national product plus net real interest payments

Inflation Rate

(10) $$\frac{\Delta UPGNP}{UPGNP(-1)} = \underset{(7.26)}{.02248}\ LOG(UCU) + \underset{(15.54)}{.048106}\ LOG\left[UCU(-1)\right] - \underset{(-10.41)}{.032238}\ LOG\left[UCU(-2)\right]$$

OBS = 28 $\bar{R}^2$ = .937 SER = .0003

(11) $$\frac{\Delta UPGNP}{UPGNP(-1)} = \underset{(11.43)}{.17689}\ UPIE - \underset{(NA)}{.10243}\ UPIE(-1) + \underset{(7.71)}{1.4634}\left(\frac{\Delta UPGNP(-1)}{UPGNP(-2)}\right)$$

$$- \underset{(-3.24)}{.53786}\left(\frac{\Delta UPGNP(-2)}{UPGNP(-3)}\right)$$

OBS = 28 $\bar{R}^2$ = NA SER = .000004

(12) $$\frac{\Delta RPGNP}{RPGNP(-1)} = \underset{(16.20)}{.052141}\ LOG(RCU) - \underset{(-9.51)}{.03061}\ LOG\left[RCU(-1)\right]$$

OBS = 28 $\bar{R}^2$ = 929 SER = .0003

(13) $$\frac{\Delta RPGNP}{RPGNP(-1)} = \underset{(NA)}{.42959}\ RPIE + \underset{(5.37)}{.57041}\left(\frac{\Delta RPGNP(-1)}{RPGNP(-2)}\right)$$

OBS = 28 $\bar{R}^2$ = NA SER = .0002

Demand for Money

(14) $$\text{LOG}\left(\frac{\text{UM1}}{\text{UP}}\right) = \underset{(9.13)}{.259104}\ \text{LOG(UGNP)} + \underset{(3.20)}{.090481}\ \text{LOG}\left[\text{UGNP}(-1)\right] - \underset{(-11.97)}{.339518}\left(\frac{\text{URS}}{100}\right)$$

$$- \underset{(-3.97)}{.130362}\left[\frac{\text{URS}(-1)}{100}\right] + \underset{(12.67)}{.616057}\text{LOG}\left[\frac{\text{UM1}(-1)}{\text{UP}(-1)}\right]$$

OBS = 41 $\bar{R}^2$ = .938 SER = .028

(15) $$\text{LOG}\left(\frac{\text{RM1}}{\text{RP}}\right) = \underset{(374.7)}{.224386}\ \text{LOG(RGNP)} - \underset{(-866.4)}{.518863}\left(\frac{\text{RRS}}{100}\right) + \underset{(993.6)}{.72497}\ \text{LOG}\left[\frac{\text{RM1}(-1)}{\text{RP}(-1)}\right]$$

OBS = 43 $\bar{R}^2$ = .999 SER = .0006

Trade Equations

(16) $$\text{LOG(UX)} = \underset{(2595.7)}{.789928}\ \text{LOG(RA)} - \underset{(-85.21)}{.03281}\ \text{LOG}\left[\text{UX}(-1)\right]$$

OBS = 25 $\bar{R}^2$ = .999 SER = .0003

(17) $$\text{LOG(UX)} = \underset{(3.72)}{.023369}\ \text{LOG(E}\cdot\text{RPGNP/UPGNP)} + \underset{(38.88)}{.106272}\ \text{LOG}\left[\text{E}(-1)\cdot\text{RPGNP}(-1)/\right.$$

$$\left.\text{UPGNP}(-1)\right] + \underset{(16.83)}{.882563}\ \text{LOG}\left[\text{UX}(-1)\right]$$

OBS = 25 $\bar{R}^2$ = .988 SER = .063

(18) $$\text{LOG(UI)} = \underset{(62.23)}{1.37984}\ \text{LOG(UA)} - \underset{(-9.32)}{.722509}\ \text{LOG}\left[\text{UA}(-1)\right] + \underset{(13.32)}{.717355}\ \text{LOG}\left[\text{UI}(-1)\right]$$

OBS = 25 $\bar{R}^2$ = .994 SER = .022

$$\text{(19)}\quad \text{LOG(UI)} = -\underset{(-33.17)}{.070178}\ \text{LOG(E}\cdot\text{RPGNP/UPGNP)}$$

$$-\underset{(-6.82)}{.016281}\ \text{LOG}[\text{E}(-1)\cdot\text{RPGNP}(-1)/\text{UPGNP}(-1)]$$

$$-\underset{(-41.92)}{.099974}\ \text{LOG}[\text{E}(-2)\cdot\text{RPGNP}(-2)/\text{UPGNP}(-2)]$$

$$+\underset{(48.24)}{.761522}\ \text{LOG}[\text{UI}(-1)] - \underset{(-6.09)}{.087122}\ \text{LOG}[\text{UI}(-3)]$$

OBS = 25 $\quad \overline{R}^2$ = .998 $\quad$ SER = 0.21

IV. Simulation Results

This section presents the results of several simulation experiments using MINIMOD. The simulations were performed with both an adaptive expectations version of the model and a consistent expectations version. In the adaptive expectations version, expectations of next period's inflation rates and long-term bond rates in each of the two economies as well as of the exchange rate are formed on the basis of current and past movements of the respective variables (as generated by the model), with adaptation parameters, which, though chosen arbitrarily, roughly replicate the simulation properties of the MCM (see Appendix I for a list of the parameters used). In the consistent expectations version of the model, this period's expectations of these variables are made to equal to the model's solution values for next period.

The consistent expectations solution--a more common, but somewhat misleading term is rational expectations solution--was calculated using a version of the algorithm described by Fair and Taylor (1983). This algorithm is iterative, and, starting from initial guesses, it revises expectations of a variable on the basis of what the model calculates for that variable in a later period. Furthermore, it successively extends the horizon for the formation of expectations until variables differ between iterations by less than some predetermined tolerance. After experimentation, we discovered that the solution path could be quite sensitive both to the tolerance and to the values provided as initial guesses each time the horizon is extended. Consequently, we created a steady-state version of the model, calculated the long-run effects of the policy changes, and used these values each time the horizon was extended. In order for the steady-state solution to exist, we had to impose some further assumptions: from 1991 on, taxes are assumed to adjust so that eventually a given value for the stock of government bonds as a ratio to GNP is obtained; the two economies are assumed to settle down eventually to the same real growth rate; and the wealth elasticity of consumption spending in the United States is set equal its long-run value in the ROW (.0438), rather than its estimated value (.0049).

We begin by analyzing the results of fiscal shocks applied to each economy under the two alternative expectational assumptions. We then examine monetary and exchange rate shocks in a similar fashion. The policy simulations are done with all other policies fixed; thus the fiscal shocks assume all monetary aggregates are unchanged while the monetary shocks treat real government expenditure as unchanged. The results of all of the simulations are discussed relative to a common baseline for 1985-1990 that reflects instructions of the organizers of a conference at the Brookings Institution. 1/

The fiscal shocks

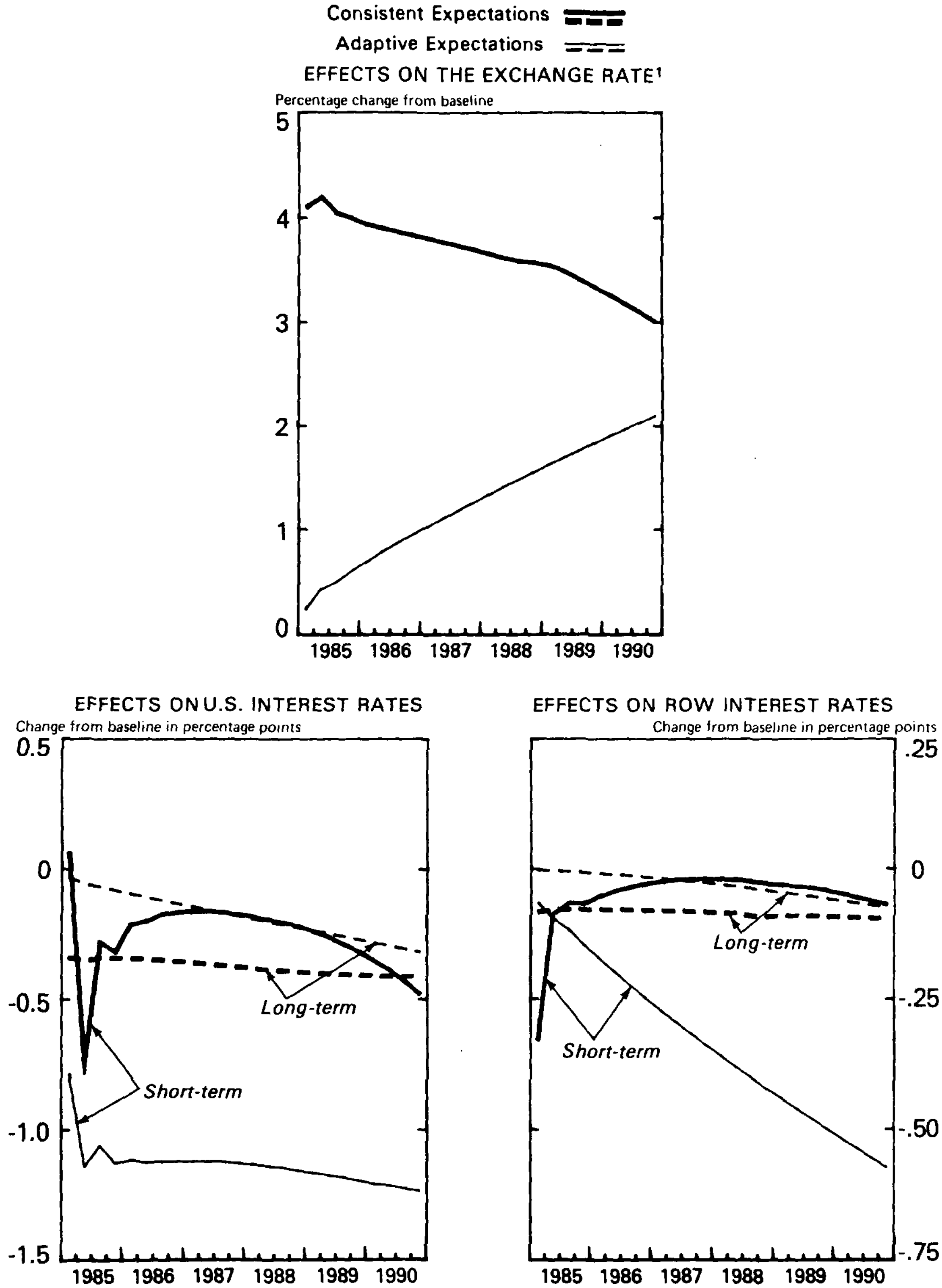
Chart 1 shows the response of the major macroeconomic variables to a previously unexpected, sustained U.S. fiscal contraction that is implemented when it is announced. Specifically, U.S. real government expenditure was permanently decreased by an amount equal to 1 percent of U.S. GNP in 1985, first quarter. Narrowly defined money in both economies was fixed while interest rates were allowed to vary.

The differences in the two versions of the model are perhaps most apparent in the behavior of the exchange rate. In the consistent expectations (CE) version, the exchange rate jumps a good deal--the dollar depreciates about 4 percent on impact--and then appreciates gradually. In the adaptive expectations (AE) version, the exchange rate depreciates very little initially, but continues to depreciate throughout the simulation. The long-run effects are, however, identical for both versions of the model, and a non-dynamic version of the model was used to calculate them; they involve an appreciation, not a depreciation, of the dollar, by about 2 percent. The dollar appreciates in the long run because the fiscal contraction brings about an increase in net claims on foreigners, and hence an improvement of the balance on investment income: this is consistent with a lower trade balance and, ultimately, an appreciation of the dollar's real exchange rate. As can be seen from Chart 1, even after six years of simulation the exchange rate is far from its long-run level. The CE simulations in fact solve the model some 10 years beyond that point, using as terminal values for the expectations variables their calculated steady-state values.

1/ "Empirical Macroeconomics for Interdependent Economies," March 10-11, 1986. The simulation results differ from those presented at that conference, because here they have been redone with a tighter convergence criterion. For most shocks, differences are minor, however.

CHART 1

REDUCTION IN U.S. GOVERNMENT PURCHASES



[1] Increase indicates dollar depreciation.

CHART 1 (Cont.)

REDUCTION IN U.S. GOVERNMENT PURCHASES

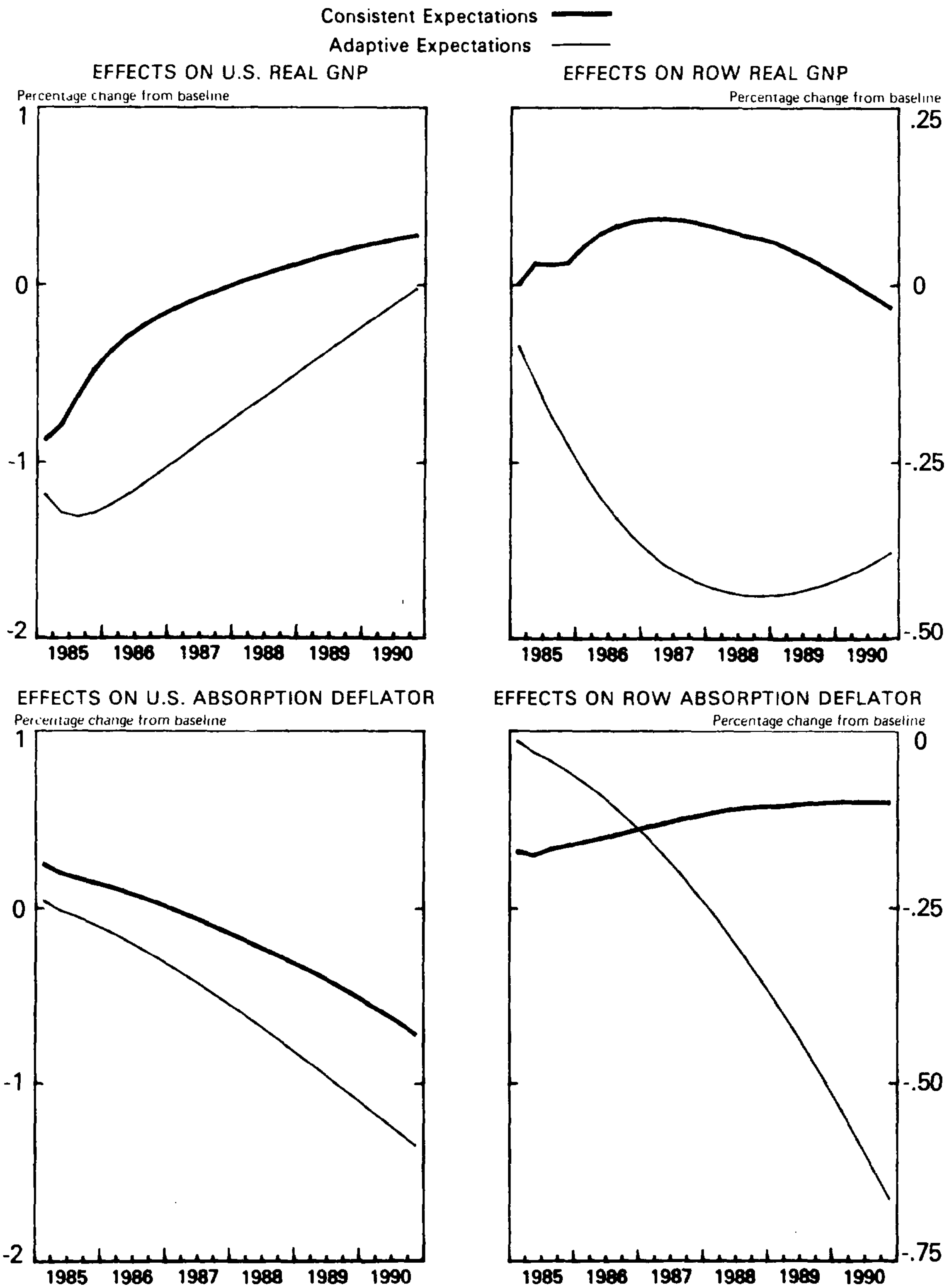
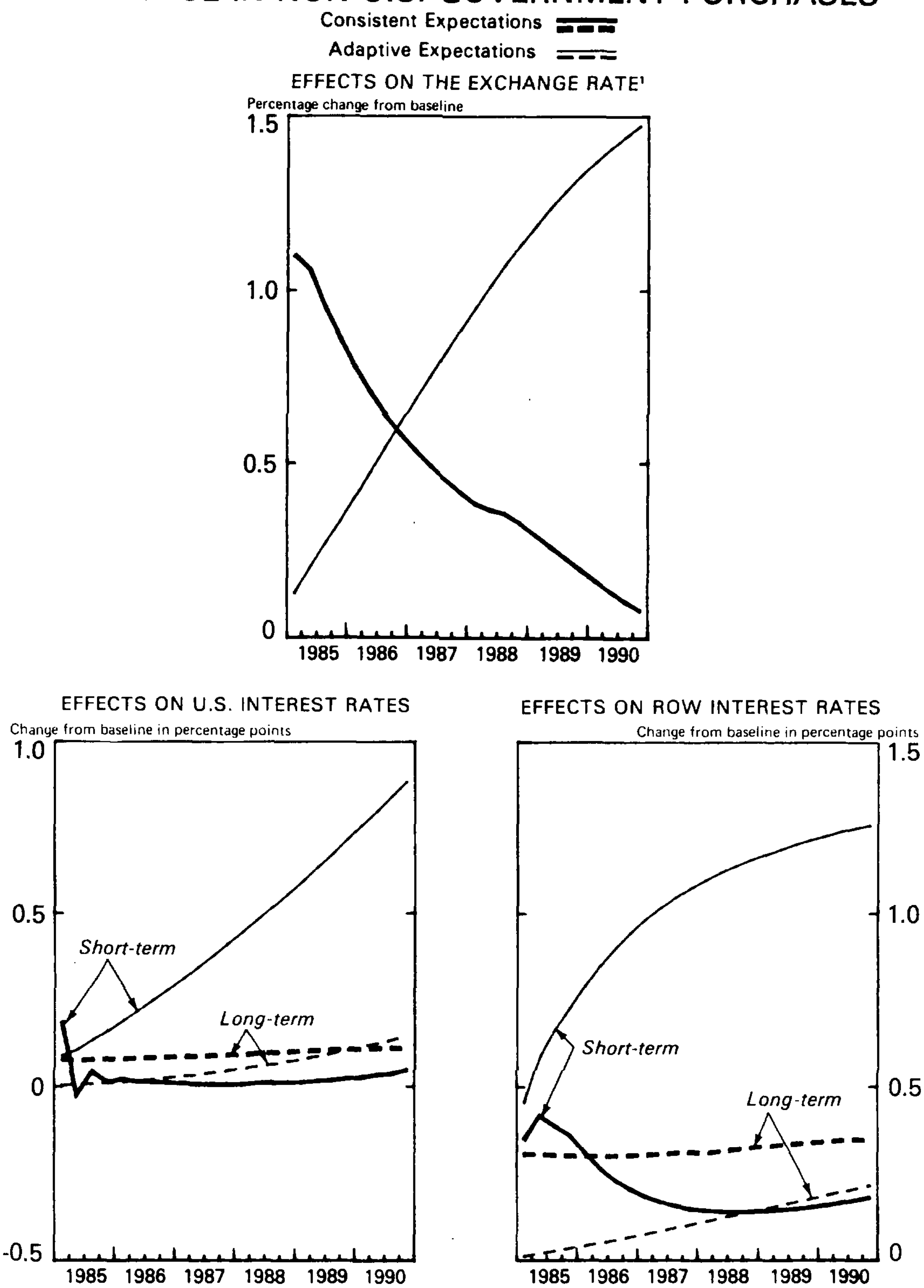


CHART 2

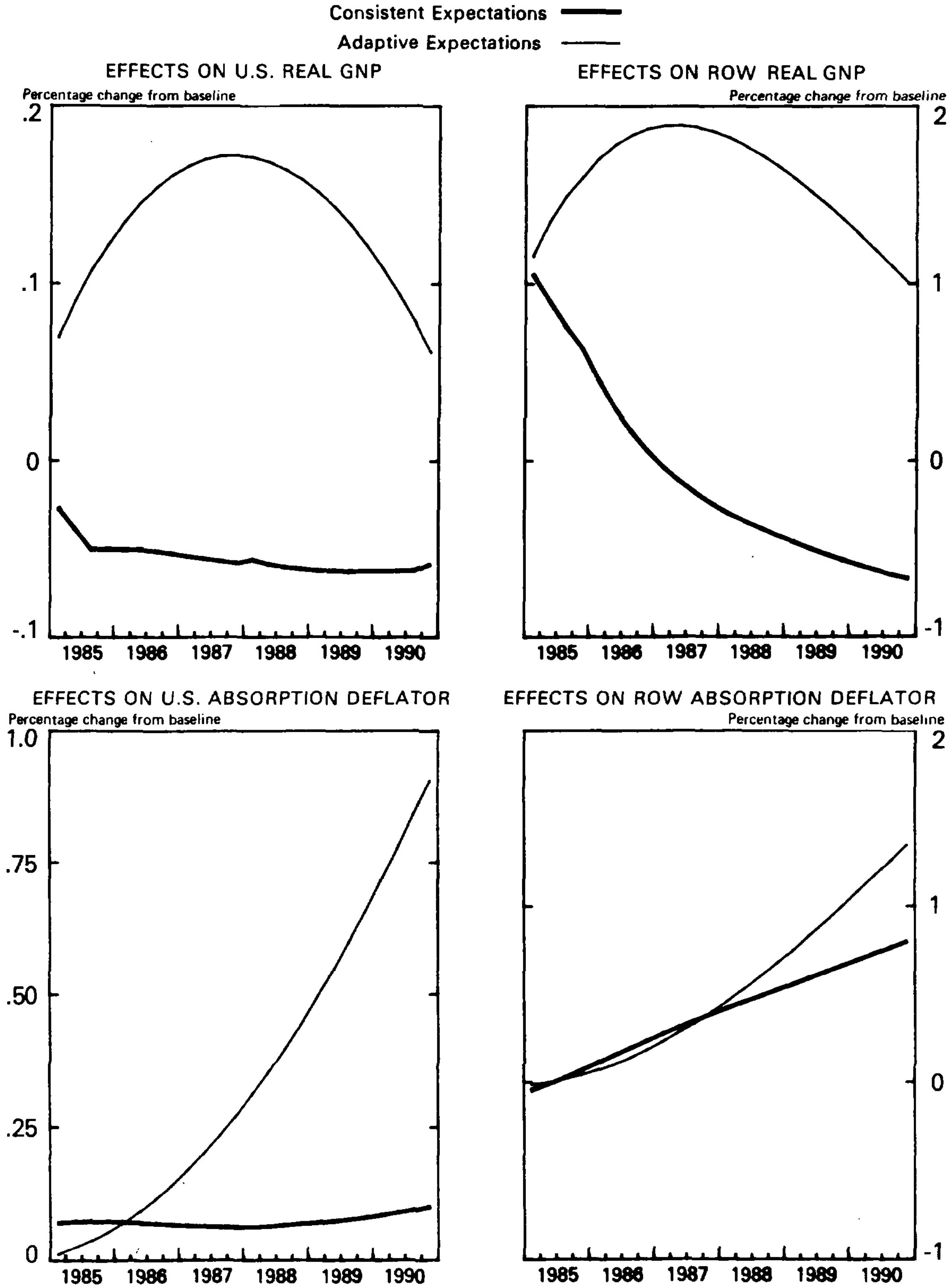
INCREASE IN NON-U.S. GOVERNMENT PURCHASES



[1] Increase indicates dollar depreciation.

CHART 2 (Cont.)

INCREASE IN NON-U.S. GOVERNMENT PURCHASES



The difference in dynamic paths for the exchange rate has a simple intuitive explanation. The interest parity condition implies that the expected rate of change of the exchange rate must equal the interest differential, or roughly $(ee-e)/e = r - r^*$, where ee is the exchange rate expected for next period. The fiscal contraction in the United States, by lowering r more than r^*, implies that ee-e must be negative--that is, the dollar is expected to appreciate in effective terms between this period and next. Under consistent expectations, actual exchange rate changes must therefore also be negative, after an initial jump when the previously unanticipated policy change occurs. Since in the long run the exchange rate appreciates, in principle the initial jump could be positive or negative, as long as it did not exceed the long-run appreciation. Given the size of the cumulative interest differential, however, an initial depreciation is required in our model. Under adaptive expectations, exchange rate expectations are given by

$$ee - ee(-1) = \eta(e-ee(-1))$$

For ee-e to be negative, e must continually be greater than the value that was expected last period to prevail this period, since

$$ee - e = (\eta-1)(e-ee(-1))$$

and $\eta < 1$. At the beginning of the simulation, e = ee; since subsequent changes in ee are a fraction, between zero and one, of movements in e, it must therefore be the case that e continually increases under adaptive expectations, even though it is expected to decrease. 1/

Long-term interest rates in both economies also exhibit different dynamic patterns under consistent and adaptive expectations. In the CE version long-term interest rates fall more on impact and continue with a flatter trajectory than in the AE version. Steady-state effects on both long-term and short-term interest rates, in both countries, imply a fall of 16 basis points in response to the cut in U.S. fiscal expenditure, whether expectations are formed rationally or adaptively. The larger initial dollar depreciation and forward-looking inflationary expectations in the CE version of the model cause prices to move more quickly than in the AE version, but inflation rates in the long run are unaffected since money growth has been held fixed.

The relatively larger dollar depreciation and larger declines in the long-term U.S. interest rate early in the simulations cause the decrease in U.S. GNP in the CE version to be smaller initially and to reverse itself sooner than in the AE version. In the long run, output will be higher in both the United States and the rest of the world as lower interest rates

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1/ Of course, since in the long run e is lower than in the baseline solution, at some point there must be a reversal, and this occurs when the interest differential moves in favor of the United States.

lead to capital accumulation by their private sectors. In the meantime, however, the U.S. fiscal contraction has a different effect on the rest of the world depending on the expectations formation assumption: under consistent expectations long-term bond rates fall enough in the ROW that output actually rises in the short run.

Chart 2 shows the results of a fiscal shock in ROW. This shock is an unanticipated, sustained increase in real ROW government expenditure that is the mirror image of the U.S. spending decrease discussed above--the expenditure change is equal to 1 percent of ROW real GNP. The charts show that the resulting macroeconomic effects differ somewhat quantitatively--for example, the dollar depreciation is less than in Chart 1--but the qualitative results are similar. The dollar depreciates more on impact--but by less at the end of the simulation--in the CE version of the model than in its AE version; the initial increase in long-term interest rates is larger under CE, in both the United States and the ROW. These effects serve to moderate the changes in both ROW and U.S. output in the CE version of the model relative to the AE version. In the new steady state, the dollar appreciates by 2.5 percent, and interest rates decline by roughly 50 basis points.

The monetary shocks

Monetary expansions were simulated in each economy; in both cases the shock was a permanent increase in the money supply of 4 percent distributed evenly over the first four quarters of the simulation. The increase in the money supply was assumed to be unexpected prior to the initial period of simulation, but, under consistent expectations, agents are assumed thereafter correctly to anticipate the subsequent path of the money supply as well as of the other macroeconomic variables. Real government expenditure in both economies, as well as M1 in the economy not receiving the shock, were unchanged during the simulation.

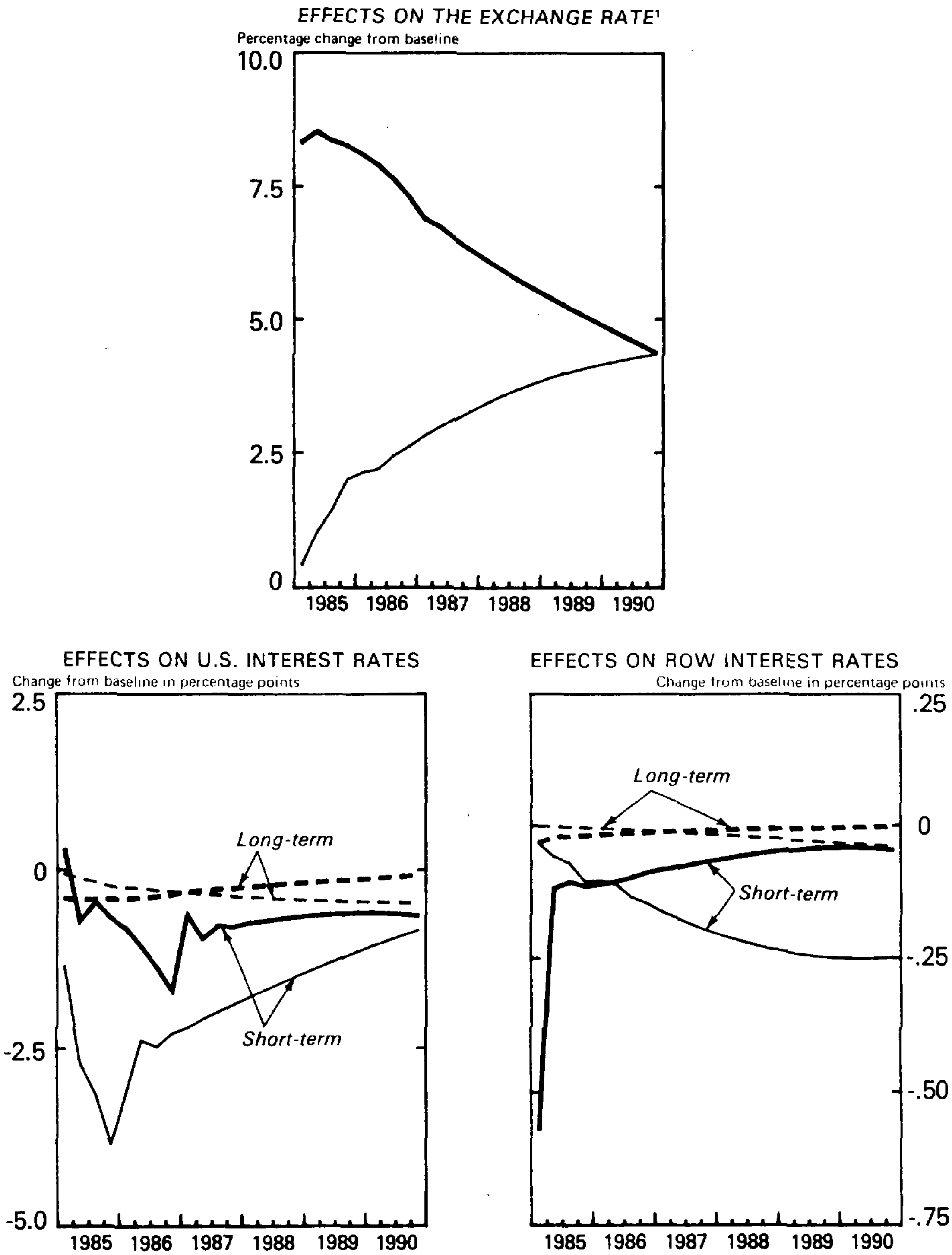
Chart 3 presents the simulation results for the U.S. monetary shock. In the AE version the exchange rate begins to depreciate monotonically to its new long-run level, which is a 4 percent dollar depreciation. The same shock applied to the CE version of the model displays the well-known property of exchange rate overshooting. In this case sticky prices combined with perfect asset substitutability lead to an impact depreciation in excess of the amount required in the long run, as an interest differential opens up in favor of the ROW currency that must be compensated by an expected dollar appreciation. Long-term interest rates fall more on impact in the CE version than in the AE version, but, by the end of the simulation, their decrease relative to baseline is less than in the adaptive version. These interest and exchange rate effects are reflected in the behavior of U.S. output; the relatively large--but temporary--real exchange rate effects in the CE version of the model combined with the initial sharp fall in long-term interest rates lead to a more pronounced

CHART 3

U.S. MONETARY EXPANSION

Consistent Expectations

Adaptive Expectations



[1] Increase indicates dollar depreciation.

CHART 3 (Cont.)

U.S. MONETARY EXPANSION

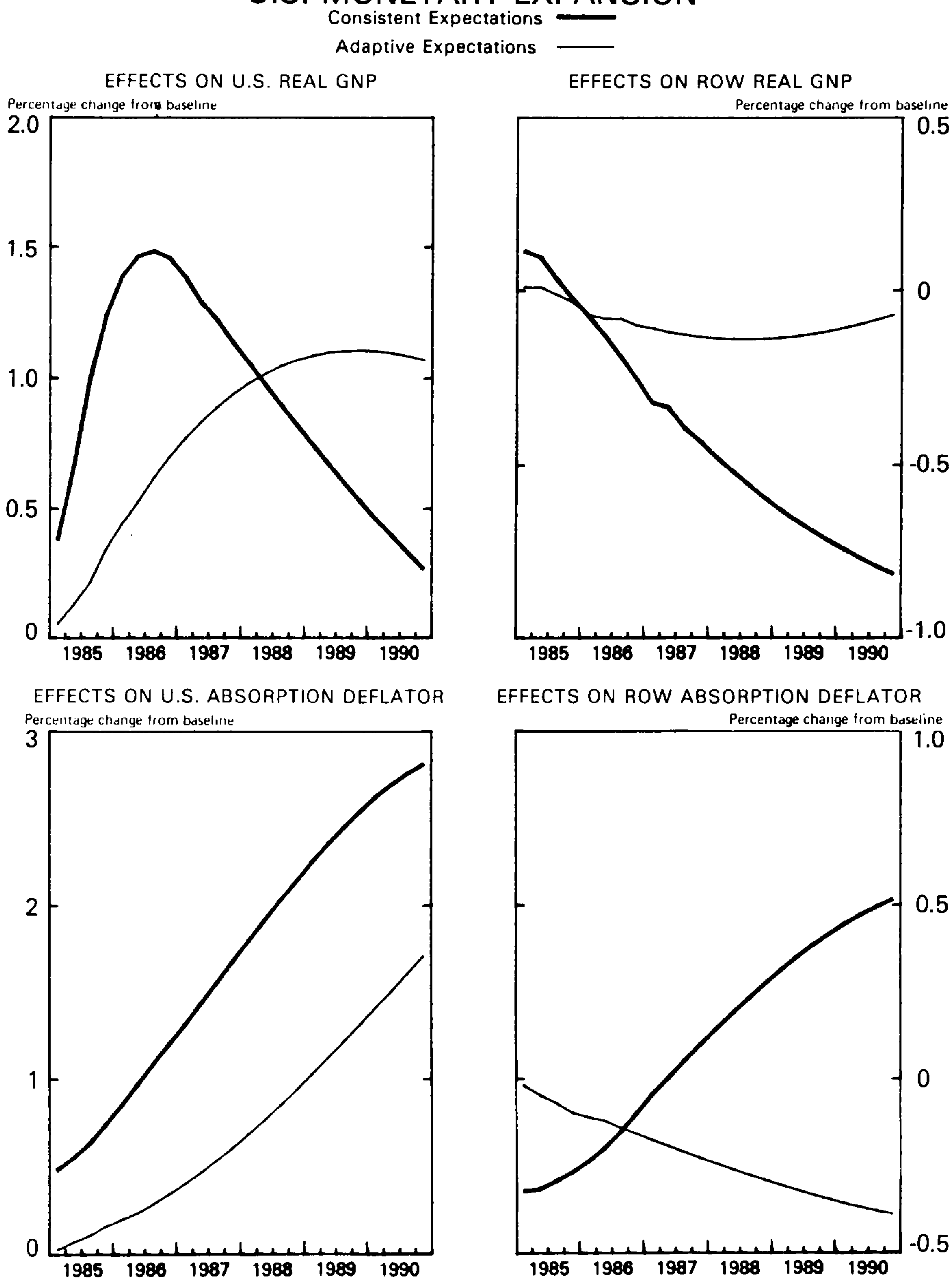
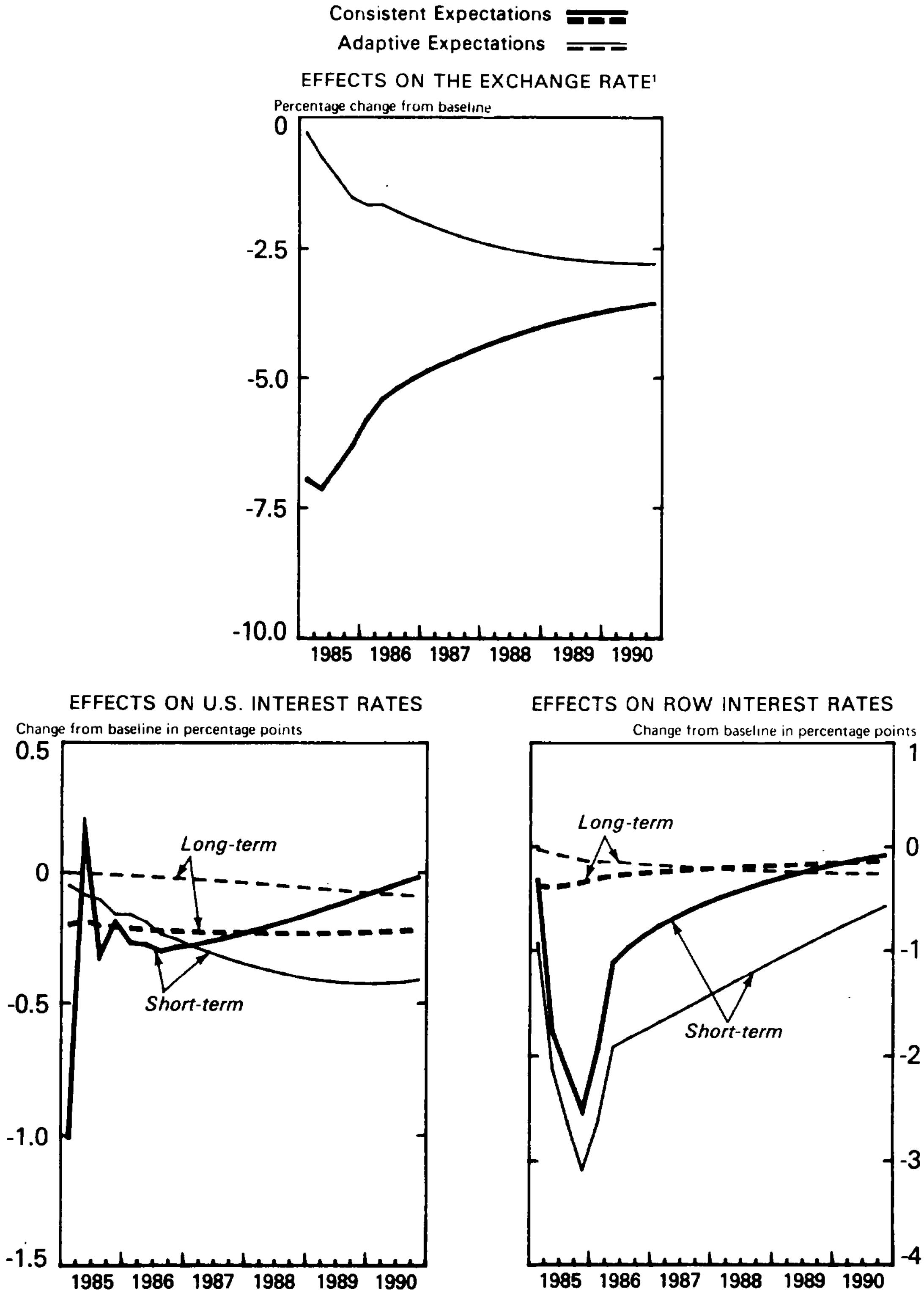


CHART 4

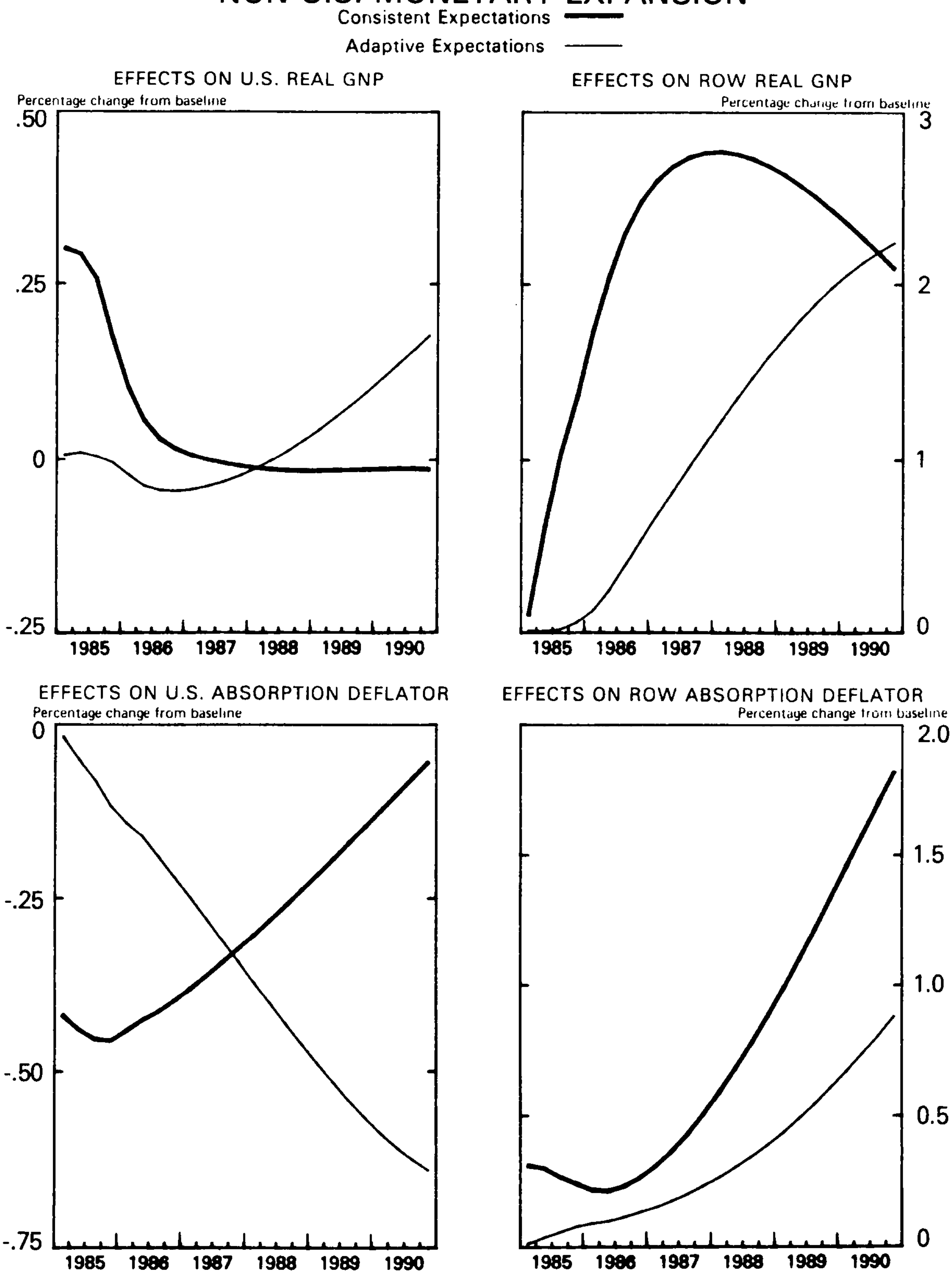
NON-U.S. MONETARY EXPANSION



[1] Increase indicates dollar depreciation.

CHART 4 (Cont.)

NON-U.S. MONETARY EXPANSION



increase in output, but it is reversed more quickly. Output in ROW increases in the short run in response to a rise in U.S. activity; soon, however, in both versions of the model, output falls--more in the CE version where the exchange rate effects are larger. In other words, MINIMOD shows a negative transmission of U.S. monetary shocks abroad, as does a simple version of the Mundell-Fleming model. In the long run, money is neutral in the model, so prices in the United States go up by 4 percent and output in both countries return to their baseline levels. By 1990, in the CE simulation, U.S. absorption prices have increased by about three quarters of their ultimate change.

Chart 4 shows the results of the same monetary experiment in ROW. Qualitatively the results are nearly the same as those discussed above in the case of a U.S. monetary expansion. The exchange rate overshoots in the CE version of the model, although by less than in the U.S. case; long-term interest rates drop more in ROW on impact--but less at the end of the simulation period--in the CE case than in the AE case. Consequently, output in ROW increases more initially, but less by the end of the simulation, in the CE version of the model. One difference worth noting is the behavior of U.S. GNP. In the AE version of the model alone, the eventual decrease in the U.S. long-term interest rate is enough to cause U.S. output after four years to be higher in response to a ROW monetary expansion than in the baseline.

The exchange rate shock

In MINIMOD the exchange rate is an endogenous variable, and assets denominated in the two currencies are perfect substitutes for one another. These two facts make it impossible to view an exchange rate shock to the model in the same way as the two policy shocks discussed above or in the same way that an interest rate shock resulting from open market operations might be assessed. In order to examine the behavior of the model in response to an imposed exchange rate change, however, a residual was introduced into the open parity relationship that equates expected returns on dollar and non-dollar assets. The simulation can be interpreted as an increase in the perceived risk of holding dollar-denominated assets that induces investors to demand an annual return on dollar assets that is higher by 1 percentage point.

Chart 5 shows the result of imposing this constant risk premium of 1 percent starting in 1985. In the long run, the risk premium will require U.S. real interest rates--short term and long term--to rise relative to ROW interest rates by 1 percentage point, as there will be no ongoing nominal or real exchange rate changes relative to baseline once the new steady state is reached. In equilibrium, this will result from an 80 basis point rise in U.S. rates, and a 20 basis point decline in ROW rates. In the meantime, however, U.S. rates do not rise by the full amount relative to foreign rates, and the increased expected return on dollar assets is obtained through expected dollar appreciation. In the CE version, this requires an initial large depreciation--a 6.5 percent decline in the value of the dollar--followed by a gradual appreciation. In the AE version, for the reasons discussed above, the exchange rate exhibits a continual depreciation during the simulation period.

Long-term interest rates rise substantially under CE from the start of the simulation, and this more than offsets the stimulative effect on U.S. output of a fall in the real value of the dollar. Under AE, in contrast, output declines by a negligible amount in the first two quarters, and then remains slightly above its baseline value. ROW output is lower under both CE and AE after the first few periods, as the decline in interest rates does not offset the negative effects of ROW appreciation.

Announcement effect simulations 1/

Additional experiments were undertaken with the consistent expectations version of MINIMOD in order to gauge the announcement effects of credible policy. Specifically, the same U.S. fiscal and monetary policies described above were applied to the model three years after the simulations began; this has the effect of providing agents with full knowledge of the policies, and their consequences, three full years before the policies take effect. Thus these agents can alter their behavior, and thereby affect macroeconomic variables, before the actual implementation of the policy changes.

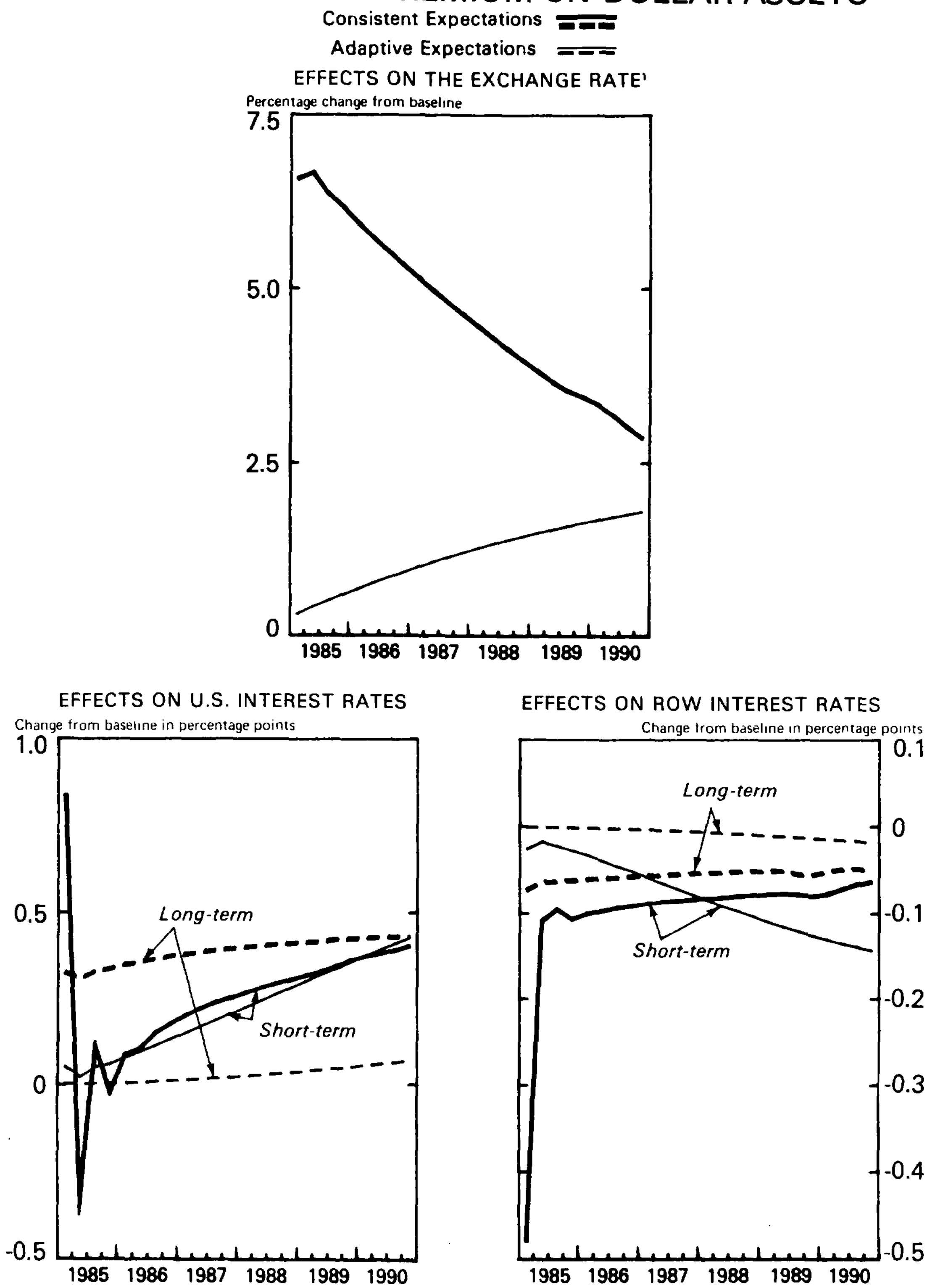
Chart 6 reproduces the U.S. fiscal shock already discussed and shows, in addition, the results of the same shock announced at the beginning of the simulation period (1985, quarter 1) but taking effect in 1988. Fiscal contraction causes the dollar to depreciate and long-term U.S. interest rates to fall. The announcement of a future fiscal contraction combined with the assumption of consistent expectations brings some of the future effects forward to the present; thus the expansionary effects of the exchange rate depreciation and the interest rate decline come into play before the contractionary effect of the actual decrease in government expenditure. Consequently, U.S. GNP rises for twelve quarters and by 1987, fourth quarter, is about 1/2 percent above its baseline value. In the next period of the simulation output declines in response to the government spending cut, but by a smaller amount than if the announcement and implementation were contemporaneous. The long-run effects of the two shocks are the same, however, and they were discussed above in the context of Chart 1. As for ROW output, it is higher throughout both simulations, aside from a few quarters near the end, but when there is an implementation lag, the stimulative effects are larger.

Chart 7 presents the results of a similar monetary experiment; the U.S. monetary expansion described above is applied to the CE version of the model both with and without a three-year lag between the announcement and the implementation of the policy. The announcement of the policy is

1/ These simulations were performed in response to a request from John Taylor.

CHART 5

INCREASE IN RISK PREMIUM ON DOLLAR ASSETS



[1] Increase indicates dollar depreciation.

CHART 5 (Cont.)

INCREASE IN RISK PREMIUM ON DOLLAR ASSETS

Consistent Expectations

Adaptive Expectations

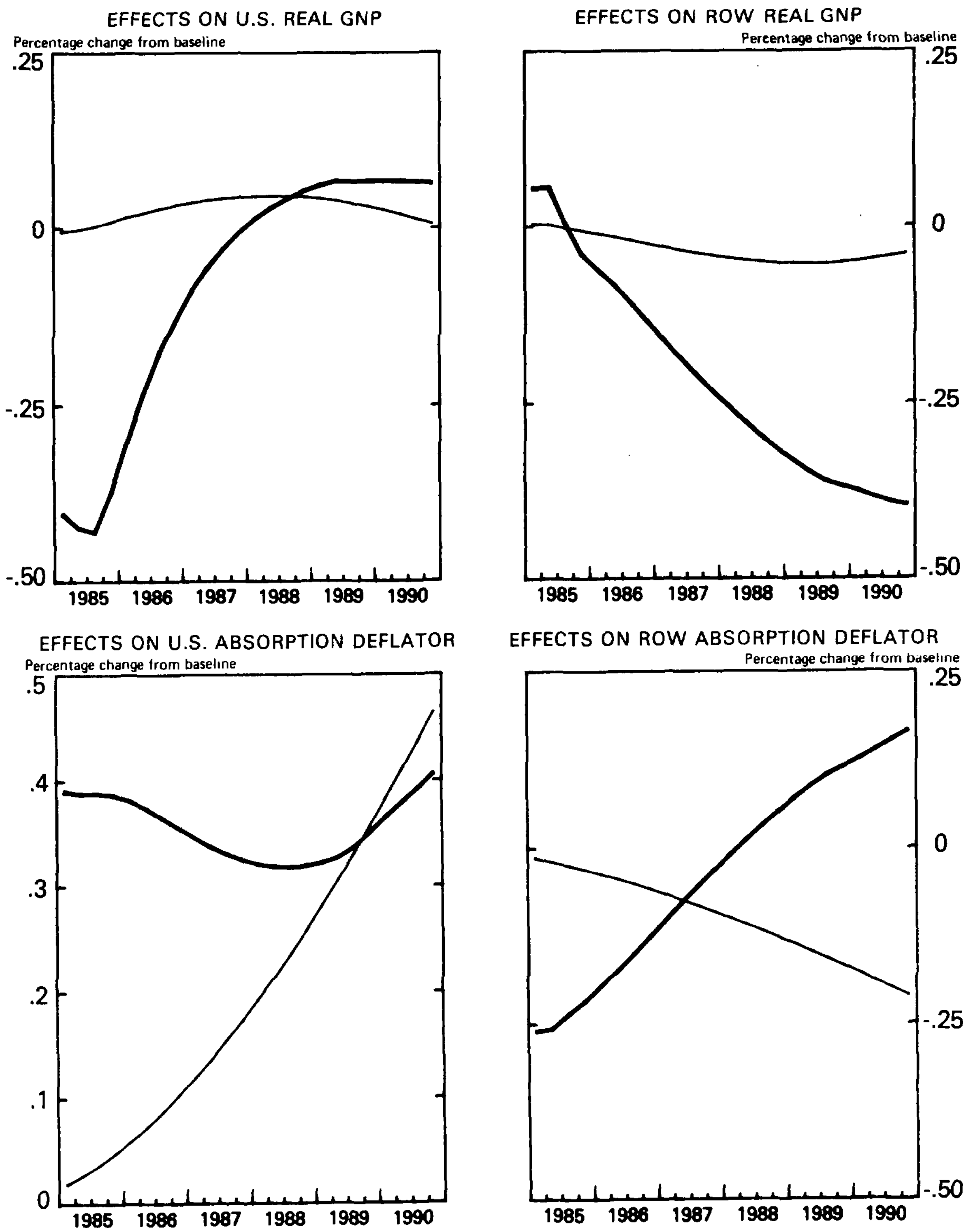
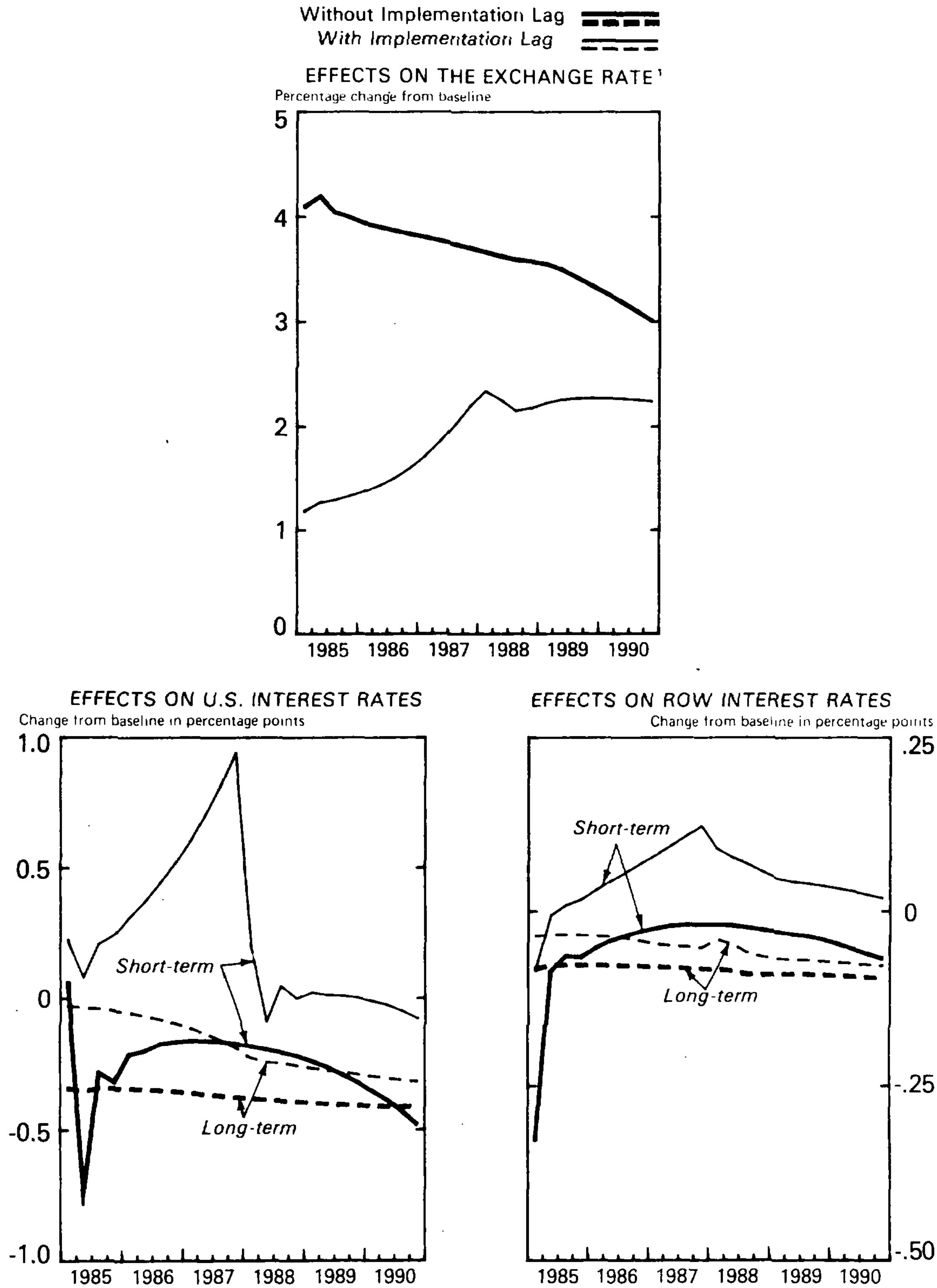


CHART 6

REDUCTION IN U.S. GOVERNMENT PURCHASES, UNDER CONSISTENT EXPECTATIONS



[1] Increase indicates dollar depreciation.

CHART 6 (Cont.)

REDUCTION IN U.S. GOVERNMENT PURCHASES, UNDER CONSISTENT EXPECTATIONS

Without Implementation Lag ━━

With Implementation Lag ——

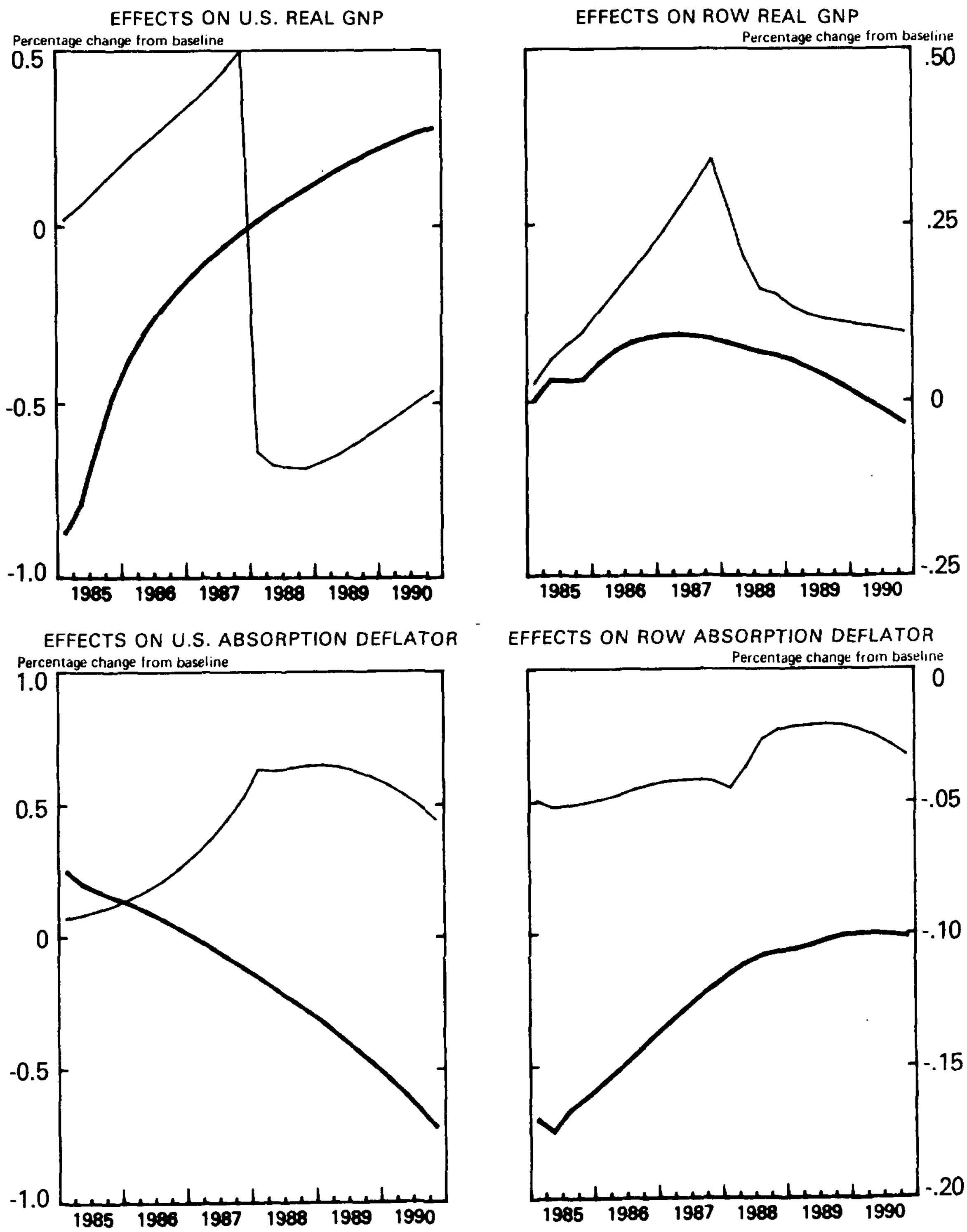
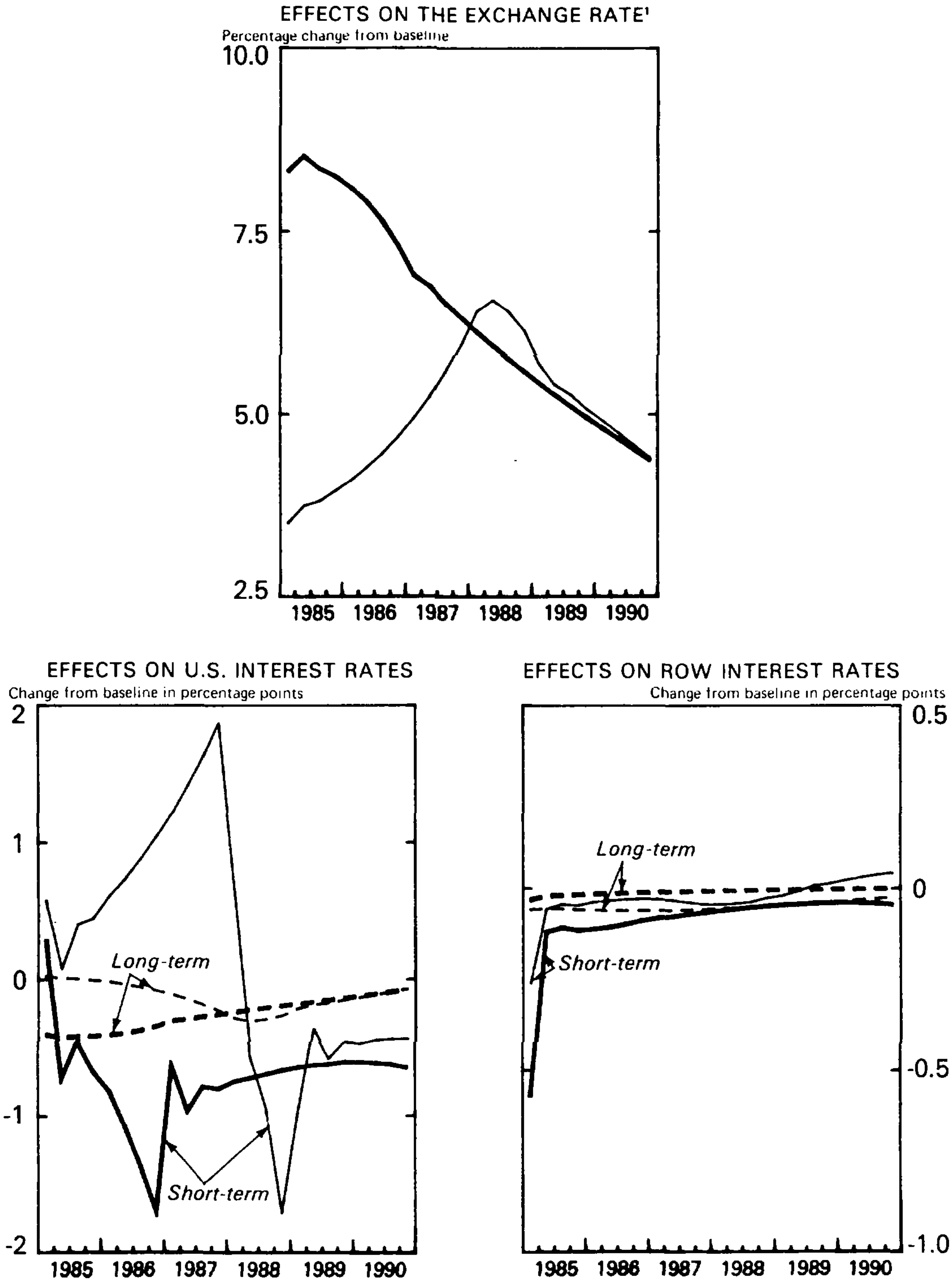


CHART 7

U.S. MONETARY EXPANSION, UNDER CONSISTENT EXPECTATIONS

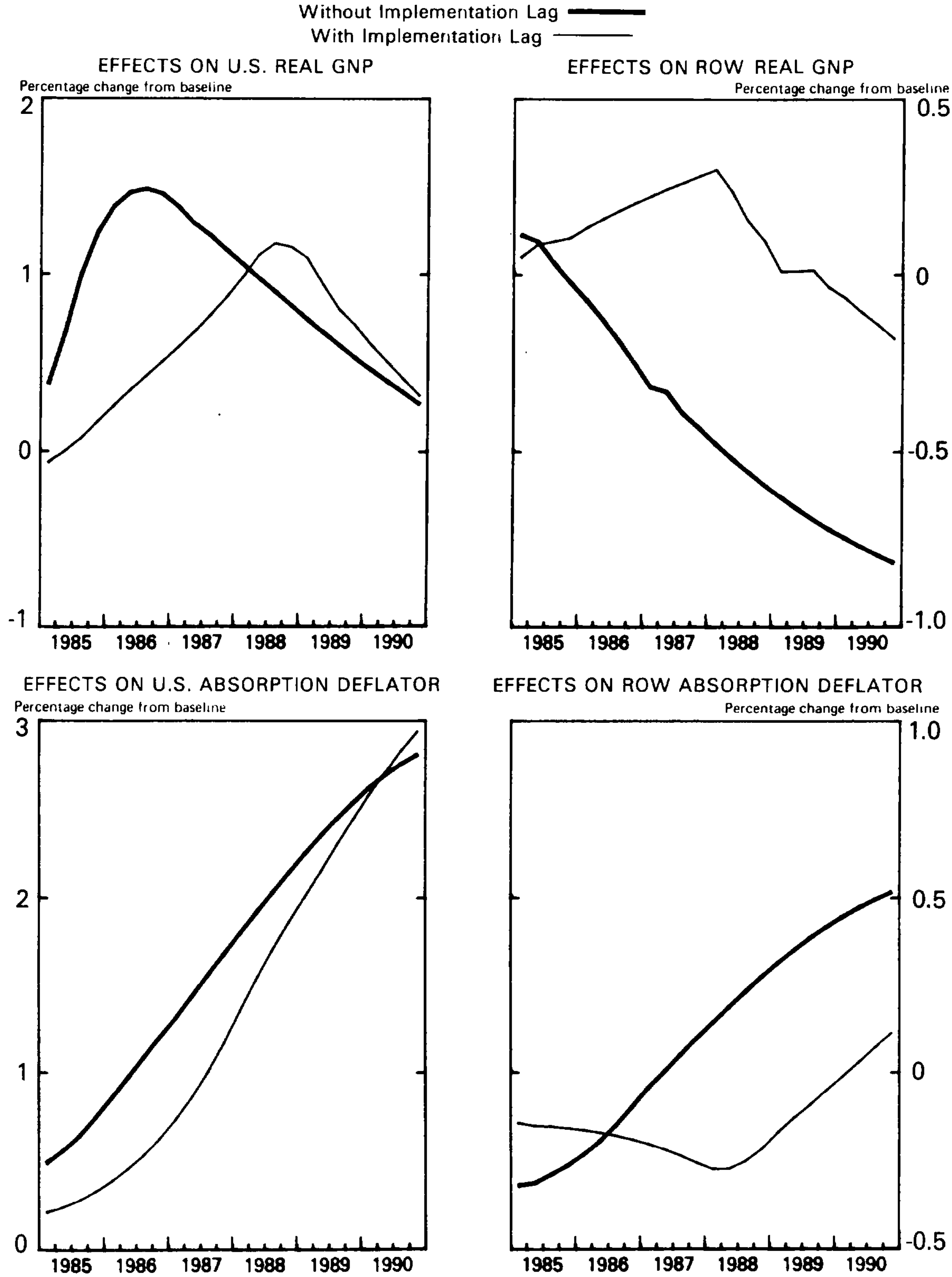
Without Implementation Lag
With Implementation Lag



[1] Increase indicates dollar depreciation.

CHART 7 (Cont.)

U.S. MONETARY EXPANSION, UNDER CONSISTENT EXPECTATIONS



enough to cause the exchange rate to depreciate immediately even though monetary expansion occurs later--indeed the exchange rate overshoots its long-run value before the money supply is actually increased, at which point it depreciates further for two periods, then appreciates to its long-run level. As a result, the expansionary effect on U.S. output begins before the policy is implemented. The peak output effect in the United States is lower when the policy is announced beforehand, however. 1/

V. Conclusions

In the construction of MINIMOD, we have deliberately sacrificed some detail from the Federal Reserve's multicountry model, but have endeavored to capture the essential features of the larger model, including the non-expectational dynamics and the key stock-flow relationships. We have not presented a direct comparison of full model simulations of MINIMOD and MCM because our purpose was not to create a replica of the MCM. For example, we have added some structure to the non-U.S. model to make it symmetric with the U.S. model, updated the wage/price block, and used demand-for-money functions as well as production functions from other sources. Nevertheless, the adaptive expectations version of MINIMOD seems to behave similarly to the MCM.

In contrast, the consistent expectations version of MINIMOD, in which expectations of the exchange rate and of U.S. and ROW long-term bond rates and inflation rates are made equal to their realized values next period, behaves quite differently. In general, this version exhibits greater flexibility of financial prices and, to a lesser extent, of goods prices, and the initial output effects of monetary shocks are smaller than for the adaptive version. In addition, the dynamics are quite different in the two versions.

The consistent expectations version, because of its forward-looking expectations, allows experiments in which announcement of a policy change

---

1/ The fact that U.S. output actually falls slightly in the first quarter of simulation in the delayed policy scenario, but not in the contemporaneous policy scenario, can be traced to the behavior of prices. In both simulations real disposable income falls because output is valued at output prices but deflated at absorption prices and because absorption prices are more sensitive to exchange rate changes. The fall in real disposable income causes consumption and thus GNP to fall. This effect is transitory, and in the case of the contemporaneous shock, it is more than offset by the effect of lower long-term interest rates on investment. In the case of the delayed policy, higher prices cause all U.S. interest rates to rise, not fall. Short-term rates fall in the United States only when the money supply is actually increased.

which is to be implemented later can have effects now (provided the announcement is believed). In these simulations, future contractionary fiscal policy can have an expansionary effect now because the exchange rate depreciates and long-term interest rates decline immediately. In contrast, future deflationary monetary policy has contractionary effects now, because interest rates rise and the exchange rate appreciates from the outset. However, in both cases, these real effects depend on expectations of future demand not having an offsetting effect on current demand. An obvious extension is to allow some middle ground between lack of forward-looking expectations (as is the case in the adaptive version) and complete credibility of future policy (which we now impose in the consistent version). An example would be a situation where announced fiscal policy was unsustainable because it involved a continual increase in the ratio of government debt to income. In these circumstances expectations would likely reflect the probability of an eventual change in policy, and a judgment as to what form the change might take.

Future work with MINIMOD centers on two areas: simulation work with the existing version of the model and further developmental work. There are a number of questions that can be addressed with the current version of MINIMOD. Among those is one that was just mentioned, the sustainability of fiscal deficits. The fact that financial wealth as well as the capital stock and foreign indebtedness are all endogenous variables in the model suggests that it is well suited to address policy questions of this sort. Further development of the model will likely focus on disaggregating the ROW sector to make macro models of Germany and Japan explicit as well as the inclusion of an abbreviated developing countries model.

```
ADAPTIVE EXPECTATIONS VERSION OF MINIMOD.
CONSISTENT EXPECTATIONS VERSION OMITS EQS 68-72 AND FORCES EE=E(+1),ETC.


SYMBOL DECLARATIONS


ENDOGENOUS:
   DELRP_PI - PARTIAL EFFECT OF INFLATION EXPECTATIONS ON ROW INFLATION
   DELUP_PI - PARTIAL EFFECT OF INFLATION EXPECTATIONS ON U.S. INFLATION
   E        - EXCHANGE RATE ($ PER FOREIGN CURRENCY)
   EE       - EXPECTED VALUE OF E NEXT PERIOD
   EPSILONE - EXPECTED RATE OF CHANGE OF E AT ANNUAL RATES
   F        - NET CLAIMS OF U.S. ON ROW (ASSUMED DENOMINATED IN U.S. $)
   RA       - ROW REAL ABSORPTION
   RB       - ROW NOMINAL STOCK OF GOVT. DEBT
   RC       - ROW REAL CONSUMPTION EXPENDITURE
   RC_W     - PARTIAL EFFECT OF ROW WEALTH ON CONSUMPTION
   RC_Y     - PARTIAL EFFECT OF DISPOSABLE INCOME ON ROW CONSUMPTION
   RCU      - RATE OF CAPACITY UTILIZATION IN ROW
   RGDP     - ROW REAL GROSS DOMESTIC PRODUCT
   RGNP     - ROW REAL GROSS NATIONAL PRODUCT
   RK       - ROW REAL PHYSICAL CAPITAL STOCK
   RMONE    - ROW MONEY SUPPLY (M1)
   RP       - ROW ABSORPTION DEFLATOR
   RPGNP    - ROW GNP DEFLATOR
   RPIE     - EXPECTED RATE OF CHANGE OF ABSORPTION PRICE IN ROW NEXT PERIOD
   RRL      - ROW LONG-TERM BOND RATE
   RRLE     - EXPECTED ROW LONG-TERM BOND RATE NEXT PERIOD
   RRS      - ROW SHORT-TERM BOND RATE
   RTAX     - NOMINAL ROW TAX RECEIPTS
   RT1      - ROW TAX PARAMETER
   RW       - ROW REAL PRIVATE SECTOR NET WEALTH
   RYCAP    - ROW CAPACITY OUTPUT
   RYD      - ROW REAL DISPOSABLE INCOME
   UA       - U.S. REAL ABSORPTION
   UB       - U.S. NOMINAL STOCK OF GOVERNMENT DEBT
   UC       - U.S. REAL CONSUMPTION EXPENDITURE
   UC_R     - PARTIAL EFFECT OF REAL INTEREST RATE ON U.S. CONSUMPTION
   UC_Y     - PARTIAL EFFECT OF DISPOSABLE INCOME ON U.S. CONSUMPTION
   UCU      - RATE OF U.S. CAPACITY UTILIZATION
   UGDP     - U.S. REAL GROSS DOMESTIC PRODUCT
   UGNP     - U.S. REAL GROSS NATIONAL PRODUCT
   UI       - VOLUME OF U.S. IMPORTS OF GOODS AND NON-FACTOR SERVICES
   UI_ACT   - PARTIAL EFFECT OF U.S. ABSORPTION ON U.S. IMPORTS
   UI_E     - PARTIAL EFFECT OF REAL EXCHANGE RATE ON U.S. IMPORTS
   UK       - STOCK OF U.S. REAL PHYSICAL CAPITAL
   UMONE    - U.S. MONEY SUPPLY (M1)
   UP       - U.S. ABSORPTION DEFLATOR
   UPGNP    - U.S. GNP DEFLATOR
   UPIE     - EXPECTED RATE OF CHANGE OF THE U.S. ABSORPTION DEFLATOR NEXT PERIOD
   URL      - U.S. LONG-TERM BOND RATE
   URLE     - EXPECTED U.S. LONG-TERM BOND RATE NEXT PERIOD
   URS      - U.S. SHORT-TERM BOND RATE
   UTAX     - U.S. NOMINAL TAX RECEIPTS
   UT1      - U.S. TAX PARAMETER
   UT2      - U.S. TAX PARAMETER
   UW       - U.S. REAL PRIVATE SECTOR NET WEALTH
   UX       - VOLUME OF U.S. EXPORTS OF GOODS AND NON-FACTOR SERVICES
```

```
18:     UA = UC+4*DEL(1 : UK)+UDELTA*UK(-1)+UG+RES1
```

U.S. REAL GROSS DOMESTIC PRODUCT:

```
19:     UGDP = UA+UX-UI+RES2
```

PARTIAL EFFECT OF DISPOSABLE INCOME ON U.S. CONSUMPTION:

```
20:     UC_Y = (1-UC3)/(UC1+UC2)*(UC1*UYD+UC2*UYD(-1))+UC3*UC_Y(-1)+RES3
```

PARTIAL EFFECT OF REAL INTEREST RATE ON U.S. CONSUMPTION:

```
21:     UC_R = UC4*(1-UC6)/(UC4+UC5)*100*((1+URL(-2)/100)/(1+UPIBAR(-2))-1)+UC5*(1-UC6)/(UC4+UC5)*100*((1+URL(-3)/
        100)/(1+UPIBAR(-3))-1)+UC6*UC_R(-1)+RES4
```

U.S. REAL CONSUMPTION EXPENDITURE:

```
22:     UC = (-UALPHA)*UG+(UC1+UC2)/(1-UC3)*UC_Y+(UC4+UC5)/(1-UC6)*UC_R*UGNP/MUGNP+UC7*UW(-1)+RES5
```

U.S. REAL NET PRIVATE SECTOR WEALTH:

```
23:     UW = ULAMBDAM*(UM/UP)+ULAMBDAB*(UB/UP)+F/UP+UK+RES6
```

U.S. REAL DISPOSABLE INCOME:

```
24:     UYD = UGDP*UPGNP/UP-UDELTA*UK(-1)-UTAX/UP+URSR*(UB(-1)+F(-1))/UP-(1-ULAMBDAB)*DEL(1 : UB)/UP+RES7
```

U.S. REAL NET CAPITAL FORMATION:

```
25:     DEL(1 : UK) = UIN1*DEL(1 : UK(-1))+UIN2*(UBETA/UUCSTCAP*UGDP-UK(-1))+UN*UK(-1)+RES9
```

U.S. GENERAL GOVERNMENT BUDGET CONSTRAINT:

```
26:     DEL(1 : UB)+DEL(1 : UM) = URSQ*UB(-1)+(UP*UG-UTAX+UGEXOG)/4+RES10
```

U.S. GENERAL GOVERNEMNT TAX RECEIPTS NET OF TRANSFERS:

```
27:     UTAX = UT1*(UPGNP*UGDP-UDELTA*UK(-1)*UP+URS(-1)/100*(UB(-1)+F(-1))-ULAMBDAT*UPI*(1+URSR)*(UB(-1)+F(-1)))+
        UT2*(UPGNP(-1)*UGDP(-1)-UDELTA*UK(-2)*UP(-1)+URS(-2)/100*(UB(-2)+F(-2))-ULAMBDAT*UPI(-1)*(1+URSR(-1))*(UB(
        -2)+F(-2)))+RES11
```

NATIONAL ACCOUNTS IDENTITY IN NOMINAL TERMS:

```
28:     UPGNP*(UGNP-UX) = UP*UA-UI*E*RPGNP+URS(-1)/100*F(-1)+RES12
```

PARTIAL EFFECT OF INFLATION EXPECTATIONS ON U.S. GNP DEFLATOR:

```
29:     DELUP_PI = UP4*((1+UPIE)**0.25-1)+UP5*((1+UPIE(-1))**0.25-1)+UP6*DELUP_PI(-1)+UP7*DELUP_PI(-2)+RES14
```

RATE OF CHANGE OF U.S. GNP DEFLATOR:

```
30:     DEL(1 : UPGNP)/UPGNP(-1) = UP1*LOG(UCU)+UP2*LOG(UCU(-1))+UP3*LOG(UCU(-2))+DELUP_PI+RES16
```

MULTIPLIER RELATIONSHIP BETWEEN THE BASE AND U.S. M1 MONEY SUPPLY:

```
31:     UMONE = UMULT*UM+RES15
```

PRODUCTION FUNCTION FOR U.S. CAPACITY OUTPUT:

```
32:     LOG(UYCAP) = USCALE+LOG(1+UN)*T*(1-UBETA)+LOG(UK(-1))*UBETA
```

RATE OF U.S. CAPACITY UTILIZATION:

```
33:     UCU = 100*UGDP/UYCAP+RES18
```

PARTIAL EFFECT OF FOREIGN ACTIVITY ON U.S. EXPORTS:

```
34:     UX_ACT = (1-UX2)*LOG(RA)+UX2*UX_ACT(-1)+RES19
```

PARTIAL EFFECT OF COMPETITIVENESS ON U.S. EXPORTS:

```
35:     UX_E = (1-UX5)*UX3/(UX3+UX4)*LOG(E*RPGNP/UPGNP)+UX5*UX_E(-1)+(1-UX5)*UX4/(UX3+UX4)*LOG(E(-1)*RPGNP(-1)/
        UPGNP(-1))+RES20
```

VOLUME OF U.S. EXPORTS OF GOODS AND NON-FACTOR SERVICES:

```
36:     LOG(UX) = UX1/(1-UX2)*UX_ACT+(UX3+UX4)/(1-UX5)*UX_E+RES21
```

PARTIAL EFFECT OF U.S. ACTIVITY ON U.S. IMPORTS:

```
37:     UI_ACT = (1-UI3)/(UI1+UI2)*(UI1*LOG(UA)+UI2*LOG(UA(-1)))+UI3*UI_ACT(-1)+RES22
```

PARTIAL EFFECT OF COMPETITIVENESS ON U.S. IMPORTS:

38: UI_E = UI4/((UI4+UI5+UI6)/(1-UI7-UI8))*LOG(E*RPGNP/UPGNP)+UI5/((UI4+UI5+UI6)/(1-UI7-UI8))*LOG(E(-1)*RPGNP(-1)/UPGNP(-1))+UI6/((UI4+UI5+UI6)/(1-UI7-UI8))*LOG(E(-2)*RPGNP(-2)/UPGNP(-2))+UI7*UI_E(-1)+UI8*UI_E(-3)+RES23

VOLUME OF U.S. IMPORTS OF GOODS AND NON-FACTOR SERVICES:

39: LOG(UI) = (UI1+UI2)/(1-UI3)*UI_ACT+(UI4+UI5+UI6)/(1-UI7-UI8)*UI_E+RES24

U.S. CURRENT ACCOUNT BALANCE (EQUAL TO THE CHANGE IN U.S. NET CLAIMS ON FOREIGNERS):

40: DEL(1 : F) = URSQ*F(-1)+(UPGNP*UX-RPGNP*UI*E)/4+RES26

INTEREST PARITY CONDITION EQUATING EX ANTE RETURNS ON U.S. AND ROW SHORT-TERM BONDS:

41: 1+URS/100 = (1+RRS/100)*(1+EPSILONE)+RES27

ARBITRAGE CONDITION EQUATING EX ANTE RETURNS ON U.S. SHORT-TERM AND LONG-TERM BONDS:

42: URS/100 = URL/100-((URLE/URL)**4-1)+RES28

DEMAND FUNCTION FOR U.S. M1:

43: LOG(UMONE/UP) = UM1*LOG(UGNP)+UM2*LOG(UGNP(-1))+UM3*0.01*URS+UM4*0.01*URS(-1)+UM5*LOG(UMONE(-1)/UP(-1))+RES29

ROW REAL ABSORPTION:

44: RA = RC+4*DEL(1 : RK)+RDELTA*RK(-1)+RG+RES30

ROW REAL GDP (NET EXPORTS ARE SCALED DOWN BY THE SHARE OF MCM COUNTRIES IN U.S. TRADE):

45: RGDP = RA+(UI-UX)/TRADSCAL+RES31

PARTIAL EFFECT OF DISPOSABLE INCOME ON ROW CONSUMPTION:

46: RC_Y = (1-RC2)*RYD+RC2*RC_Y(-1)+RES32

PARTIAL EFFECT OF WEALTH ON ROW CONSUMPTION:

47: RC_W = RC3/((RC3+RC4)/(1-RC5-RC6))*RW+RC4/((RC3+RC4)/(1-RC5-RC6))*RW(-1)+RC5*RC_W(-1)+RC6*RC_W(-2)+RES33

ROW REAL CONSUMPTION EXPENDITURE:

48: RC = (-RALPHA)*RG+RC1/(1-RC2)*RC_Y+(RC3+RC4)/(1-RC5-RC6)*RC_W+RES34

ROW REAL NET PRIVATE SECTOR WEALTH:

49: RW = RLAMBDAM*(RM/RP)+RLAMBDAB*(RB/RP)-F/E/TRADSCAL/RP+RK+RES35

ROW REAL DISPOSABLE INCOME:

50: RYD = RGDP*RPGNP/RP-RDELTA*RK(-1)-RTAX/RP+RRSR*(RB(-1)/RP)-((1+URS(-1)/100)/(1+EPSILON)/(1+RPI)-1)*(F(-1)/E/TRADSCAL)/RP-(1-RLAMBDAB)*DEL(1 : RB)/RP+RES36

ROW REAL NET CAPITAL ACCUMULATION:

51: DEL(1 : RK) = RIN1*(RBETA*RGDP/RUCSTCAP)+RIN2*(RBETA*RGDP(-1)/RUCSTCAP(-1))+RIN3*(RBETA*RGDP(-4)/RUCSTCAP(-4))+RIN4*RK(-1)+RIN5*RK(-2)+RIN6*RK(-5)+RN*RK(-1)+RES38

ROW GENERAL GOVERNMENT BUDGET CONSTRAINT:

52: DEL(1 : RB)+DEL(1 : RM) = (RP*RG-RTAX)/4+RRSQ*RB(-1)+RES39

ROW NOMINAL GOVERNMENT TAX RECEIPTS NET OF TRANSFERS:

53: RTAX = RT1*(RPGNP*RGDP-RDELTA*RK(-1)*RP+RRS(-1)/100*RB(-1)-((1+URS(-1)/100)/(1+EPSILON)-1)*(F(-1)/E/TRADSCAL)-RLAMBDAT*RPI*((1+RRSR)*RB(-1)-(1+URS(-1)/100)/(1+EPSILON)/(1+RPI)*F(-1)/TRADSCAL/E))+RT2*RTAX(-1)+RES40

PARTIAL EFFECT OF INFLATION EXPECTATIONS ON THE RATE OF CHANGE IN ROW GNP DEFLATOR:

54: DELRP_PI = RP3*((1+RPIE)**0.25-1)+(1-RP3)*DELRP_PI(-1)+RES43

RATE OF CHANGE OF ROW GNP DEFLATOR:

55: DEL(1 : RPGNP)/RPGNP(-1) = RP1*LOG(RCU)+RP2*LOG(RCU(-1))+DELRP_PI+RES44

```
   NATIONAL ACCOUNTS IDENTITY IN NOMINAL TERMS FOR ROW:
56:      RPGNP*(RGNP-UI/TRADSCAL) = RP*RA-UX/TRADSCAL*UPGNP/E-URS(-1)/100*F(-1)/E/TRADSCAL+RES45

   ARBITRAGE CONDITION EQUATING EX ANTE RETURNS ON SHORT-TERM AND LONG-TERM BONDS IN ROW:
57:      RRS/100 = RRL/100-((RRLE/RRL)**4-1)+RES46

   DEMAND FUNCTION FOR ROW M1:
58:      LOG(RMONE/RP) = RM1*LOG(RGNP)+RM2*0.01*RRS+RM3*LOG(RMONE(-1)/RP(-1))+RES47

   PRODUCTION FUNCTION EXPLAINING ROW CAPACITY OUTPUT:
59:      LOG(RYCAP) = RSCALE+LOG(1+RN)*T*(1-RBETA)+LOG(RK(-1))*RBETA

   ROW RATE OF CAPACITY UTILIZATION:
60:      RCU = 100*RGDP/RYCAP+RES49

   MULTIPLIER RELATIONSHIP BETWEEN ROW MONETARY BASE AND M1:
61:      RMONE = RMULT*RM+RES50

   U.S. REAL GROSS NATIONAL PRODUCT:
62:      UGNP = UGDP+URS(-1)/100*F(-1)/UPGNP+RES57

   ROW REAL GROSS NATIONAL PRODUCT:
63:      RGNP = RGDP-URS(-1)/100*F(-1)/E/TRADSCAL/RPGNP+RES58

   EXPECTED RATE OF CHANGE OF THE EXCHANGE RATE:
64:      EPSILONE = (EE/E)**4-1+RES59

   TAX CHANGES TO STABILIZE U.S. GOVT.DEBT RATIO:
65:      UT1 = UT1BAR+UTAU*DUM*(UB(-1)/UPGNP(-1)/UGNP(-1)-UBRATIO)+RES60

   U.S. TAX PARAMETERS MOVE TOGETHER:
66:      UT1/UT2 = UT1BAR/UT2BAR

   TAX CHANGES TO STABILIZE ROW GOVT. DEBT RATIO:
67:      RT1 = RT1BAR+RTAU*DUM*(RB(-1)/RPGNP(-1)/RGNP(-1)-RBRATIO)+RES61

   ADAPTIVE EXPECTATIONS OF THE RATE OF CHANGE OF U.S.
   ABSORPTION PRICE:
68:      UPIE = ETAUPI*UPI+(1-ETAUPI)*UPIE(-1)+RES51

   ADAPTIVE EXPECTATIONS OF THE RATE OF CHANGE OF ROW
   ABSORPTION PRICE:
69:      RPIE = ETARPI*RPI+(1-ETARPI)*RPIE(-1)+RES52

   ADAPTIVE EXPECTATIONS OF THE EXCHANGE RATE:
70:      EE = ETA*E+(1-ETA)*EE(-1)+RES53

   ADAPTIVE EXPECTATIONS OF THE U.S. LONG-TERM BOND RATE:
71:      URLE = ETAURL*URL+(1-ETAURL)*URLE(-1)+RES54

   ADAPTIVE EXPECTATIONS OF THE ROW LONG-TERM BOND RATE:
72:      RRLE = ETARRL*RRL+(1-ETARRL)*RRLE(-1)+RES55
```

ETA	0.3	ETARPI	0.3	ETARRL	0.3
ETAUPI	0.3	ETAURL	0.3	MUGNP	1361.01
RALPHA	0.	RBETA	0.383	RC1	0.201055
RC2	0.56857	RC3	0.00634	RC4	0.022152
RC5	0.24771	RC6	0.101305	RGAMMA	0.908193
RIN1	0.0023	RIN2	0.00534	RIN3	0.00252
RIN4	0.6545	RIN5	-0.66217	RIN6	-0.00252
RMRT	0.39995	RM1	0.22	RM2	-0.518863
RM3	0.72497	RN	0.00579	RP1	0.052141
RP2	-0.03061	RP3	0.241165	RP4	-0.182324
RTAU	0.1	RT1	0.332668	RT2	0.168227
UALPHA	0.	UBETA	0.326	UC1	0.252096
UC2	0.021982	UC3	0.594668	UC4	-3.05811
UC5	-0.456031	UC6	0.42672	UC7	0.043768
UGAMMA	0.934533	UIN1	0.27905	UIN2	0.02937
UI1	1.37984	UI2	-0.722509	UI3	0.711355
UI4	-0.070178	UI5	-0.016281	UI6	-0.099974
UI7	0.761522	UI8	-0.087122	UMRT	0.494668
UM1	0.259104	UM2	0.099481	UM3	-0.339518
UM4	-0.130362	UM5	0.616057	UN	0.00579
UP1	0.02248	UP2	0.048106	UP3	-0.032238
UP4	0.17689	UP5	-0.10243	UP6	1.4634
UP7	-0.53786	UTAU	0.1	UTRATIO	0.962673
UT1	0.242634	UT2	0.252042	UX1	0.78992
UX2	-0.03281	UX3	0.023369	UX4	0.106272
UX5	0.882563				

The Methodology of Creating Minimodels from Large Structural Macroeconomic Models

This appendix gives a formal description of the procedure of reducing a large model to a "minimodel." 1/ Let us represent the large model, which may be non-linear, by a vector of functions F, each of which has as arguments the vector of endogenous variables Y, their lagged values Y_{-1}, and exogenous variables X:

$$F(Y, Y_{-1}, X) = U \tag{1}$$

where U is vector of additive errors, some of whose elements may be identically zero. Though seemingly restricted to one-period lags, the state-space representation in equation (1) in fact allows for lags of any order, through suitable definition of the vector Y. The above formulation is convenient because it allows a compact representation of any nonlinear model (provided error terms enter additively).

Obtaining a smaller version of this model involves simulating the model to see how it responds to a once-and-for-all change in the exogenous variables. Since the model may be non-linear, in general these responses will depend on the levels of all the variables. 2/ We assume that a control solution has somehow been chosen; the simulation responses will be relative to this control solution.

If the aim were to generate a reduced-form representation of the model, each of the exogenous variables X could be separately shocked by one unit at the beginning of the simulation period only (call this period 0), and the response of the endogenous variables Y observed over a number of periods (say T periods) sufficient for the endogenous variables to have returned to their control solution values, on the basis of some pre-set tolerance level. If there are M endogenous and N exogenous variables, then these simulation results can be grouped into matrices $J_0, J_1, \ldots, J_T$ where $(J_k)_{ij}$ represents the deviation from control of the i'th endogenous variable when the j'th exogenous variable was subject to a unit shock k periods earlier. In effect this implies that we have a reduced-form (or,

1/ This formalization draws on the description in Maciejowski and Vines (1984), but extends it to consider creating structural representations of the larger model.

2/ Our method is useful on condition that behavioral relationships (as opposed to identities) are not strongly dependent on the control solution.

more properly, a final form) representation of the model, which we can write:

$$\delta Y = \sum_{i=0}^{T} J_i \; \delta X_{-i} \tag{2}$$

where here the δ operator stands for deviation from the control solution.

In order to reduce the resulting model to a more manageable size, it may be desired either to aggregate linearly some of the endogenous variables, or to drop some of them. Both can be represented by a matrix A of dimension mxM, where m is the number of endogenous variables (y) in the aggregated model, and where:

$$y = AY \text{ and } \delta y = A\delta Y \tag{3}$$

Linear aggregation is not severely restrictive, as the original variables Y can usually be defined such that the aggregated model contains the appropriate variable. For instance, the Y vector could contain the logarithms of selected variables, and the non-linear F functions redefined to be consistent with them; consequently the aggregated model could include geometric averages of the underlying variables, that is, arithmetic averages of the logarithms of those variables. The aggregated model takes the following form:

$$\delta y = A\delta Y = A\sum_{i=0}^{T} J_i \; \delta X_{-i} = \sum_{i=0}^{T} K_i \; \delta X_{-i}, \qquad \text{where } K_i = AJ_i$$

In a similar fashion one may want to aggregate the exogenous variables, but here a problem arises. Suppose that we want to create a composite exogenous varable, x, from the exogenous variables X_1 and X_2, and that the latter affect the endogenous variables Y according to matrices $J_0, J_1, \ldots, J_T$. If the relative effect of X_1 and X_2 on each of the Y's is the same, i.e.,

$$(J_k)_{i1}/(J_k)_{i2} = (J_\ell)_{j1}/(J_\ell)_{j2} \text{ for all } i,j,k, \text{ and } \ell \tag{4}$$

then there is a simple and unambiguous way to aggregate X_1 and X_2: they should be added together using weights that represent their unique relative effect on the endogenous variables. The likelihood of condition (4) holding is however very slight, and so in general by aggregating the exogenous variables one is creating a model which will not replicate exactly the underlying model. This will not be a problem when the exogenous variables under consideration move together; in this case the variables can be weighted together whatever their relative effects on the endogenous variables. In the example cited above, suppose $\delta X_1 = \delta X_2 = b$, and let us define a new variable x as some arbitrarily weighted average of X_1 and X_2:

$$\delta x = w\delta X_1 + (1-w)\delta X_2 \tag{5}$$

so that $\delta x = b$ also. If in the original model

$$\delta Y = \sum_{i=0}^{T} J_i \, \delta X_{-i}$$

then the aggregated model becomes

$$\delta y = \sum_{i=0}^{T} J_i \cdot b \cdot \underset{\sim}{1} = \sum_{i=0}^{T} J_i \cdot \underset{\sim}{1} \, \delta x_{-i} \tag{6}$$

where $\underset{\sim}{1}$ is the unit vector. The coefficient to be applied to δx is thus the sum of the coefficients for the variables X_1 and X_2. Note that the weighting implicit in the new variable x does not enter the solution in any way.

The procedure is somewhat more complicated when, instead of generating an aggregated version of the reduced form, one wants to contruct a small structural model. This is in fact our goal: the minimodel is to be a smaller version of the large structural model, with fewer variables and simplified structure. Instead of simulating the whole model, in this case we must divide the large structural model into blocks, and apply the procedures discussed to each of the blocks separately. Variables that are endogenous to the whole model are necessarily exogenous to some of the blocks; in those blocks they are shocked as if they were exogenous variables and multipliers generated for them. Because of their dual role, the problem in aggregating these variables is more severe; whereas above the assumption that the exogenous variables moved together was sufficient to permit us to aggregate them together, this assumption is no longer tenable for those variables which, while exogenous to the sector, are endogenous to another block. Except in the most unlikely of circumstances, even if their driving variables move together they themselves will not move together.

The method of aggregating first involves identifying the sets of equations in the larger models that correspond to single equations in the minimodel, that is identifying sectors. In most cases this is straightforward. For instance, the wage/price block might be summarized by an expectations-augmented Phillips curve. Its counterpart in a disaggregated model might be equations for contract wages and unit labor costs, and markup pricing equations. Similarly, it is generally straightforward to identify those equations explaining aggregate demand components.

There are however some equations that fall in the middle, and deciding in what sector they should be will depend on the specification of the mini-model. For instance, the minimodel's inflation equation may contain a gap between actual and potential output, instead of an unemployment rate, which typically appears in wage equations in large macromodels. Consequently, the demand for employment equation (assuming there is one in the large model) should be simulated in the wage/price block, so that the consequences of shocking aggregate demand will include affecting the unemployment rate and hence contract wages.

Once the sectors have been isolated, each sector is reduced to a simplified structural equation by simulation, with care taken that definitions of the aggregated variables are compatible across sectors. For example, suppose that there are M and N endogenous and exogenous variables, respectively, but that the model is to be reduced to two sectors, with endogenous variables y_1 and y_2 defined as

$$y_1 = u'Y_1 \text{ and } y_2 = v'Y_2$$

where Y_1 and Y_2 are composed of elements of Y. It is assumed that Y_1 and Y_2 are disjoint, that is, they do not have common elements. Furthermore, we assume that the exogenous variables are to be aggregated into one variable x using a vector of weights w:

$$x = w'X$$

In each case the weights sum to unity, so that

$$\underset{\sim}{1}'u = 1 \qquad \underset{\sim}{1}'v = 1 \qquad \text{and } \underset{\sim}{1}'w = 1$$

The two sectors of the original model can be written as follows:

$$F_1(Y,\ Y_{-1},\ X) = U_1$$

$$F_2(Y,\ Y_{-1},\ X) = U_2$$

We arbitrarily choose to normalize F_1 on y_1 and F_2 on y_2. Each of these sectors is simulated separately; for sector 1 we perform one simulation where we shock all the elements Y_2 by one unit, and another simulation where we shock the elements of X by one unit. The response of the endogenous variables Y_1 allows us to construct an aggregate model with the following form:

$$\delta y_1 = \sum_{i=0}^{T} J_i^1(\delta y_2)_{-i} + \sum_{i=0}^{T} K_i^1 \delta x_{-i} \tag{7}$$

For sector 2 a similar procedure is followed, with each of the elements of Y_1 shocked by one unit in one simulation, and each of the elements of X shocked in the other simulation. This yields a second equation for the aggregated model of the form

$$\delta y_2 = \sum_{i=0}^{T} J_i^2(\delta y_1)_{-i} + \sum_{i=0}^{T} K_i^2 \delta x_{-i} \tag{8}$$

These two equations, (7) and (8), then constitute an aggregated version of the original model. These equations are not very useful in their present form because the number of lags needed before each of the J_i and K_i matrices converges to zero may be quite large. It is therefore desirable to express the lag distributions in a more parsimonious way.

Just as in classical regression models with real data, the lag distributions embodied in (7) and (8) can be summarized in a number of ways. We have chosen to represent a lag distribution by the ratio of two low-order polynomials 1/. This is most convenient when the model is to be used for optimal control or solved under the assumption of consistent expectations, because it gives a model with a small number of state variables. If we rewrite equations (7) and (8) using polynomials of order T in the lag operator L, where $LX_t = X_{t-1}$, then

$$\delta y_1 = J_1(L)\delta y_2 + K_2(L)\delta x \tag{7'}$$

$$\delta y_2 = J_2(L)\delta y_1 + K_2(L)\delta x \tag{8'}$$

These polynomials are expressed in rational lag form as follows, where the G, H, P, and Q are lag polynomials of low order:

$$\delta y_1 = \frac{G_1(L)}{H_1(1)}\delta y_2 + \frac{P_1(L)}{Q_1(L)}\delta x \tag{9}$$

$$\delta y_2 = \frac{G_2(L)}{H_2(L)}\delta y_1 + \frac{P_2(L)}{Q_2(L)}\delta x \tag{10}$$

In practice, specifying polynomials in the numerator and denominator at most of second order usually allows enough flexibility to capture adequately the lag patterns in (7') and (8').

1/ See Jorgenson (1966).

It seems natural to use regression methods to fit these rational lags, as the aim is to approximate the whole lag distribution and not just a few points along it: ordinary least squares is consistent with a quadratic loss function applied to deviations from each of the T+1 lag coefficients. Of course the lag distribution is in principle infinite, but we suppose that the model is stable so that the J_i and K_i matrices converge to the zero matrix as i goes in infinity. We must verify that the denominators $H_j(L)$ and $Q_j(L)$ do have stable roots and hence the fitted lag distributions also converge to zero.

Fitting the lag distributions requires a separate regression for each right hand side variable in equations (9) and (10). For example, to approximate the lagged effect of y_2 on y_1 in (9), we would take the simulation output, a time series for δy_1, in response to a unit shock to y_2 in period 0. The time series for δy_2 would be a vector with unity in period 0 and zeros thereafter. Using this data, we would fit the model

$$\delta y_1 = a_1(\delta y_1)_{-1} + a_2(\delta y_1)_{-2} + b_0(\delta y_2) + b_1(\delta y_2)_{-1} + b_2(\delta y_2)_{-2} + u_1 \qquad (11)$$

No constant term should be included; this will ensure that if y_2 is at its control solution value, so will be y_1, in the absence of other disturbances. Another regression of the form of equation (11) would be needed to capture the lagged effect of x on y_1.

In many cases equation (11) is unnecessarily general, so it may be that some of the regressors can be dropped. The most general model should be fitted first, and zero coefficients subsequently imposed on the basis of some measure of goodness of fit. There may be reasons for preferring parsimonious to more complicated models that maximize R^2, however. Unlike a classical regression problem, we are not observing data generated from a model with fixed coefficients and additive disturbances, since we are not performing stochastic simulations. The coefficients of the larger model are themselves subject to uncertainty and there are implicit confidence intervals around them. However, we do not capture this uncertainty as we are simulating the model with error terms set to zero. A more satisfying procedure would consist of performing the shocks to the exogenous variables as above but let the error terms result from a drawing each period from a joint probability distribution. However, this would be considerably more expensive than what we chose to do.

The results of the preferred regression equations can then be substituted back into (9) and (10). However, it is more convenient to create new variables that correspond to the dynamic response of δy_1 and δy_2 to each of the right hand side variables. For equation (9), suppose we call these variables δz_1 and δz_2. If we normalize these these variables such that in the long run they equal δy_2 and δx, respectively, then δz_1 and δz_2 will have coefficients in the equation for δy_1 which capture the equilibrium impacts of δy_2 and δx on δy_1. Let these equilibrium impacts be $\bar{J}_1$ and $\bar{J}_2$, respectively, where

$$\bar{J}_1 = G_1(1)/H_1(1) \text{ and } \bar{J}_2 = P_1(1)/Q_1(1)$$

Then the relevant equations can be written as:

$$\delta y_1 = \bar{J}_1 \, \delta z_1 + \bar{J}_2 \, \delta z_2 \tag{12}$$

$$H_1(L)\delta z_1 = [G_1(L)/\bar{J}_1]\delta y_2 \tag{13}$$

$$Q_1(L)\delta z_2 = [P_1(L)/\bar{J}_2]\delta x \tag{14}$$

Similar equations would result from equation (10) above.

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