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Monetary Instruments and Targets under Alternative Bank Market Structures in Liberal Financial Systems

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1. Introduction

This paper examines the operation of monetary policy instruments in a liberal financial system and compares the responses under different competitive structures of the banking industry. This subject is of interest because of the recent trend toward the liberalization of direct controls on commercial bank balance sheets, and on the level of commercial bank interest rates in several industrial and developing countries. It is frequently suggested that a freer financial environment can enhance savings mobilization and the efficient utilization of financial resources. In this freer environment it is also essential to know how monetary policy will operate and, in particular, whether the effectiveness of monetary policy is affected by the competitive structure of the banking industry and if institutional changes will be necessary to implement monetary policy effectively.

Two main subjects are examined in the paper. First, the responses of market interest rates to central bank policy instruments and thus the effectiveness of policy instruments in controlling expenditure are analyzed under different competitive banking structures. These topics are examined in section 3. The policy instruments include open market operations by the central bank in bond and money markets and the levels of the reserve ratio and the discount rate. Open market operations depend on the existence of adequate market institutions, which are frequently absent in developing countries. For the purposes of analysis, it is assumed that markets are well developed, but an issue examined is whether the development of money and bond markets has to accompany the liberalization of controls over commercial banks' balance sheets for effective monetary control. With adequate markets, the central bank can choose between the alternative money market operating regimes of fixing the money market rate or controlling the stock of the base money and can also use bond market transactions as an additional instrument of monetary policy.

The second main subject is an examination of the behavior of various financial aggregates under different competitive banking structures and monetary policy instruments. This topic, examined in section 4, is of interest because the movement in financial aggregates is often cited as an indication of the stance of monetary policy. However, as section 4 makes clear, broad money and credit aggregates may not be useful intermediate target variables when the structure of interest rates is changing. Section 4 also examines how the use of targets for more than one financial aggregate imposes constraints on the mix of policy instruments available to the monetary authorities. Multiple targeting may not be feasible when the financial system is poorly developed.

The behavioral relationships and the balance sheet identities which are the basis for the results derived in sections 3 and 4 are provided in section 2 of the paper. This model follows the portfolio framework

presented, for example, by Tobin (1969), but it departs from his basic framework in two main respects. First, an expenditure equation is included. This equation describes how changes in market rates influence expenditures and thus whether monetary policy is effective in expenditure control. Since expenditures influence holdings of transaction money balances as well as the demand for loans, interest rates influence the portfolio allocation decisions both directly as well as indirectly through their effect on expenditures. Secondly, the portfolio model is somewhat less general than Tobin's model in that some a priori restrictions are imposed on parameter values. A mathematical appendix provides the detailed working of the model and derives the results reported in sections 3 and 4. Section 5 is a summary of the main conclusions.

2. The model

The model is based on balance sheet identities for the nonbank private sector, commercial banks and the central bank. The nonbank private sector is assumed to hold government bonds, bank deposits and currency, and to borrow from commercial banks. Commercial banks make loans to the nonbank private sector and finance lending by accepting deposits from that sector. Commercial banks are also required to hold noninterest-bearing reserve balances at the central bank against bank deposits. An excess (deficit) of commercial bank loans and holdings of reserves over the supply of nonbank deposits is financed (utilized) by borrowing (placing) funds in the money market. The transactors in the money market are commercial banks and the central bank. The central bank issues currency and is the depository of bank reserves and holds an inventory of government bonds, which acts as backing for the note issue, and a net position in the money market. The balance sheet identities are written:

a. Consolidated private sector net financial wealth:

$$NW = B_p + D + C - L \qquad (1)$$

b. Commercial banks:

$$L = (1 - r)D + A \qquad (2)$$

c. The central bank:

$$B_{cb} + A = C + rD \qquad (3)$$

where NW = private net financial wealth

L = bank loans, with loan interest rate i_L set by commercial banks

B_p and B_{cb} = government bond holdings by the private sector and the central bank, respectively; the bond interest rate i_s is freely determined in the open market. The outstanding stock of government bonds is assumed to be predetermined.

D = bank deposits, with deposit rate i_d set by commercial banks

C = currency, noninterest bearing

A = the net claim of the central bank in the money market, which may be positive or negative, influencing the money market rate i_a 1/

r = the reserve ratio; reserves are assumed to be non-interest bearing and the ratio is set by the central bank.

Thus the aggregate financial wealth constraint is

$$NW = B$$

where B is the predetermined stock of bonds.

Equation (2) for the commercial banks does not allow for commercial bank holdings of bonds. Quite often, of course, central bank open market operations are conducted in instruments which are held mainly by the nonbank private sector, e.g., long-term government bonds. The implications of commercial bank bond holdings are commented upon briefly in the competitive model.

The behavioral equations explaining the nonbank private sector's demand for different financial assets reflects general portfolio considerations and also some prior assumptions about the substitutability between assets. The specific behavioral equations, where the a_{ij}'s are fixed positive parameters, are shown in first difference form as follows. 2/

$$dC = -a_{cd}di_d + \alpha_1 dE \tag{4}$$

$$dD = a_{dd}di_d - a_{ds}di_s - a_{dL}di_L + \alpha_2 dE \tag{5}$$

where $\alpha_1 + \alpha_2 = \alpha$.

$$dB_p = -a_{es}di_r + a_{ss}di_s - a_{sL}di_L - a_{ds}di_d \tag{6}$$

$$dL = (a_{ss} - a_{ds})di_s - (a_{sL} + a_{dL})di_L + (a_{dd} - a_{cd} - a_{ds})di_d + \alpha dE \tag{7}$$

$$dE = a_{ee}di_r - a_{es}di_s - a_{eL}di_L \tag{8}$$

1/ The term "money market rate" is used to denote the interest rate on lending between the central bank and commercial banks. Depending on the context, this rate could be the central bank discount or advances rate or the rate freely set in the money market.

2/ For any variable Y, dY reflects the absolute change in the variable.

Equation (4) states that demand for currency is positively related to expenditure (E), reflecting the transactions role of currency holdings, and negatively related to the interest rate on bank deposits (i_d), reflecting its close substitutability with currency. Although currency is noninterest bearing it may be held instead of interest-bearing bank deposits because of transaction costs and for convenience. 1/ Equation (5) states that the demand for bank deposits is positively related to its own interest rate and to expenditures and negatively related to the securities and loan rates which are gross substitutes for bank deposits. 2/ Equation (6) states that the demand for securities is positively related to its own interest rate and negatively to the implicit rate on expenditures, i_r, and to the interest rates on loans and bank deposits which are gross substitutes for bond holdings. The implicit rate on expenditures, i_r, reflects the return on holding real assets, inflation and the real return on capital, and is assumed exogenous to the model. Thus equations (4)-(6) imply that holdings of monetary assets (currency and bank deposits) are complementary with expenditures while securities and expenditures are substitutes.

The nonbank private sector's demand for bank loans is determined by the adding-up constraint, equation (1). When the yield on real assets is fixed, this can be written as equation (7). Assuming that own interest rate responses are large relative to individual cross interest rate responses, the demand for bank loans is positively related to the security and bank deposit rates and expenditures, and negatively related to the lending rate.

The system is completed by equation (8) which relates expenditures to financial variables. Expenditure is negatively related to the interest rates on securities and loans and positively related to the return on real assets. The deposit rate is assumed not to affect expenditures directly.

The equations (4)-(8) represent a system where financial variables influence expenditures (equation (8)) and expenditures feed back onto financial variables via the demand for transaction balances (equations (4) and (5)) and, as a result of the constraint that net financial wealth is fixed (equation (1)), also the demand for bank loans (equation (7)). These equations are not completely general, 3/ but might represent the pragmatic type of relationships that are quite often assumed or observed in practice.

1/ See Johnston (1984), for example, for a discussion of the demand for currency in the United Kingdom.

2/ For a discussion of the factors influencing the demand for broader liquidity aggregates see, for example, Johnston (1985).

3/ For example, expenditures do not enter the bond equation and the bank deposit rate does not enter the expenditure equation.

Commercial banks are assumed to maximize profits

$$\Pi = i_L L - i_d D - i_a A - C(L,D) \tag{9}$$

where C(L,D) is the cost of bank intermediation (including an allowance for "normal" bank profit), subject to the balance sheet constraint (2).

Under perfect competition, profit maximization implies

$$i_a = i_L - C_L = \frac{i_d + C_D}{(1 - r)} \tag{10}$$

where C_L and C_D are the partial derivatives of C(L,D) with respect to L and D, respectively. If banks also hold bonds, the security rate is determined by

$$i_s - C_s = i_a \tag{10a}$$

where C_s is the marginal resource cost of purchasing bonds. Thus profit maximization with perfect competition results in a set of interest arbitrage equations relating the advances rate and loan, deposit and bond rates as well as the marginal costs of intermediation.

Some implications of competitive banking structures are evident from equations (10) and (10a): first, bank deposits will be implicitly or explicitly interest bearing since

$$i_d = i_a(1 - r) - C_D.$$

An exception would be the case where the costs of servicing bank deposits vary with the money market rate but this does not seem very plausible. The payment of interest on current accounts is usually an indication that the financial system is competitive, and hence in such a system the non-interest-bearing component of money is essentially holdings of currency. Second, if banks hold bonds, the securities rate will be tied to the advances rate and central bank bond purchases will not have an independent influence on interest rates. By forcing banks to borrow from the central bank, bond sales can however increase the effectiveness of discount rate policy. Third, the lending rate is independent of the reserve ratio (equation (10)) and is simply tied to the money market rate and the marginal resource cost of making additional bank loans.

Under perfect monopoly, profit maximization implies bank loan and deposit supply functions:

$$L = -(i_L - c_L - i_a)\frac{dL}{di_L} + [i_D + C_D - i_a(1 - r)]\frac{dD}{di_L}$$

and $$D = (i_L - c_L - i_a)\frac{dL}{di_d} - [i_D + C_D - i_a(1 - r)]\frac{dD}{di_d}.$$

Assuming that the bank knows the private sector asset demand equations (5) and (7), the bank loan and deposit supply equations can be rewritten:

$$L = (i_L - c_L - i_a)(a_{eL} + a_{sL} + a_{dL} + \alpha a_{eL}) - [i_D + C_D - i_a(1 - r)](a_{dL} + \alpha_2 a_{eL}) \quad (11)$$

and $$D = (i_L - C_L - i_a)(a_{dd} - a_{cd} - a_{ds}) - [i_d + C_D - i_a(1 - r)]a_{dd} \quad (12)$$

The assumption of perfectly competitive or perfect monopolistic banking structures obviously constitute an extreme. Banking structures in many countries probably fall somewhere between these polar cases and Startz (1983), for example, has examined a model of commercial bank interest rate setting behavior under monopolistic competition. The assumptions of profit maximization may also be inappropriate, and these simple models of bank behavior ignore important questions of risk and uncertainty (Baltensperger (1980)). Models of perfect competition or perfect monopoly are, however, analytically more tractable within general portfolio frameworks and two earlier papers by Aftalion and White (1977) and Van Hoose (1983) have compared, in a different context the monetary policy implications of competitive versus monopolistic banking structures. 1/ In developed economies the liberalization of financial systems has quite often been associated with increasing banking competition as commercial banks have been able to expand their balance sheets at the expense of other financial institutions unconstrained by direct controls. The growth of international banking has also led to increased banking competition (see Johnston (1983)). In many developing countries, however, the domestic financial system is small and dominated by a few large banks and the assumption of monopolistic behavior may be more appropriate in these countries.

1/ Aftalion and White's (1977) model is not set in a general portfolio framework which allows for adjustments in all financial markets; Van Hoose's (1983) model, based on the work of Benavie and Froyen (1982), is designed specifically with the United States in mind and does not allow for a direct impact of monetary policy instruments on expenditures as we do in this paper. A more recent paper by Van Hoose (1985) has also sought to address the question of the implications of different competitive banking structures within a macroeconomic model. His paper is more general than this paper in the treatment of banking competition, but less general in the analysis of monetary policy instruments and in the treatment of the portfolio restrictions on the private sector. A survey of the implications of different competitive banking structures for monetary policy is contained in Faig-Aumelle (1986).

The monetary policy framework in the model is based on the following: the monetary authorities are assumed to operate in three markets, those for bonds, bank reserves, and the money market. We distinguish between central bank operations with the nonbank private sector and with commercial banks. Bonds are taken to refer to any instrument which is held by nonbank investors but not by commercial banks. Money market instruments and reserves are traded only by the central bank and commercial banks. In most industrial countries central banks do not intervene to control the level of interest rates in the bond market, but rather purchase or sell a quantity of bonds and accept the bond yield freely determined in the open market. This feature is reflected in the model specification. Many central banks, however, operate to fix the interest rate in the money market. The central bank might do this by announcing or signalling its discount rate in advance and by operating to force the commercial banks to borrow at the discount window. Where money markets exist, the central bank can intervene by buying or selling bills to the commercial banks, and thereby control the money market rate. Alternatively, the central bank can control the monetary base and allow the money market rate to be freely determined. The regime where the money market rate is exogenously controlled by the central bank, and a regime of monetary base control are examined. If money and bond markets do not exist, the central bank would have to rely on administrative changes in the reserve ratio to force commercial banks to borrow from the central bank, making discount rate policy effective. This may, however, raise problems by creating incentives for avoidance of the reserve requirement in the liberal financial system. Reserves are assumed to be noninterest bearing and the reserve ratio is exogenously set by the central bank. The situation where the discount rate and the reserve ratio are used as the main instruments of monetary policy reflects the initial position in many developing countries which are in the process of liberalizing their financial systems and where money markets are not well developed. In some developing countries, where money market development was emphasized at an early stage in the liberalization process, the policy of intervening in the money market to influence the rate has become feasible.

The changes in the monetary instruments--changes in the reserve ratio, the money market rate or the stock of base money, and purchases and sales of bonds by the central bank--determine the movement in three endogenous interest rates, the rate on bonds (securities), the bank deposit rate, and the bank lending rate. When the monetary base is the operational variable, the advances rate becomes a fourth endogenous interest rate. The type of relationship could be written

$$\overline{di} = A\ \overline{dP}$$

where $\overline{i}$ is a vector of endogenous interest rates, $\overline{P}$ the vector of policy instruments and A the matrix of responses. The way in which the endogenous interest rates adjust to the exogenous monetary policy instruments (the A matrix) reflects the competitive structure of the banking industry. As we have seen above, perfect competition implies

a set of interest rate arbitrage equations, while monopoly implies loan and deposit supply functions and that bank deposit and loan rates respond to interest rates in all markets. Depending on the movement in the endogenous interest rates, adjustments occur in expenditures, (equation (8)) and various financial aggregates, for example holdings of currency, bank deposits, credit demand, etc. The movements in expenditure influence the demand for financial aggregates. The next section derives the A matrix of responses and discusses the influence of the policy instruments on interest rates and expenditures, under perfectly competitive and perfect monopolistic banking structures. Throughout this section it is assumed that the authorities have control over money market liquidity, and there is an established bond market.

3. Monetary policy responses and expenditure control

a. Perfect competition

i. An exogenous money market rate regime

Equation (10) determines bank loan and deposit rates in terms of the policy instruments, i_a and r in the perfectly competitive model:

$$di_L = di_a,$$

and

$$di_d = (1 - r)di_a - i_a dr \qquad (15)$$

That is, the lending rate moves with the money market rate, and the deposit rate is negatively related to the reserve ratio and rises with the advances rate.

The securities rate, assuming commercial banks do not hold bonds, can be derived from the bond market equilibrium condition $dB_{cb} = -dB_p$ that the central bank purchases of bonds have to be matched by private sector sales. Using equation (6) for private sector bond holdings determines the securities rate as:

$$di_s = -\frac{dB_{cb}}{a_{ss}} + \frac{(a_{sL} + (1 - r)a_{ds})}{a_{ss}} di_a - \frac{a_{ds}}{a_{ss}} i_a dr \qquad (16)$$

A central bank bond purchase or a rise in the reserve ratio reduces the securities rate while a rise in the money market rate increases it. These results are conventional, except perhaps for the effect of a change in the reserve ratio. Under the exogenous money market rate regime, a rise in the reserve ratio leads commercial banks to reduce the deposit rate in order to equalize the marginal cost of nonbank deposits with the interbank money market rate, and this leads to a shift out of bank deposits into securities.

Therefore, the matrix relating the endogenous interest rates i_d, i_L and i_s to the central bank policy instruments i_a, B_{cb} and r is,

$$\begin{vmatrix} di_d \\ di_L \\ di_s \end{vmatrix} = \begin{vmatrix} (1 - r) & 0 & -i_a \\ 1 & 0 & 0 \\ \dfrac{(a_{sL} + (1 - r)a_{ds})}{a_{ss}} & -\dfrac{1}{a_{ss}} & -\dfrac{a_{ds}}{a_{ss}} i_a \end{vmatrix} \begin{vmatrix} di_a \\ dB_{cb} \\ dr \end{vmatrix} \qquad (17)$$

The movement in expenditure (equation (8)) depends inversely on the responses of the loan rate and securities rate to the policy instruments. All endogenous rates rise with the money market rate; the securities rate falls and the loan (and deposit) rate is unchanged with bond purchases; the securities (and deposit) rate falls with a rise in the reserve ratio, while the loan rate is unchanged. Thus all three monetary policy instruments are effective in expenditure control, although the increase in the reserve ratio has a somewhat counterintuitive effect of raising expenditures.

ii. Monetary base control

When the central bank operates to control the monetary base, the money market rate becomes endogenous and is determined by the supply and demand for base money.

$$dMB = dC + rdD + drD \qquad (18)$$

This base money identity is solved for the money market rate by substituting equations (4) and (5) for the nonbank private sector's demand for currency and deposits. The solution of the model is described in the Appendix, section 1. Because the securities rate enters the demand for deposits, equation (18) is solved simultaneously with equation (16) for the securities rate and the interest rate responses are derived in terms of the three policy instruments, dMB, dB_{cb}, and dr.

The solution of the model gives the following matrix of signs of interest rate responses:

$$\begin{vmatrix} di_d \\ di_L \\ di_s \end{vmatrix} = \begin{vmatrix} - & + & ? \\ - & + & + \\ - & - & + \end{vmatrix} \begin{vmatrix} dMB \\ dB_{cb} \\ dr \end{vmatrix}$$

As might be expected, an expansion in the monetary base reduces interest rates; bond purchases reduce the securities rate, but the fall in the securities rate raises the demand for bank deposits and

thus base money, and it therefore raises the money market rate and bank deposit and loan rates. A higher reserve ratio raises the demand for base money and thus the money market rate and bank loan rates, and by decreasing the demand for securities it also raises the securities rate. However, the response of the deposit rate to the change in the reserve ratio is ambiguous. While a higher reserve ratio raises the money market rate, it increases the cost of holding noninterest-bearing reserves against deposits which may lead to a fall in the deposit rate.

A contraction in the monetary base raises interest rates and will be an effective instrument for controlling expenditures. This is also true for a rise in the reserve ratio which has the normal contractionary effect, although this result depends partly on the absence of a deposit rate in the expenditure equation. Bond purchases by the central bank have, however, a somewhat indeterminate impact on expenditures since loan and securities rates may move in opposite directions. The overall response depends on the level of the reserve ratio (see the Appendix, section 1). For a low ratio there will be little increased demand for base money following the fall in the securities rate and thus little upward pressure on loan rates and the bond purchase is therefore more likely to be expansionary. Thus, if the authorities wish to use open market bond sales or purchases under a monetary base control regime, it is advisable to set reserve ratios at a low level or alternatively to abandon the base control regime and intervene to fix the money market rate instead.

b. Perfect monopoly

i. An exogenous money market rate

Under the monopolistic banking structure, equilibrium in the deposit and loan markets is described by equations (5), (12), (7) and (11) representing, respectively, the demand and supply of bank deposits and demand and supply of bank loans. These equations can be written in a matrix form which facilitates solution of the equilibrium positions (see the Appendix, section 2). The monopolistic model is solved in stages to help isolate the main factors influencing the responses. Initially the securities rate is treated as exogenous (Appendix, section 2a) and then the full model with an endogenous securities rate is examined (Appendix, section 2b).

The following assumptions are used in deriving the matrix of responses: the reserve ratio is relatively small, the α coefficient determining the transactions demand for money is not substantially greater than unity and currency is relatively interest insensitive. It is also assumed that an equal rise in all interest rates leaves the demand for deposits and securities approximately unchanged reflecting a common assumption in portfolio models of homogeneity of interest rate response. This latter assumption is not strictly valid when the portfolio includes a noninterest-bearing asset (currency), the demand for which is interest

sensitive, but given the assumption that currency is relatively interest insensitive, it is probably not an unreasonable approximation. The assumption about the size of r is needed to avoid uncertainty about the direction of the response of deposit and loan rates to a change in the money market rate, while the assumption about the size of α avoids uncertainty about the direction of the responses to a change in the securities rate. Under the assumptions the following matrix of signs for the direction of the response of deposit and loan rates to exogenous changes in the securities and money market rates and the reserve ratio can be derived.

$$\begin{vmatrix} di_d \\ \\ di_L \end{vmatrix} = \begin{vmatrix} + & + & - \\ & & \\ + & + & + \end{vmatrix} \begin{vmatrix} di_a \\ di_s \\ dr \end{vmatrix} \qquad (19)$$

Deposit and loan rates rise with the money market rate and also with the securities rate. Interestingly in the monopolistic model, with an exogenous securities rate, a rise in the reserve ratio raises the loan rate and reduces the deposit rate. This follows because the monopolist's loan supply depends on the costs of bank deposits. A higher reserve ratio raises deposit costs and this is recouped partly by raising loan rates and partly by reducing deposit rates. A "common" assumption that a higher reserve ratio will raise loan rates is thus dependent on an element of monopoly in the banking system. However, as we observe below, if the securities rate is also endogenous it becomes much more difficult to sign the responses to a change in the reserve ratio and thus the "common" assumption does not generally hold even for a monopolistic banking system.

The solution of the monopolistic model when the securities rate is also endogenous (Appendix, section 2b) yields the following matrix for the signs of the responses under the above assumptions. Without the assumptions the response parameters are generally indeterminate and the system is likely to be unstable.

$$\begin{vmatrix} di_d \\ di_L \\ di_s \end{vmatrix} = \begin{vmatrix} + & - & ? \\ + & - & ? \\ + & - & ? \end{vmatrix} \begin{vmatrix} di_a \\ dB_{cb} \\ dr \end{vmatrix} \qquad (20)$$

One result which has already been noted is the indeterminacy of the impact of a change in the reserve ratio on the endogenous interest rates. Starting from the situation where the securities rate is exogenous, we have noted that a rise in the reserve ratio raises the loan rate and reduces the deposit rate. If the securities rate now becomes endogenous, the lower deposit rate encourages the private sector to buy and the higher loan rate to sell securities and the securities rate may therefore rise or fall. The solutions for the deposit and loan rates

in response to a change in the reserve ratio depend on the relative shifts in the deposit and loan demand schedules, which in turn reflect the combination of the effects of the reserve ratio on deposit and loan rates when the securities rate is fixed, and the impact of changes in the securities rate. These terms cannot be signed unambiguously.

The implications of the results for expenditure control are that changes in the money market rate or bond purchases are effective control instruments, while the reserve ratio is not an unambiguous control instrument when the banking system is monopolistic and the authorities set the money market rate exogenously.

ii. Monetary base control

The results for the endogenous interest rate responses in the monopolistic model when the monetary base is the exogenous policy instrument, derived in the Appendix, section 3 are

$$\begin{vmatrix} di_d \\ di_L \\ di_s \end{vmatrix} = \begin{vmatrix} - & - & ? \\ - & - & ? \\ - & - & ? \end{vmatrix} \begin{vmatrix} dMB \\ dB_{cb} \\ dr \end{vmatrix} \qquad (21)$$

These results are similar to those derived for the regime of exogenous money market rate with perfect monopoly (equation (20)) and continue to exhibit the indeterminancy of response of endogenous interest rates and thus expenditure to changes in the reserve ratio. Moreover, unlike the competitive model, bond purchases now have an unambiguous impact in reducing bank deposit and loan rates as well as the securities rate, reflecting the tendency of the monopolist to reduce bank deposit and loan rates in response to a fall in the securities rate. Bond purchases thus remain an effective instrument of expenditure control in the monopolistic model.

c. Some policy implications

To sum up the main policy implications of this section, the authorities' control over the money market rate or the monetary base is likely to be an effective instrument of expenditure control regardless of the competitive structure of the banking industry. In the monopolistic model it is necessary, however, to assume that the reserve ratio is relatively small to avoid some uncertainty in interest rate responses. Central bank bond sales are an effective instrument of expenditure control in the competitive model with an exogenous money market rate, but there is some uncertainty about the direction of interest rate responses under monetary base control in the competitive model when reserve ratios are high, and also in the monopolistic model if borrowing to finance holdings of transaction money balances rises more than proportionally with expenditures. The implications of changes in the reserve ratio are closely dependent on the banking structure and are generally indeterminate when the banking system is monopolistic.

Without a money or bond market, a rise in the reserve ratio could be used to force borrowing from the central bank to make the discount rate effective. The results for control of expenditures would then depend on a combination of the responses to the change in the reserve ratio and the discount rate. In the competitive model a rise in the reserve ratio with a fixed discount rate is expansionary. A contractionary monetary policy therefore requires a larger movement in the discount rate than if money market rates could be raised without a rise in the reserve ratio. This may also be true in the monopolistic model. However, a concern with this policy is that noninterest-bearing reserve ratios create incentives for avoidance which erodes their effectiveness. Over time, it therefore becomes necessary to establish market mechanisms of monetary control through the development of, and open market operations in, bill or money markets. Active open markets give the option to use monetary base control rather than interest rate setting as the policy instrument, in addition to avoiding the uncertainty of monetary policy response associated with changing the reserve ratio.

4. The behavior of financial aggregates

Section 3 has determined the direction of interest rate responses to the monetary policy instruments

$$\overline{di} = A\ \overline{dp}.$$

This section examines how the endogenous interest rates influence different financial aggregates under different competitive banking structures and monetary policy instruments. The responses can be written as:

$$\overline{dT} = B\ \overline{di},$$

where T is the vector of financial aggregates consisting of currency, the monetary base (when this is not a monetary policy instrument), money defined as M = D + C, and bank credit. The B matrix, defined as

$$B = \begin{vmatrix} -a_{cd} & -\alpha_1 a_{eL} & -\alpha_1 a_{es} \\ (ra_{dd}-a_{cd}) & -(ra_{dL}+(\alpha_1+r\alpha_2)a_{eL}) & -ra_{ds}+(\alpha_1+r\alpha_2)a_{es}) \\ (a_{dd}-a_{cd}) & -(a_{dL}+\alpha a_{eL}) & -(a_{ds}+\alpha a_{es}) \\ (a_{dd}-a_{cd}-a_{ds}) & -(a_{eL}+a_{sL}+a_{dL}) & (a_{ss}-a_{es}-a_{ds}) \\ 0 & -a_{eL} & -a_{es} \end{vmatrix}$$

describes how the endogenous interest rates i_d, i_L and i_s, are transformed into the financial aggregates. This transformation is of interest since it shows which financial aggregates may be potentially useful as intermediate target aggregates. The relationship between the target aggregates and the policy instruments is given by

$$\overline{dT} = AB\ \overline{dp}.$$

In assessing the usefulness of different financial aggregates as target variables we use the criterion that a target aggregate would be useful if it provided an unambiguous signal about the movement in expenditure. If the response of the chosen financial aggregates is ambiguous in response to a change in one of the central bank's operational variables, this may lead to adverse expectational effects or a loss of credibility in policy since the authorities would have to justify apparently perverse movements in the target aggregates in response to changes in policy instruments. This is, of course, only a partial, but nevertheless potentially important criterion for the selection of a target aggregate. In the deterministic framework used here, there is no information gain from using a financial aggregate as an intermediate target. Friedman (1975) discusses the importance of uncertainty and information lags in the selection of target aggregates. Nevertheless, the results based on a deterministic model allow us to draw some conclusions about the use of multiple targets and the selection of consistent target ranges. The detailed results are provided in the Appendix and summarized in Table 1 which also summarizes the direction of response of the endogenous interest rates and expenditure to the monetary policy instruments.

a. Perfect competition

With perfect competition and an exogenous money market rate, central bank bond purchases increase currency, the monetary base, money and expenditures. Money will respond by the largest amount to central bank bond purchases followed by the monetary base and currency. 1/ The response of credit to the bond purchase reflects two offsetting forces: 2/

1/ The exact relationships between the monetary aggregates and expenditure under the policy of central bank bond purchases shown in Appendix Table A2 are:

$$\frac{dE}{dC} = \frac{1}{\alpha_1}\ ,\ \frac{dE}{dMB} = \frac{1}{r(a_{ds}/a_{es}) + \alpha_1 + r\alpha_2},\ \frac{dE}{dM} = \frac{1}{(a_{ds}/a_{es}) + \alpha}$$

and $\frac{dE}{dC} > \frac{dE}{dMB} > \frac{dE}{dM}$.

2/ The response depends on $\frac{-(a_{ss} - \alpha a_{es} - a_{ds})}{a_{ss}}$. This can also be written as $\frac{-a_{sL} + \alpha a_{es}}{a_{ss}}$, where a_{sL} is the coefficient of subsitution between loan and securities.

Table 1. Summary of the Responses to Monetary Policy Instruments under Competitive and Monopolistic Banking Structures

A. Exogenous Money Market Rate

Policy instrument	Competitive banking structure			Monopolistic banking structure		
	Central bank bond purchases	Increase in money market rate	Increase in reserve ratio	Central bank bond purchases	Increase in money market rate	Increase in reserve ratio
Interest rates						
Deposit rate	0	+	-	-	+	?
Loan rate	0	+	0	-	+	?
Securities rate	-	+	-	-	+	?
Objective variable						
Expenditure	+	-	+	+	-	?
Financial aggregates						
Currency	+	-	+	+	-	?
Monetary base	+	-(?)	+	+	-	?
Money stock	+	?	?	?	?	?
Bank lending	?	?	?	?	?	?

B. Exogenous Monetary Base

Policy instrument	Competitive banking structure			Monopolistic banking structure		
	Central bank bond purchases	Increase in monetary base	Increase in reserve ratio	Central bank bond purchases	Increase in monetary base	Increase in reserve ratio
Interest rates						
Deposit rate	+	-	?	-	—	?
Loan rate	+	-	+	-	—	?
Securities rate	-	-	+	-	—	?
Objective variable						
Expenditure	?	+	-	+	+	?
Financial aggregates						
Currency	?	+	?	+	+	?
Money stock	?	?	?	?	?	?
Bank lending	?	?	?	?	?	?

an increased demand for bank loans to finance larger money balances associated with the higher expenditures and lower borrowing because of the decrease in the yield on securities. The aggregate response cannot be determined unambiguously. If the transactions response dominates, as might follow in a model with financial deepening, bank lending will also expand with the bond purchase. In a developed country context where the yield arbitrage effect dominates, bank lending might, however, contract with the bond purchase and lending would not be a good indicator of movements in expenditure.

An increase in the advances rate reduces expenditure and the demand for currency. The impact on the monetary base is slightly ambiguous but we might generally conclude that it will also be negative. The influences on money and credit are more ambiguous. The monetary response is complicated because a rise in the money market rate raises the own rate on bank deposits, although this may be more than offset by cross substitution effects and by a decline in expenditures which reduces money demand. The credit response again reflects the two influences noted above. Lending rates rise but so do deposit and securities rates which may lead to some increased borrowing, while there is a fall in expenditure, which leads to reduced borrowing to finance transactions demand for money.

An increase in the reserve ratio expands expenditure because, as noted above, it will depress the securities rate. Currency and the monetary base also expand with a rise in the reserve ratio. Signing the direction of the change in money is again complicated because the rise in the reserve ratio depresses the own rate on bank deposits, which may lead to substitution out of money, although this may again be offset, in this case because of the increase in the holdings of currency, the fall in the securities rate, and the rise in expenditure. The response of credit is again also ambiguous.

Under monetary base control none of the financial aggregates, apart from currency, a major component of the monetary base, respond unambiguously to the central bank policy instruments. The value of using the monetary base to target a particular aggregate is therefore in question, with the implication that monetary base control may require the abandonment of intermediate targeting of other monetary or credit aggregates.

b. Perfect monopoly

The responses of financial aggregates under perfect monopoly with an exogenous money market rate differ in two major respects from those obtained for the competitive model. First, central bank bond purchases no longer have an unambiguous impact on the money stock. In the competitive model the own rate on bank deposits is tied to the advances rate, but in the monopolistic model the own rate also adjusts with the securities rate and thus the money stock may expand when the central bank sells bonds. Secondly, changes in the reserve ratio do not provide an unambiguous control mechanism, either for expenditure or any of the

financial aggregates. As in the competitive model, currency and the monetary base provide correct signals about the movement in expenditures when the advances rate or central bank bond purchases are used as policy instruments. Similar results occur in the monopolistic model under monetary base targeting.

To sum up, the only financial aggregates which generally provide an unambiguous signal about the movement in expenditures are the non-interest-bearing financial aggregates, currency and the monetary base. The money stock provides an unambiguous signal only in the situation where the banking system is competitive, the authorities set the money market rate exogenously and operate policy through central bank bond sales. In other circumstances, the own rate on money responds to the changes in monetary policy instruments which may lead to substitution into money balances even though expenditures are reduced. Bank credit never provides an unambiguous signal about movements in expenditure because it is affected both by borrowing to hold transaction money balances, which rises with expenditure, and borrowing to hold other financial instruments which tend to decline as expenditure rises and interest rates on securities and bank deposits are reduced.

c. Multiple target ranges

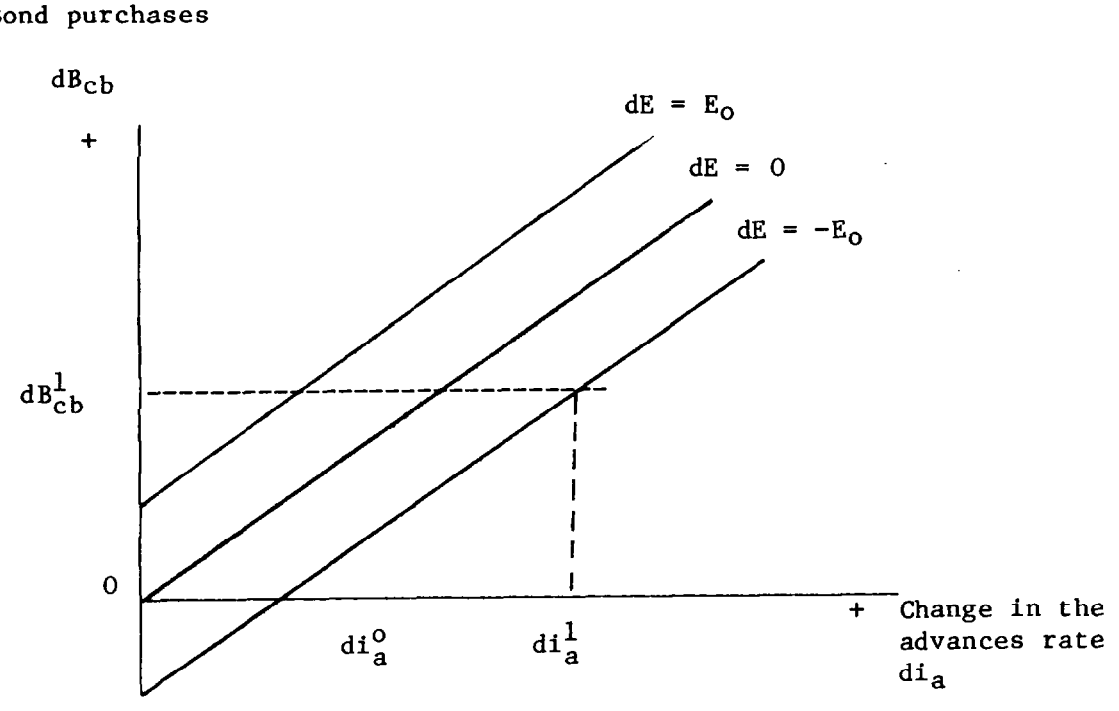
The results for the competitive model with an exogenous money market rate can be used to illustrate some properties of the selection of multiple targets. The results in Appendix Table A2 can be used to derive a set of changes in monetary policy instruments which lead to the same expenditure objectives. Examining two instruments, central bank bond purchases and changes in the advances rate, the relationship between these monetary policy instruments needed to hold private expenditure constant is

$$dB_{cb} = \frac{1}{a_{es}}[a_{sL} + (1 - r)a_{es} + a_{eL}a_{ss}]di_a.$$

This is shown in Figure 1. The same decline in expenditure, say E_o in the figure, can be achieved with an increase in the advances rate of di_a^o and no change in central bank bond purchases or a larger increase in the advances rate of di_a^1 and central bank bond purchases of dB_{cb}^1. Any combination of di_a and dB_{cb} along the $dE = 0$ line will achieve this expenditure objective. Similarly, it is also possible to derive the relationship between central bank bond purchases and changes in the advances rate which hold constant the level of currency

$$dB_{cb} = \left[\frac{a_{ss}}{\alpha_1 a_{es}}[(1 - r)a_{cd} + \alpha_1 a_{eL}] + [a_{sL} + (1 - r)a_{ds}]\right]di_a$$

Figure 1.



and the level of money

$$dB_{cb} = \left[[a_{sL}+(1-r)a_{ds}] - \frac{[(a_{dd}-a_{cd})(1-r)-a_{dL}-\alpha a_{eL}]a_{ss}}{(a_{ds}+\alpha a_{es})} \right] di_a .$$

These lines are illustrated in Figure 2. The slope of the dM = 0 line is drawn negatively, reflecting the possibility of a perverse response of money to a change in interest rates.

With two monetary policy instruments it is possible to target only two variables. An expenditure objective of $dE = E_o$ can be achieved with any combination of di_a and dB_{cb} along the $dE = E_o$ line and the policy instruments can be selected to meet a currency target, $dC = C_o$, but the resulting outcome for money, $dM = M_o$, may be inconsistent with the money target (Figure 3).

Figure 2.

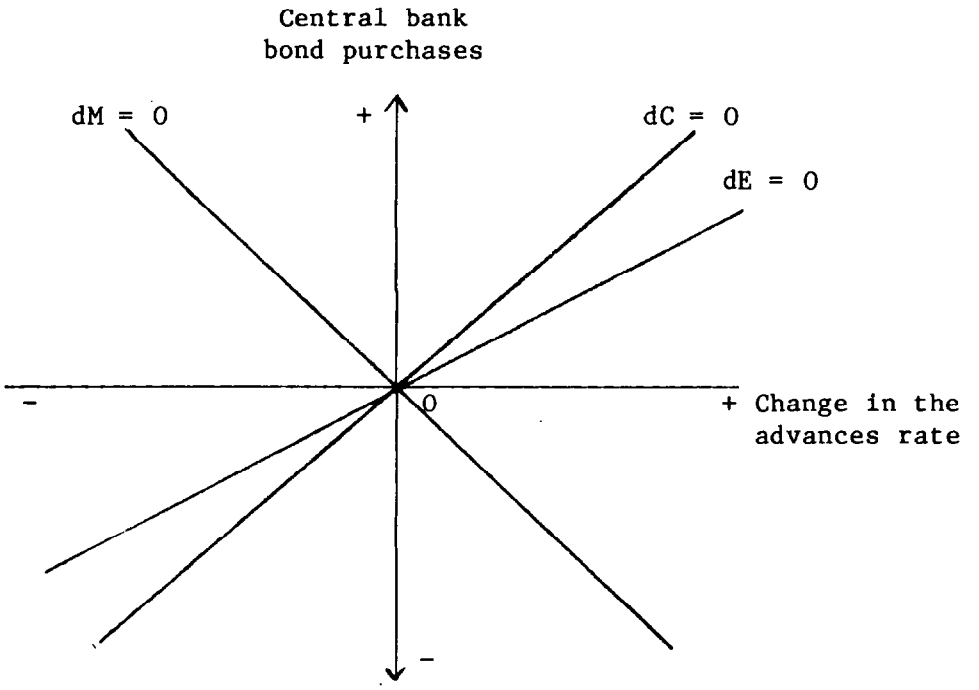
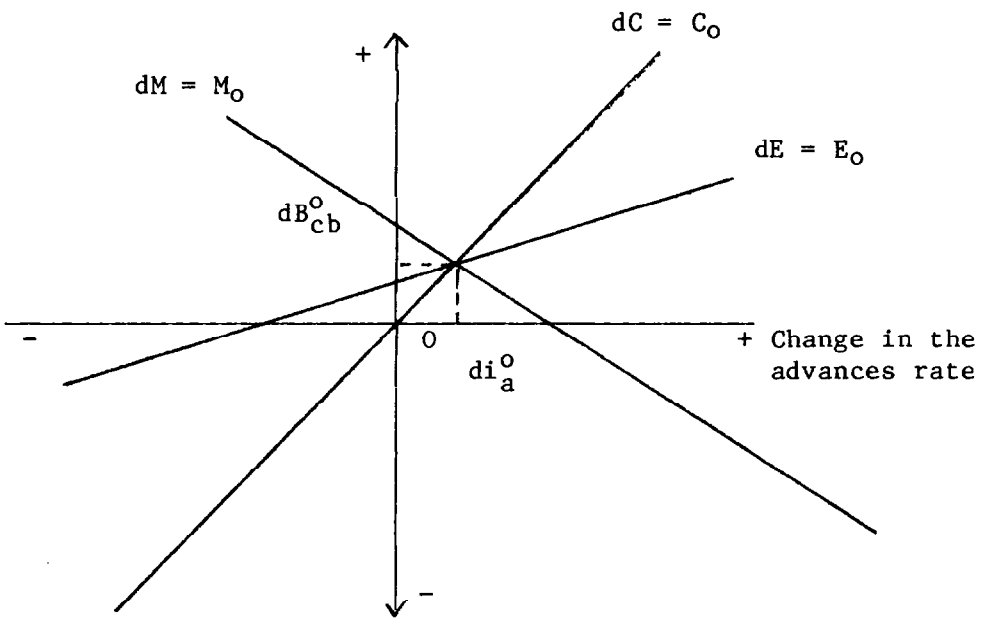


Figure 3.



Alternatively the authorities may be able to choose di_a and dB_{cb} to achieve the currency and money targets and the outcome for expenditures will be the result of meeting these targets. To target all three variables directly it would be necessary to use a third monetary instrument, e.g., changes in the reserve ratio. In practice, the potential conflict between the objectives can be resolved by the selection of the target ranges. The selection of $dC = C_o$ and $dM = M_o$ as targets in Figure 3 will determine the necessary change in the monetary policy instruments, dB^o_{cb} and di^o_a, and the change in the expenditure objective $dE = E_o$: dC and dM act as intermediate target variables. Notice, however, that the same target level of expenditure $dE = E_o$ can be achieved by any number of possible outcomes for di_a and dB_{cb} along the $dE = E_o$ line and thus any number of possible outcomes for dM and dC. Therefore, the selected ranges should reflect the instruments of monetary policy chosen to achieve the targets. A greater reliance on interest rates to stimulate expenditure in the competitive system with an exogenous money market rate would imply a higher target for currency and a lower one for money than if central bank bond purchases are the main policy instrument.

5. Summary and main conclusions

This paper examines the operation of monetary policy instruments in a liberal financial system and compares the response under different competitive structures of the banking industry. The subject is of interest not only because of the recent trend toward the liberalization of direct controls over commercial bank balance sheets in several industrial and developing countries, but also because of the policy prescription that the efficient utilization of financial resources requires a freer financial environment. In freeing up the controls on commercial banks it is necessary to know how monetary policy will operate in the more liberal financial environment, how it depends on the competitive structure of the banking sector and if institutional changes are necessary to ensure effective monetary control.

A portfolio framework is used to examine the operation of monetary policy instruments. The potential instruments available to the central bank are changes in the reserve ratio, the discount rate, and open market operations in bond and money markets; however, open market instruments are not always available in developing countries because of the lack of established markets.

The paper shows that in the liberal financial system the authorities will not generally be able to rely solely on changes in the reserve ratio as an effective monetary policy instrument, especially if the banking system is monopolistic. Changes in the reserve ratio may be used to make discount rate policy effective, but movements in the discount rate may have to be larger for effective expenditure control than if open market instruments are used. In a liberal financial environment high reserve ratios also create incentives for avoidance and thus become less effective over time, making it necessary to rely on nondiscriminatory methods of monetary control.

Central bank purchases and sales of bonds with the nonbank private sector are effective instruments of monetary control when the banking system is competitive and money market rates are set exogenously. There is some uncertainty about the response of interest rates under monetary base control when the banking system is competitive and the reserve ratio is high, and when it is monopolistic, there is some uncertainty of response if loan demand to finance transactions money demand rises more than proportionally with expenditures.

Central bank open market operations in the money market, which are used to control either the money market rate or the monetary base, are effective instruments of monetary control with a competitive banking industry; and also when the banking system is monopolistic, providing that reserve ratios are relatively low. In the competitive model, under monetary base control, a rise in the reserve ratio has a contractionary influence, whereas it tends to be expansionary when the money market rate is set exogenously. The potential importance of money market operations in monetary control in the liberal system and the risks and uncertainties associated with high reserve ratios indicate that if the authorities are to be able to operate monetary policy flexibly and effectively, a policy of liberalizing direct controls on commercial banks may have to be accompanied by the development of money markets where these do not already exist.

The paper also examines the behavior of financial aggregates in the liberal system under different competitive banking structures and monetary control instruments. The results indicate that:

- *noninterest-bearing forms of money can be helpful intermediate* target variables in that they generally provide unambiguous signals about the movement in expenditures.

- if money is defined more broadly to include interest-bearing deposits, it may expand as the general level of interest rates and the own rate on money rises. In these circumstances interest-bearing money does not usually provide an unambiguous signal about movements in expenditures. An exception is the case where the banking system is competitive, the money market rate is set exogenously and monetary policy is conducted through sales of bonds to the nonbank private sector.

- bank credit is a poor intermediate target in the liberal financial system and in none of the cases studied does it provide an unambiguous signal about movements in expenditure. This reflects the opposite effects on bank credit of increased borrowing to hold transactions money balances and reduced borrowing to hold financial assets when interest rates on financial assets are reduced and expenditure increases.

- the setting of target ranges for more than one financial aggregate is feasible in the liberal financial system when the institutional structure is reasonably sophisticated, but the selected target ranges impose constraints on the mix of policy instruments available to the authorities.

APPENDIX

Solution Equations for Monetary Policy Responses

1. Perfect competition with a policy of operating on the monetary base

The solution equations are:

$$\begin{vmatrix} [(ra_{dd} - a_{cd})(1 - r) - ra_{dL} - (\alpha_1 + r\alpha_2)a_{eL}] & -[ra_{ds} + (\alpha_1 + r\alpha_2)a_{es}] \\ (a_{sL} + (1 - r)a_{ds}) & -a_{ss} \end{vmatrix} \begin{vmatrix} di_a \\ di_s \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & [(ra_{dd} - a_{cd})i_a - D] \\ 0 & 1 & a_{ds}i_a \end{vmatrix} \begin{vmatrix} dMB \\ dB_{cb} \\ dr \end{vmatrix}$$

and the solution of the model is of the form:

$$\begin{vmatrix} di_a \\ di_s \end{vmatrix} = \frac{1}{H} \begin{vmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \end{vmatrix} \begin{vmatrix} dMB \\ dB_{cb} \\ dr \end{vmatrix}$$

where the variables and their signs are:

Variable	Sign
$H = -a_{ss}[(ra_{dd} - a_{ed})(1 - r) - ra_{dL} - (\alpha_1 + r\alpha_2)a_{eL}] + [ra_{ds} + (\alpha_1 + r\alpha_2)a_{es}][a_{sL} + (1 - r)a_{es}]$	> 0
$F_{11} = -a_{ss}$	< 0
$F_{12} = ra_{ds} + (\alpha_1 + r\alpha_2)a_{es}$	> 0
$F_{13} = -a_{ss}[(ra_{dd} - a_{cd})i_a - D] + a_{ds}i_a[ra_{ds} + (\alpha_1 + r\alpha_2)a_{es}]$	> 0
$F_{21} = -(a_{sL} + (1 - r)a_{ds})$	< 0
$F_{22} = (ra_{dd} - a_{cd})(1 - r) - ra_{dL} - (\alpha_1 + r\alpha_2)a_{eL}$	< 0
$F_{23} = -(a_{sL} + (1 - r)a_{ds})[(ra_{dd} - a_{cd})i_a - D] + [(ra_{dd} - a_{cd})(1 - r) - ra_{dL} - (\alpha_1 + r\alpha_2)a_{eL}]a_{ds}i_a$	> 0

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In signing the variables it is assumed that for a small value of r

$$(ra_{dd} - a_{cd})(1 - r) - ra_{dL} - (\alpha_1 + r\alpha_2)a_{eL} < 0$$

and

$$(ra_{dd} - a_{cd})i_a - D < 0$$

These assumptions make the signing of F_{23}, di_s/dr, problematic however, since the first term of F_{23} is positive and the second negative. If the first term dominates by virtue of D being large relative to other terms then $di_s/dr > 0$ and this is assumed in what follows.

The relationship between the bank deposit and loan rates and the securities rate and the policy instruments, the A matrix, under perfect competition is:

$$\begin{vmatrix} di_d \\ di_L \\ di_s \end{vmatrix} = \frac{1}{H} \begin{vmatrix} (1 - r)F_{11} & (1 - r)F_{12} & (1 - r)F_{13} - i_a \\ (-) & (+) & (?) \\ F_{11} & F_{12} & F_{13} \\ (-) & (+) & (+) \\ F_{21} & F_{22} & F_{23} \\ (-) & (-) & (+) \end{vmatrix} \begin{vmatrix} dMB \\ dB_{cb} \\ dr \end{vmatrix}$$

The sign of di_d/dr is somewhat indeterminate, as noted in the text. Using the behavioral equations (4)-(7), expenditures and financial aggregates can be written in terms of the policy instruments as

$$\begin{vmatrix} dC \\ dM \\ dL \\ dE \end{vmatrix} = \frac{1}{H} \begin{vmatrix} -a_{cd} & -\alpha_1 a_{eL} & -\alpha_1 a_{es} \\ (a_{dd} - a_{cd}) & -(a_{dL} + \alpha a_{eL}) & -(a_{ds} + \alpha a_{es}) \\ (a_{dd} - a_{cd} - a_{ds}) & -(\alpha a_{cL} + a_{sL} + a_{dL}) & (a_{ss} - \alpha a_{es} - a_{ds}) \\ 0 & -a_{eL} & -a_{es} \end{vmatrix} \begin{vmatrix} (1 - r)F_{11} & (1 - r)F_{12} & (1 - r)F_{13} - i_a \\ F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \end{vmatrix} \begin{vmatrix} dMB \\ dB_{cb} \\ dr \end{vmatrix}$$

The expenditure response to a central bank bond purchase is:

$$dE = - a_{eL}[ra_{ds} + (\alpha_1 + r\alpha_2)a_{es}]$$

$$- a_{es}[(ra_{dd} - a_{cd})(1 - r) - ra_{dL} - (\alpha_1 + r\alpha_2)a_{eL}]$$

Assuming that 1/ $a_{dd} \simeq a_{dL} + a_{ds}$, this can be rewritten as

$$dE = a_{es}(1 - r)a_{cd} - r[a_{eL}a_{es} + a_{es}a_{ds} + ra_{dd}]$$

which will be positive for a small value of r.

2. Perfect monopoly with an exogenous money market rate

a. A partial model

The equations representing the supply and demand for bank loans and deposits can be written in the following matrix form:

$$\begin{vmatrix} a_{dd} & -(a_{ds}+\alpha_2 a_{es}) & -(a_{dL}+\alpha_2 a_{eL}) \\ (a_{dd}-a_{cd}-a_{ds}) & (a_{ss}-a_{ds}-\alpha a_{es}) & -(a_{sL}+a_{dL}+\alpha a_{eL}) \end{vmatrix} \begin{vmatrix} di_d \\ di_s \\ di_L \end{vmatrix}$$

$$= \begin{vmatrix} (a_{dd}-a_{cd}-a_{ds}) & -a_{dd} \\ (a_{sL}+a_{dL}+\alpha a_{eL}) & -(a_{dL}+\alpha_2 a_{eL}) \end{vmatrix} \begin{vmatrix} di_L - di_a \\ di_d - (1-r)di_a + dri_a \end{vmatrix}$$

The solution of the model when the securities rate is treated exogenously is of the form:

$$\begin{vmatrix} di_d \\ di_L \end{vmatrix} = \frac{1}{J} \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{vmatrix} \begin{vmatrix} di_s \\ di_a \\ dr \end{vmatrix}$$

and where the terms and their signs are:

1/ This assumption imposes the common restriction of homogeneity of response to interest rate movements. This is not, however, strictly valid in a model which includes noninterest-bearing assets. Thus, the approximate sign is used.

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Variable	Sign
$J = (a_{dd} - a_{cd} - a_{ds} + a_{dL} + \alpha_2 a_{eL})^2$	
$-4a_{dd}(a_{sL} + a_{dL} + \alpha a_{eL})$	< 0
$b_{11} = -2(a_{sL} + a_{dL} + \alpha a_{eL})$	< 0
$b_{12} = (a_{dL} + \alpha_2 a_{eL} + a_{dd} - a_{cd} - a_{ds})$	> 0
$b_{21} = -(a_{dd} - a_{cd} - a_{ds} + a_{dL} + \alpha_2 a_{eL})$	< 0
$b_{22} = 2a_{dd}$	> 0
$c_{11} = a_{ds} + \alpha_2 a_{es}$	> 0
$c_{12} = -(ra_{dd} - a_{cd} - a_{ds})$	> 0
$c_{21} = -(a_{ss} - a_{ds} - \alpha a_{es})$	< 0
$c_{22} = -(a_{eL}(\alpha_1 + \alpha_2 r) + ra_{dL} + a_{sL})$	< 0
$c_{13} = -a_{dd} i_a$	< 0
$c_{23} = -(a_{dL} + \alpha_2 a_{eL}) i_a$	< 0

The signs of the variables are derived under the assumptions

$a_{dd} - a_{ds} - a_{dL} \simeq 0$,	an equal rise in interest rates and no change in expenditure leaves the demand for deposits unchanged. 1/
$a_{ss} - a_{es} - a_{sL} - a_{ds} \simeq 0$,	an equal rise in interest rates leaves the demand for securities unchanged. 1/
$2a_{dL} + \alpha_2 a_{eL} > a_{cd}$	currency is relatively interest insensitive.

r is relatively small, $ra_{dd} < a_{cd} + a_{ds}$, and α is not substantially greater than unity. The assumption about the size of α is needed in the signing of C_{21}, which determines the response of the loan and deposit rates to the securities rate, and the assumption about the size of r is used in the signing of C_{12}, which determines the responses to the money market rate. Even under these assumptions the sign of J is not determined but for stability it is assumed that $J < 0$. Under the assumptions the response of the deposit and loan rates to changes in the

1/ See footnote 1 on page 24.

securities and advances rate and the reserve ratio are: 1/

Table A1.

Change in the:	Response of: Deposit rate	Response of: Loan rate
Securities rate	$\frac{1}{J}(b_{11}c_{11} + b_{12}c_{21}) > 0$	$\frac{1}{J}(b_{21}c_{11} + b_{22}c_{21}) > 0$
Advances rate	$\frac{1}{J}(b_{11}c_{12} + b_{12}c_{22}) > 0$	$\frac{1}{J}(b_{21}c_{12} + b_{22}c_{22}) > 0$
Reserve ratio	$\frac{1}{J}(b_{11}c_{13} + b_{12}c_{23}) < 0$	$\frac{1}{J}(b_{21}c_{13} + b_{22}c_{23}) > 0$

b. The complete model

Using the condition for equilibrium in the securities market, the complete system of equations defining equilibrium in the deposit, loan and security markets in terms of the exogenous policy variables, di_a and dB_{cb}, and dr is given by:

$$\begin{vmatrix} 1 & 0 & -\frac{1}{J}(b_{11}c_{11} + b_{12}c_{21}) \\ 0 & 1 & -\frac{1}{J}(b_{21}c_{11} + b_{22}c_{21}) \\ a_{ds} & a_{sL} & -a_{ss} \end{vmatrix} \begin{vmatrix} di_d \\ di_L \\ di_s \end{vmatrix}$$

$$= \begin{vmatrix} \frac{1}{J}(b_{11}c_{12} + b_{12}c_{22}) & 0 & \frac{1}{J}(b_{11}c_{13} + b_{12}c_{23}) \\ \frac{1}{J}(b_{21}c_{12} + b_{22}c_{22}) & 0 & \frac{1}{J}(b_{21}c_{13} + b_{22}c_{23}) \\ 0 & 1 & 0 \end{vmatrix} \begin{vmatrix} di_a \\ dB_{cb} \\ dr \end{vmatrix}$$

1/ The sign of the response of the deposit and loan rates to the reserve ratio is not immediately apparent from the signs of the b_{ij} and c_{ij} terms, but the terms can be signed by substituting out and using the assumptions.

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The solution of the complete model is of the form:

$$\begin{vmatrix} di_d \\ di_L \\ di_s \end{vmatrix} = \frac{1}{G} \begin{vmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{vmatrix} \begin{vmatrix} di_a \\ dB_{cb} \\ dr \end{vmatrix}$$

where the variables and their signs are:

Variable	Sign
$G = - a_{ss} + \frac{1}{J} [a_{sL}(b_{21}c_{11} + b_{22}c_{21}) + a_{ds}(b_{11}c_{11} + b_{12}c_{21})]$	< 0
$d_{11} = [-a_{ss} + \frac{1}{J}(b_{21}c_{11} + b_{22}c_{21})a_{sL}].[(\frac{1}{J}(b_{11}c_{12} + b_{12}c_{22})]$ $- [\frac{1}{J}(b_{11}c_{11} + b_{12}c_{21})].[\frac{1}{J}(b_{21}c_{12} + b_{22}c_{22})a_{sL}]$	< 0
$d_{12} = \frac{1}{J}(b_{11}c_{11} + b_{12}c_{21})$	> 0
$d_{21} = - [\frac{1}{J}(b_{21}c_{11} + b_{22}c_{21})a_{ds}].[\frac{1}{J}(b_{11}c_{12} + b_{12}c_{22})]$ $+ [- a_{ss} + \frac{1}{J}(b_{11}c_{11} + b_{12}c_{21})\ a_{ds}].[\frac{1}{J}(b_{21}c_{12} + b_{22}c_{22})]$	< 0
$d_{22} = \frac{1}{J}(b_{21}c_{11} + b_{22}c_{21})$	> 0
$d_{31} = - \frac{1}{J}(b_{11}c_{12} + b_{12}c_{22})a_{ds} - a_{sL}[\frac{1}{J}(b_{21}c_{12} + b_{22}c_{22})]$	< 0
$d_{32} = 1$	
$d_{13} = [- a_{ss} + \frac{1}{J}(b_{21}c_{11} + b_{22}c_{21})\ a_{sL}].[\frac{1}{J}(b_{11}c_{13} + b_{12}c_{23})]$ $- [\frac{1}{J}(b_{11}c_{11} + b_{12}c_{21})].[\frac{1}{J}(b_{21}c_{13} + b_{22}c_{23})]$	?

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Variable	Sign
$d_{23} = -[\frac{1}{J}(b_{21}c_{11} + b_{22}c_{21})a_{ds}].[\frac{1}{J}(b_{11}c_{13} + b_{12}c_{23})]$	
$+ [-a_{ss} + \frac{1}{J}(b_{11}c_{11} + b_{12}c_{21})a_{ds}][\frac{1}{J}(b_{21}c_{13} + b_{22}c_{23})]$	?
$d_{33} = -\frac{1}{J}(b_{11}c_{13} + b_{12}c_{23})a_{ds} - a_{sL}[\frac{1}{J}(b_{21}c_{13} + b_{22}c_{23})]$	?

Where in signing these variables we have used the signs of the responses reported in Table A1, and assumed that the responses di_d/di_s and di_L/di_s are not substantially greater than unity. Notice that the response of the securities rate to a central bank purchase of bonds is given by:

$$di_s/dB_{cb} = 1/[-a_{ss} + a_{ds}(di_d/di_s) + a_{sL}(di_L/di_s)].$$

The fall in the securities rate following the purchase of bonds by the central bank will, however, be greater than in the competitive case $(-1/a_{ss})$ since in the monopolistic model the banking system will react by reducing the loan and deposit rates.

The relationship between money and credit aggregates and expenditure and the authorities' policy variables is given by:

$$\begin{vmatrix} dc \\ dMB-drD \\ dM \\ dL \\ dE \end{vmatrix} = -\frac{1}{G}\begin{vmatrix} -a_{ed} & -\alpha_1 a_{eL} & -\alpha_1 a_{es} \\ (ra_{dd}-a_{cd}) & -(ra_{dL}+(\alpha_1+r\alpha_2)a_{eL}) & -ra_{ds}+(\alpha_1+r\alpha_2)a_{es}) \\ (a_{dd}-a_{cd}) & -(a_{dL}+\alpha a_{eL}) & -(a_{ds}+\alpha a_{es}) \\ (a_{dd}-a_{cd}-a_{ds}) & -[a_{eL}+a_{sL}+a_{dL}] & [a_{ss}-a_{es}-a_{ds}] \\ 0 & -a_{eL} & -a_{es} \end{vmatrix}$$

$$\begin{vmatrix} d_{11} & d_{12} & d_{13} \\ (+) & (-) & ?(-) \\ d_{21} & d_{22} & d_{23} \\ (+) & (-) & ?(+) \\ d_{31} & d_{32} & d_{33} \\ (+) & (-) & ? \end{vmatrix} \begin{vmatrix} di_a \\ dB_{cb} \\ dr \end{vmatrix}$$

3. Perfect monopoly with a policy of operating on the monetary base

Using the commercial banks' deposit supply equation (12) and the central bank balance sheet, the advances rate can be written in terms of the monetary base and other interest rates as:

$$(a_{cd} + ra_{dd})di_d + \alpha_1 a_{es} di_s - [r(a_{dd} - a_{cd} - a_{ds}) - \alpha_1 a_{eL}]di_L$$

$$+ r[(a_{dd} - a_{cd} - a_{ds}) - (1 - r)a_{dd}]di_a = (D - ra_{dd}i_a)dr - dMB$$

Using the matrix solution for the case of an exogenous money market rate in the monopolistic model and rearranging to place the advances rate on the left-hand side as an endogenous variable allows us to write the four endogenous interest rates, bank deposit and loan rates, and the security and money market rates in terms of the three central bank policy variables, dB_{cb}, dr and dMB

$$\begin{vmatrix} 1 & 0 & 0 & -\frac{1}{G}d_{11} \\ 0 & 1 & 0 & -\frac{1}{G}d_{21} \\ 0 & 0 & 1 & -\frac{1}{G}d_{31} \\ (a_{cd}+ra_{dd}) & -[r(a_{dd}-a_{cd}-a_{ds}) - \alpha_1 a_{eL}] & \alpha_1 a_{es} & r[(a_{dd}-a_{cd}-a_{ds}) - (1-r)a_{dd}] \end{vmatrix} \begin{Vmatrix} di_d \\ di_L \\ di_s \\ di_a \end{Vmatrix}$$

$$= \begin{vmatrix} \frac{d_{12}}{G} & \frac{d_{13}}{G} & 0 \\ \frac{d_{22}}{G} & \frac{d_{23}}{G} & 0 \\ \frac{d_{32}}{G} & \frac{d_{33}}{G} & 0 \\ 0 & (D-ra_{dd}i_a) & -1 \end{vmatrix} \begin{Vmatrix} dB_{cb} \\ dr \\ \\ dMB \end{Vmatrix}$$

The solution of this matrix system is of the form:

$$\begin{vmatrix} di_d \\ di_L \\ di_s \\ di_a \end{vmatrix} = \frac{1}{M} \begin{vmatrix} N_{11} & 0 & 0 & N_{14} \\ 0 & N_{22} & 0 & N_{24} \\ 0 & 0 & N_{33} & N_{34} \\ N_{41} & N_{42} & N_{43} & 1 \end{vmatrix} \begin{vmatrix} \frac{d_{12}}{G} & \frac{d_{13}}{G} & 0 \\ \frac{d_{22}}{G} & \frac{d_{23}}{G} & 0 \\ \frac{d_{32}}{G} & \frac{d_{33}}{G} & 0 \\ 0 & (D-ra_{dd}i_a) & -1 \end{vmatrix} \begin{vmatrix} dB_{cb} \\ dr \\ \\ dMB \end{vmatrix}$$

where the variables and their signs are:

Variable	Sign
$M = r[(a_{dd} - a_{cd} - a_{ds}) - (1 - r)a_{dd}] + \frac{d_{31}}{G}\alpha_1 a_{es} - \frac{d_{21}}{G}[r(a_{dd} - a_{cd} - a_{ds}) - \alpha_1 a_{eL}] + \frac{d_{11}}{G}(a_{cd} + ra_{dd})$	> 0
$N_{11} = r[(a_{dd} - a_{cd} - a_{ds}) - (1 - r)a_{dd}] + \frac{d_{31}}{G}\alpha_1 a_{es} - \frac{d_{21}}{G}[r(a_{dd} - a_{cd} - a_{ds}) - \alpha_1 a_{eL}]$	> 0
$N_{14} = \frac{d_{11}}{G}$	> 0
$N_{22} = r[(a_{dd} - a_{cd} - a_{ds}) - (1 - r)a_{dd}] + \frac{d_{31}}{G}\alpha_1 a_{es} + \frac{d_{11}}{G}(a_{cd} + a_{dd})$	> 0
$N_{24} = \frac{d_{21}}{G}$	> 0
$N_{33} = r[(a_{dd} - a_{cd} - a_{ds}) - (1 - r)\, a_{dd}] - \frac{d_{21}}{G}[r(a_{dd} - a_{cd} - a_{ds}) - \alpha_1 a_{eL}] + \frac{d_{11}}{G}(a_{cd} + a_{dd})$	> 0

Variable	Sign
$N_{34} = \frac{d_{31}}{G}$	> 0
$N_{41} = -(a_{cd} + ra_{dd})$	< 0
$N_{42} = r(a_{dd} - a_{cd} - a_{ds}) - \alpha_1 a_{eL}$	< 0
$N_{43} = -\alpha_1 a_{es}$	< 0

In signing the variables it is assumed that r, the reserve ratio, is small and the signs of G and the d_{ij} terms are derived in Appendix, section 2b.

The matrix of the direction of interest rate responses can thus be determined as:

$$\begin{vmatrix} di_a \\ di_L \\ di_s \\ di_a \end{vmatrix} = \begin{vmatrix} - & ? & - \\ - & ? & - \\ - & ? & - \\ ?(+) & ? & - \end{vmatrix} \begin{vmatrix} dB_{cb} \\ dr \\ dMB \end{vmatrix}$$

Table A2. Response of Different Objective Variables to Monetary Policy Instruments Under Perfect Competition with an Exogenous Money Market Rate

Policy variable	Central bank bond purchases	Increase in the advances rate	Increase in the reserve ratio
Objective variable			
Currency	$\frac{\alpha_1 a_{es}}{a_{ss}}$	$-(1-r)a_{cd}-\alpha_1\left[a_{eL}+a_{es}\frac{(a_{sL}+(1-r)a_{ds})}{a_{ss}}\right]$	$\left[a_{cd}+\frac{\alpha_1 a_{es}a_{ds}}{a_{ss}}\right]i_a$
	(+)	(-)	(+)
Monetary base	$\frac{[ra_{ds}+(\alpha_1+r\alpha_2)a_{es}]}{a_{ss}}$	$-(a_{cd}+ra_{dL}+(\alpha_1+\alpha_2 r)a_{eL})+r(a_{cd}+a_{dd})$ $-(ra_{ds}+(\alpha_1+\alpha_2 r)a_{es})\frac{(a_{sL}+(1-r)a_{ds})}{a_{ss}}$	$[a_{cd}-ra_{dd}+\frac{(ra_{ds}+(\alpha_1+\alpha_2 r)a_{es})a_{ds}}{a_{ss}}]i_a+D$
	(+)	(?, -)	(+)
Money	$\frac{(a_{ds}+\alpha a_{es})}{a_{ss}}$	$-\frac{(a_{ds}+\alpha a_{es})(a_{sL}+(1-r)a_{ds})}{a_{ss}}$ $+(a_{dd}-a_{cd})(1-r)-a_{dL}-\alpha a_{eL}$	$-[a_{dd}-a_{cd}-\frac{(a_{ds}+\alpha a_{es})}{a_{ss}}a_{ds}]i_a$
	(+)	(?)	(?)
Credit	$-\frac{[a_{ss}-\alpha a_{es}-a_{ds}]}{a_{ss}}$	$\frac{(a_{ss}-\alpha a_{es}-a_{ds})(a_{sL}+(1-r)a_{ds})}{a_{ss}}$ $-(\alpha a_{eL}+a_{sL}+a_{dL})+(1-r)(a_{dd}-a_{cd}-a_{ds})$	$-\left[a_{dd}-a_{cd}-a_{ds}+\frac{(a_{ss}-\alpha a_{es}-a_{ds})}{a_{ss}}a_{ds}\right]i_a$
	(?)	(?)	(?)
Expenditure (Income)	$\frac{a_{es}}{a_{ss}}$	$-\left[\frac{a_{sL}+(1-r)a_{ds}}{a_{ss}}+a_{eL}\right]$	$\frac{a_{es}a_{ds}}{a_{ss}}i_a$
	(+)	(-)	(+)

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