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The Optimal Monetary Aggregate and Monetary Policy

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Summary

The recent upsurge in financial innovations and deregulation in many countries has highlighted the difficulties currently faced by central banks in measuring the monetary aggregates. This paper presents an empirical approach to defining and measuring money that builds upon the concept of an optimum monetary aggregate as one that contributes to the predictability of nominal income. The contribution of the paper is to present a generalized approach to the optimum monetary aggregate and to examine the operational relevance of the concept for monetary policy.

The optimal monetary aggregate is defined as a set of weighted stocks of financial assets, where the weights are derived so as to minimize the forecast variance in nominal income. A weighted aggregate of this type is derived from Australian quarterly monetary data for the sample period 1962(2)-1983(2). The properties of the weighted and unweighted monetary aggregates are compared, with particular emphasis upon criteria relevant for monetary policy. The empirical results suggest that a potentially large reduction in forecast income variance could be obtained by controlling a weighted monetary aggregate, compared with the variance when the unweighted aggregate (M3) is controlled. The major implications of the paper for various issues relating to monetary policy are also discussed, including the stability of money demand, monetary targeting, and financial innovation.

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I. Introduction

In recent years, the acceleration of financial innovation and deregulation have brought into sharp focus the difficulties that currently face central banks in defining and measuring the monetary aggregates. The purpose of this paper is to present a generalized empirical approach to defining money that is tied directly to the goals of monetary policy. The analysis applies the concept of an optimum monetary aggregate discussed in Roper and Turnovsky (1980) and shows how the resulting weighted monetary aggregate can be estimated and made operational for the purposes of monetary policy.

Several empirical approaches have been applied to defining money 1/ including the approach adopted by Friedman and Schwartz (1970), who argue that the definition of money should be related to a particular problem of monetary policy, for example the prediction of nominal income. However, the Friedman-Schwartz method of choosing the best definition of money by regressing income against various monetary aggregates in several sets of equations lacks a strong theoretical basis. In a recent paper, Roper and Turnovsky (1980) approach the problem of defining money in a similar spirit to Friedman and Schwartz but in a way that ensures that the resulting weighted monetary aggregate will satisfy a specified set of conditions that define optimality. Roper and Turnovsky define the optimal monetary aggregate as that which minimizes the forecast variance in nominal income. By using optimal control techniques, a set of weights is derived that meets this condition. However, the contribution by Roper and Turnovsky is of limited operational usefulness to central banks for two reasons. First, the discussion is confined to the two-asset case. Second, the paper is intended as a theoretical contribution and does not offer any guidance as to how a set of empirical weights can be derived from time series data based upon the underlying theory.

Of more direct operational relevance for monetary policy have been several important contributions to improving the informational content of the monetary aggregates, in particular by Tinsley, Spindt, and Friar (1980), Kareken, Muench, and Wallace (1973), and Mitchell (1980). Tinsley et al. have shown that intermediate monetary targeting is equivalent to using an indicator approach in which the noise components of the monetary aggregate control variables are filtered to forecast nominal income.

1/ The problem of defining money has a long history and has been approached from two main perspectives; "a priori" and empirical. The "a priori" approach takes as its departure point the functions of money, emphasising in particular its use as a medium of exchange and as a store of liquidity. The main difficulty with the "a priori" approach is that there may exist only a weak relationship between the theoretical definition of money, the measured monetary aggregates, and monetary policy.

These approaches--in particular, that adopted by Roper and Turnovsky--have several advantages over alternative weighting methods adopted by Chetty (1969) and Barnett (1982). One important potential advantage of the Roper and Turnovsky approach is that if the authorities use intermediate monetary targets to minimize nominal income variance, the problem of choice among alternative monetary aggregates is resolved in principle, since the optimum monetary aggregate will always theoretically satisfy this condition. Because the weights reflect the relation between income and financial shocks, this approach also offers an alternative criterion for "redefining" money in the face of significant financial innovation.

The approach adopted by Chetty (1969) derives liquidity weights by estimating the elasticity of substitution between various assets and a reference asset "money". The main advantage of that approach is that the weights are derived from utility maximization and hence have a clear economic interpretation. The main disadvantage of the Chetty approach is that the resulting weighted monetary aggregate is not tied directly to the goals of monetary policy. An additional disadvantage is that the weights are fixed and, since they are estimated from a reasonably long span of data, require the usual statistical stability properties. Policy shifts such as deregulation during the sample period and the process of financial innovation are likely to destabilize the substitution parameters.

An alternative weighting approach adopted by Barnett (1980, 1982) also derives liquidity weights but uses statistical index number theory. One advantage of the Barnett approach is that, unlike a regression method, it requires only the current observations to compute the weights, allowing the weights to alter as interest rates change over time. However, the Barnett method shares the same limitation of the Chetty approach, namely that the derived weighted monetary aggregate is not tied directly to the goals of monetary policy.

The framework of the paper is as follows. Section II presents the underlying theoretical framework that shows how the optimum weights can be derived and interpreted. Section III applies the theoretical analysis to deriving a set of optimal weights from quarterly Australian monetary data for the sample period 1962(2)-1983(2). The use of Australian monetary data offers a rich field for analysis that has been less extensively studied in comparison with the United States. The sample period is characterized by several important financial innovations, 1/ and two important monetary policy developments, in particular the introduction of announced monetary projections in 1976, and the deregulation of all bank

1/ In particular, two important new deposit instruments were introduced in Australia in 1969, negotiable certificates of deposit and saving-investment accounts by trading and saving banks respectively.

deposit interest rates in 1980(4). A variety of tests are performed that compare the properties of the unweighted and weighted monetary aggregates. In Section IV, the approach set out in Section II is compared with three alternative methods of deriving monetary aggregates; the combination policy approach (Roper and Turnovsky (1980), Poole (1970)), the indicator and intermediate target approach (Tinsley, Spindt, and Friar (1980)), and the informational variable approach (Kareken, Muench, and Wallace (1983)). In the final section, the main implications of the analysis for various issues relating to monetary policy are discussed. These issues include the definition and measurement of money, the stability of money demand, monetary targeting, and financial innovation.

II. Derivation of Optimal Weighted Aggregates

The concept of an optimal monetary aggregate lies at the core of the analysis, and hence the definition adopted and the assumptions that lie behind it need to be stated carefully. At the outset, it is assumed that it is optimal for the authorities to have a monetary rule. The general optimal rule is defined as one in which the control variables are chosen in such a way as to minimize the sum of the squared deviations between the values of the actual and target variables.

Choice of the target, control, and exogenous variables will depend upon the type of problems likely to confront the policymaker. For example, if the policymaker attempts to choose an intermediate monetary target to minimize fluctuations in nominal income, then the target vector will contain nominal income as an argument, while the control variables will consist of either asset stocks or a combination of both asset stocks and controlled asset prices. The vector of the exogenous variables in this case would then contain those variables that are thought to be important in influencing income but that are assumed to be outside the control of the policymaker. If, on the other hand, the policymaker attempts to control the stock of money, then the target vector will contain the stock of money whereas the control vector will contain such arguments as the level of reserves and the rate of interest. Finally, the vector of exogenous variables will contain variables that are considered to be important in affecting both money demand and money supply functions. In the following analysis, it is assumed that the target variable is nominal income and the control variables are a set of asset stocks. 1/ The optimal monetary aggregate is defined as a set of weighted asset stocks where the weights are derived such that the forecast variance in nominal income is minimized.

1/ The model that is specified and estimated is a vector autoregression system in which each variable is assumed to be a lagged function of itself and all other variables.

Consider a simple general equilibrium model with a goods market and n asset markets. In general, the system can be written as:

$$y_t = f_o(y_{t-1}, \ldots; \; A_{1t}, A_{1t-1}, \ldots)$$

$$A_{1t} = f_1(y_t, y_{t-1}, \ldots; \; A_{1t-1}, \ldots, \; A_{2t}, \ldots)$$

$$A_{nt} = f_n(y_t, y_{t-1}, \ldots; \; A_{nt-1}, \ldots) \tag{1}$$

where y_t is nominal income and A_{it} ($i = 1, \ldots n$) is the nominal stock of asset i.

In addition to the above set of equations, the expression defining the optimal monetary aggregate is given by (2):

$$\Psi_t = \mu \sum_{i=1}^{n} \lambda_i A_{it} \tag{2}$$

and

$$\sum_{i=1}^{n} \lambda_i = 1$$

where ψ_t is the optimal monetary aggregate, μ is an arbitrary scale parameter, and λ_i are the optimal weights whereby (n-1) relative weights are assigned to the n assets.

These weights satisfy the property that their sum is unity but, unlike the Chetty and Barnett weights, need not be non-negative. Since the weights reflect cross-correlations between shocks in the target variable and shocks in the control variables, they may assume positive or negative values. As defined earlier, this control aggregate is optimal in the sense that the weights are based on the policymaker choosing the control variables in such a way that the sum of the squared deviations between the actual and target variables is minimized. For example, suppose there is a choice of two control variables, M1 and M2. If the optimal policy is to control only M1 (M2), this implies $\lambda_1 = 1$ ($\lambda_2=1$) and $\lambda_2 = 0$ ($\lambda_1=0$). A combination policy that is optimal is represented by both weights being non-zero.

The reduced-form solution is obtained by solving the model for the n+1 current values of the variables so that each variable is expressed as a lagged function of itself and all other lagged variables. Thus, equation (3) can be interpreted as an unconstrained vector autoregression (VAR) system.

$$y_t = g_1(y_{t-1}, \ldots A_{1t-1}, \ldots A_{nt-1}, \ldots)$$

$$A_{1t} = g_2(Y_{t-1}, \ldots A_{1t-1}, \ldots A_{nt-1}, \ldots)$$

$$A_{nt} = g_{n+1}(Y_{t-1}, \ldots A_{1t-1}, \ldots A_{nt-1}, \ldots) \tag{3}$$

Let V be an n+1 vector of random error terms such that $V \sim N(0,\Sigma)$ where Σ is the variance-covariance matrix. The linearized version of the VAR system is:

$$y_t = \pi_{00} + V_{0t}$$

$$A_{1t} = \pi_{10} + V_{1t}$$

$$A_{nt} = \pi_{n0} + V_{nt} \tag{4}$$

where π_{i0} (i=0, ... n) are the projections of income in period t based upon information at time t-1, and the V_{it} (i=0, ... n) are "noise" components.

The predictor of the target variable income is defined to be optimal in the sense that all available information is being utilized. The available information can be divided into two sets. The first set contains all current and lagged values of the control variables and all lagged values of the target, control, and error variables. The second set contains the current values of the error or noise components of the control variables. Splitting the information set in this way highlights the importance of updating information. The first set consists of variables before information on current control variables is available, while the second set contains those variables that can be observed only after the control variables have been observed. More specifically, the noise components in the second set represent the forecasts errors of the control variables based on information in the first set.

Consider a two-asset model. The first information set at time t is defined as Ω_t' and the total information set as Ω_t. Then, the target variable is determined as:

$$y_t^* = E_t[y_t] \tag{5}$$

$$= P[y_t | \Omega_t] \tag{6}$$

$$= P[y_t | \Omega_t', V_{1t}, V_{2t}] \tag{7}$$

where $E_t[\quad]$ is the expectations operator based upon information at time t, and $P[\quad]$ is the projection operator. 1/

To process the information on the shocks in the control variable equations, the Kalman filter is utilized. The Kalman filter technique has been widely used in control theory and more recently in work on the control of monetary aggregates (see, for example, LeRoy and Waud (1977), Tinsley, Spindt and Friar (1980) and Mitchell (1982)).

Equation (7) can be rewritten as

$$y_t^* = P[y_t|\Omega_t'] + P\{y_t-P[y_t|\Omega_t']|V_{1t}-P[V_{1t}|\Omega_t']|V_{2t}-P[V_{2t}|\Omega_t']\} \tag{8}$$

Equation (8) shows that the prediction of the target variable can be potentially improved by adding on the projection of the noise in the target variables conditional on the noise in the control variables. Now:

$$\begin{aligned} &P\{y_t - P[y_t|\Omega_t'|V_{1t} - P[V_{1t}|\Omega_t'],V_{2t} - P[V_{2t}|\Omega_t']\} \\ &= \phi_1 V_{1t} + \phi_2 V_{2t} \end{aligned} \tag{9}$$

where $\phi_1$and ϕ_2 are the least squares parameters, referred to hereafter as the Kalman coefficients, derived by solving the following system of equations:

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} \sigma_{01} \\ \sigma_{02} \end{bmatrix} \tag{10}$$

The term σ_{0i} is the covariance between the noise in income and the ith asset, σ_i^2 is the variance of the noise in the ith asset, and $\sigma_{12} = \sigma_{21}$ is the covariance between the noise in asset 1 and the noise in asset 2.

The conditional estimate (optimal forecast) of y_t is given by (11).

$$E[y_t] = \pi_{00} + \phi_1 V_{1t} + \phi_2 V_{2t} \tag{11}$$

1/ If it is assumed that the variables are normally distributed, expected values equal projected values.

which can be compared to the unconditional estimate given by (12).

$$E[y_t] = \pi_{00} \tag{12}$$

Comparing (11) and (12) indicates that by exploiting information arising from cross-correlations between the noise in income and the noise in asset variables, one should be able to improve the income forecasts. If the pertinent cross-correlations are zero, it is clear that the conditional and unconditional estimates are equivalent. Furthermore, it may be possible that the asset shocks combine in such a way that the conditional and unconditional estimates are equivalent; i.e., $\phi_1 V_{1t} + \phi_2 V_{2t} = 0$.

By certainty equivalence, we have:

$$y_t^* = E[y_t] \tag{13}$$

where y_t^* is the optimal target level of income.

If we define the Kalman filter least squares regression as:

$$V_{0t} = \phi_1 V_{1t} + \phi_2 V_{2t} + \eta_t \tag{14}$$

where $\eta \sim N(0, \sigma^2)$, we can write:

$$y_t^* = y_t - \eta_t \tag{15}$$

From equation (4), one can write:

$$\phi_1 A_{1t} = \phi_1 \pi_{10} + \phi_1 V_{1t} \tag{16}$$

$$\phi_2 A_{2t} = \phi_2 \pi_{20} + \phi_2 V_{2t} \tag{17}$$

Adding (16) and (17); and solving out V_{1t} and V_{2t} by equation (14):

$$\begin{aligned} \phi_1 A_{1t} + \phi_2 A_{2t} &= \phi_1 \pi_{10} + \phi_2 \pi_{20} + \phi_1 V_{1t} + \phi_2 V_{2t} \\ &= \phi_1 \pi_{10} + \phi_2 \pi_{20} + V_{0t} - \eta_t \end{aligned} \tag{18}$$

Solving (18) for asset 1, we obtain:

$$A_{1t} = \left[\frac{\phi_1\pi_{10} + \phi_2\pi_{20} + V_{0t} - \eta_t}{\phi_1}\right] + \left[\frac{-\phi_2}{\phi_1}\right]A_{2t} \tag{19}$$

i.e. $A_{1t} = \alpha_0 + \alpha_1 A_{2t}$

From the definition of the optimal monetary aggregate (equation (2)), we have:

$$A_{1t} = \frac{\psi_t}{\mu\lambda_1} - \frac{\lambda_2}{\lambda_1} A_{2t} \tag{20}$$

From equations (19) and (20), 1/

$$\alpha_1 = -\frac{\lambda_2}{\lambda_1}$$

i.e., $\lambda_2 = -\alpha_1 \lambda_1$

Using the condition that the weights sum to unity, i.e.,

$\lambda_2 = 1 - \lambda_1$, we have:

$$\alpha_1 = \frac{\lambda_1 - 1}{\lambda_1} \tag{21}$$

i.e., $\lambda_1 = \dfrac{1}{1-\alpha_1} \qquad \lambda_2 = -\alpha_1 \lambda_1$ (21)'

where $\alpha_1 = \dfrac{-\phi_2}{\phi_1}$

1/ Recall that μ is an arbitrary scale parameter; it may be defined such that $\alpha_0 = \dfrac{\psi_t}{\mu\lambda_1}$.

A summary of the solutions for the optimal weights for the case of 2, 3, and 4, n assets is given in Table 1. As shown by Table 1, the general solution for the n asset case is given by (22).

$$\lambda_1 = \frac{1}{1 - \sum_{i=1}^{n-1} \alpha_i} \tag{22}$$

$$\lambda_i = -\alpha_{i-1} \lambda_1 \qquad \forall_i > 1$$

2. Economic interpretation

In general, the larger the Kalman filter coefficient on a particular control variable, the greater the monetary weight placed on that variable. In the above model, the magnitudes of the two Kalman coefficients (ϕ_1, ϕ_2) reflect the responsiveness of the reduced-form income disturbances to the reduced-form asset shocks. For example, consider the limiting case where $\lambda_1 = 1$ and $\lambda_2 = 0$ (α_1=0). In this case, a value of unity is attached to the first control variable. This result occurs if either:

a) $\phi_2 = 0$ or

b) $\phi_1 \rightarrow \infty$

In case (a), shocks in the second control variable contribute nothing toward improving income forecasts. This situation may arise, for example, if the covariances between the noise in the second asset variable and the noise in both income and the first asset is zero. Thus, there is no reason to weight the second asset, since it exerts no effect on the aggregate control variable. In case (b), any shocks in the first asset are amplified into instability in income. This can occur, for example, when the covariance between income and the first asset is very large. In this case, to minimize minimize fluctuations in income, the first asset should be assigned a weight of unity. By placing further restrictions on the structural parameters and disturbance terms, the more familiar cases of the monetary targeting literature emerge (see Roper and Turnovsky (1980), Poole (1970)), which are discussed in Section IV.

Table 1. Solutions to Optimal Weights

Weights	2 Assets	3 Assets	4 Assets	n Assets
λ_1	$\frac{1}{1 - \alpha_1}$	$\frac{1}{1 - \alpha_1 - \alpha_2}$	$\frac{1}{1 - \alpha_1 - \alpha_2 - \alpha_3}$	$\frac{1}{1 - \sum_{i=1}^{n-1} \alpha_i}$
λ_2	$- \alpha_1\lambda_1$	$-\alpha_1\lambda_1$	$-\alpha_1\lambda_1$	$-\alpha_1\lambda_1$
λ_3		$-\alpha_2\lambda_1$	$-\alpha_2\lambda_1$	$-\alpha_2\lambda_1$
λ_4			$-\alpha_3\lambda_1$	$-\alpha_3\lambda_1$
λ_i				$-\alpha_{i-1}\lambda_1$

where,

$$\alpha_1 = -\phi_2/\phi_1$$

$$\alpha_2 = -\phi_3/\phi_1$$

$$\alpha_3 = -\phi_4/\phi_1$$

$$\alpha_{n-1} = -\phi_n/\phi_1$$

III. An Empirical Application

1. Determination of the optimal monetary weights

The purpose of this section is to derive a set of Kalman coefficients and optimal weights from Australian monetary data utilizing the method set out in Section II. To generalize the empirical results, an unconstrained vector autoregression (VAR) system is estimated (see Sims (1980)) and Sargent (1979a)). The VAR approach provides a convenient starting point for empirical estimation since it is unnecessary to specify any specific structural model. However, sets of exclusionary restrictions as implied by most economic theories can be easily imposed and tested by using Granger causality procedures (see Sargent (1979a)). The VAR system is estimated with each variable assumed to have the same lag length. To reduce problems of misspecification from inadequately modeling the dynamic structure of the model, several lag structures are chosen and the sensitivity of the empirical weights to different lag lengths is examined. 1/

The variables chosen in the VAR system consist of one target variable--nominal income, and four control variables--currency, demand deposits, other trading bank deposits, and savings bank deposits. In deriving the weights, we have followed the asset groupings used by the Reserve Bank of Australia for the unweighted M1, M2, and M3 aggregates. This procedure does not address the fundamental issue concerning which set of assets should be included in the definition of money. 2/ Ideally, an approach that begins with a very broad grouping of assets and works backward is preferable. However, quarterly data on non-bank liabilities of the Australian financial sector are not available on a consistent basis until after mid-1976. Further, our approach offers the advantage that the properties of the weighted aggregates can be compared with an unweighted M3 aggregate that until recently was used for targeting purposes in Australia. 3/

1/ An alternative approach that allows for varying lag lengths in a VAR system is to use Hsiao's method (see McMillin and Fackler (1984)). However, use of the Hsiao procedure when there are several variables becomes not only impractical, but it may not lead to the correct lag structure (see Judge et al. (1982)).

2/ The exclusion of assets other than currency, demand deposits, other trading bank deposits, and savings bank deposits implies that an implicit weight of zero has been placed upon them.

3/ Announced target ranges of M3 were abandoned in Australia in January 1985.

The sample period chosen is 1962(2) to 1983(2). The data are quarterly; where in particular, the data on asset stocks are based on monthly averages in order that both nominal income and the asset stock data are centered in the middle of the quarter. 1/ The data have not been seasonally adjusted. Instead, a set of seasonal dummies has been included in each regression. Other deterministic variables such as a constant and a time trend have also been added to the set of regressions. Each VAR system is estimated by OLS; since the lag structure is the same in each equation, OLS is asymptotically efficient and consistent (see Sargent (1979a)). 2/ Excluding the one quarter lag length, the higher order lags satisfy the normal diagnostic tests of absence of serial correlation and maximum explanatory power. Tables 2, 3, and 4 give the optimal weights and the Kalman filter coefficients for the 2, 3, and 4 asset groupings respectively.

A consistent finding to emerge from the empirical results is that the greatest weight tends to be attached to the most liquid asset, currency. 3/ In the simple two-asset case, this result stems directly from the relatively greater magnitude of the currency Kalman coefficient which is about three times as large as the demand deposit coefficient. For the higher-order asset models, the relative ranking of the magnitude of the Kalman coefficients corresponds to the relative weights placed on each asset. 4/

2. Properties of the weighted monetary aggregates

a. Testing the optimality of the weighted monetary aggregate

For policy purposes, the usefulness of the optimal monetary aggregate depends upon whether a sizable reduction in forecast income variance

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1/ The source for all monetary data is the Reserve Bank Bulletin (various issues).

2/ Assuming other conditions are also met. All data are transformed into log-levels. We have not directly taken into account problems of autocorrelation by formulating an explicit stochastic model of the error structure. Instead, we have adopted a more general approach by extending the lag structure and testing for first-order autocorrelation with the usual h-statistic. This approach has the advantage of not imposing the implied within and across-equation restrictions that arise when data are filtered to achieve "white noise" error terms.

3/ In the three asset case with lag = 3, the weight on currency lies slightly below that of demand deposits (see Table 3).

4/ For the two-asset case, $\lambda_1 = \frac{1}{1+\phi_2/\phi_1}$. Thus, the larger is ϕ_1/ϕ_2, the larger is λ_1. In the higher-order asset models, the larger are ϕ_1/ϕ_2, ϕ_1/ϕ_3, and ϕ_1/ϕ_4, the larger is λ_1. Given λ_1, the larger the ratios, ϕ_2/ϕ_1, ϕ_3/ϕ_1, ϕ_4/ϕ_1, the larger are λ_2, λ_3, and λ_4, respectively.

Table 2. Derived Weights and Kalman Coefficients
Two Assets: 1962(2)-1983(2) 1/

Lags	Weights		Kalman Coefficients 2/	
	Currency	Demand deposits	ϕ_1	ϕ_2
2	0.73	0.27	0.38 (0.22)	0.14 (0.12)
3	0.74	0.26	0.33 (0.21)	0.11 (0.11)
4	0.76	0.24	0.33 (0.21)	0.10 (1.11)

1/ Standard errors in parentheses.
2/ Many of the Kalman coefficients are statistically insignificant at a 95 percent level. However, tests of significance cannot be applied to the optimal weights because the equations used to derive the weights involve reciprocals.

Table 3. Derived Weights and Kalman Coefficients
Three Assets: 1962(2)-1983(2) 1/

Lags	Weights			Kalman Coefficients		
	Currency	Demand deposits	Other trading bank deposits	ϕ_1	ϕ_2	ϕ_3
2	0.17	0.16	0.68	0.20 (0.22)	0.19 (0.11)	0.83 (1.84)
3	0.39	0.43	0.19	0.11 (0.21)	0.12 (0.10)	0.05 (5.00)
4	0.43	0.32	0.25	0.15 (0.20)	0.11 (0.10)	0.08 (0.05)

1/ Standard errors in parentheses.

Table 4. Derived Weights and Kalman Coefficients
Four Assets: 1962(2)-1983(2) 1/

	Weights				Kalman Coefficients			
Lags	Currency	Demand deposits	Other trading bank deposits	Savings bank deposits	ϕ_1	ϕ_2	ϕ_3	ϕ_4
2	0.82	0.45	0.20	-0.46	0.32 (0.21)	0.17 (0.11)	0.07 (0.06)	-0.17 (-0.12)
3	1.36	0.87	0.26	-1.49	0.22 (0.21)	0.14 (0.10)	0.04 (0.05)	-0.20 (-0.13)
4	1.06	0.38	0.33	-0.78	0.25 (0.21)	0.09 (0.11)	0.07 (0.05)	-0.18 (-0.15)

1/ Standard errors in parentheses.

can be achieved through targeting the weighted monetary aggregates. In order to assess the relative superiority of the weighted monetary aggregate over the unweighted aggregate, four within-sample tests relevant to monetary policy and monetary targeting are used. 1/ The first three tests are concerned with establishing whether use of the optimal monetary aggregate results in any sizable reduction in forecast income variance. The last test compares the controllability of the weighted and unweighted monetary aggregates.

The first test is concerned with establishing the reduction in variance in the target variable arising from the inclusion of the noise variables into the VAR system. This test compares the non-optimal forecast income variance from the VAR system (V_{0t}) with the optimal forecast income variance (η_t) from the Kalman filter regression. 2/ If the cross-correlation between the noise in income and the noise in the asset variables is small, we can expect only a marginal reduction in forecast income variance. On the other hand, if the asset noise variables are important, a significant reduction in forecast income variance can be expected.

The second test compares the optimal forecast income variance from the Kalman filter regression with the forecast variance of income from a VAR system consisting of income and the unweighted monetary aggregate. This test is a measure of the loss in efficiency that results from aggregating across assets assuming equal weights. 3/

As argued in Friedman and Schwartz (1970), a prerequisite of a suitable monetary aggregate for targeting purposes, whether weighted or unweighted, is that it predicts reliably the target variable, nominal

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1/ Out-of-sample tests based upon an analysis of the dynamic impulse functions of a simulated VAR system (see Sims (1980)) can also be performed. For example, the two bivariate systems, income and money (weighted and unweighted) can be shocked by a one standard deviation shock to both the income and asset noise components. As a dynamic test of the superiority of the optimal aggregate, we would expect to find this bivariate system would show greater dampening in response to all shocks. The results are in general consistent with this prediction and are available from the authors upon request.

2/ The non-optimal variance in income is the variance that results from regressing income against its own lagged values and the lagged values of each asset. The optimal income variance is obtained by regressing V_{0t} against the asset "noise" variables.

3/ As an additional test, a comparison between the reduced-form income variance and the income variance when assets are aggregated with equal weights would also be useful. This type of testing procedure was adopted by Tinsley, Spindt, and Friar (1980), where it was generalized to a multi-target multi-control variable model.

income. The third test compares the forecast variance of income determined from the VAR system containing both income and the unweighted monetary aggregate, with the forecast variance of income derived from a VAR system containing both income and the optimal monetary aggregate as regressors. If the optimal monetary aggregate is a superior predictor of income, we can expect a reduction in the forecast variance of income when this variable is used as a regressor to explain income. 1/

The occurrence of feedback from noncontrol variables to control variables reduces the controllability of these control variables and hence the controllability of the target variables. If the relative importance of these noncontrol variables can be diminished by using the redefined monetary aggregate, an improvement in the controllability of the target variable is also achieved. To test for feedback from the target variable to the control variable, the Granger test of causality is used. If the optimal monetary aggregate is more controllable, then "a priori" we expect the sum of the lags on nominal income in the optimal monetary aggregate equation to be relatively less statistically significant than in the equation with the unweighted monetary aggregate as the dependent variable.

The results of the first three tests are given in Table 5. A consistent finding from all of the above tests is a considerable reduction in forecast income variance ranging from 3-8 percent (first test) to 17-40 percent (second test). In particular, the second test shows that the method presently used by the Australian monetary authorities of attaching equal weight to all monetary assets results in a considerable efficiency loss in predicting nominal income.

The final test is given in Table 6. The results of the Granger causality tests demonstrate that although both the weighted and unweighted monetary aggregates are exogenous in the Granger sense, and are therefore both statistically controllable, the F-statistics show that the lagged values of nominal income in the unweighted monetary aggregate equations are relatively more statistically significant than in the optimal monetary aggregate. By this criterion, the optimal monetary aggregate is marginally more controllable than the unweighted monetary aggregate.

b. Stability analysis

In their discussion of the earlier approaches to weighting the monetary aggregates adopted by Chetty and others, Friedman and Schwartz (1970)

1/ Note that the results of the third test are specific to the estimated VAR model. For example, it is possible to obtain poor results using the estimated VAR model yet obtain satisfactory results using a structural model (and vice versa).

Table 5. Reduction in Forecast Income Variance: 1962(2)-1983(2)

Lags	Number of Assets	Percentage Reduction in Income Variance		
		Test 1	Test 2	Test 3
2	2	5.23	17.67	9.59
3	2	4.18	20.69	10.45
4	2	4.22	23.57	13.93
2	3	7.01	26.97	7.96
3	3	3.04	33.73	8.79
4	3	4.50	34.32	12.24
2	4	8.21	35.26	1.26
3	4	5.91	36.26	6.61
4	4	6.20	39.92	7.88

Table 6. Granger Causality Test

Monetary Aggregate	M3 (unweighted)	M3 (weighted)
F statistic	1.76	0.97

argue that a desirable property of these weighting methods is that the weights are stable. However, in the event of significant institutional changes and financial innovation, it is highly likely that the weights will alter. 1/ Unless explicit allowance is made for variable weights within the Chetty framework as in Boughton (1981), the assumption of fixed weights is a drawback under such conditions. 2/ A priori, it is difficult to predict how the weights will alter in response to financial innovations and policy shifts. The results reported in Table 7 compare the weights obtained for the whole period with four subsamples based upon two important structural shifts that occurred in the Australian financial sector. These shifts were the deregulation of certificates of deposit in 1973(2) (included in other trading bank deposits) and the deregulation of all interest rates on bank deposits in 1980(4). Although the estimated model does not explicitly take into account substitution between assets, these changes are likely to be reflected in the residual shocks to the financial sector and thereby are likely to alter the optimal weights. Table 7 shows that while currency retains its dominant weighting for all subperiods except 1962(2)-80(3), the weights alter among the remaining assets. In particular, the effect of deregulating all interest rates appears to shift the weights more in the direction of their liquidity rankings that is obtained for the whole period. In the earlier period, the less liquid assets, especially savings bank deposits, have a much heavier weight in the aggregate reflecting to some extent the "distortions" induced by regulation. This result suggests that savings bank deposits, that were subject to an interest rate ceiling prior to 1980(4), tended to be used for transactions purposes during the earlier period of regulation.

c. Behavior of Monetary Aggregates

In Chart 1, the quarterly rates of growth of M3 based upon the weights derived for the sample period 1963-83 are compared with the growth of unweighted M3 for the period 1970-83. When the stance of Australian monetary policy is measured by a weighted M3 aggregate, it appears consistently more restrictive than when measured by unweighted M3. The reason is that the above weighting method reflects to a large extent liquidity characteristics and gives highest weight to currency. The sample period 1970-83 was characterized by high and variable inflation rates and nominal interest rates with private agents substituting

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1/ For example, Kane (1964) argues that the weights will alter with secular changes in financial institutions.

2/ Boughton (1981) introduces a dummy variable to allow for the effects of extending Regulation Q to saving and loan associations in September 1966. This institutional change is shown to exercise a significant effect on the weights of saving and loan shares and other financial assets. Horne, Martin and Bonetti (1985) show that the deregulation of returns on bank liabilities in Australia in 1980(4) had the effect of increasing the degree of substitution between bank and nonbank liabilities.

out of the more liquid to less liquid assets. Thus, the weighted monetary aggregate shows a slower growth rate. For this sample period, the weighted monetary aggregate series also show considerably less volatility than the unweighted series.

Table 7. Stability of Weights 1/

Period	Currency	Demand deposits	Other trading bank deposits	Savings bank deposits	Kalman coefficients 2/ ϕ_1	ϕ_2	ϕ_3	ϕ_4
1962(2)-83(2)	1.06	0.38	0.33	-0.78	0.25	0.09	0.07	-0.18
					(0.21)	(0.11)	(0.06)	(-0.15)
1962(2)-73(4)	0.98	-0.49	-0.28	0.78	0.75	-0.38	-0.21	0.59
					(0.37)	(-0.26)	(-0.16)	(0.45)
1962(2)-73(2)	1.00	-0.49	-0.28	0.78	0.74	-0.37	-0.21	0.58
					(0.62)	(-0.27)	(-0.14)	(0.53)
1962(2)-80(3)	0.55	-0.56	0.25	0.76	0.13	-0.13	0.06	0.17
					(0.22)	(-0.14)	(0.14)	(0.43)
1962(2)-81(1)	0.59	0.16	0.09	0.16	0.09	0.02	0.02	0.03
					(0.21)	(0.09)	(0.15)	(0.23)

1/ Maximum lag is four quarters.
2/ Standard errors in parentheses.

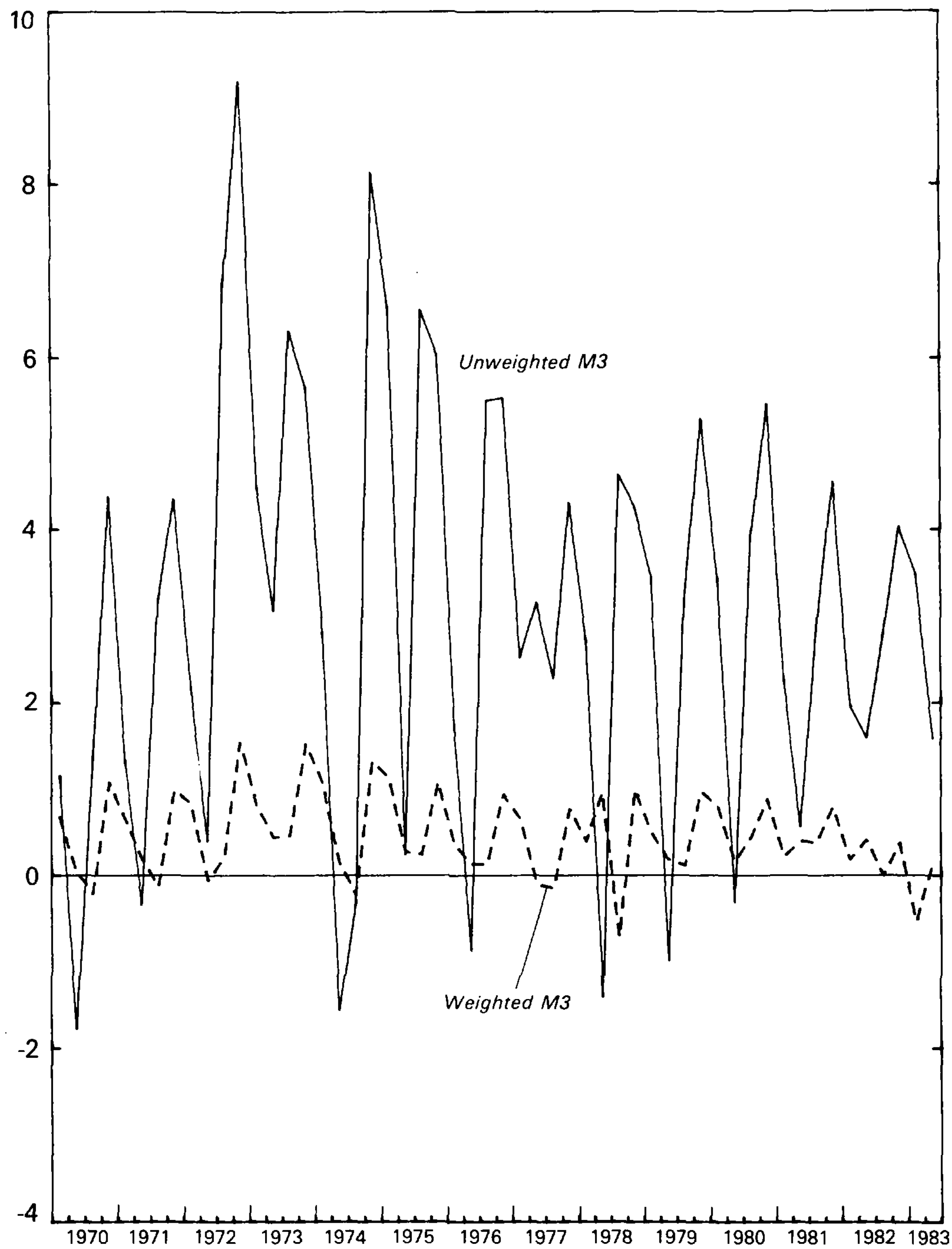
In practice, it may prove difficult for the central bank to achieve the gains in income predictability noted above by using a weighted monetary series. A weighted series available to the central bank would normally be based upon weights estimated from earlier data. 1/ As discussed above,

1/ The Barnett weighted series offer the advantage that they are constructed from current observations. However, for forecasting purposes, forecasts of both interest rates and asset stocks are required.

CHART 1

AUSTRALIA

BEHAVIOR OF WEIGHTED AND UNWEIGHTED MONETARY AGGREGATES: QUARTERLY PERCENTAGE CHANGE, 1970-83



if institutional change and shocks occur between the two periods, the weights may not be stable. Hence, a series based upon the weights derived from an earlier period may not necessarily achieve an improvement in income prediction. In order to determine whether a reduction in forecast variability of income could be achieved in practice, one extension of the analysis would be to run the model each period (for example, each quarter), allowing the weights to vary, and to derive a series based upon an averaging of the weighted and unweighted monetary aggregate.

IV. Relationship to Previous Work on Monetary Aggregates

In this section the method of deriving optimal weights set out in Section II is related to three areas of monetary theory and policy. The first area concerns the derivation of combination monetary policies based upon optimal control techniques; the second area is the use of indicators and intermediate targets in deriving forecasts of target variables; while the third area shows how information variables can be used in achieving targets.

1. Combination policies

The importance of the Roper-Turnovsky (1980) contribution is to show how the weights of a monetary aggregate can be derived from optimal control theory where the chosen weights are optimal in the sense that an income loss function is minimized. The present study shows how the weights can still be derived without resorting to constrained optimization techniques.

To highlight the relationship between the theoretical results of this paper with the results presented in the Roper-Turnovsky study, we re-derive some of their results by making use of the following structural model. The target variable y_t is again assumed to be nominal income and the two control variables are assumed to be the nominal stock of money (x_{1t}) and the price of bonds (x_{2t}).

$$y_t = a_0 + a_1 x_{1t} + a_2 x_{2t} + u_{0t} \tag{23}$$

$$x_{1t} = b_0 + b_1 y_t + b_2 x_{2t} + u_{1t} \tag{24}$$

$$x_{2t} = c_0 + u_{2t} \tag{25}$$

where equation (23) represents the IS schedule with wealth effects (assuming fixed prices), equation (24) represents the LM schedule, and equation (25) shows that the price of bonds does not directly depend upon current values of either nominal income or the money supply. This assumption is used only to close the model and simplify the derivations, and in no way is it supposed to represent a realistic approximation to the behavior of asset markets. Finally, it is assumed that the structural error terms satisfy the property $u_{it} \sim N(0, \sigma_i^2)$, and $i = 0, 1, 2$.

Solving for the current values of the target and control variables yields the reduced form:

$$y_t = \pi_{00} + v_{0t} \tag{26}$$

$$x_{1t} = \pi_{10} + v_{1t} \tag{27}$$

$$x_{2t} = \pi_{20} + v_{2t} \tag{28}$$

where

$$\pi_{00} = \frac{a_0+a_1b_0}{1-a_1b_1} + \left(\frac{a_1b_2+a_2}{1-a_1b_1}\right)c_0 \tag{29}$$

$$\pi_{10} = \frac{b_0+b_1a_0}{1-a_1b_1} + \left(\frac{b_1a_2+b_2}{1-a_1b_1}\right)c_0 \tag{30}$$

$$v_{0t} = \frac{a_1u_{1t}+u_{0t}}{1-a_1b_1} + \left(\frac{a_1b_2+a_2}{1-a_1b_1}\right)u_{2t} \tag{31}$$

$$v_{1t} = \frac{b_1u_{0t}+u_{1t}}{1-a_1b_1} + \left(\frac{b_1a_2+b_2}{1-a_1b_1}\right)u_{2t} \tag{32}$$

$$v_{2t} = u_{2t} \tag{33}$$

The reduced-form (innovation) covariances are defined as:

$$\sigma_{01} = \frac{(a_1b_2+a_2)(b_1a_2+b_2)\bar{\sigma}_2^2 + a_1\bar{\sigma}_1^2 + b_1\bar{\sigma}_0^2}{(1-a_1b_1)^2} \tag{34}$$

$$\sigma_{02} = \frac{(a_1b_2+a_2)\bar{\sigma}_2^2}{(1-a_1b_1)^2} \tag{35}$$

$$\sigma_{12} = \frac{(b_1 a_2 + b_2)\bar{\sigma}_2^2}{(1-a_1 b_1)^2} \tag{36}$$

$$\sigma_1^2 = \frac{(b_1 a_2 + b_2)^2}{(1-a_1 b_1)^2}\bar{\sigma}_2^2 + \frac{b_1^2\bar{\sigma}_0^2}{(1-a_1 b_1)^2} + \frac{\bar{\sigma}_1^2}{(1-a_1 b_1)^2} \tag{37}$$

$$\sigma_2^2 = \bar{\sigma}_2^2 \tag{38}$$

where we have assumed for simplicity that the shocks in the goods, money, and interest equations are independent.

We consider two special cases of this model: an IS curve without wealth effects and a simple monetarist model in which the demand for money is interest inelastic.

Case 1: A simple IS schedule without wealth effects

If there are no wealth effects in the goods market, then we have $a_1 = 0$. The expression for α_1 simplifies to:

$$\alpha_1 = -\frac{a_2\bar{\sigma}_1^2}{b_1\bar{\sigma}_0^2} + b_2 \tag{39}$$

The value of α_1 is negative if:

$$\frac{a_2\sigma_1}{b_1\bar{\sigma}_0^2} > b_2$$

and in this case, using equation (21), we have $0 < \lambda_1, \lambda_2 < 1$. That is, if the ratio of the financial shock to the output shock is greater than $b_2 b_1/a_2$, then the optimal policy is a Poole combination policy where both money and interest rates are used to minimize fluctuations in the target variable, nominal income. Moreover, a positive weight is assigned to both money and the interest rate so that both control variables move in the same direction.

If on the other hand we have:

$$\frac{\bar{\sigma}_1^2}{\bar{\sigma}_0^2} < \frac{b_1 b_2}{a_2} \tag{40}$$

Then α_1 is positive, which implies $\lambda_1 > 1$ and $\lambda_2 < 0$. A Poole combination policy is again optimal but the control variables must move in different directions (see Roper and Turnovsky (1980)).

Case 2: A pure monetarist model

In a simple monetarist model, the price of bonds does not appear in the demand for money function ($b_2 = 0$) and there are no wealth effects in the goods market ($a_1 = 0$). Setting $b_2 = 0$ implies that the value of α_1 in equation (39) is unambiguously negative and therefore a combination policy is again optimal. However, if the money supply is known with certainty ($\bar{\sigma}_2^2 = 0$) then $\alpha_1 = 0$ and the optimal policy is to peg the money supply, i.e., $\lambda_1 = 1$ and $\lambda_2 = 0$. This is a familiar result since it is well known that when the demand for money is solely transactions based, there is a one-to-one relationship between the nominal stock of money and nominal income, and this is a predictable relationship providing that velocity is stable (i.e., $\bar{\sigma}_2^2 = 0$).

If the interest elasticity of investment is zero ($a_2 = 0$), then the value of α_1 simply depends upon the interest elasticity of the demand for money. In the pure monetarist model, however, we have $b_2 = 0$ and therefore $\alpha_1 = 0$, which implies the optimal policy is to peg the money supply.

Other special cases familiar in the literature on monetary targeting and the contribution of Poole (1970) can easily be derived and are given in the Appendix.

2. Indicators and intermediate targets

In a recent paper by Tinsley, Spindt, and Friar (1980), the authors compared the forecasting performance of several filtering techniques. In the simplest case, only the control variables are used to make forecasts of the target variables. This approach is referred to as the method of indicators, and the control variables are called the indicator variables.

Without loss of generality, we assume the policymaker uses only one indicator. If X_{1t} is the chosen indicator variable, the indicator projection equation is given by using the equation for the second control variable to substitute out X_{2t} and can be written as:

$$y_t = \gamma X_{1t} + W_{1t} \tag{41}$$

where for simplicity we have suppressed the deterministic variables. The parameter γ is the filter and W_{1t} is an error term with zero mean and unit variance. The filter, γ, is chosen so as to minimize the variance in the target variable subject to the Kalman filter equation:

$$V_{0t} = \gamma V_{1t} + W_{2t} \tag{42}$$

where W_{2t} is an error term with zero mean and unit variance. This is a "mixed" regression problem with solution: 1/

$$\hat{\gamma} = \frac{E[X_{1t} y_t] + E[V_{0t} V_{1t}]}{E[X_{1t}^2] + E[V_{1t}^2]}$$

$$= \frac{\pi_{10}\,\pi_{00} + \sigma_{01}}{\pi_{10}^2 + \sigma_1^2} \tag{43}$$

The indicator projection equation is:

$$\hat{y}_t^I = \gamma\ P[X_{1t} | \Omega_t']$$

$$= \left[\frac{\pi_{10}\,\pi_{00} + \sigma_{01}}{\pi_{10}^2 + \sigma_1^2}\right] \pi_{10} \tag{44}$$

Tinsley et al. have defined intermediate targeting to be equivalent to using the indicator projection equation plus a feedback term to capture the noise in the indicator variable. The intermediate target projection equation is: 2/

1/ See Tinsley, Spindt, and Friar (1980), p. 65. As Tinsley et al. note, this equation represents the minimum average risk linear (MARL) estimator discussed in Swamy and Tinsley (1980).

2/ See Tinsley, Spindt, and Friar (1980), p. 68.

$$y_t^T = \hat{\gamma}\,\pi_{10} + \hat{\gamma}\,V_{1t} \tag{45}$$

where $\hat{\gamma}$ is defined in equation (43).

From equation (11), the projection equation based on the Kalman filter with only one control variable is:

$$y_t^* = \pi_{00} + \phi_1 V_{1t} \tag{46}$$

The projection equation simply based on the information set Ω_t' is given by the reduced-form equation,

$$y_t^R = \pi_{00} \tag{47}$$

Comparing the optimal projection, y_t^*, with the reduced-form projection, y_t^R, it is clear that these projections will be equal if there is no cross-correlation between the target and control noise components ($\sigma_{01} = 0$). The Kalman filter is zero such that any information in the control variables cannot be used to improve forecasts of the target variable. Equation (44) shows that the indicator forecast $\hat{y}_t^I$ will also be equal to the reduced-form forecast when the indicator variable is deterministic ($\sigma_1^2 = 0$, hence $\sigma_{01} = 0$). 1/

Now suppose the indicator filter equals the Kalman filter. From equation (43) and using the definition of the Kalman filter, we have the following identity:

$$\frac{\sigma_{01}}{\sigma_1^2} = \frac{\pi_{10}\,\pi_{00} + \sigma_{01}}{\pi_{10}^2 + \sigma_1^2}$$

Using the solution for $\phi_1 = \dfrac{\sigma_{01}}{\sigma_1^2}$ (assuming one control variable) from equation (10) and rearranging this expression leads to:

1/ This is a stronger condition since for y_t^* to equal y_t^R we only require $\sigma_{01} = 0$, while still allowing X_{1t} to be stochastic.

$$\phi_1 = \frac{\pi_{00}}{\pi_{10}} \tag{48}$$

Equation (48) highlights several important relationships. First, this equation shows that the Kalman filter regression can be interpreted as an indicator equation. Second, the intermediate targeting as defined by Tinsley et al. and Kalman filtering strategies are shown to be equivalent in this simple case. To see this, substituting (48) into (45) with $\hat{\Upsilon} = \phi_1$ gives equation (46). Therefore, the optimal target strategy given by (46) can be considered as an intermediate target strategy when both the Kalman and indicator filters are equal.

3. Information variables

The information variable approach was first introduced into monetary theory by Kareken, Muench, and Wallace (1973). The main feature that distinguishes their approach from the Poole combination policy approach is that the set of control variables is divided into two sets. The first set contains the policy variables that are given preset values, while the second set contains the information variables that provide information about shocks in the system that in turn lead to revisions in the policy instruments.

To show the relationship between the information approach and that represented here, recall the one-target two-control variable model set out in subsection 1 of Section IV. Without loss of generality, designate the first control variable to be the policy variable (X_{1t}) and the second to be the information variable (X_{2t}). It is assumed that no shocks arise in the information variable ($\bar{\sigma}_2 = 0$), and there are no wealth effects in the goods market ($a_1 = 0$). From equation (14) the optimal Kalman filter regression is:

$$V_{0t} = \phi_1 V_{1t} + \eta_{1t} \tag{49}$$

where

$$\begin{aligned} \phi_1 &= \sigma_{01}/\sigma_1^2 \\ &= b_1\bar{\sigma}_0^2/(b_1^2\ \bar{\sigma}_0^2+\bar{\sigma}_1^2) \end{aligned} \tag{50}$$

The optimal target is now given by:

$$y_t^* = a_0 + a_2X_{2t} + \phi_1 V_{1t} \tag{51}$$

Solving this equation for X_{2t} gives the optimal adjustment equation of the information variable.

$$X_{2t} = \frac{y_t^* - a_0}{a_2} - \frac{\phi_1}{a_2} V_{1t}$$

$$= \theta_0 + \theta_1 V_{1t} \tag{52}$$

where $\theta_0 = (y_t^* - a_0)/a_2$ and $\theta_1 = -\phi_1/a_2$

The restriction that the information variable is deterministic means that (19) can be rewritten as:

$$X_{1t} = \pi_{10} + \frac{V_{0t} - \eta_{1t}}{\phi_1} \tag{53}$$

The optimal trade-off between the policy and information variables is found by substituting equation (49) and (50) into (53):

$$X_{1t} = \pi_{10} + \frac{X_{2t} - \theta_0}{\theta_1} \tag{54}$$

$$= \frac{\theta_1(b_0 + b_1 a_0) - \theta_0}{\theta_1} + (b_1 a_2 + b_2 + \frac{1}{\theta_1}) X_{2t} \tag{55}$$

Equation (55) is the optimal policy trade-off schedule presented in Kareken, Muench and Wallace (1973), that has been shown to be equivalent to both the Poole combination policy (see LeRoy and Waud (1977)) and the Roper-Turnovsky weighted monetary aggregate (see Roper and Turnovsky (1980)).

If the model is extended to allow for shocks in the information variable, this will require extending the Kalman filter regression equation (see Mitchell (1982)). Provided that the covariances between the noise and the information variable are positive together with the additional restriction $\sigma_1^2 \sigma_{02}^2 > \sigma_{21} \sigma_{01}$, information about shocks in the information variable will help stabilize the target variable. The information approach in general will be superior to the Roper-Turnovsky approach when the monetary aggregate excludes stochastic monetary variables that provide information on the target variable.

V. Implications, Extensions, and Conclusions

1. Stability of money demand

The issue of stability of money demand has played a central role in the monetary policy of many countries that emphasise control of the monetary aggregates. More recently, it has been suggested that money demand has become unstable because of financial innovation, possibly reversing the Poole argument for monetary aggregates as intermediate targets. The above analysis shows that for the general case, stability of demand for particular assets (structural shocks) is a critical factor but not the sole one that determines the optimal monetary aggregate. The determination of the optimal aggregate depends on both the variance in asset shocks and the ratio of the cross-correlation between the asset and income shocks to this variance. Nevertheless, stability of demand for particular assets remains of key importance for monetary policy in general and for monetary targeting in particular. In the absence of stable asset demands, the case for a weighted monetary aggregate is considerably weakened.

2. Monetary targets

The method of construction of the optimal monetary aggregate used in this analysis is of direct relevance to the policy of intermediate monetary targeting currently followed in many countries. One important practical problem facing central banks is that of choice among various monetary aggregates when their growth rates diverge significantly (see Howard and Johnson (1983)). 1/ Much of the previous theoretical and empirical work on this issue has been unsatisfactory and inconsistent. For example, the theoretical approach taken by Argy (1983) uses a small structure-specific model. However, the empirical work has not used structural models but relied upon either reduced-form single equation regressions or more general VAR systems (see Friedman (1982), Valentine (1982/3), Horne and Monadjemi (1985), and McMillin and Fackler (1984)). The rationale behind the optimal monetary aggregate is to augment the limited informational content of a single aggregate. The Poole case of attaching all the weight to a single monetary aggregate is shown to be justified only under very restrictive conditions. The optimal monetary aggregate approach offers a solution to the problem of choice of monetary target that is both theoretically and empirically consistent.

1/ In an open economy, the difficulties of defining money and the effectiveness of monetary policy are compounded. See Bryant (1980).

If a weighted monetary aggregate constructed in the above manner were adopted by the central bank, its usefulness would be subject to at least two limitations. The first limitation is that a weighted monetary aggregate may prove more difficult to control than an unweighted aggregate. Although the within-sample test discussed in Section III shows that the optimal monetary aggregate is marginally more controllable than the unweighted aggregate, this result is specific to the particular sample set and may not hold out-of-sample. Further, there is no guarantee that policymakers can adjust the instruments of monetary policy so as to closely control the weighted monetary aggregate. The second limitation of using a weighted monetary aggregate is that the switch to a new monetary regime may invoke Lucas' critique, that is, the induced changes in the variances of asset shocks and covariances between income and asset shocks may destabilize the optimum weights.

3. Definition of money and financial innovation

The optimal monetary aggregate to a certain extent offers a means of resolving a problem currently besetting monetary policy, which is how to redefine the monetary aggregates in the face of considerable financial innovation. For example, in February 1980, the Federal Reserve redefined the U.S. monetary aggregates by including some important financial innovations such as NOW accounts in M1 and by ignoring the difference between types of depository institutions (see Simpson (1980)). However, this strategy is not only unsatisfactory because it need not be consistent with minimizing variability in the target variable but also because it leaves unresolved the issue of when to further redefine the set of assets in the face of continuing and future innovation. In addition, the issue of the correct measurement of the selected set of assets remains. If the optimal monetary aggregate approach discussed above is to be used, it is necessary to obtain sufficient data on the financial innovations before they can be included in the aggregate. Innovations alter the weights of assets already included in the aggregate, thereby affecting the optimal aggregate.

4. Relationship to other weighting methods

While most observers agree that a weighted monetary aggregate is theoretically superior to an unweighted one (see Friedman and Schwartz (1970)), there is less agreement concerning the practical implications, including the method of weighting to be adopted. The chief advantage of the information approach derived above is that it is tied directly to the goals of monetary policy. One disadvantage is that, except in the limiting cases discussed above, it is difficult to give a clear-cut economic interpretation to the weights. Further, the weights need to be estimated from a fairly long span of data although as we have emphasised the method need not be structure-specific. With respect to the first limitation, the Chetty (1969) method is superior to alternative weighting methods since the weighted monetary aggregate is derived explicitly from microeconomic consumer demand theory. The estimated weights have a clear economic interpretation and reflect the elasticity of substitution between various assets and a reference asset "money". However, the Chetty weights need to be estimated, and unlike the weights derived when new information is utilized, are expected to show stability (see Horne, Martin, and Bonetti (1985)). Alternatively, the main advantage of the Barnett index weights is that they are constructed from current observations and hence can change over time. However, these liquidity weights do not have a rigorous economic interpretation, being derived from statistical index number theory. It would then seem to be an advantage to combine these alternative approaches and define money in a way that is both theoretically consistent in terms of economic theory and policy targets, as well as being empirically consistent.

5. Extensions

The empirical work of Section III represents an application of the theoretical framework developed in this paper to a simple model of the monetary sector. It is clear that these results, while promising, suggest that further analysis and tests may be required before a weighted monetary aggregate derived in the above manner can prove to be of direct operational use. The empirical work can be extended to include multiple targets, a wider range of control variables such as asset prices, and the imposition of economic restrictions either in the form of zero restrictions or Bayesian restrictions (see Litterman (1982)). Prior weights can also be assigned to different target variables in order to reflect the relative importance to the policymaker of achieving each policy target. Furthermore, it may also be necessary to introduce prior weights on the control variables to avoid problems of instrument instability (see Sims (1974) and Lane (1984)). Further tests, especially on the "out-of-sample" reduction in forecast income variance that can be achieved by using an optimal monetary aggregate are also desirable.

Although this paper has been concerned primarily with deriving weighted monetary aggregates, the theoretical framework can be applied to various areas. One such application would be to derive the fixed-weighted basket exchange rate in accordance with a policy goal (such as minimizing fluctuations in foreign reserves) and comparing the resulting effective exchange rate with the conventional approach whereby the weights are assumed to be equal to the trade shares of each country. Another possible application is the computation of composite indexes to be used to predict business cycles (see Zarnowitz and Boschan (1975)). Existing composite indexes are based on the aggregation of series where the weights are derived from simple ad hoc procedures. Applying the above method avoids the adoption of ad hoc procedures and leads to weights that are optimal in the sense of minimizing the forecast variance of a given policy target.

6. Conclusions

The purpose of this paper has been to provide a systematic approach to deriving a set of weights for the optimal monetary aggregate that are both theoretically and empirically consistent. The weights are optimal in the sense that the forecast error in the target variable is minimized. The empirical results obtained for the particular sample set and model appear very promising and suggest that a potentially large reduction in forecast income variance could be obtained from using a weighted monetary aggregate constructed in the above manner. However, further extensions of the analysis and tests are desirable to determine whether the optimal monetary aggregate is likely to be of operational use to central banks. Although the approach has been applied to weighted monetary aggregates, there also exist many other potential areas in which the technique can be applied. The challenge for future research in weighted monetary aggregates is to combine the alternative approaches to defining money in a way that captures the best features of each and that is of relevance for monetary policy.

Further Special Cases of the Monetary Model

Case 3 - A pure Keynesian model

In a simple Keynesian model, the interest elasticity of investment is hypothesized to be 'low', whereas the interest elasticity of the demand for money is hypothesized to be 'high.' In the limiting case the interest elasticity of investment is zero, $a_2 = 0$, while the interest elasticity of the demand for money is infinite, $b_2 \to \infty$ (a liquidity trap). Given that there are no wealth effects in the goods market, this implies that $\alpha_1 \to \infty$ and hence the optimal policy is to control the interest rate.

Case 4 - No output shocks, no wealth effects

If there are no disturbances originating in the goods market, $\bar{\sigma}_0^2 = 0$ and given $a_1 = 0$, then $\alpha \to -\infty$ and the optimal policy is to stabilize the interest rate. If the only direct linkage between income and the asset market is through the price of bonds, minimization of output variance can be achieved by controlling the interest rate. In particular, the optimal variance will be zero. In the case of independent shocks, the optimal variance of income is given by:

$$\sigma_0^2 = \frac{\bar{\sigma}_0^2 + a_1^2\bar{\sigma}_1^2 + (a_1b_2+a_2)^2\bar{\sigma}_2^2}{(1-a_1b_1)^2}$$

and this is equal to zero when there are no wealth effects in the goods market, there are no shocks originating from the goods market, and the interest rate is pegged. This result was originally derived by Poole (1970), and later by Roper and Turnovsky (1980).

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