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DM/85/62

INTERNATIONAL MONETARY FUND

Research Department

The Stability of the Demand for International Reserves

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October 1, 1985

Since the availability of international reserves and liquidity can have an important influence on the timing, duration, and intensity of balance of payments adjustment policies, it has been long argued "that the satisfactory working of the adjustment process can be hampered by an imbalance in either direction between the supply and demand for world reserves." 2/ One of the principal elements in measuring the adequacy of a given stock of international reserves is therefore to identify the levels of reserves that would be demanded under alternative exchange rate arrangements and macroeconomic conditions. In this regard, the key empirical question is the stability of the global demand for reserves.

If, on the one hand, it is reasonably predictable on the basis of a few determinants, and if these determinants themselves can be forecast with reasonable reliability, then the level of reserves that is needed in a given situation can be predicted fairly well, and there will be a presumption in favor of devising institutional mechanisms to ensure that the supply of reserves grows in step with demand. If, on the other hand, the demand for reserves is not at all predictable, or depends on determinants that cannot themselves be predicted, the question naturally arises whether the benefits of control over international liquidity are worth the institutional costs of putting the necessary control mechanisms in place. In such circumstances, an alternative approach might be to have an elastic supply mechanism that enabled countries to have the reserves they desired. 3/

1/ This paper has benefited from the comments and suggestions of Andrew Crockett, Michael Dooley, Mohsin Khan, and Franco Spinelli. Kellett Hannah provided his usual excellent computational and programming assistance.

2/ Hirsch (1979), p. 146.

3/ Crockett (1978), p. 8.

Empirical studies undertaken in the 1970s and 1980s generally found well-established statistical relationships for the demand for international reserves, apart from the period surrounding the collapse of the Bretton Woods system. ^{1/} However, since these studies were based on data samples from the mid-1960s to the late 1970s, it is unclear whether these estimated demands remained stable during the period of disturbances in international financial markets during the early 1980s. These disturbances may have had a significant effect on reserve holdings since throughout the 1960s and 1970s many countries resorted to borrowing in international financial markets as a means of accumulating reserves. ^{2/} While this borrowing provided a relatively low cost means of augmenting reserves, it has also increased the vulnerability of the reserve system to disturbances in financial markets such as occurred in 1981 and 1982. The reduced access of many developing countries to international financial markets during 1981 and 1982 contributed to a sharp decline in existing international liquidity and prospects for the future availability of borrowed reserves. The impact of the disturbances in financial markets on the availability of reserves therefore raises the issue of whether the estimated models of the demand for reserves, which are based on data samples from the 1960s and 1970s, have remained stable for all countries or country groups during the early 1980s.

This paper examines the recent stability of the demand for international reserves by comparing the estimation results obtained for a set of representative models of this demand for a sample period encompassing the 1960s and 1970s with the results generated by using an enlarged sample period including data from the early 1980s. These results are used to consider whether the relative importance of the various determinants of the long-run demand for reserves has changed over time and whether the speed of adjustment of actual to desired reserves has been altered by the emergence of disturbances in international financial markets in the early 1980s. To examine these issues, the rest of this paper is divided into three sections. Section I first describes the models of the demand for reserves that are used in our analysis and then considers the estimates obtained for these representative models of the demand for reserves for samples of data for the 1960s and 1970s. These models include both

^{1/} For example, Frenkel (1983, p. 86) has argued that:

The evidence indicates that countries have continued to hold and use international reserves and that they have chosen to manage their exchange rates rather than let them float freely. This suggests that the move to a floating exchange rate regime has not reduced significantly the need for international reserves; nor has it removed the need to establish clearly the means of and mechanisms for providing such reserves.

^{2/} See Masera (1983) for a discussion of this point.

equilibrium and disequilibrium formulations of the demand for reserves and provide estimates for different groups of developed and developing countries. In order to examine the stability of the demand for international reserves, the second section re-estimates these models on an extended sample which includes data through 1982. The final section summarizes our results.

Three general conclusions emerge from our analysis. First, the shifts in the demands for reserves associated with the disturbances in international financial markets during the early 1980s for many country groups were as significant as those experienced during the collapse of the Bretton Woods system in the early 1970s. Second, for the equilibrium formulations of the demand for reserves (the demand equals the supply of reserves at all times), this structural instability reflected an increase in the sensitivity of the demand for reserves to balance of payments variability and the economy's openness (as measured by the ratio of imports to income). Finally, for disequilibrium formulations of the demand for reserves (which allow desired and actual reserves to diverge), instability appears to be more closely associated with changes in the speed of adjustment of actual reserves to the desired level than with changes in the structure of the long-run demand for reserves. In the disequilibrium formulations, there is more evidence of instability in the demand for reserves on the part of developing countries during the early 1980s than for developed countries.

I. Empirical Models of the Demand for International Reserves

Although there have been numerous empirical studies of the demand for international reserves, most analyses have used a fairly standard specification of the determinants of a country's desired stocks of reserves and have employed one of two basic hypotheses about how rapidly actual reserve holdings adjust to desired holdings. One hypothesis has been that actual reserves always adjust to desired holdings during the observation period (continuous equilibrium). ^{1/} In contrast, many recent studies have assumed that the behavior of reserve holdings can be described by a stock adjustment (disequilibrium) process in which changes in reserves during a given period are related to the gap (at the beginning of the period) between desired and actual reserves. ^{2/} ^{3/}

^{1/} See, for example, Frenkel (1974), Frenkel and Hakkio (1980), and Frenkel (1983).

^{2/} See, for example, Bilson and Frenkel (1979a, 1979b), Edwards (1980), Frenkel (1983), and Edwards (1984). Theoretical discussions on the speed of adjustment are presented in Clark (1970a) and Claassen (1975).

^{3/} Surveys of the literature on the demand for reserves includes Grubel (1971), Williamson (1973), and Cohen (1975). See also International Monetary Fund (1970) for a collection of useful papers.

Both the equilibrium and disequilibrium formulations have used relatively similar specifications of the determinants of the long-run demand for reserves. In general, it has been assumed that the long-run demand for international reserves is determined by such factors as the size of the country's international transactions (usually proxied by either the country's level of income or imports), the average propensity to import (to represent either the economy's openness or its marginal propensity to import), 1/ and some measures of the variability of external transactions. Efforts were sometimes made to incorporate the opportunity cost of holding reserves, but these studies generally found it difficult to measure this cost or were unable to identify significant parameter estimates. 2/

As long as a distinction was made between the behavior of various country groups, empirical studies based on either equilibrium or disequilibrium formulations have generally yielded estimated demands for international reserves that have been regarded as relatively stable, except for the period surrounding the collapse of the Bretton Woods system. 3/ While the most common division has been between developed (industrial) and developing countries, some studies have adopted more detailed breakdowns for the developing countries especially to distinguish between the

1/ The sign of the coefficient on the average propensity to import in the demand for reserves could potentially be positive or negative. If the average propensity represents the economy's openness, then a positive coefficient would be expected since greater openness would cause a country to hold more reserves. If the average propensity represents the marginal propensity to import, then a higher marginal propensity would allow for smaller reserve holdings, since relatively small adjustments in income would lead to relatively large changes in imports. For example, Heller and Khan (1978) find the coefficient to be negative and interpret the average propensity to import as a proxy for the marginal propensity to import. In contrast, Frenkel (1983) finds a positive coefficient and interprets the average propensity to import as a measure of the openness of the economy.

2/ See, for example, Kenen and Yudin (1965), Heller (1966), Clark (1970b), Iyoha (1976), and Hipple (1979). Some positive results were obtained by Frenkel and Jovanovic (1981).

3/ For discussions on the characteristics of the demand for reserves under alternative exchange rate systems, see Makin (1974), Suss (1976), Crockett (1978), Heller and Khan (1978), Bilson and Frenkel (1979b), Saidi (1981), and Frenkel (1983).

behavior of oil exporting countries and non-oil developing countries. ^{1/} Even with such a country breakdown, there was some evidence of structural instability in the demands for reserves during the early 1970s. In studies which assumed continuous equilibrium between actual and desired reserve holdings, it has generally been argued that the move towards greater exchange rate flexibility in the early 1970s was accompanied by reduced holdings of reserves for both industrial and developing countries and greater sensitivity of these holdings to the variability of external transactions. In contrast, studies of the demand for reserves based on a stock adjustment process suggest that the structural instability has also reflected changes in the speed of adjustment of actual to desired reserves. Apart from this evidence of structural instability of the demands for reserves during 1972-73, these studies found that the demand relationships remained stable within each country group throughout the rest of the 1970s.

There have been a number of developments during the late 1970s and early 1980s, however, which raised the prospect of further structural changes in the traditional demand for reserve equations. In particular, disturbances in financial markets in 1981 and 1982 fundamentally altered the access of many developing countries to these financial markets and thereby sharply reduced their ability to obtain borrowed reserves. These countries were thus confronted with the high real cost of acquiring reserves through current account surpluses. Any empirical formulation of the demand for reserves which does not reflect the effects of changes in the cost of acquiring reserves would be unlikely to remain stable during periods such as the early 1980s.

In this paper, the structural stability of the demand for reserves is examined by comparing the estimated parameters for a set of representative models of this demand that are first obtained using data samples from the 1960s to the late 1970s with those estimates derived from expanded data sets which encompass observations through 1982. In considering the estimates for the period of the 1960s and 1970s, there is also a comparison between the results reported in the original studies with the estimates obtained using the latest revised data for that period. This two-step comparison is used since there are two potential reasons why new estimates may differ from the original estimates. One possibility is that data revisions may lead to new estimated parameters that differ from the original parameter estimates even if the same sample period is selected. In general, our results indicate that the original conclusions regarding the structural stability of the demands for reserves and the relative importance of the determinants of these demands are supported by the results based on revised data. In some cases, however, data

^{1/} See Heller and Khan (1978) and von Furstenberg (1982).

revisions have created a situation where the measures of balance of payments instability appear to be less significant than in the original studies. A second reason why parameter estimates may change over time is that the underlying structure of the demand function may shift. Expanding the original samples to include data through 1982 leads to test statistics which imply that there were such shifts in the structural parameters of the long-run demand for international reserves and the speeds of adjustment of actual to desired reserve holdings for a number of country groups.

In selecting representative models of the demand for reserves, our analysis focuses on models that have been estimated on the basis of data reported in the International Financial Statistics (IFS). Since a variety of equilibrium and disequilibrium models have been estimated on the basis of such samples, this does not seriously constrain our analysis; but it does simplify the problems of maintaining the comparability of data and the updating of the original data sample.

1. Equilibrium models of the demand
for international reserves

While there have been a large number of studies which have employed equilibrium models of the demand for international reserves, those by Heller and Khan (1978) and by Frenkel (1983) provide representative results. ^{1/} The Heller-Khan analysis assumed that the demand for international reserves (R_t) for various country groups depended on the level of imports (I_t), the average propensity to import (I_t/Y_t), and the variability of the country's balance of payments (σ_t^2). Thus,

$$(1) \ln R_t = \alpha_0 + \alpha_1 \ln(I_t/Y_t) + \alpha_2 \ln I_t + \alpha_3 \ln \sigma_t^2 + u_t,$$

where

\ln = natural logarithm,

$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$, and

ε_t = a random error term.

This formulation allowed for correlated error terms and was estimated on a sample of quarterly observations from 1964 to 1976 for the world, the

^{1/} See Table 1 for a summary of the structures of the models used in this analysis.

Table 1. Structure of Representative Models of the Demand for Reserves ^{1/}

Equilibrium Models

Heller-Khan (1978)

$$\ln R_t = \alpha_0 + \alpha_1 \ln(I/Y_t) + \alpha_2 \ln I_t + \alpha_3 \ln \sigma_t^2 + u_t$$

where $u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$

R_t = stock of reserves

I_t = imports

Y_t = income

σ_t^2 = measure of balance of payments variability

ε_t = random error term

Frenkel (1983)

$$\ln R_t = \beta_0 + \beta_1 \ln \alpha_t + \beta_2 \ln Y_t + \beta_3 \ln m_t + \varepsilon_t$$

where R_t = stock of reserves

α_t = measure of balance of payments variability

Y_t = income

m_t = average propensity to import

ε_t = random error term

Disequilibrium Models

Bilson-Frenkel (1979)

- a. Country (n) Target Reserve Holdings (estimated as a cross-section regression using averages [across time] for dependent and independent variables)

$$\ln R_n^D = \beta_0 + \beta_1 \ln \alpha_n + \beta_2 \ln Y_n + \beta_3 \ln m_n + \varepsilon_n$$

where R_n^D = average target level of reserves for country n

α_n = average balance of payments variability for country n

Y_n = average income of country n

m_n = average propensity to import of country n

ε_n = specific factor in the demand for reserves of country n

- b. Stock Adjustment Process (pooled time series and cross-section regression)

$$\ln R_{n,t} - \ln R_{n,t-1} = \alpha + \gamma (\ln R_{n,t}^D - \ln R_{n,t-1}) + \varepsilon_{n,t}$$

where $\varepsilon_{n,t}$ = random error term

Frenkel (1983)

- a. Country (n) Target Reserve and Money Holdings (estimated as cross-section regressions using averages [across time] for dependent and independent variables)

$$\ln R_n^D = \beta_0 + \beta_1 \ln \alpha_n + \beta_2 \ln Y_n + \beta_3 \ln m_n + \varepsilon_n$$

$$\ln (M/P)_n^D = \gamma_0 + \gamma_1 \ln Y_n - \gamma_2 r_n + \varepsilon'_n$$

where $(M/P)_n$ = average real stock of money in country n

r_n = average rate of interest in country n

ε_n = specific factor in the demand for reserves of country n

ε'_n = specific factor in the demand for money in country n

- b. Stock Adjustment Process (pooled time series and cross-section regression)

$$\begin{aligned} \ln R_{n,t} - \ln R_{n,t-1} &= \lambda_1 (\ln R_{n,t}^D - \ln R_{n,t-1}) \\ &+ \lambda_2 (\ln (M/P)_{n,t}^D - \ln (M/P)_{n,t-1}) + \varepsilon_{n,t} \end{aligned}$$

where $\varepsilon_{n,t}$ = random error term

^{1/} See Appendix for detailed definition of variables used in each study.

world excluding oil-exporting countries, the world excluding oil-exporting countries and the United States, the industrial countries, industrial countries excluding the United States, and the developing countries. ^{1/}

Table 2 compares the results obtained in the original Heller-Khan study, a re-estimation of the Heller-Khan model for the original sample period but on the basis of the latest revised data for that period, and the estimation results obtained with a sample that includes observations from 1974 to 1982. For the sample period from the first quarter of 1964 to the fourth quarter of 1976, the re-estimation of the Heller-Khan model leads to estimates of the parameters associated with the average propensity to import and levels of imports that are of the same sign and significance as in the original study. However, the parameter on the variability measure is less significant than in the original analysis, and the structure of the variability measure is different.

In the Heller-Khan model, the variability measure was constructed using a two-step procedure. In the first stage, auto-regressive integrated moving average (ARIMA) models for reserves were estimated for the six country groupings. This involved transforming the level of reserves (R_t) into a stationary series R_t^* (by taking the logarithmic first differences) and then fitting the ARIMA model of the form $\Phi(L)R_t^* = \Theta(L)v_t$ where $\Phi(L)$ and $\Theta(L)$ are polynomial functions of the lag operator, L , and v_t are serially uncorrelated, white noise errors. In the second stage, equation (1) was estimated in the form

$$(2) \ln R_t = \alpha_0 + \alpha_1 \ln(I_t/Y_t) + \alpha_2 \ln I_t + \alpha_3 \sum_{i=0}^{12} a_i \ln \hat{v}_{t-i}^2 + u_t$$

where the \hat{v}_t s are the residuals from the ARIMA model. The a_i s, which are the weights attached to the current and lagged values of \hat{v}_t^2 s, were estimated as a second degree polynomial with a constraint of zero at the twelfth lag. The variability measure was then constructed as a weighted average of the \hat{v}_t^2 s, using the weights obtained from this estimation, starting at the period when the weight became significantly positive at the 5 percent level and stopping at the last statistically significant weight.

In the original analysis, the time series behavior of reserves for five of the six country groups was characterized by first-order auto-regressive and first-order moving average processes; whereas the relevant model for the less developed countries was an auto-regressive process

^{1/} The correlation in the error term (u_t) could be interpreted as allowing for lagged adjustment in reserves. See Heller and Khan (1978).

Table 2. Estimates of the Heller-Khan Model of the Demand for International Reserves ^{1/}

$$\ln R_t = \alpha_0 + \alpha_1 \ln(I/Y)_t + \alpha_2 \ln I_t + \alpha_3 \ln \sigma_t^2$$

Country Grouping	α_0	α_1	α_2	α_3	ρ_1	ρ_2	SEE	\bar{R}^2	DW
<u>Original Sample: (First Quarter 1964--Fourth Quarter 1976)</u>									
World	0.192 (0.27)	-0.713 (7.06)	0.854 (8.64)	0.999 (4.12)	1.399 (10.80)	-0.473 (3.60)	0.026	0.72	2.04
World (excluding oil exporting countries)	1.151 (1.76)	-0.608 (6.79)	0.691 (8.17)	1.064 (4.80)	1.442 (11.71)	-0.555 (4.47)	0.026	0.77	2.27
World (excluding oil exporting countries and the United States)	2.211 (4.15)	-0.498 (7.05)	0.600 (8.50)	1.052 (6.28)	1.522 (13.38)	-0.643 (5.74)	0.029	0.82	2.30
Industrial countries	-0.620 (0.60)	-0.770 (4.12)	0.852 (5.98)	0.789 (2.14)	0.916 (18.92)	--	0.037	0.50	1.72
Industrial countries (excluding the United States)	0.428 (0.51)	-0.635 (3.67)	0.757 (5.74)	0.686 (2.35)	0.918 (15.92)	--	0.047	0.43	1.53
Less developed areas	0.522 (1.24)	-0.290 (4.88)	0.687 (6.32)	0.395 (0.84)	1.300 (9.18)	-0.360 (2.60)	0.040	0.45	1.84
<u>Revised Sample: (First Quarter 1964--Fourth Quarter 1976)</u>									
World	-2.077 (2.02)	-1.067 (7.26)	1.116 (9.01)	0.041 (1.54)	1.312 (9.94)	-0.394 (3.11)	0.027	0.73	2.13
World (excluding oil exporting countries)	-2.023 (1.89)	-1.027 (6.46)	1.030 (7.21)	0.007 (0.30)	1.354 (10.34)	-0.423 (3.30)	0.029	0.53	2.21
World (excluding oil exporting countries and the United States)	-0.421 (0.52)	-0.892 (7.05)	0.913 (8.58)	0.057 (2.45)	1.435 (11.54)	-0.529 (4.52)	0.031	0.70	2.20
Industrial countries	-1.892 (1.33)	-0.978 (4.28)	0.974 (5.09)	-0.004 (0.13)	0.941 (27.33)	--	0.038	0.33	1.38
Industrial countries (excluding the United States)	0.329 (0.29)	-0.779 (3.92)	0.800 (4.96)	0.074 (2.13)	0.931 (18.43)	--	0.045	0.40	1.29
Less developed areas	1.425 (1.17)	-0.480 (2.31)	0.470 (2.25)	--	1.617 (13.64)	-0.629 (5.43)	0.033	0.06	1.90
<u>Extended Sample: (First Quarter 1974--Fourth Quarter 1982)</u>									
World	-0.397 (0.68)	-0.709 (5.27)	0.858 (12.58)	--	0.703 (6.06)	--	0.031	0.84	1.72
World (excluding oil exporting countries)	-2.858 (2.21)	-0.982 (5.54)	1.099 (8.84)	-0.045 (0.92)	0.771 (10.23)	--	0.034	0.77	1.73
World (excluding oil exporting countries and the United States)	-1.774 (2.740)	-0.891 (6.63)	1.020 (13.29)	-0.020 (1.05)	0.705 (7.33)	--	0.036	0.84	1.81
Industrial countries	-1.679 (2.00)	-0.865 (5.06)	0.981 (9.18)	--	0.752 (7.45)	--	0.040	0.72	1.65
Industrial countries (excluding the United States)	-1.253 (2.15)	-0.844 (6.00)	0.969 (12.08)	--	0.670 (6.02)	--	0.042	0.81	1.79
Less developed areas	-0.014 (0.01)	-0.581 (2.21)	0.768 (3.16)	--	1.337 (8.26)	-0.411 (2.53)	0.038	0.29	2.15

^{1/} t-values in parentheses below coefficients; SEE = standard error of the estimate; and DW = Durbin-Watson statistics.

with first-order, fourth-order, and fifth-order terms (Table 3). The re-estimation of these ARIMA relationships for the period from the second quarter of 1960 to the fourth quarter of 1976 using revised data yields similar results. Despite these similar ARIMA estimates, however, the measures of variability have been affected by the revision in the data base. The use of revised data for the period from the second quarter of 1960 to the fourth quarter of 1976 implies that: (1) the calculation of the variability measure employs different lags than in the original study (see last column of Table 3); and (2) the parameter estimates associated with the variability measure are generally less significant (see Table 2). Given the changes in the variability measure, it is not surprising that the parameter estimates associated with imports and the average propensity to import are also somewhat different.

In analyzing their results, Heller and Khan also concluded that there had been a significant shift in the demands for reserves for the various country groups during the period of the collapse of the Bretton Woods system. To test for structure stability of the demand for reserves, the authors used three tests: the cusums test (to detect gradual or systematic structural changes); the cusums-squared test (to detect random changes); and the Quandt log-likelihood ratio test (to identify the specific quarter the change took place). The Quandt log-likelihood ratio test implied a shift in the demand for reserves in the fourth quarter of 1973 for all country groups except the developing countries. For the latter group, the shift appeared to have occurred in the second quarter of 1972. The estimation results using the revised data yield a similar conclusion. Thus, the re-estimation of the Heller-Khan model for the original sample period but using revised data generally confirms that there is a stable demand for reserves that suffered a structural shift in the 1972-73 period; although the results regarding the variability measure are less satisfactory than in the original study.

Frenkel's (1983) results are similar to those obtained by Heller and Khan. While the Frenkel study incorporated both equilibrium and disequilibrium versions, this section focuses on the results obtained for the equilibrium model. The original sample was composed of data for 22 industrial countries for the 1963-79 period and 32 developing countries for the 1963-77 period. The model for the demand for reserves is (see Appendix for definition of terms):

$$(3) \ln R_t = \beta_0 + \beta_1 \ln \sigma_t + \beta_2 \ln Y_t + \beta_3 \ln m_t + u_t$$

where

R_t = gross reserves in real terms;

σ_t = variability of international receipts and payments;

Y_t = real income; and

m_t = average propensity to import.

Table 3. ARIMA Model Estimates for Reserves and Lags Used
in Weighted Averages for Variability Measure 1/

Country Grouping	ARIMA Model	Lags Used (in quarters)
<u>Original Sample: (Second Quarter 1960--Fourth Quarter 1976)</u>		
World	$(1 - 0.867L) \{(1 - L)\ln R_t - 0.022\} = (1 + 0.548L)u_t$ (8.52) (2.25) (3.19) SSR = 0.044 ; $\chi^2 = 9.65$; DF = 10	6-12
World (excluding oil exporting countries)	$(1 - 0.857L) \{(1 - L)\ln R_t - 0.019\} = (1 + 0.536L)u_t$ (7.89) (2.00) (2.99) SSR = 0.045 ; $\chi^2 = 12.89$; DF = 10	6-12
World (excluding oil exporting countries and the United States)	$(1 - 0.841L) \{(1 - L)\ln R_t - 0.026\} = (1 + 0.459L)u_t$ (7.75) (2.27) (2.55) SSR = 0.054 ; $\chi^2 = 16.74$; DF = 10	6-12
Industrial countries	$(1 - 0.818L) \{(1 - L)\ln R_t - 0.017\} = (1 + 0.607L)u_t$ (4.74) (1.95) (2.54) SSR = 0.075 ; $\chi^2 = 11.25$; DF = 10	7-12
Industrial countries (excluding the United States)	$(1 - 0.794L) \{(1 - L)\ln R_t - 0.025\} = (1 + 0.400L)u_t$ (5.22) (2.16) (2.28) SSR = 0.110 ; $\chi^2 = 13.22$; DF = 10	7-12
Less developed areas	$(1 - 0.530L - 0.417L^4 + 0.462L^5) \{(1 - L)\ln R_t - 0.030\} = u_t$ (5.03) (3.67) (4.09) (3.84) SSR = 0.067 ; $\chi^2 = 7.56$; DF = 9	1-5
<u>Revised Sample: (Second Quarter 1960--Fourth Quarter 1976)</u>		
World	$(1 - 0.804L) \{(1 - L)\ln R_t - 0.022\} = (1 + 0.436L)u_t$ (6.24) (2.54) (2.16) SSR = 0.043 ; $\chi^2 = 9.41$; DF = 10	4-12
World (excluding oil exporting countries)	$(1 - 0.870L) \{(1 - L)\ln R_t - 0.019\} = (1 + 0.564L)u_t$ (8.51) (1.91) (3.30) SSR = 0.045 ; $\chi^2 = 12.28$; DF = 10	3-12
World (excluding oil exporting countries and the United States)	$(1 - 0.863L) \{(1 - L)\ln R_t - 0.024\} = (1 + 0.508L)u_t$ (8.70) (2.06) (3.02) SSR = 0.054 ; $\chi^2 = 15.45$; DF = 10	4-12
Industrial countries	$(1 - 0.782L) \{(1 - L)\ln R_t - 0.022\} = (1 + 0.512L)u_t$ (4.73) (2.54) (2.15) SSR = 0.064 ; $\chi^2 = 12.76$; DF = 10	4-12
Industrial countries (excluding the United States)	$(1 - 0.854L) \{(1 - L)\ln R_t - 0.024\} = (1 + 0.539L)u_t$ (7.89) (1.78) (3.10) SSR = 0.088 ; $\chi^2 = 14.61$; DF = 10	4-12
Less developed areas	$(1 - 0.637L - 0.289L^4 + 0.350L^5) \{(1 - L)\ln R_t - 0.027\} = u_t$ (6.33) (2.39) (2.89) (3.31) SSR = 0.054 ; $\chi^2 = 9.89$; DF = 9	none
<u>Extended Sample: (Second Quarter 1960--Fourth Quarter 1982)</u>		
World	$(1 - 0.385L) \{(1 - L)\ln R_t - 0.021\} = u_t$ (3.92) (3.94) SSR = 0.090 ; $\chi^2 = 12.99$; DF = 16	none
World (excluding oil exporting countries)	$(1 - 0.734L) \{(1 - L)\ln R_t - 0.019\} = (1 + 0.413L)u_t$ (4.56) (2.66) (1.84) SSR = 0.093 ; $\chi^2 = 17.80$; DF = 15	1-12
World (excluding oil exporting countries and the United States)	$(1 - 0.752L) \{(1 - L)\ln R_t - 0.023\} = (1 + 0.400L)u_t$ (5.31) (2.70) (1.95) SSR = 0.108 ; $\chi^2 = 21.86$; DF = 15	6-12
Industrial countries	$(1 - 0.332L) \{(1 - L)\ln R_t - 0.018\} = u_t$ (3.29) (2.97) SSR = 0.130 ; $\chi^2 = 19.30$; DF = 16	none
Industrial countries (excluding the United States)	$(1 - 0.708L) \{(1 - L)\ln R_t - 0.023\} = (1 + 0.379L)u_t$ (4.26) (2.43) (1.67) SSR = 0.162 ; $\chi^2 = 19.23$; DF = 15	none
Less developed areas	$(1 - 0.335L - 0.202L^2) \{(1 - L)\ln R_t - 0.034\} = (1 - 0.354L^4)u_t$ (3.13) (1.86) (3.30) (3.29) SSR = 0.103 ; $\chi^2 = 19.36$; DF = 14	none

1/ t-values in parentheses below coefficients; SSR = sum of squared residuals; DF = degrees of freedom; L = lag operator ($Lx_t = x_{t-1}$).

The Frenkel study differs from the Heller-Khan analysis in terms of its sample period, the use of real instead of nominal reserves, the use of annual instead of quarterly data, the country groupings, the scale variable in the demand for reserves (real income rather than imports), and the variability measure. In Frenkel's samples, the variability index (σ_t) for a given year T was obtained by first estimating the regression

$$(4) \quad NR_t = \alpha + \beta_{T-1}t + u_t \quad \text{over } t = T-15, \dots, T-1$$

where

NR_t = nominal reserves.

Then, using the estimated trend $\hat{\beta}_{T-1}$, $\tilde{\sigma}_T^2$ was defined as

$$(5) \quad \tilde{\sigma}_T^2 = \sum_{t=T-14}^{T-1} (NR_t - NR_{t-1} - \hat{\beta}_{T-1})^2 / 14.$$

To obtain a measure free of scale, the variability measure for period T is defined as the ratio of the standard error of the trend adjusted changes in reserves to the value of imports (IM_T). Thus, $\sigma_T = \tilde{\sigma}_T / IM_T$.

Tables 4 and 5 compare the original estimates obtained by Frenkel with those obtained using revised data for the same period. The results for both sets of data are quite similar. There are two sets of regressions involved. The first set consists of 17 cross-section regressions for 22 developed countries for the years from 1963 to 1979 and 15 cross-section regressions for 32 developing countries for the years from 1963 to 1977. In this first set of cross-sectional equations, there is one equation for each country group in each year (Table 4). In the second set, there are pooled time series and cross-section regressions for each country group for various time periods (Table 5). The time periods selected for the pooled regressions are those that were generally regarded as having relatively stable structures for the demand for reserves.

A comparison of the cross-sectional regressions reported in Table 4 indicates that there are only minor differences between the results originally reported by Frenkel and those for the revised figures. In general, these regressions for both the old and new data sets, imply a unit elasticity of the demand for reserves with respect to the level of real income; and positive and significant effects associated with the variability measure and the average propensity to import.

Table 4. Estimates of the Frenkel Equilibrium Model of the Demand for International Reserves--Cross-Sectional Equations, 1963-79 ^{1/}

$$\ln R_t = \beta_0 + \beta_1 \ln a_t + \beta_2 \ln Y_t + \beta_3 \ln m_t$$

Year	Developed Countries (22)						Developing Countries (32)					
	β_0	β_1	β_2	β_3	\bar{R}^2	Standard Error	β_0	β_1	β_2	β_3	\bar{R}^2	Standard Error
Original Sample (1963-77)												
1963	4.081 (6.00)	0.625 (3.13)	1.063 (11.81)	1.398 (4.66)	0.88	0.434	5.724 (8.67)	0.297 (1.75)	1.244 (11.31)	1.895 (6.53)	0.82	0.523
1964	4.240 (5.44)	0.607 (2.53)	1.012 (11.24)	1.348 (4.65)	0.86	0.477	5.641 (7.95)	0.241 (1.27)	1.196 (9.97)	1.732 (5.57)	0.78	0.553
1965	4.476 (5.09)	0.492 (1.82)	1.061 (10.61)	1.373 (4.16)	0.84	0.530	5.529 (8.64)	0.189 (1.05)	1.125 (9.38)	1.412 (4.87)	0.77	0.539
1966	4.415 (4.75)	0.592 (2.11)	1.100 (10.00)	1.634 (4.07)	0.84	0.542	4.866 (6.85)	0.380 (1.90)	1.114 (7.96)	1.509 (4.19)	0.70	0.627
1967	4.201 (4.57)	0.659 (2.35)	1.147 (10.43)	1.755 (5.01)	0.84	0.557	4.108 (4.95)	0.586 (2.66)	1.253 (7.37)	1.697 (3.95)	0.67	0.753
1968	4.184 (3.64)	0.580 (1.76)	1.151 (8.85)	1.602 (4.33)	0.83	0.588	4.518 (6.11)	0.425 (2.24)	1.215 (8.10)	1.526 (4.36)	0.71	0.684
1969	4.277 (4.41)	0.519 (1.92)	0.995 (9.05)	1.266 (3.96)	0.81	0.553	4.531 (6.04)	0.297 (1.49)	1.206 (8.61)	1.244 (3.89)	0.75	0.655
1970	3.985 (4.74)	0.574 (2.30)	1.016 (9.24)	1.196 (3.74)	0.83	0.536	4.212 (5.27)	0.293 (1.33)	1.207 (8.62)	1.077 (3.26)	0.73	0.719
1971	4.378 (5.54)	0.356 (1.48)	1.017 (9.25)	0.779 (2.29)	0.84	0.540	4.810 (5.80)	0.240 (1.00)	1.237 (9.52)	1.312 (4.23)	0.75	0.684
1972	3.572 (5.67)	0.742 (3.37)	0.943 (11.79)	1.014 (3.07)	0.88	0.425	4.981 (7.02)	0.269 (1.35)	1.232 (11.20)	1.330 (5.32)	0.81	0.592
1973	3.862 (5.68)	0.716 (2.56)	0.972 (10.80)	1.272 (2.83)	0.86	0.472	3.747 (6.14)	0.549 (2.89)	1.191 (13.23)	1.148 (5.47)	0.85	0.537
1974	3.083 (3.90)	0.872 (2.64)	1.061 (10.61)	1.581 (3.29)	0.86	0.526	2.872 (3.86)	0.703 (2.70)	1.103 (9.19)	0.995 (3.55)	0.76	0.700
1975	3.311 (3.52)	0.704 (2.01)	1.190 (8.50)	1.750 (2.92)	0.83	0.603	3.328 (4.97)	0.603 (2.87)	1.070 (8.23)	1.142 (4.08)	0.72	0.737
1976	3.260 (3.54)	0.816 (2.40)	1.139 (9.49)	1.923 (3.43)	0.83	0.582	3.623 (4.83)	0.462 (2.10)	1.158 (8.91)	1.007 (3.47)	0.71	0.807
1977	3.225 (5.29)	0.825 (3.75)	1.275 (15.94)	2.222 (6.54)	0.92	0.411	3.383 (4.23)	0.569 (2.19)	1.141 (8.78)	1.072 (3.57)	0.73	0.759
1978	4.382 (5.83)	0.306 (1.19)	1.113 (11.02)	1.256 (3.33)	0.86	0.552						
1979	3.681 (5.49)	0.564 (2.54)	1.136 (13.36)	1.585 (5.32)	0.90	0.486						

Table 4 (Concluded). Estimates of the Frenkel Equilibrium Model of the Demand for International Reserves—Cross-Sectional Equations, 1963-82 ^{1/}

$$\ln R_t = \beta_0 + \beta_1 \ln \alpha_t + \beta_2 \ln Y_t + \beta_3 \ln m_t$$

Year	Developed Countries (22)						Developing Countries (32)					
	β_0	β_1	β_2	β_3	\bar{R}^2	Standard Error	β_0	β_1	β_2	β_3	\bar{R}^2	Standard Error
Revised and Extended Sample (1963-82)												
1963	3.984 (5.89)	0.644 (3.24)	1.065 (12.46)	1.403 (4.80)	0.89	0.429	4.884 (6.96)	0.447 (2.33)	1.268 (10.24)	1.856 (5.39)	0.79	0.601
1964	4.148 (5.30)	0.624 (2.61)	1.013 (11.17)	1.333 (4.58)	0.86	0.475	4.746 (6.17)	0.478 (2.27)	1.223 (8.93)	1.861 (5.16)	0.74	0.652
1965	4.352 (4.91)	0.515 (1.88)	1.063 (10.35)	1.365 (4.13)	0.84	0.528	4.614 (6.46)	0.420 (2.04)	1.167 (8.27)	1.518 (4.17)	0.72	0.659
1966	4.205 (4.50)	0.637 (2.22)	1.105 (10.34)	1.634 (4.74)	0.83	0.538	3.959 (5.33)	0.576 (2.68)	1.145 (7.26)	1.527 (3.69)	0.68	0.717
1967	4.061 (4.37)	0.676 (2.40)	1.151 (10.54)	1.743 (5.03)	0.84	0.555	3.577 (5.52)	0.805 (4.47)	1.270 (8.62)	1.925 (5.17)	0.76	0.644
1968	4.044 (3.45)	0.599 (1.84)	1.157 (9.22)	1.602 (4.41)	0.82	0.587	3.759 (6.65)	0.676 (4.28)	1.209 (9.92)	1.658 (5.73)	0.79	0.581
1969	4.225 (4.27)	0.532 (1.97)	0.999 (9.18)	1.269 (4.01)	0.81	0.552	3.761 (6.16)	0.542 (3.05)	1.208 (9.97)	1.363 (4.84)	0.79	0.598
1970	3.988 (4.67)	0.575 (2.31)	1.018 (9.64)	1.211 (3.67)	0.83	0.536	3.631 (5.67)	0.511 (2.71)	1.227 (9.63)	1.261 (4.42)	0.78	0.655
1971	4.373 (5.48)	0.359 (1.48)	1.018 (9.69)	0.787 (2.32)	0.84	0.541	4.023 (6.29)	0.502 (2.60)	1.238 (10.17)	1.457 (5.11)	0.79	0.627
1972	3.605 (5.67)	0.739 (3.40)	0.945 (11.86)	1.021 (3.17)	0.89	0.425	4.542 (7.96)	0.409 (2.43)	1.218 (12.10)	1.406 (6.03)	0.83	0.555
1973	3.873 (5.66)	0.717 (2.66)	0.974 (10.42)	1.280 (2.91)	0.86	0.471	3.323 (6.43)	0.701 (4.30)	1.177 (14.37)	1.235 (6.54)	0.88	0.478
1974	3.104 (3.90)	0.847 (2.67)	1.063 (10.62)	1.559 (3.27)	0.85	0.528	2.482 (3.55)	0.849 (3.58)	1.035 (8.87)	0.990 (3.53)	0.75	0.695
1975	3.251 (3.36)	0.700 (2.01)	1.192 (8.78)	1.751 (2.90)	0.82	0.603	3.023 (4.91)	0.705 (3.73)	0.989 (8.39)	1.073 (3.90)	0.74	0.701
1976	3.196 (3.42)	0.829 (2.47)	1.144 (9.37)	1.959 (3.48)	0.83	0.579	3.544 (5.11)	0.501 (2.39)	1.118 (8.54)	1.016 (3.55)	0.72	0.793
1977	3.141 (5.04)	0.829 (3.85)	1.281 (15.50)	2.257 (6.52)	0.92	0.410	3.237 (4.34)	0.653 (2.74)	1.105 (8.56)	1.151 (4.02)	0.74	0.743
1978	4.294 (5.64)	0.344 (1.35)	1.122 (11.15)	1.336 (3.52)	0.86	0.548	2.237 (3.24)	0.939 (4.40)	1.165 (10.55)	1.281 (5.07)	0.80	0.666
1979	3.745 (5.56)	0.547 (2.44)	1.130 (13.40)	1.585 (5.15)	0.90	0.479	1.849 (3.28)	1.022 (5.97)	1.152 (13.10)	1.171 (5.52)	0.86	0.553
1980	3.456 (5.18)	0.670 (2.87)	1.171 (14.35)	1.893 (5.53)	0.91	0.454	2.121 (3.51)	0.830 (4.52)	1.221 (11.77)	1.140 (4.57)	0.83	0.652
1981	2.430 (3.23)	1.015 (3.97)	1.048 (12.79)	1.643 (5.40)	0.90	0.448	2.032 (2.74)	0.850 (3.84)	1.296 (11.14)	1.416 (5.42)	0.80	0.746
1982	1.541 (1.66)	1.174 (3.92)	1.035 (11.93)	1.457 (4.66)	0.90	0.462	2.197 (3.42)	0.912 (4.89)	1.150 (11.91)	1.530 (7.33)	0.84	0.637

^{1/} t-values in parentheses below coefficient.

Table 5. Estimates of the Frenkel Equilibrium Model of the Demand for International Reserves--Pooled Times-Series and Cross-Sectional Equations 1/

$$\ln R_{n,t} = \gamma_0 + \gamma_1 \ln a_{n,t} + \gamma_2 \ln Y_{n,t} + \gamma_3 \ln m_{n,t}$$

Period	Developed Countries						Developing Countries					
	γ_0	γ_1	γ_2	γ_3	\bar{R}^2	Standard Error	γ_0	γ_1	γ_2	γ_3	\bar{R}^2	Standard Error
<u>Original Sample</u>												
1963-72	4.108 (16.50)	0.594 (8.03)	1.059 (27.15)	1.353 (13.53)	0.85	0.504	4.848 (21.55)	0.317 (5.11)	1.191 (29.78)	1.428 (14.42)	0.76	0.623
1973-77	3.381 (8.81)	0.750 (5.47)	1.106 (22.57)	1.619 (7.39)	0.85	0.543	3.346 (10.79)	0.575 (5.99)	1.114 (2.15)	1.020 (8.72)	0.77	0.694
1973-79	3.615 (12.47)	0.636 (6.06)	1.105 (29.08)	1.520 (9.50)	0.86	0.532						
<u>Revised and Extended Sample</u>												
1963-72	4.040 (16.04)	0.606 (8.13)	1.062 (34.81)	1.357 (13.71)	0.85	0.502	4.128 (20.72)	0.522 (9.21)	1.209 (30.73)	1.531 (15.85)	0.78	0.614
1973-77	3.358 (9.09)	0.745 (5.48)	1.108 (22.70)	1.620 (7.37)	0.84	0.543	3.099 (10.86)	0.671 (7.43)	1.064 (21.22)	1.031 (9.03)	0.77	0.677
1973-79	3.590 (12.16)	0.640 (6.13)	1.108 (28.82)	1.543 (9.52)	0.86	0.530	2.833 (11.94)	0.751 (10.11)	1.087 (26.95)	1.077 (11.61)	0.79	0.656
1980-82	2.977 (6.95)	0.769 (5.43)	1.105 (22.95)	1.630 (8.81)	0.90	0.461	2.272 (6.01)	0.816 (7.28)	1.233 (20.25)	1.365 (9.88)	0.82	0.683
1973-82	3.491 (14.10)	0.633 (7.47)	1.091 (35.53)	1.468 (11.80)	0.86	0.526	2.816 (13.71)	0.726 (11.46)	1.112 (32.44)	1.120 (14.25)	0.79	0.687

1/ t-values in parentheses below coefficients.

Table 5 presents the results for the second set of combined cross-section and time-series regressions for developed countries (in the periods 1963 to 1972, 1973 to 1977, and 1973 to 1979) and developing countries (in the 1963-72 and 1973-77 periods). ^{1/} Once again, the results in the original study as well as those based on the revised data imply the existence of a well-identified demand for international reserves. One difference between the results is that for the developing countries the coefficients on the variability measures are larger for the estimates based on the revised data than for those based on the original samples. A Chow test for the stability of regression coefficients between two periods 1963 to 1972 and 1973 to 1979 again confirms Frenkel's results that the demands for reserves for both country groups shifted at the end of 1972. ^{2/} This shift was partly reflected in a decline in the real demand for reserves (i.e., a lower constant term) during the move toward greater flexibility of exchange rates and a larger coefficient on the variability measure.

The Heller-Khan and Frenkel results thus generally support the same set of conclusions. First, apart from a shift in the demand for reserves during the period of the move to greater exchange rate flexibility from 1972 to 1973, this demand has been a relatively stable function of a scale variable, the average propensity to import, and some measure of the variability of external payments. Second, this stability is evident in the empirical results described in the authors' original studies and those obtained using revised data for the 1960s and 1970s.

2. Disequilibrium models of the demand for international reserves

The disequilibrium versions of the demand for international reserves are based on stock adjustment models which allow for a gradual adjustment of actual to desired reserve holdings. Recent works by Bilson and Frenkel (1979) and Frenkel (1983) provide representative examples of this type of model. The Bilson-Frenkel model assumes that long-run demand for reserves for any country is given by: ^{3/}

^{1/} In Table 5, the dual subscript (n and t) on the variables in the equation denote the country (n) and time period (t). $R_{n,t}$ is thereby the stock of real reserves for country n at time t.

^{2/} The values of the F statistics are $F_{4,366} = 13.0$ for developed countries, and $F_{4,536} = 8.3$ for developing countries, both significantly different from zero at the 5 percent level of significance.

^{3/} A variable $X_{n,t}$ is the value of X for country n at time t. \bar{X}_n is the average value of X for country n across time.

$$(6) \ln R_n^D = \beta_0 + \beta_1 \ln \sigma_n + \beta_2 \ln Y_n + \beta_3 \ln m_n + u_n$$

where

R_n^D, R_n = target and actual average level of reserves for country n, gross reserves measured in U.S. dollars;

σ_n = variability of international receipts and payments; ^{1/}

Y_n = GNP of country n;

m_n = average propensity to import of country n; and

u_n = country specific factor.

The stock adjustment process was given by: ^{2/}

$$(7) \ln R_{n,t} - \ln R_{n,t-1} = \alpha + \gamma (\ln R_{n,t}^D - \ln R_{n,t-1}) + e_{n,t}$$

where $e_{n,t}$ = random error.

The long-run demand for reserves was estimated with a cross-section regression (one observation per country) using the averages (across time) of the dependent and independent variables. The residuals from that regression (one residual per country) were considered to be "specific factors" of each country (i.e., $\hat{u}_n = \ln R_n - \hat{\beta}_0 - \hat{\beta}_1 \ln \sigma_n - \hat{\beta}_2 \ln Y_n - \hat{\beta}_3 \ln m_n$). The desired stock of reserves for each country for each year was then computed using the estimated coefficients of the regression and the specific factor (residual) of each country (i.e., $\ln R_{n,t}^D = \hat{\beta}_0 + \hat{\beta}_1 \ln \sigma_{n,t} + \hat{\beta}_2 \ln Y_{n,t} + \hat{\beta}_3 \ln m_{n,t} + \hat{u}_n$). These desired stocks were next used in the estimation of the stock adjustment equation (7).

Table 6 compares the results of estimating the long-run demand for reserves using the original data sample and the revised data sample. The results based on the revised data are very similar to those of the original paper except that the coefficients on the variability measure for the developing countries take on a higher value, particularly for the 1964-72 period, than in the original results.

^{1/} This measure is constructed in the same manner as described in the previous section for the Frenkel (1983) model.

^{2/} Note that $\alpha \neq 0$ implies a change in reserves, even if the gap between $R_{n,t}^D$ and $R_{n,t-1}$ is eliminated.

Table 6. Estimated Long-Run Demand for International Reserves in the
Bilson-Frenkel Disequilibrium Model 1/

$$\ln R_n = \beta_0 + \beta_1 \ln a_n + \beta_2 \ln Y_n + \beta_3 \ln m_n$$

Period	Developed Countries						Developing Countries					
	β_0	β_1	β_2	β_3	\bar{R}^2	Standard Error	β_0	β_1	β_2	β_3	\bar{R}^2	Standard Error
<u>Original Sample</u>												
1964-72	3.783 (4.61)	0.723 (2.76)	1.077 (11.97)	1.505 (4.97)	0.87	0.453	4.854 (6.66)	0.367 (1.90)	1.237 (10.66)	1.537 (5.39)	0.81	0.536
1973-77	3.139 (4.13)	0.828 (2.96)	1.141 (11.76)	1.795 (4.00)	0.88	0.468	2.783 (4.35)	0.757 (3.90)	1.140 (11.88)	1.200 (5.38)	0.83	0.559
<u>Revised and Extended Sample</u>												
1964-72	3.695 (4.51)	0.742 (2.88)	1.079 (11.99)	1.504 (4.92)	0.88	0.452	4.010 (6.37)	0.625 (3.63)	1.248 (11.04)	1.688 (6.01)	0.83	0.523
1973-77	3.083 (4.15)	0.863 (3.17)	1.146 (12.19)	1.853 (4.30)	0.89	0.459	2.703 (4.71)	0.800 (4.52)	1.090 (11.98)	1.195 (5.56)	0.85	0.537
1973-79	3.128 (4.46)	0.829 (3.29)	1.146 (13.82)	1.817 (4.93)	0.91	0.422	2.524 (4.53)	0.842 (4.88)	1.113 (12.92)	1.209 (5.81)	0.86	0.511
1980-82	2.100 (2.78)	1.055 (4.26)	1.086 (14.90)	1.699 (6.14)	0.92	0.402	1.558 (2.47)	1.033 (5.48)	1.201 (13.41)	1.475 (6.98)	0.86	0.585
1973-82	2.920 (4.01)	0.869 (3.42)	1.135 (14.93)	1.819 (5.48)	0.92	0.400	2.102 (3.84)	0.957 (5.56)	1.144 (14.48)	1.338 (6.76)	0.88	0.483

1/ t-values in parentheses below coefficients.

Table 7 provides the estimates for the parameters in the stock adjustment equations both as reported in the original paper and as given by the revised data. For the 1964-72 period, the results for the two sets of data are quite similar. Although the Bilson-Frenkel paper does not present results for the stock adjustment process given by equation (7) for the 1973-77 period, Table 7 reports the estimates obtained on the basis of the revised data for the 1973-77, 1973-79, 1980-82, and 1973-82 periods. While the estimates of γ suggest a slower speed of adjustment for the developed countries in the 1973-77 period compared with the 1964-72 period, the speed of adjustment for developing countries appears to have risen sharply between 1964 to 1972 and 1973 to 1977.

The structural stability of the demand for reserves between the 1964-72 and 1973-79 periods was tested in two stages. In the first stage, the hypothesis that the long-run demand remained stable could not be rejected for any of the country groups. 1/ In the second stage, the hypothesis that the adjustment process remained stable (under the assumption that the long run demand was stable) was tested using two F tests. 2/ The hypothesis of a stable process of adjustment was rejected for developed countries but could not be rejected for developing countries. 3/ Thus, these results suggest that despite the drastic changes in the structure of the international monetary system during 1972 to 1973, the long-run demand for reserves remained stable for both groups of countries and the adjustment process remained stable for developing countries. These conclusions stand in contrast with the results derived from equilibrium models, which implied that the demand for reserves was subject to structural changes for both groups of countries.

The conclusions from the Bilson-Frenkel disequilibrium model are based on relatively weak statistical tests. Given the manner in which the "specific factors" for each country are estimated, it is not possible

1/ The F tests have the values $F_{4,36} = 2.60$ for developed countries and $F_{4,56} = 1.25$ for developing countries.

2/ When testing for stability of the adjustment process, the presence of a lagged dependent variable in equation (7) could potentially present a problem when all the observations are included in an F test, since the first observation of the second period would not be independent from the last observation of the first period. To allow for this possibility, an additional F test was performed in which one observation at the break point was deleted. This procedure was followed with all the equations that could present a similar problem. In every case, the two F tests had the same result regarding whether the equation is stable or not.

3/ The results were $F_{2,348} = 28.48$ for developed countries and $F_{2,508} = 0.97$ for developing countries. When dropping one overlapping observation the results were $F_{2,326} = 39.23$ and $F_{2,476} = 0.28$ respectively.

Table 7. Estimated Stock Adjustment Model of the Demand for International Reserves in the Bilson-Frenkel Disequilibrium Model 1/

$$\ln R_{n,t} - \ln R_{n,t-1} = \alpha + \gamma (\ln R_{n,t}^* - \ln R_{n,t-1})$$

Period	Developed Countries				Developing Countries			
	α	γ	$\overline{R^2}$	Standard Error	α	γ	$\overline{R^2}$	Standard Error
<u>Original Sample</u>								
1964-72	0.053 (3.31)	0.540 (9.82)	0.32	0.208	0.049 (2.72)	0.415 (8.83)	0.21	0.295
1973-77								
<u>Revised and Extended Sample</u>								
1964-72	0.052 (3.25)	0.548 (9.79)	0.32	0.207	0.052 (3.25)	0.392 (8.34)	0.19	0.264
1973-77	0.024 (1.00)	0.419 (5.44)	0.21	0.254	0.046 (1.59)	0.751 (10.29)	0.40	0.330
1973-79	0.043 (2.13)	0.412 (6.68)	0.22	0.246	0.065 (2.63)	0.651 (10.84)	0.34	0.331
1980-82	-0.001 (0.05)	0.657 (4.77)	0.25	0.230	-0.030 (0.82)	0.576 (5.84)	0.26	0.347
1973-82	0.035 (2.06)	0.302 (6.04)	0.14	0.257	0.067 (2.91)	0.388 (7.46)	0.15	0.391

1/ t-values in parentheses below coefficients.

to test whether these specific factors remain constant from one subperiod to the other. As already indicated, the specific factors are estimated as the residuals from the regression of the long-run demand for reserves, which is run with one observation per country using the averages (across time) of the dependent and independent variables. When testing the stability of the long-run demand for reserves, independent regressions are run for each subperiod (one observation per country in each regression), and then another regression is run with observations from the two subperiods (two observations per country) with the coefficients constrained to be the same for the entire period. The test determines whether the constraint imposed on the coefficients in the third regression causes the sum of square residuals to be significantly higher than the sum of the square residuals of the two independent regressions. When running the third regression, however, only the coefficients β_1 s of equation (6) can be constrained to be the same for the entire period since there is no way in which the two residuals for each country can be constrained to be equal. Therefore, the test of stability of the long-run demand for reserves does not test whether the country "specific factors" remain constant. 1/ The test of the stability of the adjustment process is also affected since this test is carried out under the assumption that the long-run demand for reserves (including the country "specific factors") remained stable.

An alternative formulation that preserves the idea of a country-specific factor but permits an appropriate F test of structural stability is to allow for a different constant term for each country in the estimation of the long-run demand for reserves. Combining the long-run demand for reserves and the stock adjustment equation then yields

$$(8) \quad \ln R_{n,t} = \beta_{0n} + \gamma \beta_1 \ln \alpha_{n,t} + \gamma \beta_2 \ln Y_{n,t} + \gamma \beta_3 \ln m_{n,t} + (1-\gamma) \ln R_{n,t-1} + v_{n,t}$$

where β_{0n} is the specific constant for country n. An F test used to examine the stability of the parameters of equation (8) would involve a joint test of the stability of the parameters of the long-run demand for reserves and the stock adjustment equation. When equation (8) was estimated for both developed and developing countries, the F test implied a rejection of the hypothesis of stability between the 1964-72 and 1973-79 periods at the 5 percent level of significance, for both groups of countries. 2/ Thus, in contrast with the results obtained when using a

1/ This test was also used in Bilson-Frenkel (1979b).

2/ The F statistics equal $F_{26,300} = 2.53$ for developed countries, and $F_{36,440} = 3.57$ for developing countries. When dropping one overlapping observation the results were $F_{26,278} = 3.22$ and $F_{36,408} = 3.29$ respectively.

two-stage test for stability, the inclusion of country specific dummy variables lead to the conclusion that developing as well as developed countries experienced some instability in their demands for reserves during the collapse of the Bretton Woods system.

Frenkel's (1983) analysis extends the Bilson-Frenkel discussion to allow for the effects of domestic monetary disequilibrium on the stock adjustment process for reserves. In this analysis, there is a long-run demand for reserves (R^D), a long-run demand for real balances ($(M/P)^D$), and a stock adjustment process which relates reserve changes to excess demands for either reserves or money.

These relations are given by:

$$(9) \quad \ln R_{n,t} - \ln R_{n,t-1} = \lambda_1 (\ln R_{n,t}^D - \ln R_{n,t-1}) + \lambda_2 (\ln M_{n,t}^D - \ln M_{n,t-1}) + \mu_{n,t}$$

where

$R_{n,t}$ = stock of reserves for country n at time t; and

$M_{n,t}$ = stock of monetary base of country n at time t.

Assuming that, on average, the holdings of international reserves and real monetary bases equal to their target levels, the target levels were estimated by:

$$(10) \quad \ln R_n^D = \beta_0 + \beta_1 \ln \sigma_n + \beta_2 \ln Y_n + \beta_3 \ln m_n + \mu_n$$

$$(11) \quad \ln \left(\frac{M}{P} \right)_n^D = \gamma_0 + \gamma_1 \ln Y_n - \gamma_2 r_n + \varepsilon_n$$

where

r_n = rate of interest in country n; and

$(M/P)_n$ = real balances in country n.

The first stage of the estimation process involves estimating equations (10) and (11) from the sample averages of the time-series and cross-sectional data for a sample of 22 developed countries (Table 8). In the second stage, the estimated parameters from the first stage were used to construct estimates of the target levels of reserves and money as:

Table 8. Estimated Demand for Money and Demand for International Reserves for Developed Countries in the Frenkel Disequilibrium Model 1/

Period	Dependent Variable	Constant	$\ln \sigma_n$	$\ln Y_n$	$\ln m_n$	r_n	\bar{R}^2	Standard Error
<u>Original Sample</u>								
1963-72	$\ln R_n$	3.959 (5.01)	0.661 (2.75)	1.066 (11.84)	1.424 (4.75)		0.87	0.458
	$\ln \left(\frac{M}{P}\right)_n$	-1.105 (2.63)		1.027 (17.12)		-0.186 (3.10)	0.92	0.393
1973-79	$\ln R_n$	3.478 (4.98)	0.730 (2.78)	1.138 (12.93)	1.737 (4.40)		0.90	0.449
	$\ln \left(\frac{M}{P}\right)_n$	-2.199 (2.96)		1.001 (10.43)		-0.010 (0.19)	0.86	0.570
<u>Revised and Extended Sample</u>								
1963-72	$\ln R_n$	3.873 (4.80)	0.675 (2.74)	1.070 (11.90)	1.424 (4.76)		0.85	0.456
	$\ln \left(\frac{M}{P}\right)_n$	6.067 (15.08)		0.953 (16.76)		-0.214 (3.70)	0.93	0.354
1973-79	$\ln R_n$	3.433 (4.84)	0.738 (2.82)	1.142 (12.91)	1.765 (4.43)		0.88	0.445
	$\ln \left(\frac{M}{P}\right)_n$	4.685 (6.49)		1.001 (10.38)		-0.009 (0.18)	0.84	0.567
1980-82	$\ln R_n$	2.497 (3.51)	0.972 (4.00)	1.100 (14.61)	1.761 (5.96)		0.90	0.415
	$\ln \left(\frac{M}{P}\right)_n$	4.078 (6.00)		1.033 (9.77)		0.018 (0.81)	0.84	0.564
1973-82	$\ln R_n$	3.221 (4.44)	0.794 (3.01)	1.138 (14.17)	1.810 (5.02)		0.89	0.419
	$\ln \left(\frac{M}{P}\right)_n$	4.297 (5.71)		1.023 (9.97)		0.018 (0.42)	0.84	0.560

1/ t-values in parentheses below coefficients.

$$(12) \quad \ln R_{n,t}^D = \hat{\beta}_0 + \hat{\beta}_1 \ln \sigma_{n,t} + \hat{\beta}_2 \ln Y_{n,t} + \hat{\beta}_3 \ln m_{n,t} + \hat{\mu}_n$$

$$(13) \quad \ln M_{n,t}^D = \hat{\gamma}_0 + \hat{\gamma}_1 \ln Y_{n,t} - \hat{\gamma}_2 r_{n,t} + \ln P_{n,t} + \hat{\varepsilon}_n$$

where $\hat{\beta}_1$ and $\hat{\gamma}_1$ are the estimated parameters. $\hat{\mu}_n$ and $\hat{\varepsilon}_n$ are the country specific factors derived using these estimated parameters and the means of each country's data or: 1/

$$(14) \quad \hat{\mu}_n = \ln R_n - \hat{\beta}_0 - \hat{\beta}_1 \ln \sigma_n - \hat{\beta}_2 \ln Y_n - \hat{\beta}_3 \ln m_n$$

$$(15) \quad \hat{\varepsilon}_n = \ln \left(\frac{M}{P} \right)_n - \hat{\gamma}_0 - \hat{\gamma}_1 \ln Y_n + \hat{\gamma}_2 r_n.$$

Estimating these relationships using revised data yield parameter estimates that are similar to those obtained in the Frenkel study. 2/ Table 9 presents parameter estimates for the stock adjustment process. Although Frenkel's results imply an increase in the speeds of adjustment between the 1963-72 and 1973-79 periods for both groups of countries, this result does not appear to be supported by the estimates of the speeds of adjustment derived from the revised data. In particular, the results from the revised data suggest that, while the impact of an excess demand for reserves on the adjustment of reserve holdings remained relatively stable between the 1963-72 and 1973-79 periods, the response to an excess demand for money diminished between the 1963-72 and 1973-79 periods.

As in the Bilson-Frenkel study, the stability of the demand for reserves between the 1964-72 and 1973-79 periods can be tested in two stages. First, using the revised data it was not found possible to reject the hypothesis that the long-run demands for reserves and money were stable. 3/ However, under the assumption that the long-run demand relationships remained stable, the hypothesis that the adjustment process

1/ X_n denotes the mean of X for country n across time; whereas $X_{n,t}$ is the value of x for country n at time t.

2/ However, the constant terms in the demand for money are different. One difficulty involved in comparing our results with those of Frenkel's is that the source of the rate of interest data used in his analysis is not specified. In our analysis, we have utilized the call money rate or the discount rate. This creates the possibility that any differences between the original results obtained by Frenkel and those obtained using the revised data may not be due solely to the effects of data revision.

3/ The F statistics were $F_{4,36} = 2.39$ for the long-run demand for reserves, and $F_{2,38} = 1.83$ for the long-run demand for money. Neither of these test statistics is significantly different from zero at the 5 per-cent level of significance.

Table 9. Adjustment Dynamics for Developed Countries in the Frenkel Disequilibrium Model of the Demand for International Reserves 1/

Period	Dependent variable	$\ln R_{n,t}^* - \ln R_{n,t-1}$	$\ln M_{n,t}^* - \ln M_{n,t-1}$	\bar{R}^2	Standard Error
<u>Original Sample</u>					
1963-72	$\Delta \ln R_{n,t}$	0.370 (4.87)	0.298 (2.95)	0.13	0.240
1973-79	$\Delta \ln R_{n,t}$	0.488 (5.88)	0.357 (2.05)	0.25	0.250
<u>Revised and Extended Sample</u>					
1963-72	$\Delta \ln R_{n,t}$	0.366 (5.27)	0.379 (5.39)	0.11	0.227
1973-79	$\Delta \ln R_{n,t}$	0.321 (4.69)	0.260 (2.23)	0.14	0.262
1980-82	$\Delta \ln R_{n,t}$	0.545 (4.35)	0.483 (2.55)	0.22	0.231
1973-82	$\Delta \ln R_{n,t}$	0.222 (4.12)	0.246 (2.34)	0.09	0.267

1/ t-values in parentheses below coefficients.

remained stable was rejected at the 5 percent level of significance. ^{1/} As noted earlier, this two-stage procedure has some shortcomings, so the model was modified to allow for different constant terms for each country in both the long-run demand for reserves and the long-run demand for money. When these long-run relationships are combined with the stock adjustment equation, then

$$(16) \quad \ln R_{n,t} = \delta_{0n} + \lambda_1 \beta_1 \ln \sigma_{n,t} + (\lambda_1 \beta_2 + \lambda_2 \gamma_1) \ln Y_{n,t} + \\ \lambda_1 \beta_3 \ln m_{n,t} - \lambda_2 \gamma_2 r_{n,t} + (1 - \lambda_1) \ln R_{n,t-1} - \lambda_2 \ln M_{n,t-1}$$

where δ_{0n} is the specific constant for country n. After estimating equation (16), it was found that the hypothesis of stability of the parameters between the 1964-72 and 1973-79 periods was rejected at the 5 percent level of significance. ^{2/}

In summary, the results obtained by re-estimating the Bilson and Frenkel (1979) and Frenkel (1983) disequilibrium models utilizing revised data for the 1963-72 and 1973-79 periods generally confirm the results of the original studies. Both formulations imply structural changes in the stock adjustment processes for reserve accumulation during the 1972-73 period for developed countries. Since the two stage procedure utilized by those authors to examine the stability of the reserve accumulation process is a rather weak statistical test, a more direct test which eliminates those weaknesses was used. This test supports the view that there were structural changes in the demand for reserves during the 1972-73 period for both developed and developing countries.

II. Disturbances in Financial Markets in the 1981-82 Period and the Stability of the Demand for Reserves

The estimates of the demand for international reserves that have been considered so far in this paper have been based on data samples from the 1960s to the mid or late 1970s. Throughout this period, a growing number of countries increased their gross foreign exchange reserves by borrowing in international capital markets. During the early 1980s, however, disturbances in financial markets led to sharp reductions in the access of many developing countries to these markets. By creating

^{1/} The F statistic was $F_{2,370} = 6.12$. When dropping one overlapping observation the result was $F_{2,348} = 6.87$.

^{2/} The F statistic was $F_{28,318} = 2.12$. When dropping one overlapping observation the result was $F_{28,296} = 2.70$.

new uncertainty about the stability of borrowed reserves as a source of future reserve growth, these changes in market conditions could have increased the demand for owned as opposed to borrowed reserves and possibly the overall desired level of reserves that countries would want to hold. The speed of adjustment of actual to desired levels of reserve holdings may also have been affected. For countries with reduced market access, the rebuilding of reserves requires significant improvements in current account balances unless inflows of official transfers and lending can be obtained. As a result, the speed of adjustment of actual to desired reserves could be much slower in the period of reduced access to international markets. To consider whether these developments have influenced the structural stability of the estimated demands for international reserves, the remainder of this section examines the results obtained by re-estimating the representative equilibrium and disequilibrium models of the demand for reserves over a sample which extends through the end of 1982.

1. Equilibrium models

The results of re-estimating the Heller-Khan model over an enlarged sample, including revised data through the fourth quarter of 1982 are given in Tables 2 and 3. While the degree of an economy's openness (as measured by the ratio of imports to income) and level of imports continue to have a significant influence on the demand for reserves, the variability measure no longer has a measurable impact. This reflects a sharp change in (1) the structure of the ARIMA processes that are used to describe the movements of reserves over time; and (2) the number of squared residuals (from these ARIMA processes) that are included in the construction of the variability measures. In four out of the six country groupings there were no significant lags that could be included in the variability measure; and, in the other two cases, the overall effect of reserve variability was not significant.

These results may reflect the fact that the ARIMA processes were estimated for the entire 1960-82 period, even though it is clear that the structure of those processes has not remained constant throughout the period. To allow for the possibility that the variability measure performs better when the ARIMA processes are estimated using only data from recent years, the variability measure was recalculated for the 1974-82 period from the residuals of new ARIMA processes that were estimated using data from the 1970-82 period. 1/ First-order auto-regressive

1/ The ARIMA processes were estimated for the 1970-82 period, although we want to test the stability in the 1974-82 period because residuals from the ARIMA processes lagged up to 12 quarters are needed to construct the variability measure and additional lags are lost in the estimation of the demand function.

processes were adequate for all the country groups, with the exception of the less developed areas, for which a second order auto regressive process was appropriate. These new ARIMA processes, however, did not improve the results concerning the variability measure. In five out of six country groupings, there were no significant lags to be included in the variability measure; and in the other group (the less developed areas) the coefficient of the variability measure on the demand for reserves was insignificant and negative. Therefore, the variability measure of the Heller-Khan model does not seem to remain significant when the sample is enlarged to include observations from the 1970s and the early 1980s.

The results of estimating the cross-section regressions of the demand for reserves in the Frenkel model using the extended sample for the 1963-82 period are given in Table 4. For both developed and developing countries, the results for the 1980-82 period are somewhat different from those in the 1973-79 period. In general, the constant terms are smaller and coefficients on the variability measures are higher for regressions for 1981 and 1982 than for the previous years.

Table 5 gives the regressions for the Frenkel equilibrium model based on pooled cross-section and time-series observations for developed and developing countries for various periods. An F test of the stability of parameter estimates across time periods indicates that there were shifts in the demand for reserves for both developed and developing countries between the 1973-79 and 1980-82 periods. ^{1/} Thus, the structural instability in the demand for international reserves that was evident in the 1972-73 period also appears to be present in the 1980s. For the developed economies, the shift in the demand for reserves over the period from 1963 to 1982 has encompassed a growing sensitivity to payments instability and the degree of openness of the economy (as measured by the average propensity to import). In contrast, the shift in the demand for reserves on the part of the developing countries has reflected a growing sensitivity to payments instability but an erratic change in sensitivity to the degree of openness. For this latter group of countries, the sensitivity of the demand for reserves to the degree of openness fell sharply between the 1963-72 and 1973-79 periods but then recovered somewhat in the 1980-82 period.

2. Disequilibrium models

The parameter estimates for the disequilibrium models included in this study also indicate a pattern of structural instability during the early 1980s. As noted earlier, Tables 6 and 7 report the results of

^{1/} The relevant F statistics are $F_{4,212} = 4.36$ for developed countries, and $F_{4,312} = 6.68$ for developing countries, both significantly different from zero at the 5 percent level of significance.

estimating the disequilibrium model formulated by Bilson and Frenkel (1979) for the groups of developed and developing countries for various periods. The overall structural stability of the demand for reserves may again be analyzed using the Bilson-Frenkel two-step procedure by considering first the stability of the long-run demands for reserves and second the stability of the stock adjustment process. Although the parameter estimates for the 1973-79 and 1980-82 periods do not appear very similar (Table 6), an F test could not reject the hypothesis that the long-run demand for reserves had remained constant between the two time periods for both groups of countries. 1/

The stability of the estimated structure of the stock adjustment process can be analyzed using the results in Table 7. Under the assumption that the long-run demand for reserves was stable during the 1973-82 period, F tests of the structural stability of the stock adjustment process led to a rejection of the hypothesis of structural stability (at the 5 percent level) for both the developing countries and developed countries. 2/ A comparison of the estimated speed of adjustment in the 1973-79 period with those for the 1980-82 period suggests that the speed of adjustment of actual to desired reserves has increased for the developed countries but has declined for the developing countries. The slower speed of adjustment for the developing countries could reflect the reduced access of many developing countries to international financial markets in the period starting in 1981.

As an alternative to this two-stage procedure, we also estimated a reduced form equation derived by combining the long-run demand for reserves (allowing for a different constant term for each country) and the equation describing the adjustment process. The estimated equation was tested for stability over the 1973-82 period using a Chow test. The hypothesis of stability was rejected at the 5 percent level of significance for both developed and developing countries. 3/

The re-estimation of the disequilibrium formulation of the demand for reserves used by Frenkel (1983), however, raises some doubts about the presence of structural instability in the 1980s in the demand for reserves of developed countries. As noted earlier, this formulation is based on the hypothesis that reserve movements are affected by the

1/ The relevant F test are for developed countries $F_{4,36} = 2.07$ and developing countries $F_{4,56} = 2.50$.

2/ The F statistics are for developed countries $F_{2,216} = 12.49$ and for developing countries $F_{2,316} = 32.60$. When dropping one overlapping observation the results were $F_{2,194} = 13.13$ and $F_{2,284} = 29.19$ respectively.

3/ The relevant F tests are for developed countries $F_{26,168} = 1.69$ and for developing countries $F_{36,248} = 3.29$. When dropping one overlapping observation the results were $F_{26,146} = 1.59$ and $F_{36,216} = 3.03$ respectively.

authorities' excess demand for reserves and the private sectors excess demand for money, and its basic components consist of a long-run demand for reserves, a long-run demand for money, and a stock adjustment equation for reserves. Tables 8 and 9 provide the parameter estimates for the Frenkel model for an enlarged sample including data from the early 1980s. The long-run demand for reserves is similar in the 1973-79 and 1980-82 periods, but the response to payment instability appears to have risen over time. However, F tests of the structural stability lead to the conclusion that there was no shift in either the long-run demands for reserves or money for developed countries between the 1973-79 and 1980-82 periods. 1/

Finally, while it appears that the speeds of adjustment shifted between the 1973-79 and 1980-82 periods, an F test could not reject the hypothesis of equal speeds of adjustment in the two periods at the 5 percent level of significance. 2/ The results of the two stage F tests therefore suggest that the long-run demands for money and reserves and the speeds of adjustment in the reserve process have remained stable for the developed countries over the 1973-82 period. This would suggest that the disturbance in international financial markets during the early 1980s did not alter the reserve accumulation process for the developed countries.

Just as with the Bilson and Frenkel (1979) model, the difficulties associated with the two-stage estimation process employed by Frenkel (1983) can be avoided by including specific country dummy variables. As discussed earlier, these dummies allow for a direct rather than the "two-stage" test of the stability of adjustment process. An F test at the 5 percent level of significance applied to the results obtained by estimating this reduced form equation for the adjustment process also indicates that the structure of the reserve accumulation process for the developed countries has remained stable. 3/

Our analysis of the equilibrium and disequilibrium formulations of the demand for international reserves suggests that the structural instability that was evident during the collapse of the Bretton Woods system (in the early 1970s) was also present during the disturbances in international financial markets in the early 1980s. In the equilibrium formulations of the demand for reserves, this structural instability

1/ The F tests have the values $F_{4,36} = 1.82$ for the demand for reserves, and $F_{3,38} = 2.06$ for the demand for money, neither of them significantly different from zero at the 5 percent level of significance.

2/ The F test has the value $F_{2,216} = 1.72$. When dropping one overlapping observation, the result is $F_{2,194} = 1.73$.

3/ The F test has the value $F_{28,164} = 0.76$. When dropping one overlapping observation, the result is $F_{28,142} = 0.81$.

involved changes in the sensitivity of the demand for reserves to payments imbalances, which have tended to increase. Moreover, the sensitivity of the demand for reserves to the openness of the economy increased for developing countries but declined for developed countries. The long-run income or scale elasticity of the demand for reserves does not seem to have changed significantly and has typically been close to one. In the disequilibrium formulations, the instability appears particularly in the speeds of adjustment. There is, however, some evidence of a more stable reserve accumulation process for the developed countries. This evidence thus implies that the demand for reserves has been characterized by extended periods of relative stability coupled with sharp structural changes during relatively short periods. Moreover, the disturbances in financial markets in the early 1980s appear to have been most destabilizing for the demands for reserves by the developing countries.

While the shifts in the demands for reserves caused by both the collapse of the Bretton Woods system in the early 1970s and the disturbances in financial markets in the early 1980s have been statistically significant, there is still the question of how large these shifts have been for the different country groups. This requires a comparison of the differences between the actual and projected holdings of reserves across the country groups for different time periods. As an example, we can consider the estimated demands for reserves that were obtained from the pooled time series and cross section regressions of the Frenkel (1983) equilibrium model. As noted earlier, F tests indicated that there were statistically significant shifts in the demands for reserves for both developed and developing countries between the 1963-72 and 1973-79 periods on the one hand, and the 1973-79 and 1980-82 periods on the other. In calculating the extent of the shift in the demands of reserves from the 1963-72 to 1973-79 periods, the projected demands for the 1973-79 period are taken as those with parameters generated by the estimated demands for reserves obtained by using the 1963-72 data. Similarly, when examining the shift from the 1973-79 to 1980-82 periods, the projected demands for the 1980-82 period are taken as those with parameters generated by the estimated demands obtained by using the 1973-79 data. The extent of the shift in the demands for reserves was then calculated by first taking the sum of the squared deviations of actual and projected demands for reserves for each particular set of years and group of countries. Since there was a general upward trend in the level of reserve holdings, this sum was then divided by the sum of actual reserve holdings across years and countries.

These mean squared error measures generally indicate that the demands for reserves of developed countries were most affected by the collapse of the Bretton Woods system; whereas developing countries' demands were

shifted the most during the period of disturbances in financial markets, in the 1980s. For the 1973-79 period, the mean squared error measure had values of 5.2 percent for the developed countries and 8.4 percent for the developing countries. For the 1980-82 period, however, the measure had values of 3.6 percent for the developed countries and 9.4 percent for the developing countries. Thus, while the developing countries had the largest divergence of actual and projected reserves in both periods, they experienced the greatest divergence in the early 1980s. In contrast, developed countries experienced their largest shift in the demand for reserves following the collapse of the Bretton Woods system.

III. Conclusions

Previous empirical studies have generally concluded that the estimated demands for international reserves were relatively stable functions of a limited number of variables, apart from some instability during the period of the collapse of the Bretton Woods system. However, these studies have been based on data samples from the 1960s and 1970s. In this paper, we have expanded these samples to include data from the early 1980s which encompass disturbances in financial markets in the 1981-82 period. The estimation results for the expanded samples indicate that these disturbances in financial markets have been accompanied by changes in the structure of the demand for reserves that were as large as those that occurred during the collapse of the Bretton Woods system. These structural changes appear to have reflected the reduced access of a number of countries to international financial markets, and they have involved an increase in the sensitivity of demand for reserves to balance of payments variability and changes in the speeds of adjustment of actual to desired reserves.

Definition of Variables Used in the Individual Studies

All data taken from International Monetary Fund, International Financial Statistics (IFS).

Heller-Khan (1978)

- (R_t) Reserves = gold, foreign exchange held by monetary authorities, SDRs, and reserve position in the Fund in U.S. dollars (line 1..d in IFS).
- (I_t) Imports = measured c.i.f. in U.S. dollars (line 71..d in IFS).
- (Y_t) Income = This variable was first calculated as the sum of GNP (or GDP) of the countries comprising the particular country groups, converted into U.S. dollars at the prevailing exchange rate. Second, the annual figures were converted to a quarterly basis using a simple linear interpolation procedure.
- (σ_t^2) Measure of balance of payments variability (see discussion in text).

Bilson-Frenkel (1979) and Frenkel (1983)

- (R_t) Reserves = gold, SDRs, foreign exchange held by monetary authorities, and reserve position in the Fund in U.S. dollars (line 1..d in IFS). Frenkel (1983) deflated reserves using the U.S. GNP deflator.
- (IM_t) Imports = measured c.i.f. in U.S. dollars (line 71..d in IFS).
- (Y_t) Income = GNP (or GDP) converted to U.S. dollars using the average exchange rate for the period. Frenkel (1983) deflated income using the U.S. GNP deflator.
- (m_t) Average Propensity to Import = rates of imports to GNP (or GDP).
- (σ_t) Measure of balance of payments variability (see discussion in text).
- (P_t) Price level = U.S. GNP deflator.
- (M_t) Monetary base = line 14 in IFS converted to U.S. dollars using end of period exchange rate.
- (r_t) Short-term nominal rate of interest: not defined in original study, but defined to equal the call money rate (line 60b in IFS) or the discount rate (line 60 in IFS) for purposes of estimation.

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