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An Intertemporal General Equilibrium Analysis of Financial Crowding Out: Theory with an Application to Australia

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I. Introduction

The aim of this paper is to analyze the extent to which public spending crowds out private production and capital formation. The analysis is carried out within the context of an intertemporal general equilibrium model, and a computational version of the model is developed and applied to Australia. The approach is especially relevant for policy analysis, since it allows the consideration of disaggregated fiscal measures, such as changes in individual tax rates, and at the same time incorporates macroeconomic aspects of fiscal policy, such as rules for deficit financing and the interaction between government deficits, interest rates, and inflation. In addition, by disaggregating the private sector, a comparison can be made of the extent to which individual industries are affected by public sector spending policies. Crowding out has usually been examined in two different but related contexts. In the first, the public sector purchases large quantities of goods and finances them either by taxes or by borrowing. Insofar as the goods purchased are used to produce public goods, they will no longer be available for private sector production, which will therefore be forced to decline. The second context is "financial crowding out," when the government increases its borrowing requirements and thereby drives up the interest rate. Credit is thus made more expensive for the private sector, which is forced to curtail any capital formation that is not self-financed. 1/

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1/ Indirect crowding out may also occur as rising interest rates may cause current consumption, and hence demand for the output of the private sector, to fall.

Financial crowding out has traditionally been analyzed within the context of aggregative macroeconomic models. 1/ There are severe limitations, however, to this approach. Borrowing requirements differ among industries so one would also expect the impact of the government's borrowing on the private sector to vary. The aggregation of demand also precludes any analysis of the relative impact of fiscal policies on the welfare of different consumer groups. In addition, as the models are usually valid only for small changes, it is difficult, if not impossible, to estimate the impact of large increases in government borrowing. Finally, governments often attempt to increase tax revenue and borrowing simultaneously. Because aggregative macroeconomic models do not normally separate tax revenues from government expenditure, such policies cannot be dealt with properly. 2/

The question of resource crowding out is increasingly being examined within the framework of computational general equilibrium (CGE) models of taxation. Such models, originally inspired by the work of Harberger (1962, 1966) on tax incidence, have been developed in Shoven and Whalley (1972, 1973), Shoven (1976), Fullerton (1982, 1983), Fullerton and others (1981), Miller and Spencer (1977), Piggott and Whalley (1983), and Whalley (1975, 1977, 1982), among others, to examine incidence and welfare implications of changes in tax regimes. The advantages of these models, as compared with the macroeconomic ones, are discussed at length in Shoven (1982); among them is the ability to deal with large changes in government policies, with disaggregated taxes, and with the analysis of the welfare implications of taxation on individual consumer categories. There are, however, a number of disadvantages. These models have been almost exclusively "real," so that the public sector is constrained to have a balanced budget, owing to the absence of financial assets that could finance a deficit. Because there is no money, and hence no price level or interest rate, these models do not allow the analysis of financial crowding out.

Research in which certain types of CGE models are expanded to include financial assets has recently been carried out by several authors. Clements (1980) allows for domestic credit expansion in a model of the United States, although it is exogenous with respect to public sector expenditure and revenues. Feltenstein (1980), in a model

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1/ Among such models are those of Blinder and Solow (1973, 1974), Brunner and Meltzer (1972), Buiter (1977), Christ (1968), Cohen and McMenamin (1978), Friedman (1978), Gramlich (1971), Infante and Stein (1976), Meyer (1975), Modigliani and Ando (1976), Spencer and Yohe (1970), and Tobin and Buiter (1976).

2/ See Tanzi (1978) and Aghevli and Khan (1978) for models which do distinguish between taxes and expenditures.

of Argentina, permits the existence of domestic and foreign financial assets, whose endogeneity of supply is dependent upon the balance of payments. Slemrod (1981) constructs a CGE model incorporating portfolio choice by consumers. For the policymaker the major flaw in these models is that they do not permit both endogenous public deficits and private investment.

The model constructed here is dynamic; it has two periods with the notion of a past (before period 1) and a future (after period 2). Both consumers and firms have perfect foresight, so that the prices, tax liabilities, and transfers received from the government in period 2 are correctly anticipated in period 1. 1/ The model is closed by assuming that, in period 2, consumers save, that is, hold debt, according to an exogenous savings rate. 2/

Firms in the private sector are constrained to cover current expenditures by current revenue, while capital formation is financed by the sale of bonds. The government, on the other hand, sets its program of expenditure in real terms and is not required to cover costs from tax revenues; any deficit it incurs is covered by a combination of money and bonds. The government is sensitive, however, to the impact that its deficits may have upon interest and inflation rates. Accordingly, it will gradually cut its spending as real interest and inflation rates rise above predetermined targets. Consumers are required to hold money to cover transaction costs, and they purchase bonds in order to save for the future. With perfect foresight, there is no risk, so that private and government bonds are identical to the consumer. The The equilibrium condition on privately issued debt is that new capital produced in period 1, which comes on line in period 2, must yield a return in period 2 equal to the obligations on the bonds that financed it. The government, on the other hand, must add the debt obligations incurred in period 1 and coming due in period 2 to its current expenditures in the period.

1/ The model may thus be interpreted as generating a rational expectations equilibrium. The minimum length of time needed to introduce a dynamic framework is two periods, but there would be no difficulty in extending the model to several periods. For a study of perfect foresight equilibria, see Brock and Turnovsky (1981).

2/ This "closure" rule is made for purely technical reasons. We must allow for some future after the final period, in order to avoid the requirement that in that final period all debt be fully paid off, as no customer would hold debt. Alternative approaches would be to have infinitely lived consumers, or to introduce an overlapping generations structure. Our closure rule will, in particular, ensure that the model does not generate Ricardian debt equivalence.

Consumers maximize intertemporal utility functions and derive a demand for bonds as a method of savings. As the interest rate rises, consumers tend to satisfy the Fisherian relation and shift their consumption to the future, releasing resources to the government. The private sector may thus suffer from both resource and financial crowding out. 1/

The model includes profit, income, and sales taxes and allows for direct transfer payments by the government to consumers. The price level is endogenous, so that the inflationary impact of various government policies may be analyzed. There is also an investment function, with the level of investment being driven by the interest rate. Section II presents a formal description of the model, while in Section III we will describe a computational version of the model and its application to Australian data. This exercise should serve not only as an illustrative example of the workings of the model, but will also allow certain qualitative judgments to be made about Australian fiscal policies. Section IV is a conclusion, while a proof of existence of equilibrium is given in the Appendix.

II. The Model

1. Production

The structure of production is Leontief in intermediate and final production, while value added is produced by smooth production functions. 2/ Because the model incorporates perfect foresight in both production and consumption, production may be represented by a nonstochastic block-diagonal matrix, whose components refer to goods that are different in their dating. 3/ If goods i = 1, ..., N refer to goods produced in period 1, and goods N + 1, ..., 2N refer to goods produced in period 2, then the structure of the production matrix for intermediate and final goods is:

1/ It should be emphasized, however, that the model does not yield a mechanical one-to-one correspondence between public deficits and crowding out, because the rising interest rate will not only have the above-mentioned effects but may also increase the overall level of private savings.

2/ This formulation is used because of the eventual goal of an empirical application and has been described in greater detail in, for example, Fullerton and others (1981) and Feltenstein (1980).

3/ See Debreu (1959) for a discussion of the use of dated commodities.

$$(1)\quad \begin{array}{llllll} a_{11}, & \ldots, & a_{1N}, & 0, \ldots, & \ldots, & 0 \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ a_{N1}, & \ldots, & a_{NN}, & 0, \ldots, & \ldots, & 0 \\ 0, & \ldots, & 0, & a_{N+1,N+1}, & \ldots, & a_{N+1,2N} \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ 0, & \ldots, & 0, & a_{2N,N+1}, & \ldots, & a_{2N,2N} \end{array}$$

The upper block of the matrix refers to first-period production, and the lower block refers to second-period production. Corresponding to each activity, there is a continuous function $f_j(K_i, L_i)$, which produces value added for the j^{th} activity using capital and labor from the corresponding stocks that exist in period i. In order to be specific, assume that the value-added functions are Cobb-Douglas, hence of the form:

$$(2)\quad f_j(K_i, L_i) = K_i^{\alpha_j} L_i^{(1-\alpha_j)}$$

In addition, there are investment activities, $H_i(K_i, L_i)$, which operate in period i, using inputs of capital and labor existing in that period, and which produce capital goods for period i + 1. 1/ The investment is carried out by the private sector, and since the capital that is produced in one period only becomes available in the next, the investment firm must pay for the input costs of its production in the current period, but will receive the revenue from that capital in the next period. 2/ In order to simplify the demonstration of the existence of an equilibrium, it is assumed that the investment functions exhibit decreasing returns to scale, 3/ and are of the form:

1/ The investment function could also require intermediate and final goods as inputs, but for simplicity of exposition, it will require only capital and labor.

2/ It would also be possible to have investment activities distinguished by firms if we also had firm-specific capital, as in Fullerton (1983), and Dervis, DeMelo, and Robinson (1982).

3/ Decreasing returns to scale will allow the derivation of a single-valued investment response. If desired, we could choose the parameters such that $1-a_i-b_i = \varepsilon_i$ with ε_i arbitrarily small. Any decreasing returns to scale investment function would be equally acceptable.

$$(3)\quad H_i(K_i, L_i) = K_i^{a_i} L_i^{b_i} \;; \qquad a_i + b_i < 1 \quad a_i, b_i \geq 0$$

Capital in period 2 is then given by the depreciated initial capital stock, plus whatever new capital has been produced in period 1. If $\bar{K}_0$ is the initial stock of capital at the beginning of period 1, δ the rate of depreciation, and $\bar{L}_0$ is the initial stock of labor, then:

$$(4)\quad K_2 = (1-\delta)\bar{K}_0 + H_1(\bar{K}_0, \bar{L}_0)$$

where K_2 is the stock of capital at the beginning of period 2.

The government also produces public goods through a smooth production function that uses capital and labor of the current period as inputs. 1/ Let $Q_i(K_i, L_i)$ denote this function in period i and, for simplicity, also assume the function to be Cobb-Douglas, hence:

$$(5)\quad Q_i(K_i, L_i) = K_i^{\beta_i} L_i^{(1-\beta_i)}$$

The government is assumed to decide, at the beginning of period i, on the real level of output of public goods to be produced:

$$(6)\quad \bar{Q}_i = K_i^{\beta_i} L_i^{(1-\beta_i)}$$

where $\bar{Q}_i$ is the real quantity of public goods to be produced in period i, in such a way as to minimize the cost of production.

2. Consumption

There is a single generation of consumers who live for the entire period of the model. Since they may have initial endowments of capital and financial assets, it is implicitly supposed that they were alive

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1/ Rather than having the government operate its own production function, it would also be possible to have the government buy directly from the private sector. Introducing a government production function allows, however, the direct representation of changing public policy toward the relative importance of hiring capital or labor. If, for example, the government wished to increase employment, it could, in the model, change the weights given to capital and labor in its production function. A more sophisticated version of our model would have public goods enter consumers' utility functions directly, or increase productivity.

before period 1. The consumers perfectly anticipate all prices of period 2 while in period 1. 1/ The individual consumer maximizes a utility function, U_i, which has as arguments the levels of consumption in each of the two periods. 2/ Thus:

$$(7) \quad U(x) = U(x_1, \ldots, x_N, x_{N+1}, \ldots, x_{2N}, x_{L1}, x_{L2})$$

where x_i: $i \leq N$ refers to the i^{th} consumption good in period 1, x_i: $i > N$ refers to the i^{th} consumption good in period 2, and L_i refers to consumption of leisure in period i. In order to be specific, the utility function is assumed to be of the form:

$$(8) \quad U = x_1^{d_1} x_2^{d_2}, \ldots, x_{2N}^{d_{2N}} x_{L1}^{d_{L1}} x_{L2}^{d_{L2}} \quad : \quad d_i \geq 0$$

Suppose that $\{d_i\}$ reflect the consumer's rate of time preference, u, uniformly across goods, so that:

$$(9) \quad d_i/d_{i+N} = u : i = i, \ldots, N, \qquad d_{L1}/d_{L2} = u$$

and, in addition, u is uniform across all consumers. Hence, leisure enters the utility function, but money, bonds, and capital do not. 3/

The consumer maximizes his utility function, subject to a set of intertemporal budget constraints, and it is assumed that capital markets are imperfect in that consumers cannot borrow against future income. 4/

1/ A rational expectations equilibrium is being defined in which consumers' expectations of period 2 are perfectly fulfilled. If the model contained more than two periods, it would be quite possible that information available for the time period after period 2 might be used to determine the consumers' choices in periods 1 and 2.

2/ There are K > 0 consumers in the model; however, in order to avoid unreadable subscripts, the consumer demand parameters will not be indexed. It should be noted that these parameters, along with initial allocations, are not uniform across consumers. One might also wish to include public goods in the consumers' utility function, although we have not done so here.

3/ It should be noted that our proof of the existence of equilibrium does not depend on this form of the utility function; any continuous utility function would be valid. This particular form permits an analytic solution to the demand function.

4/ Another approach in, for example, Grandmont (1977) and Grandmont and Laroque (1975), is to have consumers borrow from the central bank against future income but to have no borrowing by the central bank. A number of technical problems are involved with allowing borrowing to go in both directions, essentially equivalent to the requirement of irreversibility of production.

A consumer has an initial allocation of money and bonds, $\bar{M}_0$ and $\bar{B}_0$, at the beginning of period 1, and, if he is a shareholder in the capital goods-producing firm, he will also hold capital $\bar{K}_0$. 1/ Let p_{Ki}, p_{Li}, p_{Mi}, p_{Bi} represent the prices of capital, labor, money, and bonds, respectively, in period i, and let TR_i represent whatever transfer payments the government pays to consumers during period i, while γ_i, represents this particular consumer's share in those transfers.

Bonds are considered to be long term, so that a consumer owning a bond receives its par value as an interest payment in each period that he owns the bond. Since this payment is made in units of money, his income from the bond in period i is p_{Mi}. He also has the possibility of selling the bond at market prices p_{Bi}. The consumer's income, $I_1(p_1)$, in period 1, is then given by:

$$(10)\quad I_1(p_1) \equiv p_{M1}\bar{M}_0 + p_{M1}\bar{B}_0 + p_{B1}\bar{B}_0 + p_{K1}\bar{K}_0 + p_{L1}\bar{L}_0 + \gamma_1 TR_1$$

In addition, the consumer has a second period budget constraint. If he has purchased a quantity, x_{B1}, of bonds in period 1, he then receives the coupon value of those bonds in terms of units of money in period 2, this being equal to $p_{M2}x_{B1}$. The consumer's income in period 2, $I_2(P_2)$, then becomes:

$$(11)\quad I_2(p_2) = p_{K2}(1-\delta)\bar{K}_0 + p_{L2}\bar{L}_0 + p_{M2}x_{M1} + p_{M2}x_{B1} + p_{B2}x_{B1} + \gamma_2 TR_2$$

where x_{M1} is the quantity of money that the consumer holds in period 1. 2/

The consumer, in solving his utility maximization problem, has two simultaneous budget constraints, as well as a closure rule to be described shortly. Suppose that the consumer faces ad valorem taxes on his purchases of consumption goods, and let:

$$(12)\quad t_1 \equiv (\tau_1, \ldots, \tau_N) \quad : 0 < \tau_i$$

$$t_2 \equiv (\tau_{N+1}, \ldots, \tau_{2N}) \quad : 0 < \tau_i$$

1/ It will be assumed that initial holdings of bonds, $\bar{B}_0$, are entirely composed of government debt. Initial private debt would be inconsistent with the specified intertemporal investment decision.

2/ The interpretation of the price of capital in period i, p_{Ki}, is that it is a rental rather than a sales, or cost of production, price. There is no secondary market for capital, as the conditions on private investment are such that the rate of return on capital is always identically equal to the interest rate. Thus all savings decisions are made by purchases of bonds, and capital gains are realized by the sale of bonds.

where t_i represents the vector of tax rates levied on the N intermediate and final goods produced in period i. Let:

(13) $\tilde{p}_1 \equiv (p_1, \ldots, p_N)$, $\tilde{p}_2 \equiv (p_{N+1}, \ldots, p_{2N})$

denote the prices of the intermediate and final goods in each of the two periods. *The consumer has, in addition, a demand for money function*, uniform across all consumers, in which demand for nominal cash balances depends on the value of current consumption and the nominal interest rate. Leisure is not included as a determinant in the demand for money, since income taxes are withheld at the source.

Suppose now that v_i, the velocity of money, is not constant, but is a function of the nominal interest rate. The nominal interest rate, r_i, in period *i* is defined by:

(14) $r_i \equiv p_{Mi}/P_{Bi} - 1$

Suppose also that:

(15) $v_i = \frac{1}{a} e^{br_i} \quad : \quad a, b > 0$

so that the velocity of money is directly related to the nominal interest rate. Hence:

(16) $p_{Mi}x_{Mi} = ae^{-br_i}(1+t_i)\tilde{p}_i \cdot x_i$

In the Appendix, it is demonstrated that the government's issuance of money in period i, $\tilde{y}_{Mi}$, is bounded for any set of prices, that is, $\tilde{y}_{Mi} < \bar{y}_{Mi}$ for some $\bar{y}_{Mi} < \infty$. In order to demonstrate the boundedness of the intertemporal excess demand functions, we will assume that the existence of this upper bound is known to consumers and that the individual consumer will therefore demand only a finite quantity of money. Hence:

$$(17) \quad x_{M1} = \frac{ae^{-br_1}(1+t_1)\tilde{p}_1 \cdot x_1}{p_{M1}} < \bar{M}_0 + \bar{y}_{M1}$$

$$x_{M2} = \frac{ae^{-br_2}(1+t_2)\tilde{p}2 \cdot x_2}{p_{M2}} < K(\bar{M}_0 + \bar{y}_{M1}) + \bar{y}_{M2} \quad \underline{1}/$$

Things may be put in a somewhat more familiar form by:

$$(18) \quad \ln(p_{Mi}x_{Mi}) = \ln a - br_i + \ln(1+t_i)\tilde{p}_i \cdot x_i$$

The total value of the consumer's consumption in period 1 and period 2 must be equal to or less than the corresponding income, hence:

$$(19) \quad (1+t_i)\tilde{p}_i \cdot x_i + p_{Li}x_{Li} + p_{Bi}x_{Bi} + p_{Mi}x_{Mi} < I_i(p_i)$$

where $p_{Mi}x_{Mi}$ is given by equation (16) and $I_i(p_i)$ is given by equations (10) and (11). In order to close the model, some assumption must be made about consumers' holding debt in period 2. Accordingly, we will suppose that demand for bonds in period 2 is equal to the long-run savings rate of the economy, assumed to be constant. Thus:

$$(20) \quad p_{B2}\ x_{B2} = (1 + t_2)\bar{p}_2 \cdot x_2/z$$

where $1/z$ is equal to the long-run savings rate. We are thus not making the assumption, equivalent to debt neutrality, that the consumer supposes that the portion of his savings held in government debt must eventually be repaid by higher taxes.

The following maximization problem for the consumer thus results:

$$(21) \quad \max\ x_1^{d_1}\ x_2^{d_2}\ \cdots\ x_{2N}^{d_{2N}}\ x_{L1}^{d_{L1}}\ x_{L2}^{d_{L2}}$$

1/ The individual consumer need not know the actual bounds $\bar{y}_{Mi}$, but only that there are such bounds and must limit his demand for money accordingly. Clearly, this restriction is not relevant in any realistic situation.

such that:

(21a) $(1+t_1)\tilde{p}_1 x_1 + p_{L1}x_{L1} + p_{B1}x_{B1} + ae^{-br_1}(1+t_1)\tilde{p}_1 x_1$

$\leq p_{M1}\bar{M}_0 + p_{K1}\bar{K}_0 + p_{M1}\bar{B}_0 + p_{B1}\bar{B}_0 + p_{L1}\bar{L}_0 + \gamma_1 TR_1$

(21b) $(1+t_2)\tilde{p}_2 x_2 + p_{L2}x_{L2} + p_{B2}x_{B2} + ae^{-br_2}(1+t_2)\tilde{p}_2 x_2$

$\leq p_{K2}(1-\delta)\bar{K}_0 + p_{L2}\bar{L}_0 + p_{M2}x_{M1} + p_{M2}x_{B1} + p_{B2}x_{B1} + \gamma_2 TR_2$

(21c) $p_{B2}\ x_{B2} = (1 + t_2)\ \tilde{p}_2\ x_2/z$

It is straightforward to solve (21) in the form of a Lagrangian to obtain the following solution: 1/

(22) $$U_1 \equiv 1 + \frac{(p_{M2}+p_{B1})p_{M1}-p_{B1}p_{M2}\ ae^{-br_1}}{(p_{M2}+p_{B1})p_{M1}}$$

$$U_2 \equiv \frac{\sum_{j=N+1}^{2N} d_j U_1 + d_{L2}U_1}{d_1} - \frac{p_{B1}\ p_{M2}\ ae^{-br_1}\sum_{j=1}^{N} d_j}{(p_{M2}+p_{B2})p_{M1}}$$

$$U_3 \equiv (1-\delta)p_{K2}\ K_o + p_{12}\ \bar{L}_o + \gamma_2 TR_2$$

$$J_1 \equiv p_{M1}\ \bar{M}_o + p_{L1}\ \bar{L}_o + p_{k1}\ \bar{K}_o + (p_{m1}+p_{B1})\bar{B}_o + \gamma_1\ TR_1$$

$$J_2 \equiv (1+ae^{-br_1})(1+\tau_1)\sum_{j=1}^{N} d_j/d_1 + d_{L1}U_1(1+\tau_1)/d_1 + U_2(1+\tau_1)$$

1/ Along with the constraint on the individual consumer's demand for money described in equation (17), a constraint is also imposed on the individual's demand for bonds. It will be shown that there is an upper bound on the supply of bonds, $\bar{y}_{Bi}$, in period i, where $\bar{y}_{Bi}$ incorporates the debt issued by both the public and the private sectors. The assumption is therefore made that this upper bound is known to individual consumers so that $x_{Bi} \leq \bar{y}_{Bi}$ where x_{Bi} are the holdings of bonds by a particular consumer in period i.

$$x_1 = \frac{\left\{ J_1 + \frac{p_{B1}\, U_3}{(p_{M2}+p_{B2})} \right\}}{J_2\, p_1}$$

$$x_j = \frac{d_j\,(1+\tau_1)\; p_1\; x_1}{d_1\,(1+\tau_j)\; p_j} \quad : \quad 1 \leqslant j \leqslant N$$

$$x_{L1} = \frac{d_{L1}\, U_1\,(1+\ \tau_1)\; p_1\; x_1}{d_1\, p_{L1}}$$

$$x_j = \frac{d_j\,(1+\tau_1)\; p_1\;(p_{M2}+p_{B2}) U_1\; x_1}{d_1\;(1+\tau_j)\; p_j\; p_{B1}\;(1+ae^{-br_2} + 1/z)} \qquad : \quad N < j \leqslant 2N$$

$$x_{L2} = \frac{d_{L2}\; U_1 (p_{M2}+p_{B2})\;(1+\tau_1)\; p_1\; x_1}{d_1\; p_{B1}\; p_{L2}}$$

The demand for money, x_{Mi}, is given by equation (17), while the demand for bonds, x_{Bi} is derived from Walras' law in each period as:

$$x_{Bi} = \left\{ I_i(p_i) - (1+\tau_i)\tilde{p}_i\; x_i - p_{Li}\; x_{Li} - p_{Mi}\; x_{Mi} \right\} / p_{bi}$$

Having calculated the individual consumer's demand for all goods plus financial assets in each period, it is appropriate to turn to the derivation of aggregate supply, and, accordingly, excess demand functions.

3. Financing the central government and the formation of capital

Using the individual industry's value-added functions given in equation (2), cost-minimizing levels of use of capital and labor for the j^{th} sector in period i are obtained:

$$(23) \quad K_j = (1+t_{Ki}) \left(\frac{(1-\alpha_j)\; p_{Ki}}{\alpha_j \quad p_{Li}} \right)^{(\alpha_j - 1)} VA_j : \quad \begin{array}{l} i = 1 \text{ if } j \leqslant N \\ i = 2 \text{ if } j > N \end{array}$$

$$L_j = \frac{1+t_{Ki}}{1+t_{Li}} \; \frac{(1-\alpha_j)}{\alpha_j} \; \frac{p_{Ki}}{p_{Li}} \; K_j$$

where t_{Ki} and t_{Li} represent the tax rates levied on capital and labor, assumed to be uniform across sectors, in the i^{th} period, and Va_j represents the required inputs of value added, in real terms, to the j^{th} sector. 1/ The nominal value added, va_j, is given by:

$$(24) \quad va_j(p) = p_{Ki}(1+t_{Ki})K_j + p_{Li}(1+t_{Li})L_j : \begin{array}{l} i = 1 \text{ if } j \leq N \\ i = 2 \text{ if } J > N \end{array}$$

and intertemporal Leontief prices, $\bar{p}(p)$ may be calculated as:

$$(25) \quad \bar{p}(p) = va(p)(I-A)^{-1}$$

Total demand for the j^{th} intermediate and final good, xL_j may be derived as:

$$(26) \quad xL_j \equiv \sum_{k=1}^{K} x_j^k$$

where x_j^k is the k^{th} consumer's demand for intermediate or final good j, as in equation (22). The vector of activity levels, w, of the 2N activities required to produce this level of demand may then be derived as:

$$(27) \quad w \equiv (w_1,\ w_2) = (I-A)^{-1} xL$$

Let y_{Kpj}, y_{Lpj} be the corresponding requirements of column j for capital and labor, and let y_{Kpi}, y_{Lpi} be the total requirements for capital and labor by private industry in period i. The total taxes, T_i, collected by the central government in each of the two periods may now be calculated.

The government uses capital and labor to produce public goods in each of the two periods. Suppose that the real quantity of these public goods is given by Q_1, Q_2. The government has a Cobb-Douglas production function, given in equation (6), and from this may be derived the cost-minimizing quantities of capital and labor, y_{KGi}, y_{LGi}, used by the government in producing Q_i, and the total cost to the government, G_i, of producing Q_i.

The deficit of the central government in period 1, D_1, is then given by:

$$(28) \quad D_1 \equiv G_1 + p_{M1}\bar{B}_0 - T_1$$

1/ The interpretation of t_{Ki} is a profit tax levied upon capital, while t_{Li} may be thought of as an income tax that is collected at the source, that is, a withholding tax.

so that if D_1 is negative, the government runs a surplus. It is assumed that a surplus is paid out as transfer payments to consumers, but in the case of a deficit, financing must take place. 1/ Here we must first make a connection between real interest rates and the level of the government's real expenditures. It is possible for a particular program of expenditures to be technologically feasible, in the sense that it does not require inputs of capital or labor beyond the capacities of the economy, yet, at the same time, it leads to a deficit representing a level of real debt greater than that which people will be willing to hold. To avoid this problem of unbounded supply of money and bonds, a functional relationship will be imposed between the real level of government expenditure and the instantaneous real interest rate and rate of inflation. Accordingly, define Q_i in the following way: let h_i be a continuous function and $\bar{Q}_i$ some fixed, target level of output of public goods, and let π_i be the i^{th} period's rate of inflation. 2/

$$(29) \quad Q_i = h_i(R_i, \pi_i) \quad : \quad R_i \equiv r_i - \pi_i \qquad i = 1, 2$$

$$Q_i = \bar{Q}_i \quad : \quad R_i \leqslant \bar{R}_i \ , \quad \pi_i \leqslant \bar{\pi}_i \ , \quad \bar{R}_i, \ \bar{\pi}_i > 0$$

$$h_i(\bar{R}_i, \ \bar{\pi}_i) = \bar{Q}_i ; \frac{\partial h_i}{\partial R_i} \ , \ \frac{\partial h_i}{\partial \pi_i} \ < 0 \quad : R_i > \bar{R}_i \ , \ \pi_i > \bar{\pi}_i$$

Thus, real output of public goods will be equal to the initial target $\bar{Q}_i$ if both the rate of inflation, π_i, as well as the real interest rate, R_i, are below corresponding target rates. As the real interest or inflation rates rise above target, the level of real government output of public goods approaches 0.

The resulting deficit, given by equation (28), is financed by a combination of money and bonds. Accordingly, let y_{BGi} be the government's bond issue in period 1 and let B_i be a continuous function such that:

1/ These transfer payments are not identically equal to the sum of the transfer payments included in the consumer's budget constraints, although at equilibrium they will be.

2/ Here $\pi_2 \equiv \dfrac{CPI_2/P_{M2}}{CPI_1/P_{M1}}$ where CPI_i is a weighted average of the period i Leontief prices. We may take π_1 to be exogenous (in practice, taken to be the actual inflation rate in that year).

(30) $p_{Bi}y_{BGi} = B_i(G_i - T_i) \quad : \quad 0 \leq B_i(G_i - T_i) \leq G_i - T_i$

$B_i(0) = 0; \; B_i(G_i - T_i) = 0 \text{ if } G_i - T_i \leq 0$

Thus, the nominal value of bond financing is a continuous function of the nominal deficit, not including debt repayment, and no sale of bonds takes place if there is a surplus. The change in the supply of money, $\tilde{y}_{M1}$, is then given by:

(31) $p_{Mi}\tilde{y}_{Mi} = D_i - B_i(G_i - T_i)$

so that debt repayment is made in money.

In period 2 the formation of the government deficit is somewhat different, since it must pay not only for its current consumption but also for its debt obligations incurred in period 1. Accordingly:

(32) $D_2 = G_2 + p_{M2}y_{BG1} - T_2$

Of course, we would have $B_1 = B_2$ if the government chooses to maintain the same financing rule in period 2 as in period 1. 1/

It is assumed that capital formation is carried out by the private sector and that it is fully financed by the sale of bonds, which are identical to the bonds sold by the government. Suppose, then, that the rate of return on capital in period i + 1 is p_{Ki+1}. 2/ The total return on a quantity, H_i, of new capital that was formed in period i and which comes on line in period i + 1 is then $p_{Ki+1}H_i$. If C_{Hi} is the cost-minimizing cost of producing H_i, then future debt obligations must be equal to the return on capital. Hence:

(33) $p_{M(i+1)}(C_{Hi}/p_{Bi}) = p_{K(i+1)}H_i$

The investment firm, having found a level of investment H_i such that H_i, C_{Hi} satisfy equation (31), then sets its sale of bonds, y_{Bpi}:

(34) $p_{Bpi}y_{Bi} = C_{Hi}$

1/ Thus, the interest obligations incurred by the government in period 1 are paid off in period 2 in units of money, rather than being rolled over in new bond sales. This form of payment is not essential to the model, but allows a simpler proof of the boundedness of the supply of bonds in period 2.

2/ For i = 2 we take $p_{K3} = (1+\pi_2)p_{K2}$.

One final assumption will be made, namely, that the investment firm recognizes the finite supply of capital and labor, and bounds its level of investment accordingly. Thus, if $H_i = H_i(y_{KHi}, y_{LHi})$ then:

$$(35) \quad y_{KH1} \leq \bar{K}_0, \ y_{LH1} \leq \bar{L}_0$$

$$y_{KH2} \leq H_1(\bar{K}_0, \bar{L}_0) + (1-\delta)\bar{K}_0, \ y_{LH2} \leq \bar{L}_0$$

4. Excess demand functions and the conditions for intertemporal equilibrium

The presence of an intertemporal input-output matrix allows the vector of excess demand functions to be confined to the space of prices corresponding to capital, labor, money, bonds, and transfer payments, indexed by their time period. Accordingly, given an arbitrary vector of prices, p, the nominal value added per unit of output may be derived for each of the 2N sectors producing intermediate and final goods, as in equation (24). Equation (25) then gives Leontief prices for each of the two periods, and equations (26) and (27) give total demand for intermediate and final goods, along with the corresponding level of production required of each activity in the Leontief matrix. The total inputs of capital and labor required in each period by the private production sector and the government may also be derived.

The requirements of capital and labor in each period also include their usage in investment which is determined by equations (3) and (33). The total supplies of the capital and labor in period i, y_{Ki}, y_{Li}, are then:

$$(36) \quad y_{K1} = -\hat{y}_{K1} + \bar{K}_0, \ y_{K2} = -\hat{y}_{K2} + (1-\delta)\bar{K}_0 + H_1$$

$$y_{L1} = -\hat{y}_{L1} + \bar{L}_0, \ y_{L2} = -\hat{y}_{L2} + \bar{L}_0$$

where $\hat{y}_{Ki}$, $\hat{y}_{Li}$ are the aggregate inputs to public and private production and investment in period i, and H_1 is the level of real investment in period 1.

The change in the money supply in period i, $\tilde{y}_{Mi}$, is given by equation (31), so that the total supply of money in each period, y_{Mi}, is:

$$(37) \quad y_{M1} = \bar{M}_0 + \tilde{y}_{M1}, \ Y_{M2} = Y_{M1} + \tilde{y}_{M2}$$

The supply of bonds in each period, y_{Bi}, is given by

$$(38) \quad y_{B1} = \bar{B}_0 + y_{BG1} + y_{Bp1} \qquad y_{B2} = y_{B1} + y_{BG2} + y_{Bp2}$$

where y_{BGi} is the government's bond issue in period i, and y_{Bpi} is private bond issue for investment financing.

An aggregate supply vector, y, has now been derived, where:

(39) $y \equiv (y_1, y_2) \equiv (y_{K1}, y_{L1}, y_{M1}, y_{B1}, y_{K2}, y_{L2}, y_{M2}, y_{B2})$

This supply vector is augmented by two additional dimensions, corresponding to transfer payments in each of the two time periods. Accordingly, define y(p), the augmented supply vector, by:

(40) $y(p) \equiv (y, \mu(D_1), \mu(D_2))$: $\mu(D_i) = D_i$: $D_i \leq 0$

$\mu(D_i) = 0$: $D_i > 0$

where D_i is the government deficit in period i.

The derivation of an augmented demand vector, x(p), is now straightforward. Since $x_{Ki} \equiv 0$, and equation (30) gives individual demands for leisure, money, and bonds in each period, summing across consumers gives the aggregate demands, x_{Li}, x_{Mi}, x_{Bi}. The aggregate demand vector, x, is then defined by:

(41) $x \equiv (x_1, x_2) \equiv (0, x_{L1}, x_{M1}, x_{B1}, 0, x_{L2}, x_{M2}, x_{B2})$

Finally, the augmented demand vector, x(p), is defined by:

(42) $x(p) \equiv (x, -TR_1, -TR_2)$

where TR_i represents the proxy for government transfer payments that enters the consumer's maximization problem, as in equation (21). The aggregate excess demand function, u(p), is then defined as:

(43) $u(p) \equiv x(p) - y(p)$

so it must be shown that there exists some price p such that:*

$u(p^*) \leq 0$. 1/

1/ The proof of this result is given in the Appendix.

III. An Application to Australia

The Australian case is suitable as an example of the model since it allows the introduction of a trade sector without modification of the theoretical structure. In addition, it is also reasonable to make the assumption of a small country that is a price taker in world trade. To use the model for simulation analysis, it is necessary to demonstrate first that it replicates reasonably accurately the Australian economy for a particular benchmark period. We will therefore first describe the data used, and will then give the results of the benchmark simulation for two successive years. We will then carry out various experiments with different rules for financing the government budget deficit, as well as for the level of real government expenditure.

Until late 1982, Australia operated under a system of fixed exchange rates and capital controls. Our model simultaneously derives a solution for two periods, so we have chosen to let 1981-82 be the years for the benchmark solution. We thereby avoid the problem of having to construct an exchange rate determination mechanism, and may treat the capital account as being exogenous. The Australian technology is represented by a 30 x 30 input-output matrix, constructed for 1977, in which activities 29 and 30 are imports of complementary and competing imports, respectively. 1/ We have not updated the matrix for 1981 and 1982. Our technology is thus represented by a 60 x 60 block diagonal matrix. The coefficients for the Cobb-Douglas functions representing production of value added in each period are given by the relative shares of capital and labor in each activity in the 1977 matrix, 2/ assuming no technological change between periods. Coefficients for capital and labor in the investment function are also derived from relative shares of capital and labor aggregated across industries, as inputs to investment. 3/ The resulting coefficients a_i, b_i, as in equation (3) are:

$$(44) \quad a_i = 0.5758, \; b_i = 0.4242$$

where the subscript 1 refers to 1981, and 2 to 1982, here and in what follows. Thus, capital has a larger share as an input to investment than does labor, and we assume no change in technology between periods. 4/

1/ See Australian National Accounts Input-Output Tables (1983), Table 12.

2/ See Australian National Accounts Input-Output Tables (1983), Table 11.

3/ See Australian National Accounts Input-Output Tables (1983), Table 18.

4/ In computation, the coefficients are sealed proportionally so as to seem to be slightly less than 1.

Targets for real government expenditure on goods and services, as a percentage of GNP, are taken to be the actual values for 1981-82. Accordingly: 1/

(45) $Q_1 = 0.3494$, $Q_2 = 0.3545$.

The coefficients for the government production of public goods function β_i, $1-\beta_i$ as in equation (5) are derived by simply deducting the government wage bill from total expenditures of the government. 2/ The corresponding coefficients for capital, β_i, in the two periods are thus:

(46) $\beta_1 = 0.4296$, $\beta_2 = 0.4096$

Finally, the rate of depreciation of capital, δ, as in equation (4) is taken to be $\delta = 0.0629$. 3/

We have chosen to use the net effective rate of sales taxes to represent the uniform sales tax rate across sectors. This simplication seems justified since the Australian tax system charges uniform sales tax rates on all goods except private motor vehicles and certain household durables. The sales tax rates, t_i, in period i across sectors are thus: 4/

(47) $t_1 = 0.141$, $t_2 = 0.138$

The tax rate on labor usage, t_{Li} (an income tax withheld at the source), is taken to be the average personal income tax rate. The tax rate on capital, t_{Ki}, is taken to be the corporate income tax rate, which is a flat rate in Australia. The corresponding rates are thus: 5/

(48) $t_{K1} = t_{K2} = 0.46$, $t_{L1} = 0.232$, $t_{L2} = 0.236$

The tariff rate on complementary imports is 0, while on competing imports it is 0.16 in 1981 and 0.15 in 1982, uniformly across goods. 6/

Our model has three consumer categories: high-income Australian; low-income Australian; rest of world. We need first to derive initial allocations of capital, labor, money, and bonds for each of these

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1/ See Australian Economic Statistics (1984), Table 2.1.
2/ See Australian National Accounts, National Income and Expenditure (1984), Tables 1 and 25.
3/ This figure is taken from the Orani general equilibrium model of Australia (see Dixon, et al. (1982)).
4/ See Australian Economic Statistics (1984), Table 2.30.
5/ See Australian Economic Statistics (1984), Table 2.29.
6/ See Customs and Excise Revenue Australia (1984), Table 13.

categories. Initial allocations are taken to be holdings at the end of 1980. Total initial allocations of capital and labor are taken to be the aggregate gross operating surplus and the total wage bill for 1980. Thus, a unit of capital or labor is defined as that which earned $A 1 in 1980. 1/ It is assumed that this income is divided entirely among Australian consumers, with shares derived from survey data giving sources of income by income level. 2/ Initial holdings of money were, for the Australian consumers, taken to be the stock of M_2 at the end of 1980. 3/ The shares in this stock of money were assumed to be equal to the relative shares of wage plus capital income for the two Australian consumer categories. The initial allocation of money for the rest of the world is taken to be the value of exports of goods in 1980. 4/ Thus, changes in the value of exports will come about only through changes in Australian export prices. There is no wealth effect for the rest of the world. In particular, we represent changes in the exchange rate by corresponding changes in the rest of the world's initial allocation of money. A 10 percent devaluation would be represented by a 10 percent increase in the rest of the world's initial holdings of money, while there would also be a corresponding increase in the quantity of domestic money needed to import one unit of foreign goods. Finally, the initial holdings of bonds by the Australian consumer categories are taken to be the interest obligations on Australian government securities on issue in 1980. 5/ Thus, we define a bond as that security which earned $A 1 of interest in 1980, corresponding to our definitions of capital and labor in terms of earning flows. The initial allocations are then:

Table 1. Initial Allocations

(In millions of 1980 Australian dollars)

	Australian 1 (Low-income)	Australian 2 (High-income)	Rest of the World
Capital	9,488	29,477	0
Labor	22,670	39,050	0
Money	15,350	40,410	11,500
Bonds	580	1,810	0

1/ See Australian Economic Statistics (1984), Table 5.5.

2/ See Income Distribution Australia, 1978-79, Individuals (1981), Table 11.

3/ See International Financial Statistics (1984).

4/ See Australian Economic Statistics (1984), Table 1.2.

5/ See Australian Economic Statistics (1984), Table 2.22. Recall that we assume that there are no outstanding initial holdings of privately issued bonds.

To obtain budget elasticities for consumption of intermediate and final goods, we have taken the addilog estimates of marginal budget shares given in Bewley (1982). These estimates are made for two categories of consumers, those with weekly household incomes of $A 50 and $A 350, based on 1976 data. The categories of low-income and high-income Australian consumers and the corresponding distribution shares are thus made to correspond to the 1980 nominal values of these income levels. There is a somewhat higher level of aggregation in these budget shares than in the input-output matrix, and, rather than constructing a transformation matrix, we have chosen to assign input-output categories to each of the consumption categories. We are also treating marginal budget shares as being equal to average shares, in order to correspond to our specification of the consumer's utility functions. Average budget shares are taken to be equal in both 1981 and 1982 and are: 1/

Table 2. *Average Budget Shares*

(In percent)

Commodity	Corresponding Input-Output Sector	Low-Income Australian	High-Income Australian
Rent	24,25	19.6	23.7
Fuel	12	2.2	0.2
Food	1,2,4,5	22.0	5.3
Alcohol, etc.	6	4.4	5.0
Clothing	7,8	8.2	5.8
Durables	9,10,11,15,16, 17,18,19,21,22	8.3	9.5
Health	27	2.7	1.0
Transport	23	15.2	20.9
Recreation	28	8.5	14.4
Miscellaneous	26	9.0	14.2

In addition, we have assumed these to be an elastic labor supply, with each Australian consumer having an elasticity of demand for leisure of 5 percent. This figure was not empirically derived but was determined by fitting the overall model to the benchmark years.

1/ Within a particular consumption category, the elasticities for the input-output sectors are derived from relative consumption shares. Budget shares for the rest of the world are taken to be the shares in total exports of goods of each input-output category.

The final element in the structure of the consumer's demand system is a money demand function, as in equation (16). We will assume that there is a partial adjustment mechanism in which the money stock adjusts with a lag to the level desired by consumers. Accordingly:

$$(49)\ \ln M - \ln M_{-1} = \beta\ (\ln M^d - \ln M_{-1})$$

where M, M_{-1} are the stocks of broad money in the current and past period, M^d is the current quantity of money demanded, and β is the unobserved speed of adjustment. We convert equation (16) into log form as:

$$(50)\ \ln M^d = a_0 + a_1 \ln C + a_2 r$$

where r is the current interest rate, and C is the nominal value of current private consumption including taxes. Substituting and setting $a_1 = 1$, the condition required for homogeneity, we obtain:

$$(51)\ \ln M/C = \beta a_0 + \beta a_2 r + (1-\beta) \ln M_{-1}/C$$

Equation (51) was estimated using annual Australian data for the period 1950-82. The interest rate is the nominal yield on short-term (two-year) Australian Treasury bonds. 1/ The resulting equation estimate is:

$$(52)\ \ln M/C = \underset{(-0.436)}{-0.0162} - \underset{(-3.186)}{1.5984}\ r + \underset{(1.734)}{0.2769} \ln M_{-1}/C$$

D.W. = 2.42 $\qquad R^2 = 0.89$

The figures in parenthesis are t-statistics. We may then identify parameters assuming $M = M_{-1}$, the required condition for long-run stability. Thus:

$$(53)\ \beta = 0.7231,\ a_0 = -0.0224,\ a_2 = -2.2106$$

Returning to the form of equation (16), $P_{Mi}\ X_{Mi} = ae^{-bri}\ (1+t_i)\ \tilde{p}_i X_i$, we obtain:

$$(54)\ a = 0.9779,\ b = -2.2106$$

We now have values for all required parameters in our model, and may turn to the simulation of the benchmark years. A key government policy instrument that we have allowed to be endogenous to the model is the choice of the mix between money and bonds used to finance its

1/ All data come from the International Financial Statistics of the International Monetary Fund.

budget deficit. It was determined upon solving the model that weights of 40 percent money and 60 percent bonds in 1981, and 60 percent money, 40 percent bonds in 1982 yielded the most accurate approximation to the actual macroeconomic variables in those years. We used a version of Merrill's (1971) fixed point algorithm to solve our model. Merrill's method is based upon a shrinking subdivision of the price simplex, and the theoretical structure of the version we use is based upon Shoven (1974). The algorithm operates on the space of excess demands for capital, labor, money, bonds, and transfer payments in each of two periods while there is identical market clearing in the markets for intermediate and final goods. A similar methodology is described in Feltenstein (1979). The algorithm stopped when all excess demands were less than 0.1 percent of the corresponding total supply.

Our initial solution yields the following equilibrium relative prices for scarce factors and financial assets.

Table 3. Equilibrium Prices
(Benchmark Case)

	Good	Price 1/
	1981	
1.	Capital	86.7
2.	Labor	80.1
3.	Money	100.0
4.	Bonds	94.6
5.	Transfer payments	0.0
	1982	
6.	Capital	68.3
7.	Labor	97.1
8.	Money	89.0
9.	Bonds	80.0
10.	Transfer payments	0.0

The interest rate in period i is then defined as p_{Mi}/p_{Bi}, while the inflation rate in period $_2$, π_2, is given by: 2/

1/ Prices are normalized, with the price of 1981 money being the numeraire. In particular, the zero price of transfer payments in both periods indicates that there is a budget deficit.

2/ There is no definition of an endogenous period 1 inflation rate, since there is no endogenous price level prior to period 1.

$$(55) \quad \pi_2 = \frac{\sum_{j=31}^{60} pL_j * w_j / p_{M2}}{\sum_{j=1}^{30} pL_j * w_j / p_{M1}}$$

Here pL_j is the Leontief price of the j^{th} intermediate or final good, w_j is the corresponding weight in the Australian GDP deflator, and p_{Mi}, p_{Bi} are the i^{th} period prices for money and bonds, respectively. 1/ We may then compare actual and simulated interest and inflation rates.

Table 4. Simulated vs. Actual
(Benchmark Case)

(In percent)

	Simulated	Actual
	1981	1981
Interest rate 2/	5.8	11.5
Inflation rate 3/	10.0	10.0
	1982	1982
Interest rate	11.3	13.1
Inflation rate	11.2	10.4

We may then define national income in period i, GDP_i as the value of period i expenditure in terms of period i money. Accordingly:

1/ Goods 1-30 refer to 1981 intermediate and final goods while 31-60 refer to 1982 goods.
2/ We are using the two-year Treasury bill rate for the actual figures. See Australian Economic Statistics (1984), Table 2.27.
3/ The 1981 simulated inflation rate is taken to have its actual value for the purpose of the conditions for boundedness in equation (29).

Table 5. Simulated vs. Actual
(Benchmark Case)

	Simulated	Actual	Simulated	Actual
	(In billions of $A)		(In percent of GDP)	
1981				
GDP	130.8 1/	130.8	100.0	100.0
Tax revenues 2/	43.0	45.2	32.8	34.6
Government expenditure 2/	48.0	49.8	36.7	38.0
Budget surplus (deficit)	-5.0	-4.6	-3.9	-3.5
Gross private investment 3/	20.8	21.7	15.9	16.6
Exports of goods 4/	20.0	18.7	15.3	14.3
Imports of goods 4/	22.8	19.2	17.4	14.7
Trade balance	-2.8	-0.5	-2.1	-0.4
1982				
GDP	149.8	147.9	100.0	100.0
Tax revenues 2/	50.9	52.6	34.0	35.6
Government expenditure 2/	57.0	57.4	38.1	38.8
Budget surplus (deficit)	-6.1	-4.8	-4.1	-3.2
Gross private investment 3/	21.4	26.9	14.3	18.2
Exports of goods 4/	18.7	19.1	12.5	12.9
Imports of goods 4/	25.9	22.4	17.3	15.1
Trade balance	-7.2	-3.3	-4.8	-2.2

We thus note that our model gives a reasonably close approximation to the actual Australian outcomes for 1981-82. 5/ It is also worth mentioning the difference between our results and those normally reported in applied general equilibrium work. Interest and inflation rates are

1/ The simulated figures are normalized so that 1981 simulated and actual GDP are equal.

2/ See Australian Economic Statistics (1984), Table 2.1.

3/ Op. cit., Table 5.7a.

4/ Op. cit., Table 1.1.

5/ The fact that simulated interest rate in 1981 is lower than the corresponding actual rate is primarily a result of our initial allocation of bonds comprising only outstanding government debt and not incorporating private debt. Inclusion of initial private debt (which, as a fixed cost, is technically not feasible) would tend to lower the 1981 price of bonds and raise the interest rate.

particular to this model as are budget deficits financed by money and bonds and balance of payments deficits financed by losses in reserves. The model, of course, also generates supply of and demand for scarce factors and financial assets, as well as intermediate and final goods, indexed by time period.

We may now turn to two counterfactual simulations. In our first example, we will suppose that a single change has taken place; in both 1981 and 1982, the government expenditure target as a percentage of GDP has risen by 10 percent. Thus, government expenditures on goods and services are targeted at 38.4 percent of GDP in 1981 and 39.0 percent in 1982. The following solution for the macroeconomic variables results:

Table 6. Counterfactual Simulation I
(10 Percent Increase in Government Spending)

	1981		1982	
Interest rate	14.7		23.0	
Inflation rate 1/	17.2		17.8	
	(In billions of $A)	(In percent of GDP)	(In billions of $A)	(In percent of GDP)
GDP	153.6	100.0	187.0	100.0
Tax revenues	51.1	33.3	64.6	34.6
Government expenditure	60.6	39.5	78.8	42.2
Budget surplus (deficit)	-9.5	-6.2	-14.2	-7.6
Gross private investment	22.1	16.1	26.9	14.4
Exports of goods	20.0	13.0	18.6	10.0
Imports of goods	25.5	16.6	30.7	16.4
Trade balance	-5.5	-3.6	-12.1	-6.4

1/ The inflation rate is calculated with respect to the price level generated in the benchmark simulation.

We thus notice that interest rates, both real and nominal, have increased sharply in response to higher government deficit financing, compared with the benchmark case. The rate of inflation has risen and the trade balance and budget deficit have deteriorated, as might be expected. The rate of growth in real GDP has risen from 3.3 percent in the initial example to 3.9 percent in this case, while gross private investment has remained approximately constant as a percentage of GDP. The reason for this apparent absence of crowding out can be seen by observing the equilibrium relative prices for factors and financial assets.

Table 7. Counterfactual Simulation I, Equilibrium Prices
(10 Percent Increase in Government Spending)

Good	Price (Benchmark)	Price (Simulation I)
	1981	
1. Capital	86.7	105.8
2. Labor	80.1	97.5
3. Money	100.0	100.0
4. Bonds	94.6	87.2
5. Transfer payments	---	---
	1982	
6. Capital	68.3	79.3
7. Labor	97.1	111.0
8. Money	89.0	79.0
9. Bonds	80.0	64.2
10. Transfer payments	---	---

The money prices of capital and labor have thus risen relative to the benchmark case. In particular, the higher wage rate has caused the supply of labor to increase. This outcome depends critically, however, on the choice of an elastic labor supply. The higher price of capital in 1982 indicates that although the cost of investment has risen, reflected by the higher interest rate, the rate of return on capital has also risen, and the two changes approximately cancel each other out. There is also a demand effect as the higher interest rate causes consumers to reduce their money holdings, increasing consumption. Thus, tax revenues have increased as a percentage of GDP, that is, an inflation tax.

As a final example, we will suppose government expenditure on goods and services stays the same in real terms as in the benchmark case, the results of which are reported in Table 5, but that its rule for financing its budget deficit changes. Under the new rule, 100 percent of the deficit will be financed by the sale of bonds, so that the deficit is not monetized. This example will indicate whether changes in financing rules can have an impact on the real economy.

Table 9. Counterfactual Simulation II
(100 Percent Bond-Financed Deficit)

	1981		1982	
Interest rate	12.7		24.5	
Inflation rate	11.0		16.6	
	(In billions of $A)	(In percent of GDP)	(In billions of $A)	(In percent of GDP)
GDP	144.5	100.0	174.1	100.0
Tax revenues	48.0	33.2	60.2	34.6
Government expenditure	52.6	36.4	67.2	38.6
Budget surplus (deficit)	-4.6	-3.2	-7.0	-4.0
Gross private investment	22.7	15.7	24.3	13.9
Exports of goods	20.0	13.9	18.6	10.7
Imports of goods	24.6	17.1	29.4	16.9
Trade balance	-4.6	-3.2	-10.8	-6.2

In this case, the rate of growth of real GDP is 3.4 percent, or approximately what it was in the benchmark case. We notice that the nominal and real interest rates have risen, the latter somewhat more than in the case of increased government spending. Gross private investment as a percentage of GDP has fallen in both years, compared to the benchmark case, primarily because there has been insufficient demand stimulus and corresponding increased returns on capital to outweigh the increased interest costs to investors. Thus, a moderate degree of crowding out of the private sector does take place in this case.

IV. Summary and Conclusion

We have constructed an intertemporal general equilibrium model that can be used to analyze crowding out of the private sector. The model is disaggregated in both production and consumption and assumes perfect foresight for all agents. To obtain a tractable empirical application, the time horizon taken is of two periods with a single generation of consumers, although there would be no difficulty in extending the model to multiple periods. Private investment is purely debt financed, and competes with government debt, issued with money to finance the deficit, for private savings. There is a fixed exchange rate and capital flows are assumed to be exogenous. A particular closing rule that is not debt-neutral is assumed, and an intertemporal equilibrium is proven to exist.

We have derived a fixed point algorithm to solve the model, and have constructed an application to Australia. A benchmark solution is calculated for 1981-82, the last two years for which the Australian exchange rate was fixed, and is shown to yield a reasonably accurate approximation to the actual outcomes of the economy for those years. Two counterfactual simulations were then carried out. In the first, an increase in government expenditure brings about a slight increase in real income, with no significant change in the proportion of income allocated to private investment. There is, however, a significant increase in the real interest rate and the govenment's budget deficit, as well as a loss in reserves via the trade account. In the second simulation, an increase in the debt-financed proportion of the government's deficit is found to have no significant impact on real income, although there is a decrease in both years in the level of private investment. Thus, a degree of crowding out does take place.

It should be noted that the results of our model are quite sensitive to the elasticity of the labor-leisure choice, as well as to the closing rule for holding debt in the second period. Some of the simulation results would also not hold in a model with a longer time horizon. In particular, continued government expenditure and deficit increases would not continue to stimulate real income as inflation accelerates. Areas for future research would thus be an extended time horizon, possibly in the form of an intergenerational model, as well as the introduction of a floating exchange rate and an endogenous capital account.

A Proof of the Existence of an Equilibrium

The proof of the existence of a competitive equilibrium in the model depends on demonstrating certain properties for the excess demand function defined in equation (43). It will first be shown that Walras' law holds at each period, and hence for the intertemporal excess demand. The value of supply in period 1, S_1, is given by:

$$(56)\quad S_1 = [\bar{p}(p_1)(I_1-A_1) - p_1 \cdot va(p_1)] - G_1 + p_{M1}\bar{M}_0 + p_{B1}\bar{B}_0$$

$$+ p_{K1}\bar{K}_0 + p_{L1}\bar{L}_0 + p_{B1}y_{Bp1} - p_{K1}y_{KH1} - p_{L1}y_{LH1}$$

$$+ p_{M1}\tilde{y}_{M1} + p_{B1}y_{BG1}$$

where A_1 denotes the upper (or first period) quadrant of the intertemporal input-output matrix A. As:

$$(57)\quad p_{B1}y_{Bp1} = p_{K1}y_{KH1} + p_{L1}y_{LH1}$$

it follows that:

$$(58)\quad S_1 = t_{K1}p_{K1}y_{Kp1} + t_{L1}p_{L1}y_{Lp1} - G_1 + p_{M1}\bar{M}_0 + p_{B1}\bar{B}_0$$

$$+ p_{K1}\bar{K}_0 + p_{L1}\bar{L}_0 + p_{M1}\tilde{y}_{M1} + p_{B1}y_{BG1}$$

The value of demand in period 1, $E_1(p_1, p_2)$, is the value of the consumers' disposable income, minus that part of their income going to sales taxes, hence: 1/

$$(59)\quad E_1 = p_{M1}\bar{M}_0 + p_{B1}\bar{B}_0 + p_{M1}\bar{B}_0 + p_{K1}\bar{K}_0 + p_{L1}\bar{L}_0 + TR_1 - \sum_{j=1}^{N} t_j xL_j$$

Thus:

$$(60)\quad E_1 - S_1 = G_1 + p_{M1}\bar{B}_0 - (t_{K1}p_{K1}y_{KP1} + t_{L1}p_{L1}y_{LP1} + \sum_{j=1}^{N} t_j xL_j)$$

$$- p_{M1}\tilde{y}_{M1} - p_{B1}y_{BG1} + TR_1$$

$$= D_1 - p_{M1}\tilde{y}_{M1} - p_{B1}y_{BG1} + TR_1$$

1/ Here, and in what follows, the K consumers have been aggregated in the model. The interested reader may easily carry out the aggregation to arrive at the equations presented.

If the first period components of $x(p)$, $y(p)$ are then denoted by $x_1(p)$, $y_1(p)$, then:

$$(61)\quad p_1 \cdot x_1(p) - p_1 \cdot y_1(p) = E_1 - S_1 - TR_1 - \mu(D_1)$$

$$= D_1 - p_{M1}\tilde{y}_{M1} - p_{B1}y_{BG1} - \mu(D_1)$$

$$= 0$$

as:

$$D_1 \geq 0 \rightarrow D_1 = p_{M1}\tilde{y}_{M1} + p_{B1}y_{BG1},\ \mu(D_1) = 0$$

$$D_1 < 0 \rightarrow \tilde{y}_{M1} = y_{BG1} = 0,\ \mu(D_1) = D_1$$

Thus, Walras' law holds in period 1.

In period 2 the value of supply, S_2, is given by:

$$(62)\quad S_2 = [\ \bar{p}(p_2)(I_2 - A_2) - p_2 \cdot va(p_2)\ ]\ y_2 - G_2$$

$$+ p_{K2}(1-\delta)\bar{K}_0 + p_{L2}\bar{L}_0 + p_{M2}\mu(x_{B1} - y_{B1}) + p_{M2}x_{M1}$$

$$+ p_{B2}x_{B1} + p_{K2}H_1 + p_{B2}y_{Bp2} - p_{K2}y_{KH2} - p_{L2}y_{LH2}$$

$$+ p_{M2}\tilde{y}_{M2} + p_{B2}y_{BG2} + p_{B2}y_{Bp2} - p_{K2}y_{KH2} - p_{L2}y_{LH2}$$

The value of the second-period initial stocks of money and bonds is taken to be equal to the values of the corresponding period 1 demands as an imposed equilibrium condition. Here, $\mu(x_{B1} - y_{B1}) \equiv x_{B1} - y_{B1}$: $x_{B1} - y_{B1} > 0$, $\mu(x_{B1} - y_{B1}) \equiv 0$: $x_{B1} - y_{B1} < 0$. 1/ Thus,

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1/ The period 2 initial stock of money will be augmented by $\mu(x_{B1} - y_{B1})$. Proof of the existence of equilibrium will ensure that $x_{B1} = y_{B1}$ at equilibrium, so that there will be no augmentation of period 2 supply. A similar augmentation of the value of demand is made in equation (64). Both of these augmentations are made in order to ensure that Walras' law holds at an arbitrary set of prices, and market clearing in period 1 will ensure that they will vanish in period 2.

$$(63)\quad S_2 = t_{K2}p_{K2}y_{Kp2} + t_{L2}p_{L2}y_{Lp2} - G_2 + p_{K2}(1-\delta)\bar{K}_0$$

$$+ p_{L2}\bar{L}_0 + p_{M2}\mu(x_{B1}-y_{B1}) + p_{M2}x_{M1} + p_{B2}x_{B1} + p_{K2}H_1$$

$$+ p_{M2}\tilde{y}_{M2} + p_{B2}y_{BG2} + p_{B2}y_{Bp2} - p_{K2}y_{KH2} - p_{L2}y_{LH2}$$

The value of demand in period 2 is, as before, given by the value of income minus tax payments.

$$(64)\quad E_2 = p_{K2}(1-\delta)\bar{K}_0 + p_{L2}\bar{L}_0 + p_{M2}\mu(y_{B1}-x_{B1})$$

$$+ p_{M2}x_{M1} + p_{M2}x_{B1} + p_{B2}x_{B1}$$

$$+ TR_2 - \sum_{j=N+1}^{2N} t_j xL_j$$

As in equation (63), $\mu(y_{B1}-x_{B1}) \equiv y_{B1}-x_{B1} : y_{B1}-x_{B1} > 0$, $\mu(y_{B1}-x_{B1}) \equiv 0 : y_{B1}-x_{B1} < 0$. Hence:

$$(65)\quad E_2-S_2 = G_2-(t_{K2}p_{K2}y_{Kp2} + t_{L2}p_{L2}Y_{Lp2} + \sum_{j=N+1}^{2N} t_j xL_j)$$

$$+ p_{M2}\mu(y_{B1}-x_{B1})-p_{M2}\mu(x_{B1}-y_{B1})+p_{M2}x_{B1}-p_{K2}H_1-p_{M2}\tilde{y}_{M2}$$

$$- p_{B2}y_{BG2}+TR_2$$

$$= G_2-(t_{K2}p_{K2}y_{Kp2}+t_{L2}p_{L2}y_{Lp2}+\sum_{j=N+1}^{2N} t_j xL_j) + p_{M2}y_{B1} - p_{K2}H_1$$

$$- p_{M2}\tilde{y}_{M2} - p_{B2}y_{BG2} + TR_2$$

By equation (33):

$$(66)\quad p_{M2}y_{B1} - p_{K2}H_1 = p_{M2}y_{B1} - p_{M2}y_{Bp1} = p_{M2}y_{BG1}$$

Thus:

$$(67)\quad E_2 - S_2 = (G_2 + p_{M2}y_{BG1} - T_2) - p_{M2}\tilde{y}_{M2} - p_{B2}y_{BG2} + TR_2$$

$$= D_2 - p_{M2}\tilde{y}_{M2} - p_{B2}y_{BG2} + TR_2$$

as $p_{M2}y_{BC1}$ represents the government's debt obligation in period 2.

Thus:

$$(68) \quad p_2 \cdot x_2(p) - p_2 \cdot y_2(p) = E_2 - S_2 - TR_2 - \mu(D_2) = 0$$

as in equation (61). Thus, Walras' law also holds for the intertemporal excess demand function u(p) defined in equation (43).

It must now be shown that the intertemporal excess demand function u(p) is continuous and bounded in prices. To demonstrate this, it may be noted that:

Lemma 1: The value added to the j^{th} sector, $va_j(p)$ is a continuous function of p, the vector of intertemporal prices for capital, labor, and financial assets.

Thus,

Lemma 2: The intertemporal Leontief prices $\bar{p}(p)$ are continuous in p.

Accordingly, the levels of demand for the individual consumer are continuous in p. Summing over all consumers:

Lemma 3: The aggregate demand for the j^{th} intermediate or final good, xL_j, given by equation (22), is continuous in p, as are x_{Ki}, x_{Li}, x_{Mi}, x_{Bi}, the aggregate demands for capital, labor, money, and bonds in period i.

The activity levels, z, for the Leontief matrix A representing private production are thus also continuous in p so that:

Lemma 4: The inputs of capital and labor in each period to private production y_{Kpi}, y_{Lpi}, i = 1;2 are continuous in p.

The rule for adjusting the real quantity of the output of public goods, Q_i, is given by

$$(69) \quad Q_i = \bar{Q}_i \qquad \text{if } R_i \leqslant \bar{R}_i,\ \pi_i \leqslant \bar{\pi}_i$$

$$Q_i = \frac{\bar{Q}_i}{1 + R_i - \bar{R}_i} \qquad \text{if } R_i > \bar{R}_i,\ \pi_i \leqslant \bar{\pi}_i$$

$$Q_i = \frac{\bar{Q}_i}{1 + \pi_i - \bar{\pi}_i} \qquad \text{if } R_i \leqslant \bar{R}_i,\ \pi_i > \bar{\pi}_i$$

$$Q_i = \min\left[\frac{\bar{Q}_i}{1 + R_i - \bar{R}_i}, \frac{\bar{Q}_i}{1 + \pi_i - \bar{\pi}_i}\right] \text{if } R_i > \bar{R}_i,\ \pi_i > \bar{\pi}_i$$

where $\bar{R}_i$ and $\bar{\pi}_i$ represent the real interest rate and rate of inflation, respectively, desired by the government in period i. 1/ By equation (69), Q_i is easily seen to be continuous in p and hence, so are y_{KGi} and y_{LGi}, the government's inputs of capital and labor, respectively, in period i. Thus:

Lemma 5: Government inputs of capital and labor in period i, y_{KGi}, y_{LGi}, respectively, as well as the cost of government production G_i, are continuous in p.

Lemmas (3-5) then gives:

Lemma 6: T_i, the tax revenues collected by the government in period i, and D_i, the government deficit in period i, are continuous in p.

Thus:

Lemma 7: y_{BGi}, $\tilde{y}_{Mi}$, the government's issuance of bonds and money in period i, are continuous in p.

Turning to private investment, it may be seen that:

Lemma 8: H_i, the real quantity of capital produced in period i, is continuous in p.

Lemma (3) and Lemma (7), give:

Lemma 9: y_{Bi}, the total supply of bonds in period i, is a continuous function of p.

Finally, by Lemmas (4-5), and Lemmas (7-8):

Lemma 10: y_{Ki}, y_{Li}, y_{Mi}, the supply of capital, labor, and money, respectively, in period i, are continuous functions of p.

Equation (43) leads to:

Lemma 10a: u(p), the augmented excess demand function, is continuous in p.

1/ $\bar{R}_i$ might, for example, be taken to be equal to the long-run real rate of interest, while $\bar{\pi}_i$ might be equal to a weighted average of past rates of inflation, as in an adaptive expectations framework. $\bar{R}_i$, $\bar{\pi}_i$ need only be finite constants for our results to hold.

The excess demand function u(p) must now be shown to be bounded. Because of the assumed bounds set on the individual consumer's demand for intermediate-final goods, as well as leisure, the aggregate demand for the j^{th} intermediate-final good is bounded also. Hence:

Lemma 11: y_{Kpi}, y_{Lpi}, the inputs of capital and labor to private industry in period i, are bounded.

Lemma 12: T_i, the taxes collected by the government in period i, are bounded.

By the fact that $Q_i \leqslant \bar{Q}_i$:

Lemma 13: y_{KGi}, y_{LGi}, the inputs of capital and labor to government production in period i, are bounded, as is G_i, the government expenditure in period i, and D_i, the corresponding deficit.

Now $D_1 \leqslant G_1 - T_1 \leqslant G_1$ and

$$(70) \quad G_1 = p_{K1}\left[\frac{(1 - B_1)}{B_1} \frac{p_{K1}}{p_{L1}}\right]^{(B_1 - 1)} \left[1 + \frac{(1 - B_1)}{B_1} \frac{p_{K1}}{p_{L1}}\right] Q_1$$

$$\equiv \lambda Q_i: \lambda \equiv p_{K1}\left[\frac{(1 - B_1)}{B_1} \frac{p_{K1}}{p_{L1}}\right]^{(B_1 - 1)} \left[1 + \frac{(1 - B_1)}{B_1}\right] \frac{p_{K1}}{p_{L1}}$$

$$= \frac{\lambda p_{B1}\bar{Q}_1}{p_{M1} - p_{B1}\bar{R}_1} \text{ if } R_1 \geqslant \bar{R}_1$$

Thus:

$$y_{BG1} \leqslant \frac{\lambda\bar{Q}_1}{p_{M1} - p_{B1}\bar{R}_1}$$

where y_{BG1} is the government's issuance of bonds in period 1. Suppose that y_{BG1} is unbounded, so that $y_{BG1} \to \infty$. Then,

$$p_{M1} - p_{B1}\bar{R}_1 \to 0$$

or $\quad R_1 \to \bar{R}_1$, so $p_{B1} > 0$

But if $R_1 = \bar{R}_1$ then:

$$y_{BG1} = \frac{\lambda\bar{Q}_1}{p_{B1}} < \infty$$

which contradicts the original assumption.

Similarly, y_{BG2}, the issuance of government bonds in period 2, is also bounded, so that:

Lemma 14: The government's issuance of bonds in period i, y_{BGi}, is bounded.

A similar result may also be demonstrated for the government's issuance of money.

We thus have:

Lemma 15: $\tilde{y}_{Mi}$, the government's issuance of money in period i, is bounded.

Let C_{Hi} denote the cost of private investment in period i. Then y_{Bpi}, the private issuance of bonds in period i, is given by:

$$y_{Bpi} = \frac{C_{Hi}}{p_{Bi}}$$

Thus, private debt obligations in period i + 1 are given by

$$p_{Mi+1}\, y_{Bpi} = p_{Mi+1} \frac{C_{Hi}}{p_{Bi}}$$

so that, as an equilibrium condition, the result must be:

$$p_{Mi+1} \frac{C_{Hi}}{p_{Bi}} = p_{Ki+1}\, H_i$$

Thus:

$$(71)\quad \frac{C_{Hi}}{H_i} = \frac{p_{Ki+1}\, p_{Bi}}{p_{Mi+1}}$$

Combining equation (71) with the cost minimization conditions for the investment function:

$$y_{KHi} = \left[\frac{p_{Ki+1}\ p_{Bi}}{p_{Mi+1}} \ \frac{p_{Ki}^{\beta_i - 1}\ p_{Li}^{-\beta_i}}{(1 + \beta_i/\alpha_i)} \left(\frac{\beta_i}{\alpha_i}\right)^{\beta_i} \right]^{\frac{1}{1 - \alpha_i - \beta_i}}$$

$$y_{LHi} = \frac{\beta_i}{\alpha_i} = \frac{p_{Ki}}{p_{Li}}\ y_{KHi}$$

Now by assumption, $y_{KH1} \leq \bar{K}_o$, $y_{LH1} \leq \bar{L}_o$. Let $\bar{H}_1 = \max\ y_{KH1}^{\alpha_1}\ y_{LH1}^{B_1}$: $y_{KH1} \leq \bar{K}_o$, $y_{LH1} \leq \bar{L}_o$. Then $y_{KH2} \leq (1-\delta)\bar{K}_o + \bar{H}_1$, $y_{LH2} \leq \bar{L}_o$ so that $C_{Hi} = p_{K1}\ y_{KHi} + p_{Li}\ y_{LHi} \leq \bar{C}_i$ for some $\bar{C}_i < \infty$. In order to show that y_{Bpi}, the private sector's issuance of bonds in period i, is bounded, it needs only to be shown that:

$$\lim_{p_{Bi} \to 0} \frac{C_{Hi}}{p_{Bi}}$$

is bounded. But

$$\frac{C_{Hi}}{p_{Bi}} = \left[1 + \left(\frac{\beta_i\ p_{Ki}}{\alpha_i\ p_{Li}}\right)\right] \left[\frac{p_{Ki+1}}{p_{Mi+1}} \ \frac{p_{Ki}^{B_i - 1}\ p_{Li}^{-B_i}}{(1 + B_i \alpha_i)} \ \frac{\beta_i}{\alpha_i}^{B_i} \right]^{\frac{1}{1 - \alpha_i - \beta_i}} p_{Bi}^{\frac{\alpha_i + \beta_i}{1 - \alpha_i - \beta_i}}$$

so that

$$\lim_{p_{Bi} \to 0} \frac{C_{Hi}}{p_{Bi}} = 0$$

Thus:

<u>Lemma 16</u>: y_{Bpi}, the private sector's issuance of bonds in period i, is bounded.

Consider now x_{Bi}^{j}, the j^{th} consumer's demand for bonds in period i. If $\bar{y}_{BGi}$, $\bar{y}_{Bpi}$ are the upper bounds on the government's and the private sector's issuance of bonds in period i, bounds already shown to exist, then, by assumption:

$$x_{B1}^{j} \leq \bar{y}_{B1} \equiv \bar{y}_{BG1} + \bar{y}_{Bp1}$$

so that: $$x_{B1} = \sum_{j=1}^{J} x_{B1}^{j} \leq J\,\bar{y}_{B1}$$

where $J > 0$ is the number of consumers in the economy.

Hence, the aggregate first-period demand for bonds is bounded.

Now, in period 2, supply of bonds, y_{B2}, is given by:

$$\begin{aligned} y_{B2} &= x_{B1} + y_{BG2} + y_{Bp2} \\ &\leq J\,\bar{y}_{B1} + \bar{y}_{BG2} + \bar{y}_{Bp2} \equiv \bar{y}_{B2} \end{aligned}$$

Thus, y_{B2} is bounded. It has also been assumed:

$$x_{B2}^{j} \leq \bar{y}_{B2}$$

so that: $$x_{B2} = \sum_{j=1}^{J} x_{B2}^{j} \leq J\,\bar{y}_{B2}$$

Thus:

Lemma 17: x_{Bi}, y_{Bi}, the aggregate demand for and supply of bonds, respectively, in period i, are bounded.

Lemma (15) and the assumption that $x_{M1}^{j} \leq \bar{M}_o + \bar{y}_{M1}$ gives:

$$x_{M1} = \sum_{j=1}^{J} x_{M1}^{j} \leq J(\bar{M}_o + \bar{y}_{M1})$$

and: $$y_{M2} = x_{M1} + \tilde{y}_{M2} \leq J(\bar{M}_o + \bar{y}_{M1}) + \bar{y}_{M2}$$

where $\bar{y}_{Mi}$ are the upper bounds on issuance of money in period i. Hence, by assumption:

$$x_{M2} \equiv \sum_{j=1}^{J} x_{M2}^{j} \leq J(\bar{M}_o + \bar{y}_{M1}) + \bar{y}_{M2}$$

and thus:

Lemma 18: x_{Mi}, aggregate demand for money in period i, is bounded.

Finally, by Lemmas (12-14):

Lemma 19: D_i, the deficit (if positive) or surplus (if negative) of the government in period i is bounded.

Recalling the augmented excess demand function u(p) leads to the conclusion:

Lemma 20: u is a non-empty, bounded, continuous function of p.

A standard line of reasoning, depending upon Brouwer's fixed point theorem, may now be applied to demonstrate the required result, namely:

Theorem: 1/ There exists a price vector p*, as in equation (63), such that $u(p^*) \leq 0$.

1/ See, for example, Shoven (1974) for a proof of this result. Brouwer's fixed point theorem may be used, rather than the usually invoked Kakutani's theorem since a single valued function, u, has been defined.

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