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The Sustainability of Fiscal Deficits

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Summary

It is now widely accepted that if the real rate of interest exceeds the economy's growth rate, the government cannot indefinitely resort to bond issues in order to finance purchases of goods and services in excess of its tax revenues; if it did, the ratio of bonds to GNP would rise indefinitely, and it is realistic to suppose that investors would at some point refuse to acquire additional government debt. In the meantime, rising debt ratios would be associated with rising real rates of interest. The present paper explores the implications, for a hypothetical economy, of a set of government tax and expenditure policies that are unsustainable in the long run, and are recognized to be so by private individuals. If the latter expect an eventual cessation of bond financing to lead to money financing, and hence to a loss in the real value of their bond and money holdings, then they are likely to demand compensation in two forms: nominal interest rates on government bonds will go up with expected inflation, and there will be a risk premium embodied in real interest rates, reflecting inflation uncertainty. By increasing the government's borrowing costs, both of these effects will add to debt levels and hasten the need for an eventual change in policies.

I. Introduction

The effects of changes in government expenditure and taxation have received much attention from economists in recent years, for several reasons. First, it seems likely that the change in the stance of fiscal policy that has occurred in the United States since 1981 is at least part

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of the reason for the conjunction of high real interest rates in that country and abroad, and of the high real value of the dollar (for a balanced discussion, see Blanchard and Summers (1984)). More generally, in many industrial countries there is increasing concern about the possible unfavorable effects on the productive capital stock of persistent and large government deficits, which inevitably lead to increases in government debt as ratios to GNP and to total private wealth.

It is now well recognized that feasible fiscal policies must be considered in a framework where the government is subject to an intertemporal budget constraint in some form (see, for instance, Buiter (1983)). If the real rate of interest is above the real growth rate of the economy, then an expansionary fiscal policy at present, whether in the form of expenditure increases or tax reductions, must involve either contractionary fiscal policy at some time in the future, or an increase in the seignorage from money creation. Otherwise, the increase in government debt will feed upon itself, as the government borrows to finance the interest payments on debt it previously incurred, and debt eventually becomes excessively large relative to other macroeconomic variables. Except in the idealized world of "Ricardian equivalence" between debt financing and taxes (see Barro (1974)), it is implausible that investors would be willing to continue to acquire government bonds indefinitely in circumstances where there was no conceivable way the government could meet its debt service payments without further borrowing. A change in the primary deficit, that is, the government deficit excluding debt service, would eventually be forced upon the government by the unwillingness of investors to acquire further debt. Therefore, such a policy could be called unsustainable, in the usual meaning of the term.

It is clear that the concept of sustainability involves a projection of future tax and spending measures, as well as an implicit forecast of the economic environment facing the government--most importantly, the rate of growth of potential output and the level of real interest rates. It is the set of present and probable future policy settings, rather than the occurrence of a deficit in any single year or series of years, that can be termed unsustainable. Of course, projections of the infinite future are almost certain to be falsified, and there is therefore a (large) grey area where judgment must be suspended concerning the sustainability of a certain set of policies. A marginal change in taxes or expenditure will not cause a reassessment of the sustainability of fiscal policy by private market participants. It is only the prospect of continuing large surpluses or deficits which would be likely to persist for the foreseeable future, given a wide range of assumptions concerning real growth and interest rates, that is likely to call into question their sustainability. Furthermore, sustainability becomes an important issue not merely because current policies, if unsustainable, must later be reversed, but because unsustainability becomes a more and more important problem as time goes on and as deficits increase due to debt accumulation. If the government is running a primary deficit, the tax rate increase necessary to stabilize government debt as a ratio of GNP will be larger the larger is the stock of debt that has been allowed to accumulate, assuming that the real rate of interest exceeds the economy's real growth rate.

If there were confidence in the ability and willingness of a government to make needed adjustments in order to satisfy its intertemporal budget constraint, then there would be no question as to the feasibility of its policies. It is precisely a conviction that governments are short-sighted in their policies and that they are biased toward overspending because of the nature of the political system that makes sustainability an issue. As a result, even if policy changes are eventually taken to allow the government to meet its intertemporal budget constraint, these changes will be viewed as having been forced on it, and through having been delayed may have more unfavorable consequences than if they had been taken sooner.

The present paper considers how uncertainty about the sustainability of a fiscal policy involving persistent government deficits may affect the behavior of the private sector. Thus, the government's tax revenues are assumed to fall short of the value of its spending; the resulting primary deficit, plus the transfer payments needed to service the outstanding debt, are initially financed by issues of bonds. As argued above, this state of affairs cannot be expected to persist forever; however, uncertainty attaches to whether in the following period the government will choose to change its financing policy (or be forced to do so), and what form the change will take--a money supply increase (i.e., "monetization of the deficit") or an increase in taxes. Individuals' subjective assessments of the various alternatives will have implications both for the level of interest rates and for their demand for government bonds--whose value is assumed fixed in nominal terms--versus their demand for real capital. The longer deficits persist, the higher will be the outstanding stock of government debt, and the greater the need for fiscal retrenchment or monetization. If private investors assign a non-negligible probability to monetization in the following period, there will be two effects on their demand for assets. First, their expectations of inflation will reflect this probability, and they will demand compensation in the form of higher nominal interest rates on bonds; second, the real return on bonds will become more uncertain relative to that of capital, and <u>real</u> returns on bonds will also tend to rise.

It is plausible to suppose that these subjective probabilities depend in a systematic way on the size of current and prospective fiscal deficits. In particular, the larger are deficits relative to national income or the tax base, the more likely it is that resolution of the problem of unsustainability may involve some degree of monetization. A reason for this is that, for a given level of per capita income, tax revenues may be constrained by some upper bound, while the real value of the outstanding debt--and of the debt service--can be reduced to a value arbitrarily close to zero by a sufficiently high rate of inflation, brought about by money supply increases in our model. This latter eventuality presumes, of course, that there is an element of surprise to the government's move so that nominal interest rates have not already fully reflected the increased inflation. Postponement of action on the deficit would thus lead to a larger and larger debt stock outstanding and to increasing worries about monetization on the part of wealth holders. The outcome might conceivably be steadily increasing real rates of interest, and an increasingly positive differential between the current return on government bonds and that on real capital. Rising real interest rates on bonds would worsen the deficit problem of the government, and eventually drastic fiscal measures would have to be taken.

The plan of the paper is as follows. The next section (Section II) develops a formal overlapping-generations model in which there is uncertainty about the eventual resolution of an unsustainable fiscal position. This section deals with the effect on interest rates of individuals' assessments of the probabilities of different government actions. Section III considers the dynamics of such a model where subjective probabilities evolve over time and depend on the amount of outstanding government debt. An illustration is provided of a possible path for interest rates, resulting from simulating the model with arbitrary parameter values. Finally, Section IV discusses the degree of generality of the results and sketches some conclusions.

II. An Overlapping-Generations Model With Uncertainty About Government Deficit Financing

In order to illustrate how expectations held by individuals concerning eventual tax increases or debt monetization might affect their current behavior, we specify a simple model where savings behavior results from utility maximization, and where inflation and tax increases have distributional effects--i.e, they do not affect all individuals equally. The context is an overlapping-generations model where individuals choose consumption in each period of their lives in order to maximize lifetime utility, subject to a lifetime budget constraint. This model is essentially that of Samuelson (1958), elaborated by Diamond (1965) and more recently by Buiter (1979, 1980). Individuals are assumed to live two periods: during the first period they work and save for their retirement years, the second period. They make their savings plans so as to maximize the expected utility of consumption in the two periods of their lives. Though all individuals are assumed to be identical, at any given time t there are two generations in existence, the old of the previous generation (labeled t-1) and the young of generation t. Government policies may affect the old and the young differently, and, in particular, government bonds are a way of transferring wealth between generations. We do not consider here the possibility of gifts or bequests between generations; Barro (1974) and Carmichael (1982) show that under certain conditions the existence of such transfers may imply that, whether the government finances its spending through tax increases or through bond issues, the outcome for output, private consumption, interest rates and prices will be the same. Buiter (1979) and Carmichael (1982) argue that these conditions are not satisfied in practice, and hence what is often called the "Ricardian equivalence" of debt and tax financing will not hold; we will not address this issue here, however.

Now, in our model, the government may finance its purchases of the composite good which is produced by this closed economy in three ways: by levying lump-sum taxes on both the old and the young currently living, by issuing bonds that mature in the next period, or by printing money. Individuals hold money for transactions purposes, but the transactions technology is not made explicit here: the young are assumed to demand money in an amount that is proportional to the price level. We do not seek to explain here why money and bonds might both be held, as do Bryant and Wallace (1979). Whether through "legal tender" laws or reserve requirements on financial intermediaries, the government ensures that it can extract seignorage. On the other hand, bonds are issued in a competitive market; their value when they mature in the following period is fixed in nominal terms, but they pay an interest rate which equilibrates the demand and supply for bonds. Ex post, their real return will also depend on the rate of inflation which, given the assumed constancy of real money balances per capita, is equal to the rate of growth of the money supply minus the rate of increase of population. Therefore, the demand for government bonds relative to the alternative saving vehicle, capital accumulation, will be affected by the probability that in the next period the government will resort to monetization to finance its deficit.

In this economy with a single composite good, current output can be stored in the form of capital equipment, which helps to produce output in the next period, in combination with the labor input from the young generation. For simplicity, the production technology is assumed to depend on only two factors, and to be subject to constant returns to scale. The old, who serve as entrepreneurs in their "retirement year," hire the young up to the point that profits, and hence their second-period consumption, are maximized, given the existing capital stock. Wages are determined in a competitive labor market where the supply of labor is assumed inelastic: the young all work. The capital stock is totally used up in production; that is, the depreciation rate is assumed to be unity.

The starting point for the formal model presented here is the framework for a closed economy used in Buiter (1979, 1980), though there are some notational differences. Buiter's work is extended by considering monetary phenomena and uncertainty about financing. The notation is as follows: lower case Roman letters refer to variables in real per capita form, that is, divided by the number of individuals in the current generation N_t (i.e., the young) and by the price level P_t. Population grows at a constant rate n, so $N_t = (1+n)N_{t-1}$. The total size of the population at t is $N_{t-1}+N_t = (2+n)N_{t-1}$. The notation is as follows:

c_t^1 = consumption of a member of generation t when young

c_t^2 = consumption of a member of generation t when old (i.e., in t+1)

τ_t^1 = lump sum tax on a member of generation t when young

τ_t^2 = lump sum tax on a member of generation t when old

k_t = capital stock acquired by a member of generation t in period t

y_t = output

w_t = real wage rate paid to young in period t

b_t = the real value of the stock of government bonds outstanding at end of period t (one period nominal bonds that mature in t+1)

g_t = government purchases of output in period t

m_t = the stock of real money balances at the end of period t (acquired by young in period t and spent in t+1)

R_t = nominal rate of interest on bonds issued in t

M_t = nominal money supply = $m_t\ P_t\ N_t$

P_t = price level

$\Pi_t=(P_t-P_{t-1})/P_t$ = rate of inflation during period t

At the beginning of his life each individual of generation t maximizes expected utility, which depends on consumption in the two periods of his life and on his holdings of real money balances: 1/

1/ Including money in an individual's utility function in the context of an overlapping-generations model has been defended by McCallum (1983).

$$(1) \qquad E\, U(c_t^1,\ c_t^2,\ m_t)$$

subject to the following budget constraints:

$$(2) \qquad c_t^1 \ = \ w_t - \tau_t^1 - k_t - b_t - m_t$$

$$(3) \qquad c_t^2 \ = \ (1+n)\ (y_{t+1} - w_{t+1}) + (1+R_t)(1-\Pi_{t+1})b_t + m_t\ (1-\Pi_{t+1}) - \tau_t^2$$

Production in t+1 is scaled by the number of persons in the following generation, as are wages, so the first term in (3) is multiplied by (1+n). Inflation Π_{t+1} both reduces the real returns on bonds, ceteris paribus, and reduces the real value of money holdings that can be spent in t+1.

Production in t+1 uses as inputs the capital stock acquired at t as well as labor hired among the young of generation t+1. If in aggregate

$$Y_{t+1} \ = \ F(K_t,\ N_{t+1})$$

where F is homogeneous of degree one, then we can write

$$(4) \quad y_{t+1} \ = \ Y_{t+1}/N_{t+1} = F(K_t/N_{t+1},\ 1) = f(k_t/1+n)$$

It is assumed that the function f satisfies the usual regularity conditions, with $f' > 0$, $f'' < 0$ and

$$\lim_{k \to \infty} f(k) \ = \ \infty, \ \lim_{k \to 0} f'(k) = \infty$$

In the absence of uncertainty, the solution to the model of equations (1) to (3) is easily found (see Buiter (1979, 1980)). An interior solution to the utility maximization problem requires that the following conditions hold, where $U_1 = \partial U/\partial c^1$, $U_2 = \partial U/\partial c^2$, and $U_3 = \partial U/\partial m$:

$$(5) \quad U_1(c_t^1,\ c_t^2,\ m_t)/U_2(c_t^1, c_t^2,\ m_t) \ = \ (1+R_t)(1-\Pi_{t+1})$$

$$(6) \quad f'(k_t/1+n) \ = \ U_1(c_t^1,\ c_t^2,\ m_t)/U_2(c_t^1,\ c_t^2,\ m_t)$$

$$(7) \quad w_{t+1} \ = \ f(k_t/1+n) - (k_t/1+n)f'(k_t/1+n)$$

$$(8) \quad U_3(c_t^1,\ c_t^2,\ m_t)/U_2(c_t^1,\ c_t^2,\ m_t) = R_t(1-\ \Pi_{t+1})$$

These equations, along with the budget constraints (2) and (3) determine each individual's, and hence the economy's, values for consumption in the two periods, the capital stock, the interest rate, the real wage rate, and the real money stock. They imply that the real interest rate is equal to both the marginal rate of substitution between period 1 and period 2 consumption and the marginal product of capital, as the real wage is equal to the marginal product of labor.

How does the rate of interest respond to increases in the debt stock? It can be shown (Appendix I) that the model can be reduced to two equations, one describing the demand for bonds and one the supply. The demand for bonds can be expressed as a function of current and lagged real interest rates (where $r_t = R_t(1 - \Pi_{t+1}) - \Pi_{t+1}$),

$$(9) \quad b_t^d = h(r_{t-1}, r_t).$$

Past interest rates are important because the amount of saving that occurred last period affects productive capacity, and hence the level of income available for saving today. The current rate is important because it directly affects the substitution of present for future consumption. Appendix I shows that with the additively separable utility function assumed there, $h_1 < 0$, but that h_2 can have either sign. It will be positive unless taxes on the old are very large. If $h_2 < 0$, the higher bond supply raises the interest rate this period; if $h_1+h_2 > 0$, it leads to a permanently higher r.

The government's budget constraint relates the sources of financing at its disposal--taxes, bond issues, or money creation--to its purchases of goods plus its debt service payments on outstanding bonds. When taxes and money creation are exogenous, it thus determines the evolution of the bond stock over time. For the economy as a whole this budget constraint can be written in nominal terms as follows:

$$(10) \quad P_tN_t(b_t+m_t) + P_t(N_{t-1}\tau_{t-1}^2 + N_t\tau_t^1) = P_tN_tg_t + P_{t-1}N_{t-1}[(1+R_{t-1})b_{t-1}+m_{t-1}]$$

After dividing (10) by P_t N_t, it can be rewritten as

$$(11) \quad b_t+m_t+\tau_{t-1}^2/(1+n)+\tau_t^1 = g_t + [(1+R_{t-1})(1-\Pi_t))b_{t-1} + (1-\Pi_t)\, m_{t-1}]/(1+n)$$

Equation (8) provides an implicit function for m_t, which we can rewrite as follows:

$$(8')\quad m_t = n(c_t^1, c_t^2, R_t(1-\Pi_{t+1})),$$

where n_1, $n_2>0$, $n_3<0$. Given the supply of nominal money M_t and the assumption that prices are freely flexible, the equality of money supply and money demand as given by (8') determines the price level. Expectations of inflation Π_{t+1} are discussed below.

When individuals are uncertain about the government's method of financing its spending next period, then the nature of the solution changes. We assume that uncertainty attaches solely to whether the government will issue bonds or money, or raise taxes to finance its deficit. In the absence of tax increases or monetization, the nominal amount of the overall government deficit next period is known in advance, since its real spending plans are assumed to be fixed, the nominal interest rate is set in the previous period, and the rate of inflation is given by the announced rate of money growth. It is assumed that the government does not default on its debt, except in the sense that inflation involves a reduction in the real redemption value of bonds and money issued in the previous period.

The situation that we consider is one where in the normal course the government would continue to finance the excess of its spending over tax revenues by bond issues, but where the real interest rate exceeds the population growth rate, so that the process is unsustainable. At some point the government will not be able to increase per capita real debt holdings, and will be forced either to issue money at a faster rate or to increase per capita taxes. However, individual investors do not know for certain when that point will be reached, or whether the government will itself choose to anticipate that occurrence by changing its financing policies before being forced to do so. There are therefore three possible states of the world for next period that are relevant for individuals alive in t, to which they are assumed to attribute subjective probabilities p_1, p_2, and p_3, respectively: (1) the status quo; (2) tax increases of amount $d_{t+1}/2$ where d_{t+1} would be next period's real per capita deficit in the absence of tax or spending changes (it is assumed that taxes fall equally on the two generations that will be alive in t+1, the old of generation t and the young of generation t+1); (3) an increase in the rate of inflation, brought about by money supply increases sufficient to increase seignorage per capita by an amount equal to d_{t+1}.

From the point of view of an individual who has already made his savings plans, if states 2 and 3 occur they will entail lower consumption than if state 1 occurs, since they involve tax increases or loss in the real value of his money and bond holdings. Ignoring the income effect, the risk of cessation of bond financing should lead to precautionary saving in order to provide for retirement. The reduction in consumption in the individual's retirement year is greater if state 3 holds because the "inflation tax" falls solely on wealth holders, which in this model excludes the young, while conventional taxation falls equally on the two generations. 1/ The risk of inflation will also tend to make capital more attractive relative to bonds because the latter are fixed in nominal terms while the former allows command over real goods.

The choice facing an individual of generation t is assumed to be made in order to maximize expected two-period utility, where we label consumption in period 2 with a tilde to emphasize that it is a random variable. Let us label the three outcomes for period 2 with a prefix taking the value 1, 2 or 3, so consumption in period 2 takes values ${}_1c_t^2$, ${}_2c_t^2$, or ${}_3c_t^2$, and similarly for inflation and taxes. In order to simplify the problem, we keep real per capita money holdings constant; this is discussed below. Then the problem can be written

$$(12)\quad \max E\, U(c_t^1, \tilde{c}_t^2) = p_1U(c_t^1, {}_1c_t^2) + p_2U(c_t^1, {}_2c_t^2) + p_3U(c_t^1, {}_3c_t^2)$$

subject to

$$(13)\quad c_t^1 = w_t - (\tau_t^1 + k_t + b_t + m_t)$$

$$(14)\quad {}_1c_t^2 = (1+n)[f(k_t/1+n) - w_{t+1}] + (1+R_t)(1 - {}_1\Pi_{t+1})\, b_t - {}_1\tau_t^2 + m_t(1 - {}_1\Pi_{t+1})$$

$$(15)\quad {}_2c_t^2 = {}_1c_t^2 + (1+n)d_{t+1}/2$$

$$(16)\quad {}_3c_t^2 = {}_1c_t^2 - (m_t + b_t(1+R_t))\,({}_3\Pi_{t+1} - {}_1\Pi_{t+1}),$$

where d_{t+1} is defined as the amount needed to be raised in increased taxes or seignorage if real per capita bond and money holdings were to remain constant:

1/ The assumption of equality is arbitrary; however, it reflects the fact that the incidence of conventional taxation is quite different from that of the "inflation tax."

$$(17)\quad d_{t+1} = g_{t+1} - {}_1\tau_t^2/(1+n) - {}_1\tau_{t+1}^1 + b_t\, R_t\, (1-{}_1\Pi_{t+1})/(1+n)$$
$$- (m_t+b_t)({}_1\Pi_{t+1}+n)/(1+n)$$

Seignorage obtained by the government from money creation takes two forms. First, by increasing the nominal money supply and spending the proceeds, the government has a command over real resources equal to $(M_t-M_{t-1})/P_t$. However, there is likely to be a maximum revenue from steady rates of money growth, since individuals will try to reduce their money holdings if they anticipate an increase in the rate of inflation. At very high rates of inflation money holdings will be negligible in real terms. Second, the government can reduce the real cost of servicing its debt by unexpected inflation. To the extent that inflation was expected, nominal interest rates will have increased, so expected inflation does not generate revenue from this source. However, if inflation is higher than expected there will be a transfer of wealth from bondholders to the government.

Our main concern here is with the possibility that a change in regime occurs and that the authorities perform a major monetization of the debt, generating an unexpected upward shift in the price level. In effect, this action by the government--which we have labelled state 3--amounts to a default on the real value of its debt. In order to simplify the analysis, we will ignore the dependence of money demand on the expected rate of inflation, which, although important for the analysis of optimal levels of inflation relative to conventional taxation, is not central to the analysis here.

Under this assumption, in order to raise additional revenue d_{t+1} through additional inflation, the amount of the increase in inflation is simply d_{t+1} divided by the "tax base", namely the sum of the outstanding holdings of bonds (including accrued interest) and money:

$$(18)\quad {}_3\Pi_{t+1} - {}_1\Pi_{t+1} = d_{t+1}\, (1+n)/[m_t+b_t(1+R_t)]$$

All individuals are assumed to be alike, except for age, so that the "representative individual" has an influence over d_{t+1} and ${}_3\Pi_{t+1}$ via (17) and (18); however, it is assumed that he acts like an "atomistic competitor" with negligible effect on the aggregate, and takes d_{t+1} and ${}_3\Pi_{t+1}$ as given.

The solution to the individual's choice problem reflects the risk of tax increases and monetization in the following period (Appendix II). Instead of the first-order conditions (5) and (6) above, we now have

$$(19)\quad EU_1(c_t^1, \tilde{c}_t^2)/EU_2(c_t^1, \tilde{c}_t^2) = (1 + R_t)(1 - E\tilde{\Pi}_{t+1})$$
$$+ p_3(1+R_t)({}_3\Pi_{t+1}-{}_1\Pi_{t+1})[1-U_2(c_t^1, {}_3c_t^2)/EU_2(c_t^1, \tilde{c}_t^2)]$$

$$(20)\quad f'(k_t/1+n) = EU_1(c_t^1, \tilde{c}_t^2)/EU_2(c_t^1, \tilde{c}_t^2)$$

Comparison of the two pairs of formulas reveals several differences: the marginal rate of substitution of consumption between the two periods is now expressed in terms of expected utility, expectations of inflation are no longer held with certainty, and there is an additional term in equation (19) reflecting the risk of inflation resulting from monetization of the deficit. It is still the case (equation (20)) that the marginal product of capital is equal to the marginal rate of substitution of first period consumption for second period consumption--now in expected value terms. However, the risk of loss on bonds has driven a wedge between this marginal rate of substitution and the expected real return on bonds in equation (19).

Isolating the effect of the risk of monetization on interest rates and on the capital stock is complicated by the fact that both monetization and tax increases have a negative income effect, because, conditional on savings decisions taken at t, both ${}_2c_t^2$ and ${}_3c_t^2$ are less than ${}_1c_t^2$. It is also the case that ${}_3c_t^2 < E\tilde{c}_t^2$; as a result, compared with the use of continued bond finance, risk of monetization will shift the marginal rate of substitution from period 1 toward period 2 consumption (provided both are normal goods), tending to favor saving. However, if we control for the income effect it can be shown that the existence of risk of monetization will tend to increase real interest rates. Given a choice between a certain real return on a bond and one with the same expected return but uncertainty as to the actual return, the consumer will prefer the former, provided his utility function displays the usual convexity property, namely, $U_{22}<0$. If the latter condition holds then ${}_3c_t^2 = E\tilde{c}_t^2$ implies that the term in square brackets in (19) is negative. Hence, the risk of monetization (p_3 strictly greater than zero) requires that the real return on bonds be greater than the return to capital, as bondholders must be compensated for the possibility of a loss in the real value of their bond holdings. Therefore, there will be a wedge driven between the real interest rate on bonds and the return to capital.

The analysis suggests that a series of primary government deficits leading to continuing increases in per capita real debt is likely to lead to rising real interest rates from two sources. First, even when there is certainty concerning the government's policies, bond demand is likely to depend positively on the interest rate (equation (9)). Hence a continued increase in the supply of bonds will require increases in their rate of return in order to induce investors to hold them. Second, if the price of these bonds is fixed in money terms, the fear that the authorities will resort to money financing at some point in the future will lead investors to demand a risk premium, also tending to lead to higher real returns. Set against this, there may be some increase in "precautionary" savings to offset the possibility that consumption may be lower in the second period, as a result of higher taxes or a reduction in the real value of money and bond holdings. However, this will tend to increase the demand for capital, whose real return is independent of the outcome for inflation in the second period, rather than the demand for bonds, which are subject to inflation risk. Increases in interest rates, from whatever source, aggravate the deficit problem by adding to the government's borrowing costs. They therefore make an unsustainable fiscal policy more unsustainable, and hasten the necessity for an adjustment of policies. In addition they may transform what appears to be a sustainable set of expenditure and revenue policies, because the real rate of interest is below the real growth rate, into one that is clearly unsustainable, because the accumulation of debt has led to continually rising interest rates.

III. A Dynamic Model

In the absence of uncertainty about financing, the model discussed above implies that primary deficits will lead to explosive growth of government debt when the equilibrium real rate of interest is above the real growth rate of the economy (here the rate of population growth n). 1/ A fiscal policy resulting in primary deficits that persisted forever would not be sustainable, because investors would not agree to acquire bonds in amounts that became unbounded relative to the economy's output and to the government's tax revenues. Though there might not be a well-defined absolute limit on debt accumulation, there would be the presumption on the part of investors that the process would have to end sometime. The second model introduced above, with subjective probabilities of tax increases or monetization, embodied that possibility.

Not only is this latter model unstable, but the actions of investors to protect themselves against the risk of inflation tend to worsen the instability, and hasten the adjustment of fiscal policy. If the government runs a primary deficit, and the real interest rate is above the real growth rate, then the financing gap d_{t+1} increases relative to the outstanding stock of bonds and money; from (18), this implies that the inflation increase necessary to close the "financing gap" also rises. Therefore, even for a given probability p_3 that monetization will occur, the risk of loss on bonds will increase, and this will tend to increase

1/ Stability is discussed in Appendix II.

the real return differential on bonds relative to capital (equations (19) and (20)). By increasing the government's borrowing costs, this will tend to increase d_{t+1} even further, accentuating the instability. A further channel that may also be important in practice is the effect of the size of the accumulated debt (or of the deficit itself) on the subjective probability assigned by investors to future fiscal adjustment. The greater the debt, the more likely that either new debt finance will not be available or that the government will be impelled to take fiscal measures to anticipate possible difficulties with bond financing; let us call α the probability of a cessation of bond financing, and β the probability if it occurs that it involves tax increases rather than monetization, so that $p_1 = (1-\alpha)$, $p_2 = \alpha\beta$, and $p_3 = \alpha(1-\beta)$. An expected end to increases in bond holdings per capita will not of itself raise real interest rates. Indeed, if the fiscal measures are expected to involve increases in lump sum taxes, with probability β equal to unity, they will lead to a fall in real interest rates in this model. However, the larger is the "financing gap", the smaller may be the probability of tax increases rather than a resort to monetization.

Though the model does not deal with these considerations, a tendency to resort to monetization as deficits rise might result from asymmetries in the incidence of fiscal levies on the one hand and of the "inflation tax" on the other. In reality, taxes are not lump sum, and income taxes involve distortions, such as disincentives to supply labor. As the size of the stock of outstanding debt grows, the increase in tax rates necessary to replace the amount raised by new issues of bonds may become extremely large, and be politically impossible to carry out. In contrast, the inflation tax falls only on those who previously acquired financial assets--bonds and money--that is, the old. Since it is the bond interest payments that are ballooning as a result of the accumulation of debt, and the inflation tax is targeted at the holders of that debt, it may be an easier option for the government. Furthermore, there are no explicit costs to collecting the inflation tax, unlike traditional taxes, where collection costs probably rise with the amount of revenue raised.

In the present model, the government can collect an inflation tax on both bonds and money, but for the former it is unexpected inflation only that provides a revenue gain, since expected inflation is reflected in the nominal interest rate. If investors knew for certain that the government was going to monetize the deficit in the following period, then they would only have acquired bonds provided they were compensated for inflation in the form of a higher nominal interest rate. However, if the probability investors attributed to monetization was less than one and monetization in fact did occur then they would lose, and the government gain, in the period when the price level increased. Therefore, the government may be tempted at some point to monetize--in effect to default--and there is an extensive literature on the costs and benefits of the latter. 1/

1/ See, for instance, Sachs (1983), and references therein.

The success of this strategy will depend on the government's ability to surprise investors. If over an extended period of time investors thought that the probability of default was non-negligible, then the government might well be paying a substantially higher interest rate, and even if it did monetize eventually it might end up worse off on balance. Though beyond the scope of this paper, it is plausible to suppose that even if the subjective probability of monetization correctly reflected the objective frequency of its occurrence, the government's financing costs would be higher than in a situation where there was no risk of monetization and revenue was raised by increased taxes. The uncertainty associated with a loss in the real value of bonds would be a net cost to the society, and could be reflected in higher real interest rates. A consequence of a loss in credibility on the part of the government is that it may face higher financing costs, whether or not the loss in credibility is justified.

A factor that is often stressed in discussion of the inflation tax is the negative dependence of real money balances on expected inflation. As a result, as the inflation rate rises and hence the tax rate grows, the tax base--the real money stock--shrinks. There is consequently a rate of inflation that maximizes the government's revenue. We have abstracted from this aspect of the inflation tax, preferring to focus on the ability of the government to bring about a once-and-for-all major change in policy regime that is at least partly unexpected. It does this by engineering a sudden rise in the price level that reduces the real value of outstanding debt--both bonds and money.

We proceed to assume that the probability of an end to new bond financing and the probability of monetization both are seen by investors as depending positively on the per capita stock of bonds outstanding, b_t,

$$(24) \quad \alpha = 1-\exp(-\alpha_0 b_t)$$

$$(25) \quad \beta = \exp(-\beta_0 b_t)$$

When debt is zero the probability α of an end to bond financing is zero (as is $p_3 = \alpha(1-\beta)$, the probability of monetization), but it approaches unity as the bond stock per capita goes to infinity. Furthermore, if there is a fiscal adjustment then the probability $(1-\beta)$ that it involves monetization goes to unity as debt becomes unbounded.

Charts 1 to 3 illustrate these points in a simple numerical version of the model we have been considering; the exact form of the equations and the parameter values are presented in Appendix III. The charts show simulations where debt is allowed to accumulate for a time, followed by an adjustment to fiscal policy in the form either of tax increases or of monetization. In each case the government initially runs a primary deficit, and though the real interest rate (2 percent) initially is less than the trend real growth rate (equal to the population growth rate, 3 percent), the former soon rises above the latter. In the simulations where a policy change occurs at t = 11, real per capita bond holdings subsequently stay constant, and the fiscal deficit (at existing tax rates) is financed either by money supply increases or higher taxes.

The first set of simulations (Chart 1) compares the continuation of an eventually unsustainable policy of bond financing with cases where monetization or tax increases take place at a certain date, arbitrarily chosen to be t = 11. The explosive growth of debt in the bond-finance case eventually causes the model to break down as real interest rates rise so high that desired holdings of capital fall to zero, implying a zero level of output. The simulation stops at that point. The simulations where a financing change occurs lead the real interest rate to stabilize, albeit at a higher level than initially. When the revenue shortfall is covered by tax increases, the real interest rate is somewhat higher, and the capital stock consequently lower, than when money supply increases finance the deficit from t = 11 on. In each case the financing change is assumed not to have been expected; hence the risk premium relating to inflation uncertainty does not operate here.

Chart 2 examines the effect of uncertainty concerning the occurrence of a financing change--a cessation of bond financing--and of what form it will take. In one of the simulations, the subjective probabilities of a policy change are assumed to be zero (as in Chart 1), so that a policy change, when it occurs, comes as a complete surprise. In the other simulation, subjective probabilities are given by equations (24) and (25); as the debt grows, expectation of a cessation to bond financing increases, as does the expectation that it will involve money financing. In the latter case, real interest rates rise above the levels that would obtain if subjective probabilities were zero. As a result, the government's borrowing costs also rise, and consequently its accumulated debt is larger when the financing charge does occur--in this case assumed to involve money financing from period t = 11 onward.

Chart 3 compares money and tax financing when there is an implicit ceiling on the real debt per capita (chosen to be b = 57). In each simulation, a policy change occurs when that debt ceiling is reached, though the public is assumed not to realize beforehand that it is a binding constraint. In one pair of simulations, it anticipates no policy change ($\alpha = p_2 = p_3 = 0$), so that the tax increase or monetization comes as a complete surprise when it occurs. In the other pair of simulations, subjective probabilities of policy change are as described in equations (24) and (25). This latter pair of simulations illustrates the idea that if individuals fear an adverse policy change by the government they may

CHART 1

SIMULATED INCREASE IN GOVERNMENT EXPENDITURE UNDER DIFFERENT FINANCING ASSUMPTIONS FROM T = 11 ON

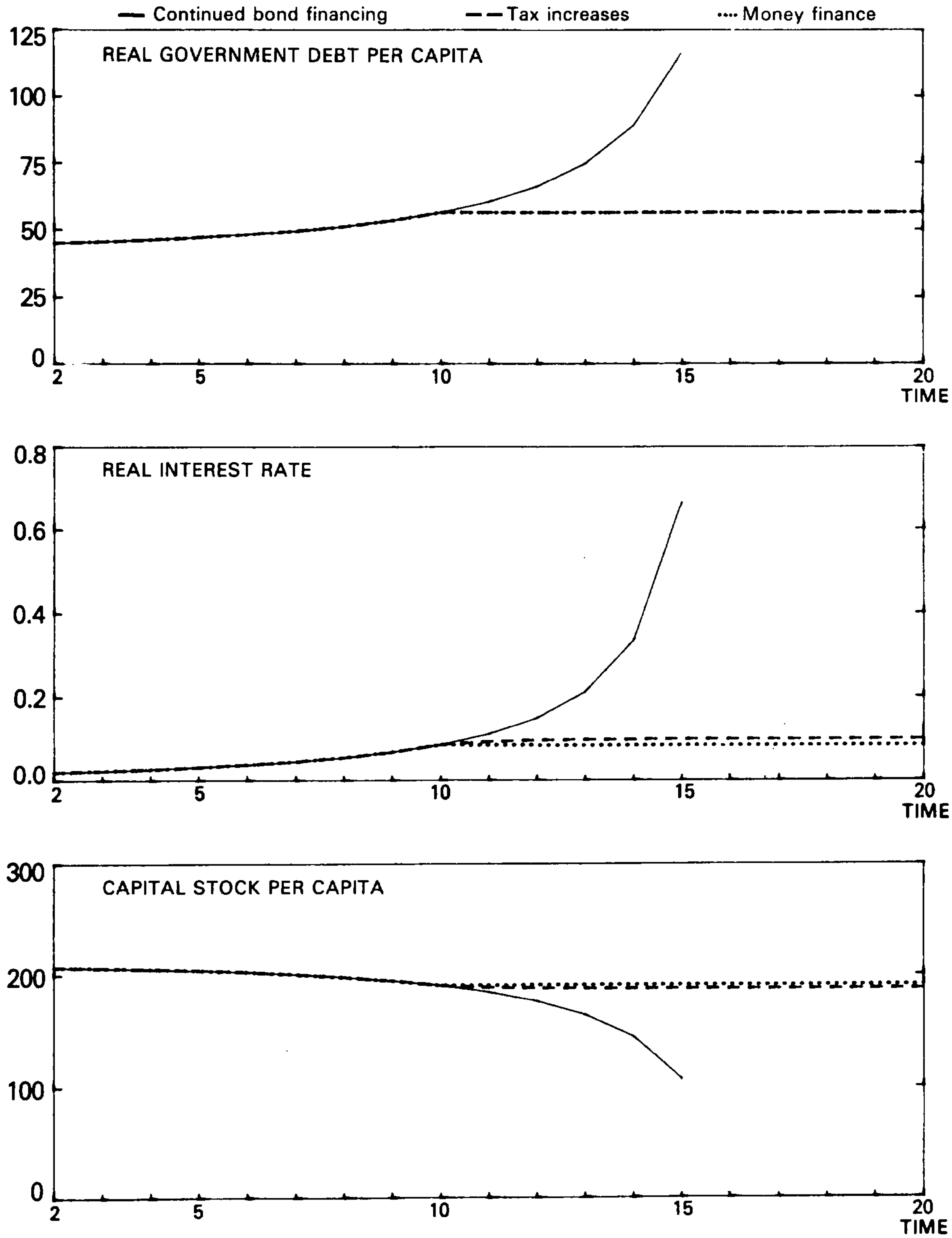


CHART 2

SIMULATED INCREASE IN GOVERNMENT EXPENDITURE, WITH MONEY FINANCING AFTER T=11: COMPARISON OF EXOGENOUS AND ENDOGENOUS PROBABILITIES

—— WITH ZERO PROBABILITY ASSIGNED TO TAX INCREASES OR MONETIZATION

– – WITH ENDOGENOUS PROBABILITIES

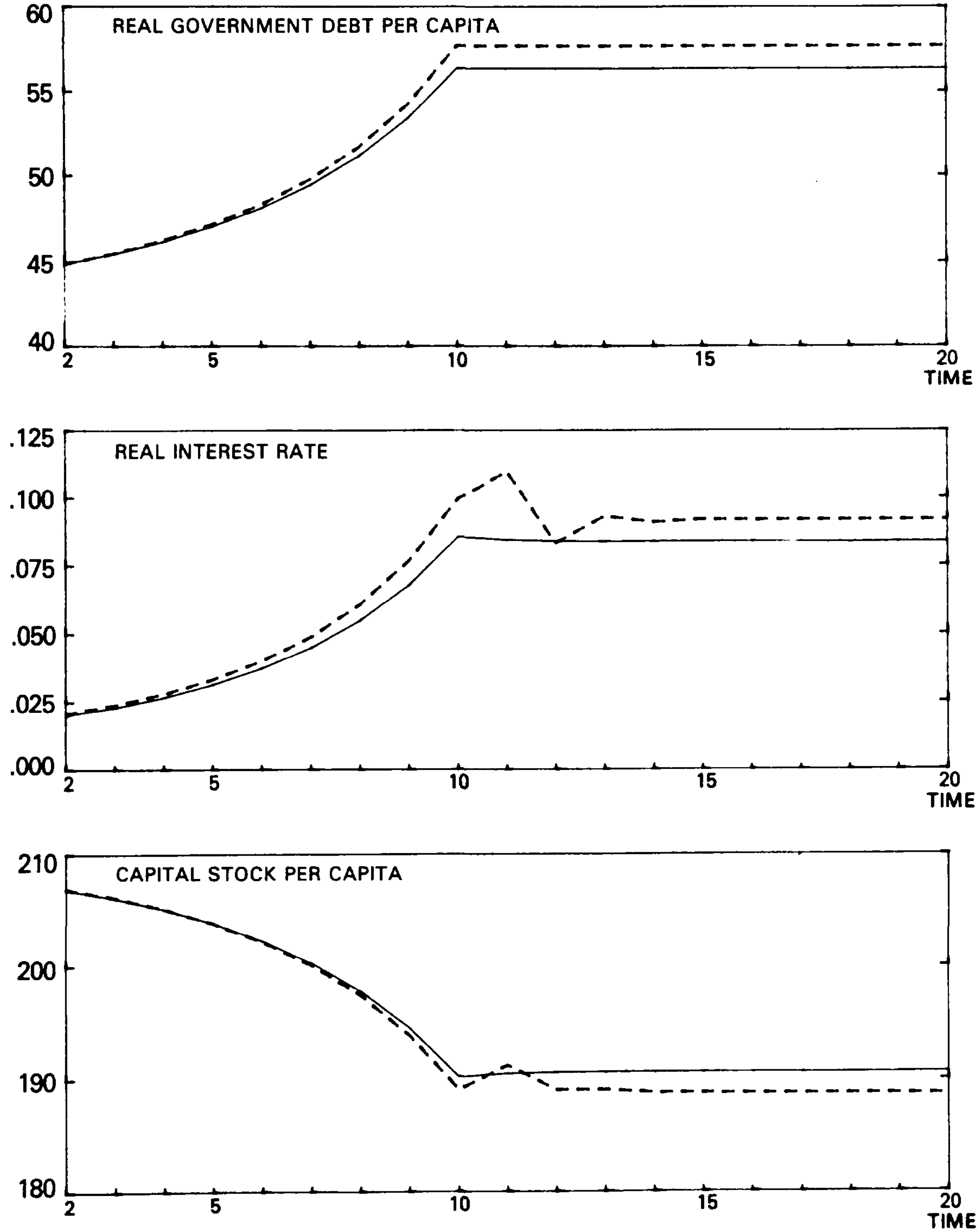
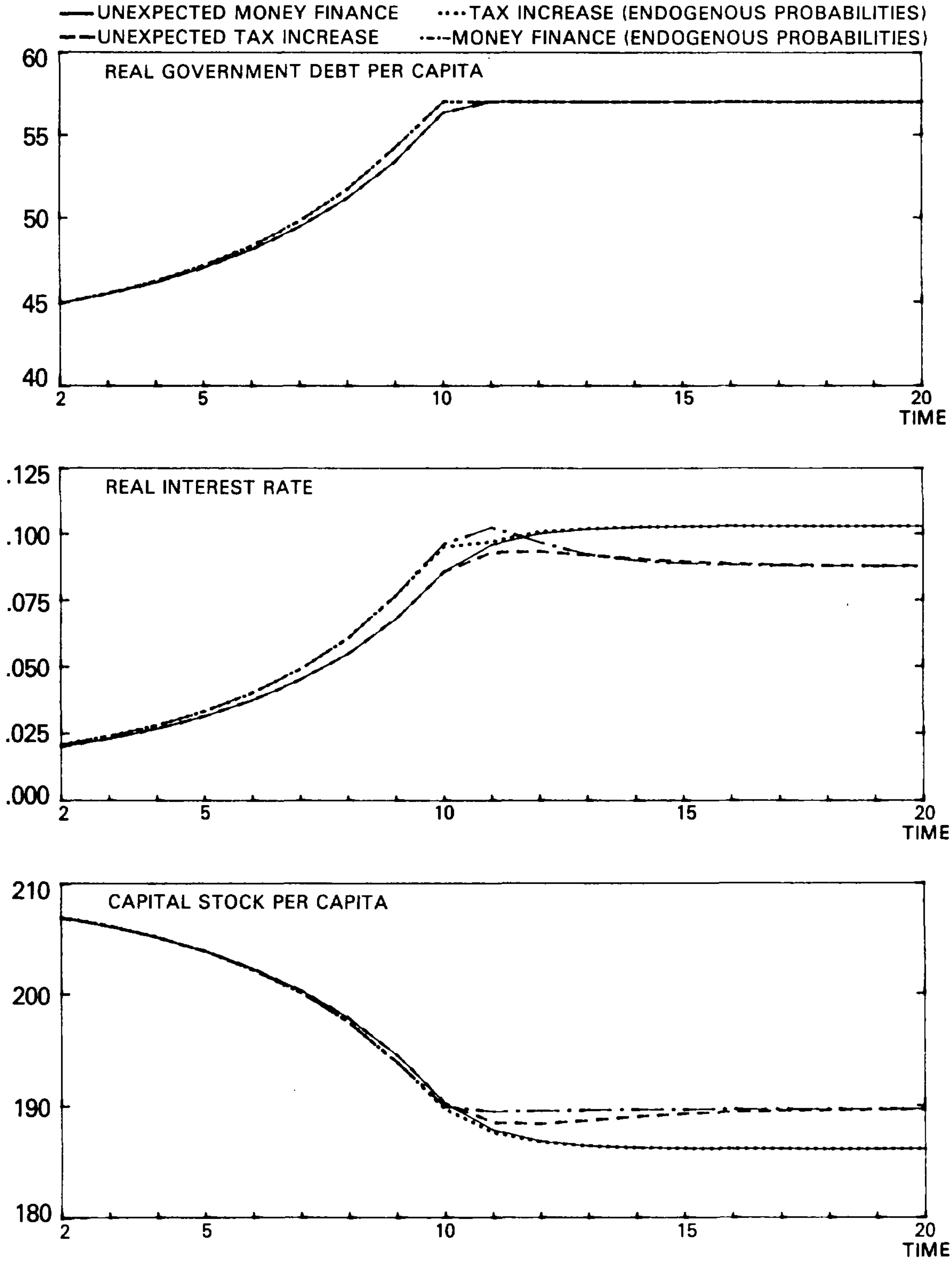


CHART 3

SIMULATED INCREASE IN GOVERNMENT EXPENDITURE WITH TAX INCREASES OR MONEY FINANCING WHEN DEBT CEILING REACHED, AND WITH EXOGENOUS OR ENDOGENOUS PROBABILITIES



act in such a way that the government's room to maneuver is reduced--in this case, by raising the government's real borrowing costs, and causing the debt ceiling to be reached sooner. 1/

IV. Concluding Remarks

The model presented here is rudimentary, but it is sufficient to illustrate the main points of the paper because it treats uncertainty about policy in an optimizing framework where the choice of government financing method has real effects. An unsustainable policy must by definition be reversed at some point, and to say anything interesting about it involves an assessment of when that reversal will take place and what form it will take. Also, by its very nature unsustainable policy does not involve a smooth adjustment: it is better modelled by a subjective probability of a discrete change on the part of investors than a continuous reaction function implying that taxes adjust to close a gap between the desired and actual bond stocks, as in Sachs and Wyplosz (1984). The paper illustrates how these subjective probability assessments influence current behavior and therefore the dynamic path of the economy. In the simulations, the view that unsustainable deficits are likely to lead to eventual monetization causes bond interest rates to rise above what they would otherwise be.

The model highlights the distinction between sustainability and stability, but also illustrates how the latter can affect the former. A set of government policies may be sustainable when viewed in isolation because the equilibrium real interest rate is below the real growth rate, permitting a primary deficit to be run indefinitely without the ratio of debt to GNP becoming unbounded. However, the persistence of primary deficits may take the economy away from this equilibrium because in the light of these policies the economy--viewed as a whole--is unstable. In particular, the resulting increase in debt may bring about a continual rise in the real interest rate, causing it to exceed the economy's growth rate. Eventually increases in debt would no longer be sustained, and a change in policies would have to result.

A simple extension to the model would involve positing an underlying objective probability distribution describing the government's actions, and making the public's subjective probabilities correspond to it. *However, it may be very difficult to evaluate future policy moves. There* may be no observations to assess the frequency of the policy actions,

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1/ There is now a large literature where private sector actions force adjustment on the authorities, which are subject to some constraint. See Henderson and Salant (1978), Krugman (1979), Obstfeld (1984), and Sargent and Wallace (1981).

but the absence of policy moves in the past is no indication that there will be none in the future. 1/ Uncertainty also attaches to the underlying economic context affecting whether the fiscal policy is sustainable or not. 2/ Nevertheless, the beliefs of investors may have an important effect on the options open to the government, and these beliefs may themselves vary in a systematic way.

1/ This is analogous to the "peso problem". See Krasker (1980).

2/ An illustration of this uncertainty is the recent exchange between Darby (1984) and Miller and Sargent (1984) concerning whether the real growth rate does in fact exceed the real interest rate.

The Stability of the Model in the Absence of Uncertainty

We will analyze the stability of the model when there is no uncertainty concerning government deficits: the government continues to finance itself through issues of bonds. We will further assume that utility is additively separable in consumption, as in Buiter (1979), and that real money balances are held constant at $\bar{m}$ and hence do not impinge on the utility maximization. We assume that utility is logarithmic. That is, it is assumed that

$$U\,(c_t^1,\ c_t^2,\ m_t) = \ell n\ c_t^1 + (1+\rho)^{-1}\ \ell n\ c_t^2 + \eta \ln(\bar{m}) \tag{26}$$

where ρ is the pure rate of time preference. Writing the model in terms of the real interest rate r_t, where

$r_t = R_t(1-\Pi_{t+1}) - \Pi_{t+1}$, equations (26) and (5) in the text imply that

$$c_t^1 = [(1+\rho)/(1+r_t)]c_t^2 \tag{27}$$

From (5) and (6), the capital stock can be expressed as a function of the real interest rate:

$$f'(k_t/1+n) = 1 + r_t$$

which we will write

$$k_t/1+n = K(r_t), \tag{28}$$

where, from the properties of f, $K' < 0$. Correspondingly, k_{t-1} and hence w_t can be expressed as functions of the real interest rate: from (7),

$$w_t = f(K(r_{t-1})) - K(r_{t-1})(1+r_{t-1}) \tag{29}$$

Now the individual's planned saving in the form of government bonds is simply his first period income net of taxes and money holdings, $\bar{m}$, minus his first period consumption and the saving that takes the form of capital accumulation:

$$b_t^d = w_t - \tau_t^1 - k_t - \bar{m} - c_t{}^1 \tag{30}$$

From this equation, the second period budget constraint

$$\begin{aligned} c_t^2 = {} & (1+n)(f(k_t/1+n)-w_{t+1}) + (1+r_t)b_t \\ & + \bar{m}(1-\Pi_{t+1}) - \tau_t^2 \end{aligned} \tag{31}$$

and equations (27) to (29) above, the demand for bonds can be derived as a function h of the real interest rate and exogenous variables:

$$b_t^d = \{f(K(r_{t-1})) - K(r_{t-1})(1+r_{t-1}) - \bar{m}[1+(1+\rho)(1-\Pi_{t+1})/(1+r_t)]$$
$$- t_t^1 + [(1+\rho)/(1+r_t)]\ \tau_t^2\}/(2+\rho) - (1+n)K(r_t)$$
$$= h(r_{t-1},\ r_t) \qquad (32)$$

The supply of bonds results from the government budget constraint, equation (11) in the text. We will call the excess of government spending on goods and services over tax revenues and seignorage from money, the primary deficit, $pdef_t$, so

$$b_t^s = pdef_t + [(1+r_{t-1})/(1+n)]b_{t-1} \qquad (33)$$

where

$$pdef_t = g_t - \tau_t^1 - \tau_{t-1}^2/(1+n) - [(\Pi_t+n)/(1+n)]\bar{m}$$

The dynamic behavior of the model is described by (32) and (33), and the condition that $b_t^s = b_t^d$: these equations determine the course over time of the real interest rate and the bond stock, given exogenous government policy variables τ^1, τ^2, g and $\bar{m}$. It is assumed in what follows that the government runs a primary deficit, so pdef > 0.

The dynamics of the model depend crucially on the response of bond demand to lagged and contemporaneous interest rates. Letting $h_1 = \partial h/\partial r_{t-1}$, $h_2 = \partial h/\partial r_t$, then

$$h_1 = -K(r_{t-1})/(2+\rho) \qquad (34)$$

$$h_2 = [\bar{m}(1-\Pi_{t+1}) - \tau_t^2](1+\rho)/[(1+r_t)^2(2+\rho)] - (1+n)K'(r_t) \qquad (35)$$

Since K > 0 and K' < 0, it is clear that $h_1 < 0$ but that h_2 can have either sign, which is a familiar result from models of savings behavior. The second term of equation (35) reflects the fall in demand for capital resulting from the higher real interest rate; ceteris paribus the demand for bonds should increase as a result. Furthermore, the greater the amount of saving in the form of real money holdings carried over from the first period of the individual's life, the more bond demand will respond positively to a rise in the real interest rate; the reverse is true for taxes levied in the second period of the individual's life. In the absence of both taxes and money holdings, $h_2 > 0$. This should be considered the normal case: only if τ_t^2 is large will bond demand respond negatively to the contemporaneous interest rate. If $h_2 > 0$, the normal case, and the government runs a primary deficit, then a phase diagram can be drawn as in Figure 1. Equation (32) for given $b_t^d = b_t$, describes the change in the interest rate:

(36) $$r_t = -(h_1/h_2)r_{t-1} + (1/h_2)\, b_t$$

If $-h_1/h_2 < 1$, then the interest rate adjusts in a stable fashion to change in b_t. In this case, since $h_1 + h_2 > 0$, a permanent increase in b causes a rise in the steady-state rate of interest. This is shown in Figure 1 as the upward-sloping locus DD. This corresponds to the (b,r) combinations that imply no change in the interest rate.

There is a locus of points, labelled SS, corresponding to no change in bond supply; from (32), this is given by

(37) $$[(n-r_t)/(1+n)]b_t = pdef_t$$

If there is a primary deficit (pdef > 0) then clearly the stock of government debt can settle down to a stable positive value only if the real rate of interest is less than the population growth rate, here the same as the economy's real growth rate. (If the government debt is negative, i.e. the government is a net creditor, then a positive primary deficit is consistent with $r > n$).

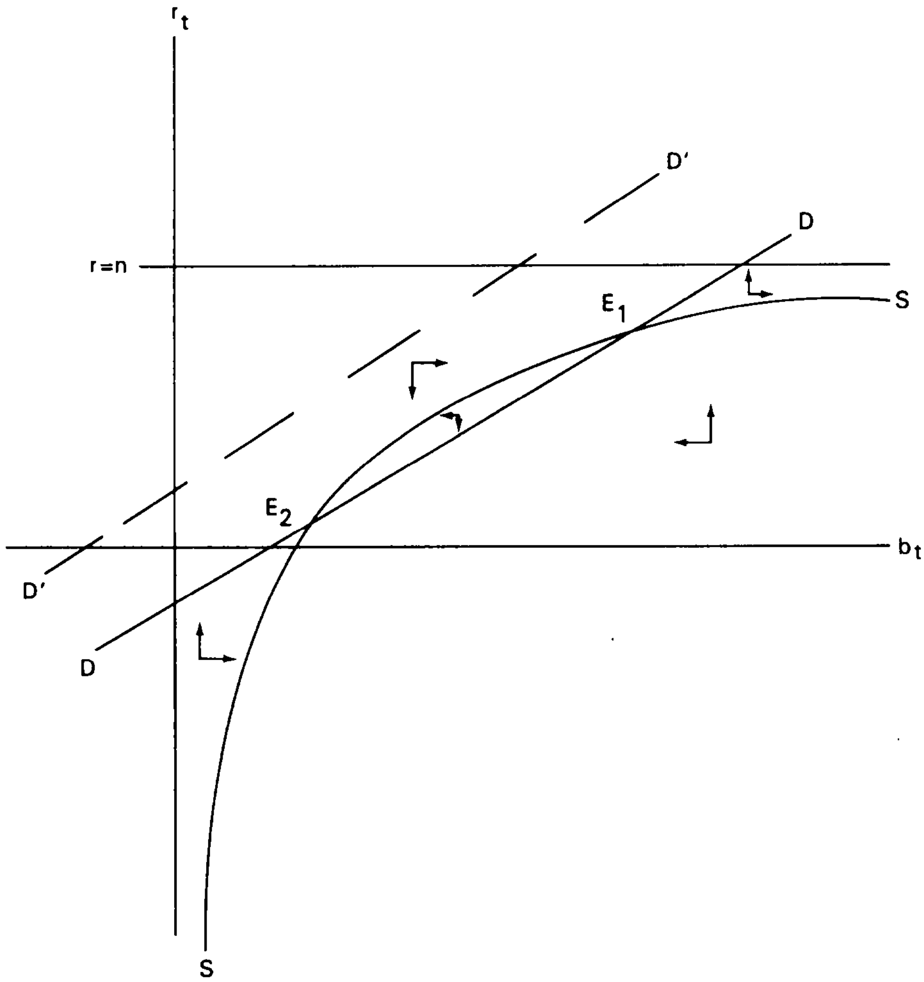
If the economy is on a point to the right of the DD locus, then interest rates rise. Similarly, if we are above the SS locus, then the bond supply increases. These directions of motion are shown by the arrows in the figure.

The two curves may or may not intersect; if they do intersect they are likely to do so at two points. From (33), the larger is the primary deficit, the further the SS shifts to the right. From (32), the greater are taxes in the first period, the more DD shifts to the left. Either of these changes makes non-intersection of the two curves more likely (such as curves D'D' and SS).

If there are two intersections as in the case with DD and SS, only the lower one, E_2, is stable. In the model there is a clear distinction between sustainability and stability. Even though, from the point of view of bond supply, an interest rate less than the real growth rate permits indefinite financing of primary deficits without causing government debt to grow without bound, such a situation is not necessarily consistent here with stability of interest rates because private saving behavior also has to be taken into account. Unless the debt stock and interest rates are low enough (and perhaps negative), saving behavior will put upward or downward pressures on the interest rate, also leading to endogenous changes in the bond stock. Point E_1 is not a stable equilibrium: a downward shock to debt will cause a cumulative fall in b and r, leading to E_2. An upward shock to debt will tend to raise both b and r, eventually leading values of r above n and to explosive increases in both r and b.

FIGURE 1

PHASE DIAGRAM, NORMAL CASE WITH PRIMARY DEFICIT



Solution to the Model with Uncertainty

The problem with uncertainty is to maximize expected utility, equation (12), with respect to bond holdings and the capital stock, and subject to constraints (13)-(16). Now, for $x = b_t$ or k_t, we set the following derivatives equal to zero:

$$\frac{\partial}{\partial x} EU(c_t^1, \tilde{c}_t^2) = 0, \text{ that is}$$

$$(38) \quad \sum_{i=1}^{3} p_i \left[U_1(c_t^1, {}_ic_t^2) \frac{\partial c_t^1}{\partial x} + U_2(c_t^1, {}_ic_t^2) \frac{\partial {}_ic_t^2}{\partial x}\right] = 0$$

It can be shown that

$$\frac{\partial c_t^1}{\partial k_t} = \frac{\partial c_t^1}{\partial b_t} = -1$$

$$\frac{\partial {}_1c_t^2}{\partial b_t} = (1 + R_t)(1 - {}_1\Pi_{t+1})$$

$$\frac{\partial {}_1c_t^2}{\partial k_t} = f'(k_t/1+n)$$

Neither b_t nor k_t affects ${}_2c_t^2$ except through ${}_1c_t^2$ (see equation (15)), since d_{t+1} is taken as given for each individual. However, b_t does affect ${}_3c_t^2$ directly: given the risk of inflation, it is in the investor's interest to reduce his bond holdings:

$$\frac{\partial {}_3c_t^2}{\partial b_t} = \frac{\partial {}_1c_t^2}{\partial b_t} - (1+R_t)({}_3\Pi_{t+1} - {}_1\Pi_{t+1})$$

From the above derivatives, it can be shown that the first order conditions with respect to b_t and k_t, respectively, can be expressed as follows:

$$\Sigma p_i U_1(c_t^1, {}_ic_t^2) = (1 + R_t)(1 - {}_1\Pi_{t+1}) \Sigma p_i U_2(c_t^1, {}_ic_t^2)$$

(39) $$- p_3(1 + R_t)({}_3\Pi_{t+1} - {}_1\Pi_{t+1}) U_2(c_t^1, {}_3c_t^2)$$

and

(40) $$\Sigma p_i U_1(c_t^1, {}_ic_t^2) = f'(k_t/1+n) \Sigma p_i U_2(c_t^1, {}_ic_t^2)$$

These equations can be rewritten as equations (19) and (20) in the text.

The Simulation Model

Simulations were performed using TROLL on the model described in the text, with arbitrary values, given below, chosen for the parameters. Utility was assumed to be additively separable in the logarithm of first and second period consumption, with the latter discounted using a rate of time preference ρ:

$$U(c_t^1, c_t^2) = \ell n\ c_t^1 + (1+\rho)^{-1}\ \ell n\ c_t^2$$

The production function was assumed to take the Cobb-Douglas form, so that per capita output was given by

$$y_{t+1} = A(k_t/1 + n)^{\theta}$$

In general, the subjective probabilities that individuals associate with states 1, 2, and 3 (continued bond financing, tax increases, or money financing, respectively), were made functions of the debt stock, as follows:

$$p_{1t} = \exp(-\ \alpha_0\ b_t)$$

$$p_{2t} = (1-p_{1t})\exp(-\beta_0\ b_t)$$

$$p_{3t} = 1 - p_{1t} - p_{2t}$$

In the simulations where expectations were exogenous (and risk of tax increases or monetization was assumed zero), α_0 and β_0 were set to zero.

From the point of view of a young person alive at t and planning his lifetime consumption and saving, consumption in period one, c_t^1, is non-stochastic:

$$c_t^1 = w_t - \tau_t^1 - k_t - b_t - m_t,$$

since the tax and financing plans of the government for the current period are known. However, consumption in period two is uncertain. That consumption level can take one of three values, ${}_ic_t^2$ (where i = 1, 2, or 3), depending on which state i of the world prevails. The level of consumption in period two depends on the levels of taxes and inflation that are chosen by the government:

$$\begin{aligned} {}_ic_t^2 = {} & (1+n)(y_{t+1}-w_{t+1}) + (1+R_t)(1-{}_i\Pi_{t+1})b_t \\ & -{}_i\tau_t^2 + m_t(1-{}_i\Pi_{t+1}) \end{aligned}$$

If state 1 occurs, tax rates and the inflation rate will be unchanged at their baseline values, and the deficit will continue to be financed by bond issues:

$$\tau_t^1 = \bar{\tau}^1$$

$$\tau_{t-1}^2 = \bar{\tau}^2$$

$$\Pi_t = \bar{\Pi}$$

$$\begin{aligned} b_t - b_{t-1} = {} & g_t - \tau_t^1 - \tau_{t-1}^2/(1+n) - m_t \\ & + m_{t-1}\ (1-\Pi_t)/(1+n) \\ & + b_{t-1}(R_{t-1}(1-\Pi_t)-\Pi_t-n)/(1+n) \end{aligned}$$

Expected values of tax rates and inflation next period, conditional on state 1 occurring, are therefore

$${}_1\tau_{t+1}^1 = \bar{\tau}^1$$

$${}_1\tau_t^2 = \bar{\tau}^2$$

$${}_1\Pi_{t+1} = \bar{\Pi}$$

The increase in the stock of bonds per capita in state 1, which is the amount of the ex ante deficit that has to be reduced to zero by tax increases in state 2 or financed by money creation in state 3 is labelled d_{t+1}; it is given by equataion (17) in the text, under the assumption that real per capita money balances m_t stay constant.

If state 2 occurs, then bonds per capita stay constant and taxes on the young and on the old, respectively, increase as follows:

$$\tau_t^1 = \bar{\tau}^1 + .5d_t$$

$$\tau_{t-1}^2 = \bar{\tau}^2 + .5d_t(1+n)$$

where d_t is calculated using the actual values of the relevant variables. Individuals' expected taxes conditional on state 2 occurring are therefore

$${}_2\tau_{t+1}^1 = \bar{\tau}^1 + .5d_{t+1}$$

$${}_2\tau_t^2 = \bar{\tau}^2 + .5d_{t+1}(1+n)$$

If state 2 occurs, the rate of inflation is unchanged, so

$$ {}_2\Pi_{t+1} = \bar{\Pi} $$

If state 3 occurs, the money supply increases such that the inflation tax raises additional revenue in amount d_t, while other tax rates remain unchanged:

$$ \Pi_t = \bar{\Pi} + d_t(1+n)/[(1+R_{t-1})b_{t-1} + m_{t-1}] $$

$$ \tau_t^1 = \bar{\tau}^1 $$

$$ \tau_{t-1}^2 = \bar{\tau}^2 $$

Similarly, individuals' expectations, conditional on state 3 occurring, are as follows:

$$ {}_3\tau_{t+1}^1 = \bar{\tau}^1 $$

$$ {}_3\tau^2{}_t = \bar{\tau}^2 $$

$$ {}_3\Pi_{t+1} = \bar{\Pi} + d_{t+1}(1+n)/[(1+R_t)b_t + m_t] $$

Now, given the logarithmic utility function described above, marginal utilities will be given by

$$ EU_{1t} = 1/c_t^1 $$

$$ EU_{2t} = (p_{1t}/{}_1c_t^2 + p_{2t}/{}_2c_t^2 + p_{3t}/{}_3c_t^2)/(1+\rho) $$

and the first-order conditions, equations (7), (19) and (20) in the text, can be written as follows:

$$ w_{t+1} = (1-\theta)y_{t+1} $$

$$ EU_{1t}/EU_{2t} = (1+R_t)(1-E\Pi_{t+1}) + p_3(1+R_t)({}_3\Pi_{t+1} - {}_1\Pi_{t+1})\{1-1/[{}_3c_t^2(1+\rho)EU_{2t}]\} $$

$$ (1+n)\theta y_{t+1}/k_t = EU_{1t}/EU_{2t} $$

The above equations constitute the simulation model, with the following parameter values assigned:

$\rho = .0152$	$A = 198.509$	$\theta = .26$
$n = .03$	$\alpha_0 = .01$	$\beta_0 = .01$

Initially, the exogenous variables take the following values:

$$\tau_t^1 = 50$$

$$\tau_t^2 = 50$$

$$g_t = 104$$

$$m_t = 40$$

$$\pi_t = .06$$

These values are consistent with the following stationary values for the endogenous variables:

$$b_t = 44.3901$$

$$c_t^1 = 241.722$$

$$c_t^2 = 242.457$$

$$k_t = 207.363$$

$$R_t = .083283$$

$$w_t = 583.476$$

The simulations involved increasing government spending by .5, to 104.5, leading to explosive growth of government debt, and making different assumptions about financing and about expectations formation.

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