DM/84/70

INTERNATIONAL MONETARY FUND

Central Banking Department

The Financial Structure of Firms and Interest Rate Policy: Macroeconomic Consequences of High Debt-Equity Ratios in Developing Countries

Prepared by V. Sundararajan 1/

November 20, 1984

Table of Contents

1/ An initial draft of this paper was prepared while the author worked in the Developing Countries Studies Division of the Research Department. He wishes to thank Linda M. Koenig, Anthony Lanyi, Vicente Galbis, Donald J. Mathieson, and D.I. Folkerts-Landau for valuable comments.

I. Introduction

The main purpose of this paper is to demonstrate that the choice between debt and equity by firms, and the institutional circumstances governing this choice have a crucial bearing on the impact of interest rate policy on saving and investment in developing countries. The reliance on debt finance has been extremely large in some developing economies because the banking system and, in some cases, the curb markets have together provided substitutes for stock issue in the form of loans, while the flow of foreign saving has been mainly in the form of debt rather than equity. In effect, the banking system, and particularly the curb markets have assumed the risk of bankruptcy of firms, and the equity instruments have remained underdeveloped.

The Korean economy provides an interesting case study of rapid economic growth, with heavy reliance on debt finance. The average debt-equity ratio (i.e., the ratio of total liabilities to net worth) of firms in the industrial sector in Korea has grown from about 100 percent in the early sixties to about 500 percent in recent years. This sharp rise is mainly due to the rapid growth of the Korean banking system following the interest rate reform in 1965, and to the large use of foreign borrowing; other contributing factors include the inadequacy of business saving in relation to investment needs, and the biases in the tax system which favor debt finance. Policymakers in Korea have generally held the view that the resulting over-leveraged financial structure restricts their macroeconomic policy options, and have, on various occasions, adopted measures to reduce the debt-equity ratio of firms as part of financial reform. 1/

Based on an analysis of corporate financial structure in Japan—another example of rapid growth based predominantly on debt finance, with interesting parallels to the Korean situation—Patrick (1972) notes that "there is no evidence that underdeveloped capital markets have had any adverse effect on the saving rate or even on the realized investment rate."

While the rapid economic growth in Japan and Korea may suggest such a conclusion, a closer analysis in a developing country context reveals that the prevalence of high corporate debt-equity ratios is detrimental to macroeconomic stability, and that the effect of interest rate policy on saving and investment is significantly altered by the size of the ratio. Interestingly, when the debt-equity ratio exceeds a critical limit, even the direction of the effect of financial policies is changed and stabilization policies involve very high costs in terms of foregone growth. These macroeconomic consequences of the financial structure of firms will become apparent when the role of interest rate policy is

---

1/ Sakong Il (1977) contains an historical account of measures taken by the Korean authorities to improve the corporate financial structure.

examined from the point of view of its effects on the cost of capital to investors, an aspect that is ignored in much of the debate on interest rate policy in developing countries.

The analysis of interest rate policy in developing countries has evolved along two distinct lines. The analytical framework pioneered by Shaw (1973) and McKinnon (1973) deals with disequilibrium systems where investment opportunities abound, but actual investment is constrained by available saving due, in part, to the financial repression fostered by high inflation coupled with controls on the monetary system. Because of controls on interest rates, short-run monetary equilibrium is achieved mainly through variations in the rate of inflation. The role of interest rate policy in this framework is to raise saving, improve allocative efficiency, spur the demand for financial assets, and facilitate stabilization. An alternative line of analysis developed in van Wijnbergen (1983) and Taylor (1983), focuses more closely on the specific characteristics of the financial markets in many developing countries. It is argued that active curb markets--or deregulated segments of the organized financial markets--exist in many countries, and that private loans in these free markets often comprise an important share in the portfolios of savers. Therefore, the interest rate in the free markets can be expected to play a role in equilibrating demand and supply of credit. In this structuralist framework, both the administered interest rate and the curb market rate (or the free rate) influence saving, investment, portfolio choice, working capital costs, and inflation. While the Shaw-McKinnon analysis deals with only two types of assets in savers' portfolios--monetary assets and inflation hedges--the structuralist model introduces a third asset, namely private loans in the free market. This extension of the asset menu significantly alters the implications of interest rate policy.

In this paper, the structuralist analysis of interest rate policy is extended by formulating an appropriate definition of the real cost of capital to investors in developing countries characterized by segmented financial markets, controls on the banking system, and substantial reliance on debt, including foreign currency debt. The earlier models ignored the important issue of how the real cost of capital to investors is influenced by interest rate policy and the financial structure. While this neglect is understandable in the Shaw-McKinnon framework, where the emphasis is on saving, not on investment, it is not valid in the structuralist model where both investment and saving respond to interest rates. Even in the Shaw-McKinnon framework, the appropriate formulation of the real cost of capital is relevant, because it is an important component of the rental-wage ratio which influences factor allocation and the efficiency of capital use. Such efficiency aspects are highlighted in many models based on the Shaw-McKinnon tradition. 1/

1/ See for example, Sundararajan and Thakur (1980), and Fry (1982).

The relationship between the cost of capital, interest rate, and the debt ratio 1/ is a subject with a long history and a voluminous literature in the field of finance. 2/ The discussion below will focus on those aspects which appear relevant in the context of developing countries, with a view to providing a heuristic explanation of why debt ratios matter in understanding the effects of interest rate policy. In its simplest formulation, the cost of capital, defined as the minimum required return on investment, can be expressed as a weighted average of the cost of equity and the cost of debt, with weights representing the shares of equity and debt respectively in total assets. Thus, the larger is the debt ratio, the greater is the impact of changes in the cost of debt on the overall cost of capital. Ignoring foreign currency debt for the moment, the cost of debt in most developing countries is simply the administratively controlled loan rate in the banking system. However, the cost of equity cannot be readily identified in developing countries with underdeveloped and fragmented financial markets. It is the opportunity cost of equity funds, or equivalently the rate of discount used by businessmen in capitalizing the net income stream from projects. By its nature, the cost of equity is likely to vary with the structure of the financial system, and the extent of financial repression. 3/ In general, however, the cost of equity is higher than the cost of debt, reflecting partly a risk premium. The gap between the two is particularly large in developing countries because of the repression of interest rates through administrative controls. Against this background, it is clear that the ultimate impact on the cost of capital of an increase in the administered interest rate depends on how this increase affects the cost of equity and the share of debt, both of which also influence the cost of capital. Indeed, a change in interest rate can either reduce or increase the cost of capital and saving depending on the initial size of the debt ratio, and on the induced adjustments in the cost of equity and in the share of debt.

In other words, the financial structure of firms, or more generally the institutional framework of the financial system that underlies it, has significant implications for interest rate policy. This point can

---

1/ Throughout the paper the term debt ratio (α) refers to the ratio of total liabilities to total assets and the term debt-equity ratio (β) refers to the ratio of total liabilities to net worth. These two terms will be used interchangeably in view of the one-to-one correspondence between the two ratios given by: $\beta = \alpha/1 - \alpha$.

2/ For a survey of the literature see Nickell (1978), and Beranek (1981). The basic reference on this subject is Modigliani and Miller (1963).

3/ For example, in a heavily repressed financial system, the major perceived alternative to using funds for fixed investment could be the acquisition of inflation hedges such as gold or inventories. If so, the expected rate of change in the price of gold or the general rate of inflation would be the relevant opportunity cost of equity. In general the average rate of return on a representative portfolio of saving instruments—the curb market loans, inflation hedges, foreign currency assets and bank deposits—is the appropriate opportunity cost of equity funds.

be illustrated by considering a dual financial structure, consisting of a controlled banking system and an unfettered curb market, where the rate in the curb market could be regarded as the relevant opportunity cost of equity. An upward adjustment in the administered rate would initially raise the cost of capital and lower investment demand. The reduction in investment demand would be larger, the greater the debt ratio, because, as already indicated, the increase in the cost of capital due to an increase in interest rate is larger, the larger the debt ratio. With a high enough debt ratio, the reduction in investment would be sharp enough to depress the demand for funds in the curb market, and thereby lower the curb market rate. 1/ If, now, saving depends positively upon real returns to available assets, then the negative impact on saving due to the fall in the curb market rate would counter the positive impact on saving due to the increase in the bank interest rate. The overall impact on saving would be negative, or weakened substantially, if the fall in the curb market rate is large owing to a high debt ratio. Thus the debt ratio used by firms significantly influences the impact of interest rate policies. This basic result remains valid when the analysis incorporates both portfolio adjustments, and adjustments in the debt ratio in response to interest rates and inflation.

The paper is organized as follows. Section II sets out the model determining saving, investment, the debt ratio, the cost of capital, and portfolio adjustments. The model emphasizes the linkages betwen debt and investment. 2/ In Section III, the Fisher effect and the effects of interest rate policy are analyzed under alternative assumptions about debt policies of firms. Subsection III.1 demonstrates how a large debt ratio can lead to macroeconomic instability and result in perverse effects of monetary policies.

Subsection III.2 deals with an aspect of debt policy which has been generally ignored in the literature, namely the impact of maturity structure of debt—the rate of amortization—on investment incentives. When the gap between the cost of equity and the cost of debt is large, as in most developing countries, it is readily shown that the choice of the maturity pattern of debt will significantly influence the present value of the project and hence investment incentives. 3/ Moreover the rate of amortization has an important bearing on how the debt ratio evolves over time. Therefore, the behavior toward amortization can significantly influence the impact of interest rates on investment and saving.

1/ The effect on the supply of curb market funds, or equity funds generally, arising from portfolio adjustments is ignored here for illustrative purposes. They are taken into account in the next section where the complete model is presented.

2/ Linkages between debt and investment are generally ignored in the theory of investment where the debt ratio is assumed to be fixed. In the theory of corporate financial behavior, the rate of investment is taken as exogenous. For a recent analysis of the interdependence between investment and financing, see Hite (1977).

3/ On the effect of maturity decisions, see Morris (1976).

Insofar as an increase in the debt ratio raises the riskiness of net returns from investment, firms might adjust their debt ratio optimally in order to balance the benefits of additional subsidized credit from banks with the associated costs arising from increased riskiness of investment. The implications of such optimal debt behavior for stability and interest rate policy are analyzed in Section III.3.

The impact of interest rates is also influenced by the choice of exchange rate regime, because this choice influences the cost of foreign currency debt and hence the cost of capital. This aspect is analyzed in Section III.4 in view of the importance of foreign currency debt in financing investment in many developing countries. Section IV contains a summary and highlights the policy implications. The algebraic details are shown in the Appendices.

II. A Model of Saving, Investment and Debt

The model specifies the determinants of saving, investment, the cost of capital, the financial structure of firms, and the asset portfolio of savers in a dual financial system characterized by a controlled banking sector and an unfettered curb market. 1/ The purpose of the model is to highlight the linkages between the debt behavior of firms and incentives for saving-investment. Special attention is paid to the determinants of the cost of capital to investors because the linkages between the financial structure of firms and investment incentives arise in part through the impact of financial policies on the cost of capital.

1. Saving and the debt ratio

It is assumed that aggregate real saving depends positively on the real returns offered by the banking system and on the real returns in the curb market.

$$S = S(\rho - \pi, R - \pi) \qquad (1)$$

where ρ = average nominal rate of return in the curb market, R = the interest rate on bank deposits, and π = the fully anticipated rate of inflation. Other variables that influence saving such as real wealth and transitory income are assumed to be fixed, and hence are suppressed for simplicity.

1/ The case of a curb market is used in this paper for illustrative purposes only. The model can be readily adapted for cases where other substitutes for bank credit exist, such as credit from nonbank financial intermediaries or equity instruments whose yields are market-determined and not controlled by the government. The market-determined free rates can be substituted for the curb market rate, and the analysis can be appropriately modified.

The interest sensitivity of aggregate saving is influenced by the distribution of saving between government, corporations, and households. For the purposes of this paper, real government saving is assumed to remain unchanged during the time span relevant for the analysis; although government saving will change due to the differential response of receipts and expenditures to changes in inflation, these considerations will be left out for simplicity, and the focus will be on private savings, which are more likely to be sensitive to interest rates.

Private savers fall into two distinct categories: those who are unaware of the full spectrum of financial alternatives and rely mainly on banks for the placement of financial savings, and those who exhibit sophisticated portfolio behavior by diversifying their savings among available assets. The saving function specified above is consistent with these two forms of behavior, if the rate of return ρ is interpreted not simply as the return in the curb market, but as the average return on the optimal portfolio of sophisticated savers.

In developing countries, this optimal portfolio will typically consist of (i) deposits in the financial system yielding risk-free returns (determined by government policy), (ii) loans supplied to the unorganized money market or deregulated segments of the organized markets (free markets for short) offering risky returns, (iii) equity holdings also offering risky returns, and (iv) holdings of physical assets reflecting savings in the form of producer durables. 1/ In a world of diversification and risk aversion, the optimal portfolio and the return on it can be derived from a mean-variance framework. 2/

In this framework, the share of various assets in the portfolio of private savers, and the mean return on the portfolio will depend, among other things, on the variances and covariances of the returns to various assets and on the debt ratio of firms. An increase in the debt ratio, insofar as it raises the riskiness of the equity streams in the portfolio, may require a higher return on the optimal portfolio in order to compensate for the additional risk. Based on this typical assumption found in the literature, the debt ratio is seen to influence saving through its effect on the return on savers' asset portfolios.

The interest sensitivity of saving is also influenced by the debt ratio: if the level of corporate debt is relatively large, an increase in interest rates will transfer significant amounts of resources from corporations to households (usually after a time lag), and this transfer may eventually depress aggregate private saving, because corporate (and government) saving falls by the full amount of additional interest costs, while household saving rises by less than the increase in interest incomes. Therefore, the interest sensitivity of saving is likely to be inversely related to the debt ratio.

1/ Saving in the form of consumer durables is treated as consumption.

2/ For a simple exposition of the mean-variance framework, see Rubinstein (1976).

2. Investment and the real cost of capital

Desired real investment depends upon the real cost of capital, real wage rate, output expectations, and the size and characteristics of the existing stock of capital. All factors, other than the real cost of capital, are assumed to be fixed, in order to focus on the short-run interactions. The real cost of capital, defined as the minimum acceptable return on investment, can be derived by assuming that firms choose the level of investment in order to minimize the total cost of producing the desired output, including the acquisition cost of capital, and debt service costs. Let Q^* denote planned output, K capital stock and L employed labor; given the production function, and the nominal wage rate W, total labor cost can be written as:

$$W_t L_t = C(Q_t^*, K_t) \tag{2}$$

where the subscript (t) denotes time. The present value of all costs is given by:

$$TC \equiv \int_0^\infty \exp(-\rho t)[C(Q_t^*, K_t) + (1 - \alpha^m) I_t P_t + (R_d + a_d) G_t + (R_f + a_f) F_t E_t] dt \tag{3}$$

where ρ = the cost of equity, I = the real gross investment, P = the price level, α^m = the proportion of investment financed by debt, i.e., the marginal debt ratio, G = total domestic debt outstanding, F = total external debt outstanding in foreign currency units, E = exchange rate measured in number of domestic currency units per unit of foreign currency, R_d = domestic interest rate, R_f = foreign interest rate, and a_d, a_f = amortization rates on domestic and foreign loans. At each point in time, total cost—cash outflows from the point of view of the owners of the firm--consists of labor cost $(C(Q^*, K))$, funds supplied by the owners to acquire and install new plant and equipment $[(1 - \alpha^m) IP]$, and the debt service payments on domestic and external debt $((R_d + a_d)G$ and $(R_f + a_f)FE$. The present value of these cash outflows is obtained by applying the rate of discount ρ, which is the opportunity cost of equity funds given by, say, the curb market rate, or the rate of return on the optimal portfolio of sophisticated savers. 1/ In some economies, the interest rate in the deregulated segment of the organized financial sector may serve as the opportunity cost of equity. In the rest of the discussion, the term discount rate will be used to refer to the cost of equity which can clearly take on a variety of forms depending upon the structure of the financial system.

The task is to minimize the total cost—the present value of all cash outflows given by equation (3)—with respect to the control variable I, subject to the constraints below:

1/ In line with this assumption, loans in the curb market are treated as equity finance and are excluded from the computation of α^m.

$$\dot{K} = I - \delta K \tag{4}$$

$$\dot{G} = \alpha^{md}\, IP - a_d G \tag{5}$$

$$\dot{F} = \alpha^{mf}\, IP/E - a_f F \tag{6}$$

where δ = the rate of economic depreciation, α^{md} = the proportion of investment financed by domestic currency loans, and α^{mf} = the proportion financed by foreign currency loans. The marginal debt ratio α^m is the sum of α^{md} and α^{mf}. The dot ($\cdot$) above a variable denotes time derivative.

Equation (4) states that the change in capital stock ($\dot{K}$) equals gross investment (I) minus depreciation. The rate of economic depreciation, stated as a proportion δ of existing capital, is assumed to remain unchanged over time.

Equations (5) and (6) describe the time path of loans outstanding, both domestic and external. They state that the change in debt outstanding--the net inflow of loans--equals total new loans minus the amortization of existing loans. Equation (6) refers to foreign loans measured in foreign currency units. Thus, the investor's external debt obligations are all denominated in foreign currency units and the investor bears the full exchange risk. This is the typical situation in developing countries. The amortization payments on both domestic and external loans are assumed to be proportional to the stock of loans outstanding, while new loans are obtained only for financing fixed investment. Loans to finance working capital requirements can be readily incorporated, but are ignored for simplicity. 1/

The problem of minimizing the expression (3) subject to constraints (4), (5) and (6) is a well-defined control problem which can be solved to characterize the path of optimal capital accumulation. The first order conditions for the cost-minimizing investment path simply reduce to the familiar rule which states that at each point in time investment should be expanded until the present value of cost reductions due to a change in investment minus the present value of debt service incurred in financing the investment equals the amount of equity finance supplied by the owners (both new and old). 2/ In order to highlight the expression for the cost of capital, this rule can be restated as follows:

1/ Borrowing for the purposes of dividend distribution and maintenance of cash reserves is also ignored.

2/ The first order conditions are analyzed in Appendix I.

$$-\frac{\partial C}{\partial K}\exp(-\pi t) = r_b \qquad (7)$$

where $-\frac{\partial C}{\partial K}$ = the reduction in nominal labor costs due to unit addition to capital stock, $\exp(\pi t)$ = the price level at time t, π = the rate of inflation, and r_b = the real cost of capital given by:

$$r_b = (1 - \alpha^m)(\rho - \pi + \delta)$$

$$+ \left[\alpha^{md}\frac{R_d + a_d}{\rho + a_d} + \alpha^{mf}\frac{R_f + a_f}{\rho + a_f - x} \right] (\rho - \pi + \delta) \qquad (8)\ \underline{1/}$$

where x is the expected rate of change in the nominal exchange rate. The expression in large brackets on the right side of equation (8) is the present value of debt service payments on α^{md} of domestic loans and α^{mf} of foreign loans.

Differentiating equation (2) with respect to capital stock, and using equations (7) and (8), investment can be expressed as:

$$I = I(Q_t^*, r_b/(W_t/P_t), K_t)$$

Assuming that real wage W/P is constant, and the desired output is predetermined, and suppressing K_t in order to focus on the short run, investment function can be written as:

$$I = I(r_b) \qquad (9)$$

An examination of the cost of capital formula (equation (8)) underscores the importance of explicit consideration of debt policies of firms in developing countries. In the special case when domestic capital markets are perfect, default risk is absent, capital is fully mobile internationally, and the exchange rate is expected to remain unchanged, all interest rates are equalized ($R_d = \rho = R_f$), 2/ and the cost of capital is given by:

$$r_b = \rho - \pi + \delta \qquad (8a)$$

Thus, only under these special assumptions, the cost of capital for investment purposes is independent of amortization rates as well as debt

1/ For an alternative derivation of the formula based on Modigliani-Miller Proposition I, see Appendix I.

2/ Domestic and foreign rates of inflation are assumed to be identical for the time being, so that the expected change in the exchange rate is zero.

ratios. Clearly, these assumptions do not adequately characterize developing economies. If, for any reason, the foreign interest rate differs from the domestic rate, then the share of external debt in investment finance will enter the cost of capital calculations. If, in addition, the interest rate on domestic debt deviates significantly from the discount rate, then debt policy assumes even greater significance. Therefore, the next section will examine the determinants of the debt ratio.

3. Determinants of the debt ratio

The marginal debt ratio that entered the cost of capital calculations is determined in part by the institutional environment—including in that term the stance of credit policy—and in part by the long-run average debt ratio (α^a) that firms regard as prudent. 1/ If financial institutions adhere to some predetermined debt-equity norms, 2/ or if the rigidities and imperfections in the financial system lock firms into some historically determined debt-equity ratios, then firms will not have much flexibility in choosing the marginal debt ratio. In these circumstances, it is best to regard the marginal debt ratio as an institutionally determined parameter. Quite often, however, firms in developing countries do have some flexibility in controlling the debt-equity mix. The debt-equity mix can be varied by adjusting the policy toward retention of earnings for reinvestment purposes, by varying the extent of use of foreign currency debt, and by accessing domestic finance from informal and equity markets.

If firms are able to adjust their financing mix, then it is reasonable to assume, on the basis of available empirical evidence, that debt decisions by firms will be guided by the long-run average debt ratio that they strive to achieve. 3/ This is because the value of the firm, or equivalently the discount rate required by the owners of the firm, is likely to depend upon the average debt ratio. It is the average debt ratio—not the marginal ratio—which is relevant for assessing the risk of equity streams of firms, such risk arising from constraints on future investment options and from the likelihood of bankruptcy. Therefore, the target value for the average debt ratio is likely to be an important determinant of the marginal debt ratio. This consideration can be formalized by solving the differential equation (5) to obtain:

$$\alpha^{md} = \alpha^{ad} \cdot \frac{g + a_d + \pi}{g + \delta} \qquad (10)\ \underline{4/}$$

1/ The average debt ratio refers to the share of debt in total assets, while the marginal ratio refers to the share of debt in financing additions to total assets.

2/ See, for example, Madan (1978) for a discussion of debt-equity norms used by financial institutions in India.

3/ For empirical evidence that firms try to achieve a target average debt ratio, see Ang (1976) and Marsh (1982).

4/ See Appendix II for the derivation of equation (10). The particular functional form has been used for analytical convenience only despite its limitation that the marginal debt ratio can exceed unity when inflation is large. A more satisfactory specification, linking the marginal ratio to the target average ratio, the rate of inflation, and other variables would complicate the analysis without materially affecting the results.

where α^{ad} = long-run target value of the average domestic debt ratio, and α^{md} = marginal debt ratio used by firms, g = expected rate of growth of real capital stock.

Equation (10) states that the share of debt in financing investment will depend not only on the target value chosen for the average debt ratio, but also on the expected rate of growth of capital stock, its rate of depreciation, the rate of amortization of debt, and the rate of inflation. For example, if there is an increase in inflation owing to expansionary credit policies, firms will choose to, and be able to raise the marginal debt ratio. Although this would raise the average debt ratio temporarily, the higher inflation will eventually reduce the average ratio to its target level by raising the value of assets in relation to debt outstanding. Another implication of equation (10) is that whenever firms are able to adjust the rate of amortization--say through frequent funding operations--to match the maturity structure of assets and the projected inflation, so that $a_d = \delta - \pi$, then the average and marginal debt ratios will be identical. 1/ If, however, the rate of amortization is fixed a priori, the fixed marginal debt ratio will fall short of the target average ratio when inflation is low (i.e., less than $\delta - a_d$) and will exceed the average ratio when inflation is high (i.e., exceeding $\delta - a_d$). This point can be illustrated by noting that for any given expected inflation, a faster rate of amortization--i.e., a shortening of loan maturities--will, by reducing the outstanding debt more rapidly than desired, induce firms to raise the marginal use of debt.

A relationship similar to equation (10) can be derived for foreign currency debt.

$$\alpha^{mf} = \alpha^{af} \cdot \frac{a_f + \pi - x + g}{g + \delta} \tag{11a}$$

$$= \alpha^{af} \cdot \frac{a_f + \pi_w - \theta + g}{g + \delta} \tag{11b}$$

where α^{mf} = marginal share of foreign currency debt, α^{af} = the long-run average share of foreign currency debt, x = the expected rate of increase in the nominal exchange rate, π_w = foreign rate of inflation, θ = the expected rate of change in the real exchange rate, given by $\theta = x - \pi + \pi_w$. It will be assumed that the rate of foreign inflation as well as the rate of amortization of foreign loans are fixed. Equation (11) implies that

1/ When inflation is high the equation $a_d = \delta - \pi$ implies a negative rate of amortization and equation (10) implies a large marginal debt ratio which could even exceed unity. These situations occur when firms build up debt far in excess of investment needs owing to the expected reduction in the real value of debt through inflation.

given the long-run target value α^{af} of the ratio of foreign currency debt to total assets, a reduction in domestic inflation, with no change in the rate of currency depreciation, will induce firms to lower the marginal share of foreign currency debt. On the other hand, if the fall in domestic inflation is expected to be offset by changes in exchange rate, so that the expected path of the real exchange rate is unchanged, then the marginal foreign currency debt ratio will remain stable. Thus, the share of foreign currency debt in financing investment will depend upon exchange rate policy, a consideration which has important implications for the effect of alternative exchange rate regimes on the average cost of capital, and hence on investment incentives.

So far the analysis of the marginal debt ratio has been based on the assumption that the target value of the average debt ratio is given a priori. The next step is to specify how this target is chosen by firms. In order to simplify the analysis and sidestep the difficult issue of determining the proper mix of domestic and foreign currency debt, it will be assumed that foreign currency debt constitutes a fixed share $1 - \beta$ of total debt. Therefore, given the average debt ratio α^a, the average foreign currency debt ratio is given by $\alpha^{af} = (1 - \beta)\alpha^a$, and the average domestic debt ratio is given by $\alpha^{ad} = \beta\alpha^a$.

The determinants of the target value of the average debt ratio can now be identified by analyzing the benefits and costs of raising the debt ratio. The benefit is simply the additional interest subsidy that can be garnered by increasing the share of the cheaper source of finance, namely loans from the financial system available at the controlled interest rate. The cost of incurring larger debt (in relation to assets) derives from the increased probability of future cash flow problems, and hence bankruptcy. This cost can be summarized by the equity cost function which links the discount rate and the average debt ratio. 1/

$$\rho = \rho(\alpha^a), \quad \rho'_\alpha > 0, \quad \rho''_\alpha > 0 \qquad (12)$$

(A dash above a variable denotes the first derivative with respect to the subscripted variable, and two dashes denote the second derivative with respect to the subscripted variables.)

Thus, the discount rate is assumed to be a strictly concave and increasing function of the average debt ratio. The optimal value of this ratio can be chosen so as to minimize the overall cost of capital by balancing at the margin the benefit of additional interest subsidy

---

1/ For example, see Feldstein et al. (1978, 1979) and Ericksson (1980). Myers (1977) and Kim (1978) contain interesting discussions of the reasons why the riskiness of returns from equity, and hence the required rate of discount, rises with increased use of debt. For a brief summary of this literature on the supply side of debt see Modigliani (1982).

with the cost of increased riskiness of investment. 1/ The optimal debt ratio so derived will be treated as the target value which firms strive to achieve.

Since the choice variable in this optimization exercise is the average debt ratio, it is necessary to express the cost of capital formula (8), which involves the marginal ratios (α^{md}, and α^{mf}), in terms of the average ratios by using equations (10) and (11). A complete expression for the cost of capital is given by:

$$r_b = (1 - c_1(\pi)\alpha^{ad} - c_2(\pi_w, \theta)\alpha^{af}(\rho - \pi + \delta) \qquad (13)$$

$$+[c_1(\pi)\alpha^{ad}\frac{R_d + a_d}{\rho + a_d} + c_2(\pi_w, \theta)\alpha^{af}\frac{R_f + a_f}{\rho + a_f - x}](\rho - \pi + \delta)$$

where $c_1(\pi) = \dfrac{g + a_d + \pi}{g + \delta}$, $c_2(\pi_w, \theta) = \dfrac{g + a_f + \pi_w - \theta}{g + \delta}$, $\alpha^{ad} = \beta\alpha^a$,

and $\alpha^{af} = (1 - \beta)\alpha^a$.

The optimal debt ratio can be obtained by substituting equation (12) into equation (13), and equating the first derivative of r_b (with respect to α^a) to zero,

$$\frac{dr_b}{d\alpha^a} = 0 \qquad (14)$$

The interpretation of the condition (14) is facilitated, if it is assumed for simplicity that the marginal and average debt ratios are identical, 2/ so that the expression for the cost of capital simplifies to the weighted average formula:

1/ Ideally the debt ratio should be treated as a control variable, along with the rate of investment, and the full optimal control problem of minimizing the present value of costs should be solved using the appropriate constraints on control variables. The problem has been simplified by assuming that the marginal and the target average debt ratios are linearly related. For an analysis of debt policy under the optimal control framework, see Ekman (1982), which also contains a detailed bibliography on this area of research. In most of these studies, the rate of interest is assumed to vary with debt, while the cost of equity is fixed. In the problem considered in this paper, the cost of equity varies with debt while the interest rate is fixed by policy. This complicates considerably an optimal control approach to the problem.

2/ Marginal and average ratios will be equal if $a_d = \delta - \pi$, $a_f = \delta - \pi + x$, so that $c_1(\pi) = c_a(\pi_w, \theta) = 1$. These conditions require that firms operate in a well developed domestic financial system, with easy access to international capital markets, so that the maturity of loans can be readily adjusted in line with inflation and the rate of depreciation of assets. Therefore, the equality of marginal and average debt ratios will be an unrealistic assumption for most developing countries.

$$r_b = (1 - \alpha)(\rho - \pi + \delta) + \alpha\beta(R_d - \pi + \delta) + \alpha(1 - \beta)(R_f - \pi + \delta)$$

where α denotes the common value of the marginal and average debt ratios ($\alpha = \alpha^a = \alpha^m$).

In this special case, the first order condition reduces to

$$(1 - \alpha)\ \rho'_\alpha = \rho - (\beta R_d + (1 - \beta)R_F) \qquad (14a)$$

The above equation serves to define implicitly the optimal target value of the average debt ratio. The right-hand side expression of equation (14a) is the implicit interest subsidy, while the left-hand side expression can be interpreted as the marginal risk premium demanded by the firm's owners. Thus, optimality requires balancing at the margin the benefits of additional subsidy with the costs of increased risk measured by the slope of the equity cost function. This balance will clearly be disturbed whenever domestic interest rate policy or foreign interest rate changes. Equally important, a change in inflation would affect the discount rate, the marginal debt ratios, and probably also the marginal risk premium (through shifts in ρ'_α) and thereby induce changes in the average debt ratio.

Some testable implications emerge from equation (14a). Regardless of the reduced form relationship linking the discount rate with other variables in the model, and insofar as the slope of the equity cost function (with respect to α) depends primarily on the debt ratio, at least over short intervals of time, the equation predicts that the gap between the cost of equity and the weighted average interest rate on debt—a measure of the interest subsidy—will be mainly a function of the debt ratio, and this relationship will be nonlinear.

4. Monetary equilibrium

The discount rate, the rate of interest and the rate of inflation are assumed to be consistent with equilibrium in the money markets. This requirement serves to capture the portfolio choices of private asset holders and can be incorporated into the model by specifying that the demand for real balances should equal supply.

$$\frac{M}{P} = f(R_d, \rho, \pi, y)$$

where M/P is the supply of real balances and the right-hand side represents the demand for real balances expressed as a function of real output (y) and returns to different types of assets in the portfolio. The money demand function specified above explicitly recognizes that individuals hold in their portfolio not only monetary assets and inflation hedges, but also loans in the curb market and claims to equity bearing similar risk.

5. Goods market equilibrium

Excess supply or demand in the goods market will clearly influence the rate of inflation and interest rates. Equilibrium in the goods market requires:

$$I = S + FS \qquad (15)$$

where FS = foreign savings assumed to be determined outside the model. The equilibrium condition (15) states that investment should equal available saving, and that such equilibrium will come about through variations in the discount rate, the rate of inflation, and the debt ratio.

6. Complete model

The model set out in Table 1 can be viewed as an adaptation of the Standard IS-LM model, with emphasis on the determinants of the cost of capital and the debt ratio in developing economies with segmented financial markets. The model determines saving, investment, the discount rate, the overall cost of capital, the debt ratio, and the rate of inflation. Before proceeding with the detailed analysis of the model it is useful to illustrate graphically the workings of the model for the simple case where the debt ratio as well as the administered interest rate are assumed to be fixed.

Table 1. A Model of Saving, Investment, and Debt

Savings

$$S = S(\rho - \pi, R_d - \pi)$$

Investment

$$I = I(r_b)$$

Cost of capital

$$r_b = (1 - a^m)(\rho - \pi + \delta) + \alpha^{md} \frac{R_d + a_d}{\rho + a_d} + \alpha^{mf} \frac{R_f + a_f}{\rho - a_f - x} \cdot (\rho - \pi + \delta)$$

Marginal debt ratio

$$\alpha^{md} = \beta\alpha^a \cdot \frac{g + a_d + \pi}{g + \delta}$$

$$\alpha^{mf} = (1 - \beta)\alpha^a \frac{g + \pi - x + a_f}{g + \delta}$$

Average debt ratio (defined implicitly)

$$\frac{dr_b}{d\alpha^a} = 0 \qquad \text{or in a special case}$$

$$(1 - \alpha)\rho_\alpha = \rho - [\beta R_d + (1 - \beta)R_f]$$

Monetary equilibrium

$$\frac{M}{P} = f(\rho, R_d, \pi), \text{ with } P = P_o(1 + \pi)$$

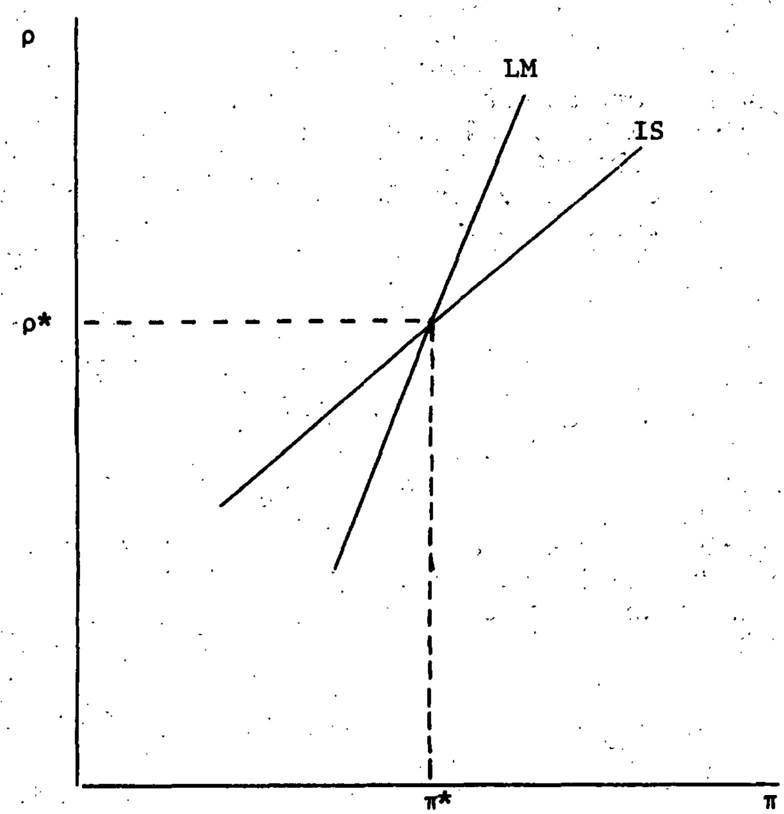
Goods market equilibrium

$$S + FS = I$$

Endogenous variables: S = domestic saving, I = domestic investment, r_b = weighted average real cost of capital, α^{md} = marginal domestic debt ratio, α^{mf} = marginal foreign currency debt ratio, α^a = average debt ratio, π = the rate of inflation, P = the general price level, ρ = the curb market rate.

Exogenous variables: R_d = domestic interest rate, R_f = the interest rate on external debt, α_f = the rate of amortization of external debt, α_d = the rate of amortization of domestic currency debt, δ = rate of depreciation of capital stock, β = share of foreign currency debt in total debt, M = nominal quantity of money, x = the rate of change of the exchange rate defined as the number of domestic currency units per unit of foreign currency, FS = foreign savings.

Figure 1. The Discount Rate and the Rate of Inflation



In Figure 1, the IS curve denotes the combinations of the discount rate and the rate of inflation that are consistent with goods market equilibrium. It is upward sloping because an increase in inflation with a fixed administered rate reduces the real administered rate and stimulates investment. In order to elicit a matching increase in saving, the discount rate rises. The usual upward-sloping LM curve represents money market equilibrium. The equilibrium values of the discount rate, and the rate of inflation are given by (ρ^*, π^*).

It will be shown in the next section that the magnitude as well as direction of the slope of the IS curve is sensitive to the size of the debt ratio, the response of the administered interest rate to variations in inflation and to other conditions affecting debt service (e.g., the rate of amortization). Since, as is well known, the stability of the system as well as the impact of policy changes depend upon the relative slopes of the IS and LM schedules, it follows that differences in the debt-equity ratio, the interest rate policy, and in other conditions governing debt will shift the impact of demand management policies. The next section illustrates that these shifts can indeed be substantial.

III. Analysis of the Model

The model is complex and therefore a general analysis of its comparative static properties is not attempted in this paper. However, useful insights can be gained by analyzing several simple special cases For example, it is found convenient to assume that the rate of inflation is determined exogenously and is not influenced by changes in interest rates. This assumption will be valid if the demand for real balances is regarded as a function of the rate of inflation alone (for any given real output), a reasonable specification in many financially repressed economies. Unless otherwise mentioned, this assumption will be maintained in order to simplify the analysis and thereby highlight the critical role of the debt-equity ratio. 1/

The effect of financial policies on the cost of capital and returns to savers will be analyzed under alternative assumptions about debt and amortization. There are several cases of interest depending upon whether the rate of amortization is fixed or variable (in response to inflation), whether the firm chooses the average debt ratio optimally or simply sticks to a target ratio, and whether foreign currency debt is significant or not. The choice of particular combinations of assumptions about debt influences the construction of the cost of capital and alters the final results. In order to underscore the impact of alternative assumptions about debt behavior, a sequence of simple models will be considered in turn.

1. The case of flexible amortization exogenously given target debt ratio, and no external debt

The importance of the debt ratio is best illustrated by considering the simplest possible model, where it is assumed that foreign currency debt is absent, the rate of amortization is varied in line with the rate of depreciation of capital assets and inflation ($a_d = \delta - \pi$) so that the marginal and average debt ratios are identical, and that the firm adheres to a target debt ratio α, and does not optimally adjust the ratio as environment changes. Foreign saving is ignored since there is no external debt in this model. Under these assumptions the model becomes:

$$S[\rho - \pi, R - \pi] = I[(1 - \alpha)(\rho - \pi + \delta) + \alpha(R - \pi + \delta)]$$

The effect of inflation on the discount rate is obtained by differentiating the equilibrium condition with respect to π, and regrouping terms, which yields:

1/ Modifications that result from using a more general money demand function--thereby allowing for variations in inflation due to changes in interest rate--are indicated in several places in the text. The impact of interest rates on working capital costs, and on short-run capacity utilization are throughout ignored for simplicity.

$$\frac{d\rho}{d\pi} = 1 + \left[\frac{(dR/d\pi - 1)(\alpha I'_r - S'_R)}{S'_\rho - (1 - \alpha)I'_r}\right] \tag{19}$$ 1/

The role of the debt ratio is brought out sharply in the special case where saving is interest inelastic ($S'_\rho = S'_R = 0$) and the controlled interest rate is not adjusted in line with inflation. In this case,

$$\frac{d\rho}{d\pi} = 1 + \frac{\alpha}{1 - \alpha} \tag{19a}$$

The interesting aspect of the formula is the implied magnitude of the Fisher effect when the debt-equity ratio ($\alpha/1 - \alpha$) is large as in many developing countries. For example, a debt-equity ratio of 3:1 implies $d\rho/d\pi = 4$; that is, the discount rate will increase by four times the increase in inflation, assuming passive interest rate policy. 2/

The size of the debt ratio also governs the impact of interest rate policy on saving under inflationary conditions. Total saving will increase—equivalently the cost of capital will fall—in response to an increase in inflation if and only if:

$$\frac{dS}{d\pi} = S'_\rho\left(\frac{d\rho}{d\rho} - 1\right) + S'_R\left(\frac{dR}{d\pi} - 1\right) > 0$$

Substituting equation (19) into the above expression and simplifying, the condition for improved saving is given by

$$\left(\frac{dR}{d\pi} - 1\right)\left[\frac{S'_\rho(\alpha I'_r - S'_R) + S'_R(S'_\rho - (1 - \alpha)I'_r)}{S'_\rho - (1 - \alpha)I'_r}\right] > 0 \tag{19b}$$

It can be verified that when

$$\frac{dR}{d\pi} > 1$$

the above inequality will hold if, and only if the debt equity ratio is less than the critical limit η, given by:

$$\eta = S'_R/S'_\rho$$

1/ In all subsequent discussions, it will be assumed that $S'_R, S'_\rho \geq 0$.

2/ This result is a special case of the more general observation that in thin markets price fluctuations will be large. A large debt-equity ratio implies that the free markets where the rate of discount is determined is quite thin.

The parameter η, which is the ratio of the response of saving to a change in the controlled rate to its response to a change in the discount rate—the interest sensitivity ratio, for brevity—is a critical determinant of the safe upper limit for the debt equity ratio. If the debt-equity ratio exceeds η, then the policy of raising the administered rate by more than the increase in inflation ($dR/d\pi > 1$) will depress saving.

Even when there is no change in inflation, the ultimate effect of interest rate policy on saving is governed by the critical limit η; the effect of interest rate on saving (dS/dR) is given by the expression within the square brackets of equation (19a) which will be positive if, and only if, the debt equity ratio is less than the interest sensitivity ratio.

The rationale of the above results is simple. The rates of saving and investment depend on both the controlled rate and the discount rate. When the controlled rate is raised, ceteris paribus, the discount rate falls. The net effect on saving or investment depends naturally on the relative size of the effects of changes in these two rates. What is interesting is that, in order to ensure a positive response of saving and investment to interest rate policy, the interest sensitivity ratio which measures the relative effect of interest rates on saving should bear an appropriate relationship to the debt-equity ratio that influences the effect of interest rate on investment.

The conditions under which the debt-equity ratio may exceed the interest sensitivity ratio are of interest. A high debt-equity ratio may induce a significant redistribution of incomes between households and businesses which, as indicated in the section II.1, is likely to dampen the size of the saving response to a change in the controlled rate, and thereby reduce the interest sensitivity ratio. In other words, a high debt-ratio will by itself serve to reduce η and raise the likelihood of a perverse saving response to interest rate policy. Even if the redistributive aspect is empirically insignificant, the debt-equity ratio could exceed the interest sensitivity ratio, if the interest sensitivity arises mainly from the response of private savings, while investment is financed to a substantial degree through foreign loans and government net lending programs.

When financial markets are segmented, the interest sensitivity ratio could be negative. For example, this is the case when the effect of controlled rate on saving is opposite in sign to the effect of the discount rate, and hence, the debt-equity ratio always exceeds the interest sensitivity ratio, thereby causing perverse saving response. This result highlights the potential shortcoming of segmented financial markets, and the advantage of unifying these markets through financial reform.

The prevalence of high debt ratios can lead to macroeconomic instability—e.g., progressively higher inflation and real discount rates—and the avoidance of such instability could involve high costs in terms of lower investment and saving. This assertion can be verified by extending the model to include portfolio equilibrium, and examining the relative slopes of the IS and LM schedules in Figure 1 as the debt ratio rises. Equation (19) implies that when the administered interest rate is kept unchanged as inflation accelerates (or raised by less than the increase in inflation), the slope of the IS curve is positive, and becomes steeper as the debt ratio rises. In other words, with passive interest rate policy, the increase in the discount rate due to a change in inflation ($d\rho/d\pi$) will be larger, the larger the debt ratio. Therefore, when the debt ratio is sufficiently large, the slope of the IS curve could become steeper than that of the LM curve, and this leads to an unstable economic system where monetary action can trigger unpredictable effects, such as accelerating inflation or deflation. 1/ Such instability in a high debt economy can be avoided through more active interest rate policy. For example, an increase in the real administered interest rate will make the slope of the IS curve negative and eliminate the source of instability. However, as already noted, an increase in the real interest rate would depress saving and investment when the debt-equity ratio exceeds a critical limit or when the interest sensitivity ratio is negative. In other words, achievement of stability in a high debt economy or in an economy with segmented financial markets is likely to involve high cost in terms of foregone growth.

2. The effect of optimal choice of debt ratio

In order to highlight the effect of optimal debt decisions, foreign currency debt will continue to be ignored. Also, as before, the rate of amortization will be assumed to be variable so that the marginal and the target average debt ratios are identical. Under these assumptions, the model determining saving, investment, and the optimal debt ratio can be stated as follows:

$$S(\rho - \pi, R - \pi) = I(r_b)$$

$$(1 - \alpha)\rho'_{\alpha} = (\rho - R)$$

$$r_b = (1 - \alpha)(\rho - \pi + \delta) + \alpha(R - \pi + \delta).$$

The first equation is the equilibrium condition between saving and investment. The second equation is the first order condition for the optimal choice of the debt ratio. The third is the expression for the real cost of capital. Noting that the discount rate is now implicitly a function of α and π, we can differentiate the two equation systems after substituting r_b into the first equation, and derive the impact

1/ In fact, with a steeper IS curve, an increase in monetary expansion will result in a new unstable equilibrium with lower inflation.

of inflation on the discount rate and the debt ratio. The differentiation yields,

$$\begin{bmatrix} S'_\rho - (1-\alpha)I'_r & I'_r(\rho - R) \\ -1 & (1-\alpha)\rho''_{\alpha\alpha} - \rho'_\alpha \end{bmatrix} \begin{bmatrix} d\rho/d\pi \\ d\alpha/d\pi \end{bmatrix} \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \quad (20)$$

where $B_1 = S'_\rho - (1-\alpha)\, I'_r + \alpha I'_r(dR/d\pi - 1) - S'_R\,(dR/d\pi - 1)$

$$B_2 = -dR/d\pi - (1-\alpha)\rho''_{\alpha\pi}$$

and $\rho''_{\alpha\pi}$ is the derivative of ρ'_α with respect to inflation.

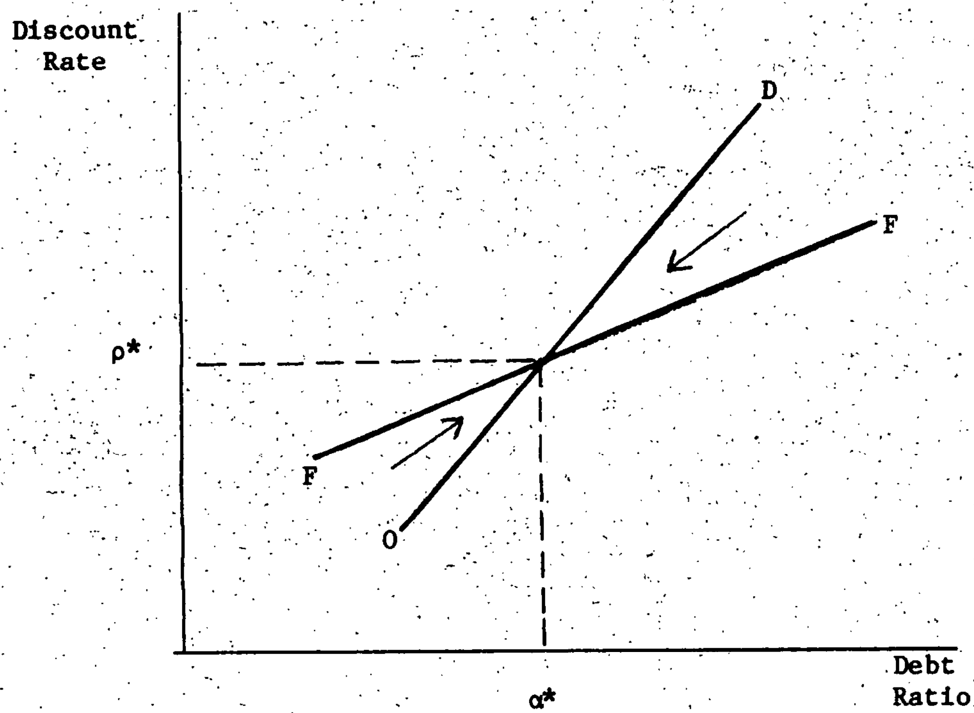
The stability condition for the model has an interesting economic interpretation that points to the possible adverse effects of large interest subsidy. Stability requires that the determinant of the matrix on the left side of equation (20) be positive. That is:

$$\Delta = [S'_\rho - (1-\alpha)I'_r][(1-\alpha)\rho''_{\alpha\alpha} - \rho'_\alpha] + I'_r(\rho - R) > 0$$

The first term is positive, because the second order condition for the optimal choice of debt ratio ensures that $(1-\alpha)\rho''_{\alpha\alpha} - \rho'_\alpha > 0$. But the second term is negative, and its magnitude depends upon the size of the implicit interest subsidy, $\rho - R$. The larger the subsidy, the greater the likelihood that instability would obtain. The instability would manifest itself in the following way. A substantial distortion in the financial market resulting in a large implicit subsidy $\rho - R$, would induce firms to raise the debt ratio so as to capture the subsidy. This, in turn, would raise the discount rate because of higher risk. The increase in the discount rate will raise the implicit subsidy, inducing firms to borrow even more. Only stringent credit rationing will limit the achievable debt ratio. Excess demand for credit will continue to persist. This possibility which is a realistic description of many economies, will be ignored, and the stability condition $\Delta > 0$ will be assumed to hold.

The model and the stability conditions are illustrated in Figure 2. The line FF shows the combinations of ρ and α which ensure flow of funds equilibrium between saving and investment. An increase in the debt ratio lowers the cost of capital, raises investment, and pushes up the discount rate. Therefore the slope is positive. The line OD shows the optimal combinations of ρ and α which are consistent with the minimization of the cost of capital. An increase in ρ raises the implicit interest subsidy obtainable on debt and induces a larger debt ratio. Again the slope is positive. The intersection of the two curves indicates the equilibrium level of the discount rate and the debt ratio (ρ*, α*). The slope of the OD curve should be steeper than the FF curve in order to ensure stability.

Figure 2. Debt Ratio and Discount Rate



Solving equation system (20), the effect of inflation can be stated as follows:

$$\frac{d\rho}{d\pi} = 1 + \frac{1}{\Delta}\left[\begin{array}{l} I'_r\,(\rho - R)(1 - \alpha)\rho''_{\alpha\pi} \\ + (dR/d\pi - 1)\left\{I'_r(\rho - R) + [(1 - \alpha)\rho''_{\alpha\alpha} - \rho'_\alpha] \right. \\ \qquad \left. \cdot\,(\alpha I'_r - S'_R)\right\} \end{array}\right] \tag{21a}$$

$$\frac{d\alpha}{d\pi} = \frac{1}{\Delta}\left[\begin{array}{l} (1 - \alpha)\rho''_{\alpha\pi}[(1 - \alpha)\ I'_r - S'_\rho] \\ + (dR/d\pi - 1)\ [\alpha I'_r - S_R + (1 - \alpha)\ I'_r - S'_\rho] \end{array}\right] \tag{21b}$$

The magnitude of the derivatives depends upon, among other things, the sign and size of $\rho''_{\alpha\pi}$, which is the shift in risk premium due to a change in inflation. If higher inflation is associated with increased uncertainty, and therefore investors perceive greater risks, then $\rho''_{\alpha\pi} > 0$. Under this assumption, equations (21a) and (21b) imply that $d\rho/d\pi < 1$ and $d\alpha/d\pi < 0$, whenever $dR/d\pi \geq 1$. Thus, an increase in the rate of inflation leads to a reduction in the optimal debt ratio—a counterintuitive result--because of increased marginal risk premiums required by investors. 1/ The decline in the debt ratio, in turn, implies a reduction in the share of the cheaper source of finance, and hence, an increase in the overall cost of capital. However, this effect on the cost of capital is partly offset by the reduction in the real discount rate.

The consequences of optimal choice of debt ratio are best illustrated if it is assumed that the real administered rate is kept unchanged (i.e., $dR/d\pi = 1$). Under this assumption, it is seen from equation (19) that when the debt ratio is fixed, and not adjusted optimally, the real discount rate remains unchanged and hence saving and investment remain unaffected. In contrast, when the debt ratio is adjusted optimally, equation (21) implies that the real discount rate falls, thereby reducing saving and investment. A matching reduction in investment obtains because of the fall in debt ratio which leads to an increase in the real cost of capital. This apparently counterintuitive result—that the real cost of capital increases in the presence of optimal debt behavior designed to minimize the cost of capital, but not so when the debt ratio is fixed—is mainly due to the effect of inflation on the risk premium required by investors, an effect which was ignored in analyzing the case of fixed debt ratio. Indeed, when the risk premium is not affected by inflation—so that $\rho''_{\alpha\pi} = 0$, equation (21) implies that as long as the real administered rate is kept unchanged, the optimal debt ratio does not change, and the real discount rate also remains unaffected just as in the case of fixed debt ratio.

If the controlled interest rate is raised by more than the increase in inflation, then the issue of how saving is affected under optimal debt policy can be analyzed by examining the sign of:

$$dS/d\pi = S'_\rho \left(\frac{d\rho}{d\pi} - 1\right) + S'_R \left(\frac{dR}{d\pi} - 1\right).$$

Substituting from equation (21a), it can be verified that saving will improve if, and only if,

$$\frac{\alpha}{1-\alpha} < \eta - \frac{\rho'_\alpha}{(1-\alpha)[(1-\alpha)\rho''_{\alpha\alpha} - 2\rho'_\alpha]}$$

From the second order condition for optimal debt ratio, the second term on the right-hand side of the above inequality is positive. Thus, when

1/ Gordon (1982) analyzes the effect of inflation on the debt ratio in the U.S. economy. Preliminary empirical tests suggest that $\rho''_{\alpha\pi} > 0$ is a valid assumption for Korea. The sign will probably depend on the level of inflation, and no a priori judgments are possible.

debt is chosen optimally, the critical upper limit on the debt equity ratio is smaller than in the case of fixed debt ratio. This tighter upper limit on the debt-equity ratio continues to apply as the condition for an increase in the controlled interest rate to improve saving, even when there is no change in inflation. In other words, the likelihood of active interest rate policy having adverse effects on saving increases when firms choose the debt ratio optimally.

3. The effect of predetermined amortization schedule

If the rate of amortization on loans cannot be adjusted when inflation accelerates, then the real value of amortization payments will be reduced and the average and marginal debt ratios will begin to diverge. To understand the implications of these developments, it is convenient to abstract from the existence of foreign currency debt and assume that all debt is denominated in domestic currency units and offered at the rate R which is lower than the discount rate ρ. However, firms strive to reach an average debt ratio in the long run, and therefore adjust the marginal debt ratio in line with inflation and growth prospects. For simplicity it will be assumed that the average debt ratio α is a predetermined target, and is not chosen optimally. Under these assumptions, the cost of capital can be expressed as:

$$r_b = (1 - \alpha c(\pi))(\rho - \pi + \delta) + \alpha c(\pi)\frac{R + a}{\rho + a}(\rho - \pi + \delta)$$

where $c(\pi) = (g + \pi + a)/g + \delta$.

The differentiation of the saving-investment equilibrium condition with respect to inflation yields:

$$\frac{d\rho}{d\pi} = 1 + \left[\frac{-I_r'(A_4 - A_2 + A_3) - S_R'(dR/d\pi - 1)}{S_\rho' - I_r'(1 - A_1 - A_4)}\right] \tag{22}$$

$$A_1 \equiv \alpha c(\pi)\frac{\rho - R}{\rho + a} \qquad A_2 \equiv \alpha c(\pi)\frac{\rho - \pi + \delta}{\rho + a}\frac{dR}{d\pi} \equiv \bar{A}_2\frac{dR}{d\pi}$$

$$A_3 \equiv \alpha\frac{1}{g + \delta}\frac{\rho - R}{\rho + a} \qquad A_4 \equiv \alpha c(\pi)\frac{R + a}{(\rho + a)^2}(\rho - \pi + \delta)$$

An analysis of the above expression reveals that the size of the Fisher effect depends not only on interest rate policy $(dR/d\pi)$, but also on whether the real discount rate initially exceeds or falls short of the expected real growth of capital stock. This result can be summarized as follows:

$$d\rho/d\pi > 1 \text{ if and only if } dR/d\pi < 1 + [AI'_r/(-S'_R + \overline{A}_2 I'_r)]$$

where

$$A = \frac{\alpha(\rho - \pi + \delta)(\rho - R)(\rho - \pi - g)}{(\rho + a)^2(g + \delta)}$$

The implication of the above result is best illustrated when $dR/d\pi = 1$. In this case,

$$\frac{d\rho}{d\pi} < 1 \text{ if } \rho - \pi < g$$

and (22a)

$$\frac{d\rho}{d\pi} > 1 \text{ if } \rho - \pi > g$$

Thus, when the administered rate is maintained in real terms, the real discount rate would rise and so would saving and investment, if initially the real discount rate was greater than the expected growth of capital stock. Otherwise, the real discount rate would fall, and with it saving and investment. These results imply that, with a fixed amortization rate, if the real discount rate was large to begin with, then inflation will make it even larger, and the slope of the IS curve will become steeper as the discount rate rises, possibly intersecting the LM curve twice; the intersection corresponding to the higher discount rate will be unstable owing to the steeper slope of the IS curve when the discount rate is large (see equations (22) and (22a)).

The dependence of the effects of macroeconomic policies on the size of the real discount rate is related to two conflicting forces acting on the real cost of capital when the rate of amortization is fixed. Assuming that as inflation rises, both the discount rate and the controlled rate increase temporarily by the same amount as inflation, it follows that the present value of debt service costs, $R + a/\rho + a$ increases, thereby raising the cost of capital. On the other hand, the increase in inflation induces a larger use of debt at the margin, and the marginal debt ratio, $\alpha(g + \pi + a)/(g + \delta)$, increases thereby lowering the cost of capital. The net effect on the cost of capital depends upon the initial size of ρ, which affects the present value of debt service in relation to the expected magnitude of the growth in assets $(g + \pi)$, which affects the marginal debt ratio.

The conditions under which saving improves in response to interest rate policy depends, as before, on the size of the debt-equity ratio, but the critical limit on this ratio is influenced not only by the initial size of discount rate, and the interest rate, but also by interest rate policy. More formally, assuming $(dR/d\pi) > 1$, the condition for improvement in saving and investment is:

$$\frac{dS}{d\pi} > 0$$

if and only if:

$$[\alpha / 1 - \alpha][1 - B] < \eta \qquad (23)\ \underline{1/}$$

where $B = \frac{(\rho - g - \pi)(\rho - R)}{(dR/d\pi - 1)(g + \delta)(\rho + a)}$.

If $\rho - \pi > g$, then B is positive and can be extremely large for small increments in the real controlled rate. In this case, the condition for improved saving will always hold because the left-hand side of the inequality (23) is negative. Moreover, for large changes in the real controlled rate, the improvement in saving depends on a much less stringent condition on the debt ratio than in the case of flexible amortization. If $\rho - \pi < g$, then B is negative, and the condition on the size of the debt ratio is much more stringent than in the case of flexible amortization. However, the earlier condition $\rho - \pi > g$ is most likely to hold under normal circumstances. This is because the larger the real discount rate, the more efficient the use of capital stock; this implies that for any given target output growth, the required growth in real capital stock is likely to be less. This association of a larger real discount rate $\rho - \pi$ with a smaller real asset expansion g, would lead to the condition $\rho - \pi > g$. In any case, the results clearly demonstrate that, depending upon the initial conditions in the financial markets and interest rate policy, the behavior with regard to amortization can make a major difference to the impact of interest rate policy.

As in previous cases, the effect of interest rate policy, unaccompanied by a change in inflation, is also governed by the size of the debt-equity ratio. An increase in the administered interest rate will improve saving if, and only if,

$$\frac{\alpha c(\pi)}{1 - \alpha c(\pi)} < \eta$$

where, as before, η is the interest sensitivity ratio. The upper limit 'η' now applies to the marginal debt-equity ratio. Because the rate of amortization is assumed to be fixed, the average and marginal debt ratios can diverge. Therefore, although the average debt ratio is not large (in relation to η), it is conceivable that a high level of inflation and the associated credit policies induce a large marginal use of debt. If so,

---

1/ The limit on the debt-equity ratio has been derived by evaluating the derivative at $a = \delta - \pi$. Without this simplifying assumption, the expressions become more complex, but the conclusions remain unaffected.

an increase in the controlled rate could lead to adverse effects on the cost of capital and saving. The analysis suggests that such adverse effects can be mitigated by allowing for a more flexible amortization schedule, and at the same time, restraining the share of debt in project finance. However, if the average debt ratio is already very large, then the adverse effects cannot be avoided.

It is legitimate to ask why firms would try to achieve the target for the average debt ratio by raising the marginal ratio in line with an increase in inflation, if the target can be readily reached by simply amortizing the existing loans at a slower rate. 1/ From the firm's point of view, the latter option may be preferable--assuming it is available--but the option chosen will depend upon particular institutional circumstances and historical practices. 2/ Often it may be easier to raise a loan which is a larger-than-normal fraction of the project's current value than to obtain rollover credits for the principal amounts falling due. Depending upon which option is the norm, or more generally, depending upon how the marginal debt ratio is determined, the impact of interest rate policy will be changed. 3/ The differences in the impact of policies can be significant, as has been demonstrated.

4. The effect of foreign currency debt, with predetermined target debt ratio, and fixed amortization rate for foreign debt

In order to highlight the effects of foreign currency debt, considerations of optimal choice of debt are ignored. The average domestic and foreign currency debt ratios are given as targets that remain fixed. Domestic and foreign interest rates are allowed to diverge by assuming that capital mobility is subject to restrictions. In view of the limited, and often uncertain, access to international capital markets in many developing countries, the rate of amortization of foreign currency loans is assumed to be fixed. However, amortization of domestic currency loans is assumed to be flexible so that the marginal and average domestic debt ratios are identical. Under these assumptions, the cost of capital is given by:

1/ The importance of such funding operations, for the validity of the weighted average cost of capital formulae, is noted in Linke and Kim (1974) and Beranek (1975).

2/ For example, many countries impose norms on debt-equity ratios for project finance, which would restrict the freedom at the margin.

3/ In this section, the marginal debt ratio was assumed to depend upon the target average ratio, the rate of inflation, the rate of growth, the rate of amortization, and the rate of depreciation. More interesting formulations are possible. For example, the rate of amortization or the average maturity of loans can be made a function of growth and interest rates. For a discussion of the determinants of the rate of amortization of corporate debt in the United States, see Morris (1975).

$$r_b = [1 - \alpha\beta - \alpha(1-\beta)C_2(\pi_w, \theta)](\rho - \pi + \delta)$$

$$+ \alpha\beta(R_d - \pi + \delta) + \alpha(1-\beta)C_2(\pi_w, \theta)\left[\frac{R_f + a_f}{\rho + a_f - x}\right](\rho - \pi + \delta) \tag{24}$$

where:

$$C_2(\pi_w, \theta) = \frac{a_f + \pi_w - \theta + g}{g + \delta}$$

The above expression reveals that the cost of capital depends, among other things, on the expected rates of increase in the nominal and real exchange rates, which influence the present value of the debt service on foreign currency loans, and also the share of such loans in investment finance. This suggests that the type of exchange rate regime influences the effect of interest rate policies.

First, suppose that the expected change in the nominal exchange rate is fixed a priori. This would be the case when the nominal exchange rate is pegged to a currency basket ($x = 0$), or, for instance, the path of the depreciation of the nominal exchange rate is preannounced ($x > 0$) independently of other relevant variables. Under such exchange rate regimes, the expected change in the real exchange rate varies with inflation, and thereby influences the marginal debt ratio (i.e., C_2 varies with θ). Differentiating the saving-investment equilibrium condition, the Fisher effect is obtained as:

$$\frac{d\rho}{d\pi} - 1 = \frac{1}{\Delta}[(dR_d/d\pi - 1)(-S'_R + \alpha\beta I'_r) + D_2 I'_r] \tag{25}$$

where,

$$\Delta = S'_\rho - [(1 - \alpha\beta - \alpha(1-\beta)C_2) + D_1]I'_r > 0$$

$$D_1 = \alpha(1-\beta)C_2 \frac{R_f + a_f}{(\rho + a_f - x)^2}(a_f + \pi - \delta - x)$$

and,

$$D_2 = \alpha(1-\beta)\frac{\rho - \pi + \delta}{g + \delta}\left[\frac{(R_f + a_f)(\rho - \pi - g)}{(\rho + a_f - x)^2} - 1\right] < 0$$

Rearranging the terms of equation (25), it can be verified that:

$$\frac{d\rho}{d\pi} > 1 \text{ if and only if } \frac{dR}{d\pi} < 1 + \left[\frac{D_2 \cdot I'_r}{S'_R - \alpha\beta I'_r}\right] \tag{26}$$

The sign of the term D_2 is, in general, negative. Only when the foreign interest rate substantially exceeds the domestic discount rate would D_2 be positive. This is quite unlikely in a developing economy. Therefore equation (26) implies that the real discount rate will increase with inflation as long as the controlled rate is not increased by significantly more than the increase in inflation. In other words, when the real controlled rate is raised slightly but within limits (set by equation (26)), saving will improve because the induced increase in the real discount rate will reinforce the positive saving impact of a rise in the controlled rate. Despite the increases in real rates, a matching increase in investment occurs because the real cost of capital actually falls owing to both an increase in the share of foreign currency loans, which is cheaper than domestic equity, and the rise in the nominal discount rate, which reduces the opportunity cost of external debt service payments. The result is illustrated in Figure 3 for the special case $dR/d\pi = 1$. When inflation rises from π_0 to π_1, the saving schedule either remains fixed or shifts to the right insofar as foreign saving (current account deficit) increases owing to the appreciation of the real exchange rate; the investment schedule also shifts to the right, thereby raising saving and investment. The upward shift in investment can be deduced from equation (24), where the present value of the projected debt service payments on external loans is given by the expression $(R_f + a_f)/(\rho + a_f - x)$. This term is clearly reduced (thereby reducing the cost of capital) when ρ increases due to higher inflation, but x remains fixed. Moreover, the share of foreign currency debt in financing investment rises due to the appreciation of the real exchange rate (Θ falls, raising C_2). 1/ This development also reduces the cost of capital and contributes to the upward shift in investment.

Conditions under which saving and investment improve can be summarized as follows. Assuming $dR/d\pi > 1$, and foreign saving is fixed 2/ saving will improve ($dS/d\pi > 0$) if and only if

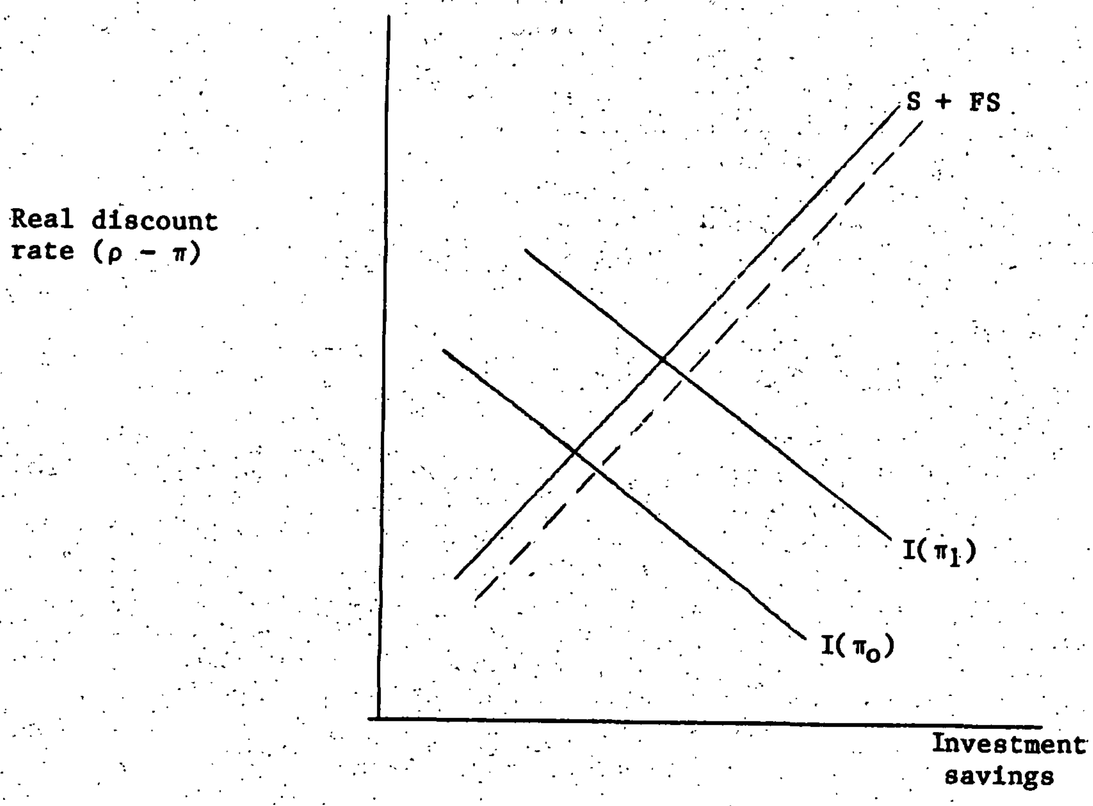
$$\alpha\beta/(1 - \bar{\alpha} + D_1) < \eta + [-D_2/(dR/d\pi - 1)(1 - \bar{\alpha} + D_1)] \qquad (27)$$

where $\bar{\alpha} = 1 - \alpha\beta - \alpha(1 - \beta)C_2$. The left-hand side of the above inequality is approximately the ratio of domestic currency debt to equity. The right-hand side is the sum of the interest sensitivity ratio and a positive term. This positive term is very large when $dR/d\pi$ is close to unity, and the above inequality always holds; therefore, saving improves

1/ The increased use of foreign currency debt in financing investment may be facilitated by the increase in foreign savings, but this need not be the case.

2/ The assumption of fixed foreign saving is for expository convenience only. Making it a function of the real exchange rate does not alter the qualitative conclusions on the impact of interest rates and exchange rate policy on investment.

Figure 3. The Effect of an Increase in Inflation with Substantial Currency Debt, Fixed Nominal Exchange Rate and Fixed Real Administered Rate



whatever the size of debt ratios. However, when the controlled rate is raised substantially in real terms the size of the debt ratio becomes binding. But the upper limit on the debt ratio now applies only to domestic debt, and moreover, the limit is larger than in the case of no foreign currency debt. Thus, the use of foreign currency debt significantly eases the conditions required for an improvement in saving and investment in response to interest rate action.

When the controlled interest rate is allowed to decline in real terms as inflation rises (or allowed to rise as inflation falls), the inequality (27) is necessary and sufficient to ensure a reduction in saving and an increase in the real cost of capital. This underscores the need for an active interest rate policy in the presence of substantial use of foreign currency debt. If foreign currency debt is substantial and hence domestic debt ratio is small enough to satisfy the inequality (27), then the emergence of a negative interest rate (due to increased inflation) will reduce investment, thus producing precisely the opposite of the effect intended.

When there is no change in inflation, interest rate policy will improve saving if, and only if, the domestic debt ratio is sufficiently small.

$$\alpha\beta/(1 - \alpha\beta - \alpha(1 - \beta)C_2 + D_1) < \eta \tag{28}$$

It is important to note that even if the average domestic debt ratio ($\alpha\beta$) is small, the above inequality may be violated, if at the margin, firms use substantial amounts of foreign currency loans to finance investment in the hope that the debt ratio would revert to target levels in the long run. In this case, the left-hand side of the above inequality could become large (i.e., C_2 is very large due to high growth and inflation expectations), thereby violating the necessary (and sufficient) condition for ensuring positive saving response. This type of adverse outcome for interest rate policy can be avoided if foreign currency loans can be amortized more flexibly, thereby permitting a reduction in marginal debt ratios.

So far, the analysis has been based on the assumption that the path of the nominal exchange rate is fixed. Similar analysis can be readily completed under the assumption that the real exchange rate (or its rate of change) is fixed a priori, so that the nominal exchange rate varies with changes in inflation. Therefore, the present value of external debt service payments varies because of changes in the nominal exchange rate, but the marginal share of foreign currency debt remains unaffected (C_2 does not change, since π_w and Θ remain fixed). In this case, the effects of interest rate policy turn out to be different from the case of fixed nominal exchange rate, often significantly so; but the safe limit on the domestic debt ratio remains as that shown in inequality (28), which now applies to both the pure interest rate action as well as interest rate responses to changes in inflation. Thus, irrespective of the exchange rate regime, the availability of foreign loans serves to ease the constraint governing the improvement of saving and investment through interest rate policy.

However, the impact of interest rate policy is influenced substantially by the exchange rate regime. For example, the policy of maintaining the real (controlled) interest rate unchanged in the face of higher inflation leaves the cost of capital and saving unchanged when the real exchange rate also is maintained unchanged, but, as already shown, lowers the cost of capital and improves saving when it is the nominal exchange rate that is fixed. Moreover, when the real interest rate is raised slightly in the face of an increase in inflation, saving and investment incentives continue to improve under fixed nominal exchange rate irrespective of the debt ratio; however, with a large share of domestic currency debt, precisely the opposite occurs when the real exchange rate is fixed. In other words, when inflation accelerates active interest rate policy can often be used to improve saving and investment incentives without regard to the size of debt ratio by simultaneously maintaining stable nominal exchange rates, but the improvement in domestic saving is obtained only at the expense of an appreciation of the real exchange rate, and the attendant weakening of the balance of payments. In these same circumstances if the real, rather than the nominal, exchange rate is held stable, then the appropriate interest rate response for improving investment incentives and saving will vary according to the share of domestic currency debt. 1/

IV. Summary and Policy Implications

In many developing countries, enterprises rely largely on debt finance while equity capital remains scarce, in part because the banking system and in some cases the unregulated segments of the financial system such as the curb markets have together provided substitutes for stock issue in the form of long- and short-term loans, while the flow of foreign saving has been mainly in the form of debt rather than equity. For example, the average debt-equity ratio of firms in the industrial sector in Korea has grown from about 100 percent in the early sixties to about 500 percent in recent years, reflecting, among other things, the rapid growth of the banking system following the interest rate reform in 1965. The resulting overleveraged financial structure of firms is often perceived to restrict the economic policy options open to the authorities. The purpose of this paper is to analyze the macroeconomic consequences that flow from enterprises financing their investment with a large share of debt in relation to equity.

1/ When the nominal exchange rate is fixed, the debt ratio becomes relevant only for large changes in the real interest rate.

For this purpose, the paper develops a model of saving, investment, portfolio adjustments, and the debt ratio in developing countries characterized by segmented financial markets, controls on the banking system, and substantial reliance on debt, including external debt. The financial structure of firms, and plausible behavioral assumptions regarding how firms adjust their financing patterns, are explicitly built into the model by appropriately defining the cost of capital in developing economies, and thereby emphasizing the linkages between debt behavior and incentives for saving and investment. The model is used to analyze the impact of interest rate policy on stability and growth.

The major conclusions are as follows. Firms' debt-equity ratio makes a sizeable difference to the impact of stabilization policies, particularly interest rate policies. When debt ratios used by firms are large, pursuit by the authorities of a passive interest rate policy--i.e., maintenance of the controlled interest rate unchanged when inflation changes--can lead to macroeconomic instability characterized by perverse effects of monetary policy and accelerating inflation or deflation. Therefore, in economies in which firms tend to have a large debt-equity ratio, appropriate adjustments in the real administered rate become necessary to achieve macroeconomic stability. However, the impact of such action on saving and investment is conditioned by the relative shares of domestic and foreign currency debt, and by the ability of firms to adjust these relative shares and to change the debt ratio in general.

In general, there exists a safe upper limit on the debt-equity ratio of firms in the aggregate, defined as the limit which, if exceeded, leads to perverse effects on saving and investment when the real interest rate is raised. This limit depends mainly on the interest sensitivity of saving, but is also influenced by a host of other considerations, including the initial conditions in domestic financial markets, the ability of firms to adjust the rate of amortization and the target debt ratio, the terms and availability of foreign currency loans, and the size of the increase in the controlled rate. For example, the safe limit on the debt-equity ratio becomes more stringent when the rate of amortization is fixed and the discount rate is low. More stringent limits also apply when firms are able to adjust the debt-equity ratio optimally in order to balance the benefits of additional subsidized credit from banks with the associated costs arising from increased riskiness of investment. In contrast, the availability of foreign capital serves to ease the constraint on the debt-equity ratio.

When the debt ratio exceeds the safe limit, appropriate increases in the real interest rate in order to ensure stability would also involve considerable cost in terms of foregone growth. In view of this high cost, in economies with firms relying on high leverage, maintenance of low and stable inflation is the optimal policy.

When firms have access to foreign currency loans the impact of interest rate policy on the cost of capital and saving is influenced substantially by the exchange rate regime. For example, when inflation accelerates, an increase in the real administered rate can improve domestic saving irrespective of the debt ratio if the nominal exchange rate is fixed, but if, so as to ensure external balance, the real exchange rate is fixed, then the increase in the real interest rate will improve saving only in the event that the share of domestic currency debt is small. Thus, the achievement of stability and external balance through necessary adjustments in the real exchange rate and real interest rate will involve a substantial loss of investment incentives if the domestic currency debt ratio exceeds a safe limit.

In view of the significant implications of the debt-equity mix used by firms, an evaluation of the financial structure of firms and the institutional framework of the financial system that underlies it is important for a proper assessment of the impact of stabilization policies. Often the effectiveness of stabilization policies, particularly interest rate policies, can be enhanced by implementing appropriate financial reform measures that include steps to reduce the debt-equity ratio of firms. Insofar as the financing patterns used by firms are conditioned by the institutional framework of the financial system, substantial changes in the debt-equity mix can be brought about only in the long run through appropriate institutional reforms (e.g., promoting corporate saving, developing capital markets, and establishing debt-equity norms). An assessment of the safe limit on the debt-equity ratio based on the macroeconomic framework suggested in the paper can help to devise debt-equity norms for project finance, and help to decide the extent to which the reforms of the financial system should emphasize a restructuring of company finance. In any event, the financial reform package should strive to reduce segmentation in the financial markets, and reduce interest subsidy because, as demonstrated in the paper, such actions can also contribute to macroeconomic stability, improve the effectiveness of interest rate policy, and eventually reduce the cost, in terms of foregone growth, of stabilization policies. In addition, appropriate adjustments in lending practices relating to rollover of credits (both domestic and foreign) and project finance and provision of adequate access to foreign capital can complement stabilization policies in a significant way.

Derivation of the Expression for the Real Cost of Capital

1. Based on optimal control approach

The problem is to minimize total cost,

$$\int_0^\infty \exp(-\rho t)[C(Q^*, K) + (1 - \alpha) I P + (R_d + a_d)G + (R_f + a_f)F.E]dt$$

with respect to the control variable I, subject to the following differential equations on the state variables, K, G, and F.

$$\dot{K} = I - \delta K \qquad (1)$$

$$\dot{G} = \alpha\beta I P - a_d G \qquad (2)$$

$$\dot{F} = 1/E\ \alpha(1 - \beta)I P - a_f F \qquad (3)$$

The notation is explained below; for convenience, the time subscript has been omitted.

ρ	=	the discount rate, or the opportunity cost of funds to the owners of the firm;
C	=	variable cost of production;
Q*	=	target output;
K	=	real capital stock;
$1 - \alpha$	=	the share of investment financed by equity (including curb market loans);
α	=	marginal debt ratio;
β	=	share of domestic currency debt in total debt;
I	=	real gross investment;
P	=	price level ($P_t = e^{\pi t}$, π is the rate of inflation);
R_d	=	domestic interest rate--the controlled rate;
R_f	=	foreign interest rate;
a_d	=	the rate of amortization of domestic debt;

a_f	=	the rate of amortization of foreign debt;
G	=	domestic currency debt outstanding;
F	=	foreign currency debt outstanding;
E	=	exchange rate, domestic currency units per unit of foreign currency ($E_t = E_o e^{xt}$, x = the rate of change in E);
$(R_d + a_d)G$	=	debt service payments on domestic curency loans;
$(R_f + a_f)F\ E$	=	debt service payments on foreign currency loans in domestic currency units.

The Hamiltonian for the control problem is:

$$H = \exp(-\rho t)[C(Q^*, K) + (1 - \alpha)\ I\ P + (R_d + a_d)\ G + (R_f + a_f)\ F\ E]$$
$$+ \lambda_1\ [I - \delta K] + \lambda_2\ [I\ P - a_d\ G]$$
$$+ \lambda_3\ [\alpha(1 - \beta)\ I\ P/E - a_f\ F]$$

The first order conditions are given by:

$$\dot{\lambda}_1 = -\partial H/\partial K = -[\exp(-\rho t)\ \partial C/\partial K - \lambda_1 \delta] \tag{4}$$

$$\dot{\lambda}_2 = -\partial H/\partial G = -(R_d + a_d)\exp(\rho t) - \lambda_2 a_d \tag{5}$$

$$\dot{\lambda}_3 = -\partial H/\partial F = -[(R_f + a_f)\ E\exp(-\rho t) - \lambda_3 a_f] \tag{6}$$

$$0 = \partial H/\partial I = (1 - \alpha)P\exp(-\rho t) + \lambda_1 + \lambda_2 \alpha\beta P + \lambda_3\ (1 - \beta)P/E \tag{7}$$

Solving the differential equations (5) and (6)

$$\lambda_2\exp(\rho t) = [\frac{R_d + a_d}{\rho + a_d}] \quad = \text{The present value of one unit of domestic currency loan} \tag{8}$$

$$\frac{\lambda_3\exp(\rho t)}{E_t} = \frac{R_f + a_f}{\rho + a_f - x} \quad = \text{The present value in foreign currency units of one unit of foreign currency loan} \tag{9}$$

It is assumed that the exchange rate at time 't' is given by $E_t = E_o\exp(xt)$ where 'x' is the expected rate of change in the nominal exchange rate.

Substituting the values of λ_2 and λ_3 given in equations (8) and (9), into equation (7) and regrouping terms, λ_1 can be expressed as:

$$\lambda_1 = -(1 - \alpha)\exp(-(\rho - \pi)t)$$
$$-\exp(-(\rho - \pi)t)[\alpha\beta \frac{R_d + a_d}{\rho + a_d} + \alpha(1 - \beta)\frac{R_f + a_f}{\rho + a_f - x}] \tag{10}$$

where the price level 'P' is entered as $\exp(\pi t)$, with π denoting the rate of inflation. The initial price level has been normalized to unity.

Differentiating both sides of the above equation with respect to time, an alternative expression for $\dot{\lambda}_1$, is given by:

$$\dot{\lambda}_1 = -(1 - \alpha)(\rho - \pi)\exp(-(\rho - \pi)t)$$
$$+ [\alpha\beta \frac{R_d + a_d}{\rho + a_d} + \alpha(1 - \beta)\frac{R_f + a_f}{\rho + a_f - x}](\rho - \pi)\exp(-\rho - \pi)t) \tag{11}$$

on substituting equations (10) and (11) into equation (4) and rewriting, it is seen that along the optimal path the following relationship should hold.

$$-\exp(-\pi t)\ \partial C/\partial K = (1 - \alpha)(\rho - \pi + \delta)$$
$$+ \alpha\beta [\frac{R_d + a_d}{\rho + a_d} + \alpha(1 - \beta)\frac{R_f + a_f}{\rho + a_f - x}](\rho - \pi + \delta) \tag{12}$$
$$\equiv r_b$$

where r_b is the real cost of capital.

In the special case when there is no foreign currency loan ($\beta = 1$), the cost of capital can be written as:

$$r_b = (1 - \alpha)(\rho - \pi + \delta) + \alpha \frac{R_d + a_d}{\rho + a_d} (\rho - \pi + \delta) \tag{13}$$

2. Based on the Modigliani-Miller theorem

An alternative approach to deriving the cost of capital formula (13) is based on the well-known Modigliani-Miller Proposition I. For simplicity, the role of foreign currency debt will be ignored.

Consider a project whose earnings stream is given by $\bar{X}\exp[(\pi - \delta)t]$ and which is financed by an initial debt D_0 which is amortized at the rate 'a'. Thus, the stream of debt outstanding over time is given by $D_0\exp(-at)$. As before, π is the rate of inflation--perfectly anticipated; but δ now stands for the rate of output decay--deriving from the real economic depreciation of the underlying equipment. Let S denote the value of equity and V the value of the project. By definition:

$$S = \int_0^\infty [\bar{X} \exp(\pi - \delta)t - (R + a)D_0 \exp(-at)]\exp(-\rho t) \tag{14}$$

where ρ is the required return to equity from the point of view of the equity investors in the project; R is the rate of interest on debt. From M-M Proposition I, the overall cost of capital c_0 is fixed given the risk characteristics of the project. Therefore the value of the project is given by:

$$V = \int_0^\infty \bar{X} \exp\left((\pi - \delta)\ t\right) \ \exp[-c_0 t] = \frac{\bar{X}}{c_0 - \pi + d} \tag{15}$$

which should equal, at the margin, the total initial cost of the project, I. Assume that a proportion α of the intitial cost of the project is debt financed; for the marginal project:

$$D_0 = \alpha V = \alpha\bar{X}/(c_0 - \pi + \delta) \tag{16}$$

Since by definition $V = S + D_0$, substituting from (14), (15) and (16) and solving for c_0,

$$c_0 - \pi + \delta = (1 - \alpha)(\rho - \pi + \delta) + \alpha \frac{R + a}{\rho + a}(\rho - \pi + \delta) \tag{17}$$

This is exactly the cost of capital expression shown in equation (13) obtained from the optimization exercise.

An interesting implication of equation (17) is that the required return to equity 'ρ' is a nonlinear function of α, for any given c_0, π, δ and $a \neq \delta - \pi$. When $a = \delta - \pi$, then the familiar linear function derived in the M-M Proposition II is obtained. Assuming $a \neq \delta - \pi$, the required return to equity ρ is the positive root of the following quadratic equation:

$$0 = (1 - \alpha)\rho^2 + \rho\,[(1 - \alpha)(\delta - \pi) + \alpha R_d + a_d - c_0]$$
$$+ [(\alpha R_d + a_d)(\delta - \pi) - c_0 a_d]$$

The Relationship Between the Marginal and Average Debt Ratios

First the relationship between the marginal and average debt ratios will be derived. Solving the differential equation (5), the level of domestic debt of a firm can be expressed as:

$$G_t = \exp(-a_d t) \cdot \alpha^{md} \int_0^t \exp[(a_d + \pi)s](\dot{K} + \delta K)ds + G_o \qquad (1)$$

where G_o is the initial level of debt assumed to be zero, and α^{md} is the marginal debt ratio for domestic loans. Integrating by parts, equation (1) can be rewritten as:

$$\begin{aligned} G_t &= \alpha^{md} K_t \cdot \exp(\pi t) \\ &\quad + \alpha^{md} \exp(-a_d t)(\delta - \pi - a_d) \int_0^t K_s \exp[(\pi + a_d)s]ds \end{aligned} \qquad (2)$$

If it is assumed for simplicity that real capital stock is expected to grow at the rate 'g', so that $K_s = K_o \exp(gs)$, then

$$\begin{aligned} G_t &= \alpha^{md} K_t \cdot \exp(\pi t) \\ &\quad + \alpha^{md} \exp(-a_d t)\left[\frac{\delta - \pi - a_d}{\pi + a_d + g}\right] K_o[\exp([\pi + a_d + g]t) - 1] \end{aligned} \qquad (2a)$$

Using equation (2a), the average debt ratio α_t^{ad} at time t can be expressed as:

$$\begin{aligned} \alpha_t^{ad} &= G_t/K_t \cdot \exp(\pi t) \\ &= \alpha^{md} + \alpha^{md} \frac{\delta - \pi - a_d}{\pi + a_d + g} - \alpha^{md} \frac{\delta - \pi - a_d}{\pi + a_d + g} \exp\left(-(g + \pi + a_d)t\right) \end{aligned} \qquad (2b)$$

In the long run, the last term of the above equation approaches zero, and the following relationship between the average and marginal debt ratios emerge.

$$\alpha^{ad} = \alpha^{md} \left(\frac{g + \delta}{\pi + a_d + g}\right)$$

This is equation (10) in the text, where α^{ad} is the limit of α_t^{ad} as t tends to ∞ .

A similar analysis for external debt (by solving equation (6)) yields:

$$F_t = \alpha^m (1 - \beta) K_t [1/E_o] \exp([\pi - x]t) \qquad (3)$$

$$+ \exp(-a_f t)\ \alpha^m(1 - \beta)[1/E_0](\delta - a_f - \pi + x)\int_0^t K_s \exp([a_f + \pi - x]s)ds$$

where E_0 is the initial exchange rate and x denotes the expected rate of increase in the nominal exchange rate. Therefore $E_0\exp(xt)$ denotes the exchange rate expected at time 't'. From equation (3), it is clear that when the rate of amortization on foreign loans, a_f, equals $\delta - \pi + x$, the average external debt ratio $F_t E_t/K_t \cdot \exp(\pi t)$ equals the marginal ratio α^{mf}. The condition $a_f = \delta - \pi + x$ reduces to $a_f = \delta - \pi_w + \theta$, if the real exchange rate is expected to change at the rate Θ, so that $x = \pi - \pi_w + \Theta$ where π_w is the foreign rate of inflation. It will be assumed that the rate of foreign inflation as well as the rate of amortization of foreign loans are fixed; proceeding as before, it can be verified that the marginal and the long-run average external debt ratios are related as follows:

$$\alpha^{mf} = \alpha^{af}\ \frac{a_f + \pi - x + g}{g + \delta} = \alpha^{af}\ \frac{a_f + \pi^w - \Theta + g}{g + \delta}$$

This is equation (11) in the text.

Bibliography

Ang, James S., "The International Behavior of Corporate Debt Policy," Journal of Financial and Quantitative Analysis, Vol. XI (November 1976), pp. 555-566.

Beranek, William, "Research Directions in Finance," Quarterly Review of Business and Economics, Vol. 21, No. 1 (Spring 1981), pp. 6-24.

______, "A Little More on the Weighted Average Cost of Capital," Journal of Financial and Quantitative Analysis, Vol. 10 (December 1975), pp. 892-896.

Ekman, Elon V., "A Dynamic Financial Model of a Managerial Firm," in Optimal Control Theory and Economic Analysis, Gustav Feichtinger, ed. (Amsterdam, North Holland Publishing Comany, 1982), pp. 79-105.

Eriksson, Gören, "The Effects of Taxation on the Firms Investment and Financial Behavior," The Scandinavian Journal of Economics, Vol. 82, No. 3 (1980), pp. 362-377.

Feldstein, Martin S., Jerry Green and Eytan Sheshinski, "Inflation and Taxes in a Growing Economy with Debt and Equity Finance," Journal of Political Economy, Vol. LXXXVI, No. 2 (1978), pp. S53-S70.

______, "Corporate Financial Policy and Taxation in a Growing Economy," Quarterly Journal of Economy, Vol. MCIII (August 1979).

Fry, Maxwell J., "Models of Financially Repressed Developing Economies," World Development, Vol. 10, No. 9 (1982), pp. 731-750.

Gordon, Roger H., "Interest Rate, Inflation and Corporate Financial Policy," in Brookings Papers on Economic Activity 2, William C. Brainard and George L. Perry, eds. (Washington, D.C.: The Brookings Institution, 1982), pp. 461-491.

Hite, Gailen L., "Leverage, Output Effects, and the M-M Theorems," Journal of Financial Economics, Vol. 4, No. 2 (1977), pp. 177-202.

Kim, Han E., "A Mean-Variance Theory of Optimal Capital Structure and Corporate Debt Capacity," The Journal of Finance, Vol. XXXIII, No. 1 (March 1978), pp. 45-63.

Linke, C.M., and M.K. Kim, "More on the Weighted Average Cost of Capital--A Comment and Analysis," Journal of Financial and Quantitative Analyisis, Vol. 9, No. 6 (September 1974), pp. 1069-1080.

Madan, B.K., Report on a Study of the Debt-Equity Ratio Norms (New Delhi: Management Development Institute, 1978).

Marsh, Paul, "The Choice Between Equity and Debt: An Empirical Study," The Journal of Finance, Vol. XXXVII (March 1982), pp. 121-144.

McKinnon, Ronald I., Money and Capital in Economic Development (Washington, D.C.: The Brookings Institution, 1973).

Modigliani, F.F., and M.H. Miller, "Corporation Income Taxes and the Cost of Capital: A Correction," American Economic Review, Vol. 53 (June 1963), pp. 433-443.

Modigliani, Franco, "Debt, Dividend Policy, Taxes, Inflation, and Market Valuation," The Journal of Finance, Vol. XXXVII (May 1982), pp. 255-273.

Morris, James R., "A Model of Corporate Debt Maturity Decisions," Journal of Financial and Quantitative Analysis, Vol. II, No. 3 (September 1976), pp. 339-357.

______, "An Empirical Investigation of the Corporate Debt Maturity Structure," Journal of Financial and Quantitative Analysis, Vol. 10 (November 1975), p. 539.

Myers, Stewart C., "Determinants of Corporate Borrowing," Journal of Financial Economics, Vol. 5, No. 2 (November 1977), pp. 147-175.

Nickell, Stephen J., The Investment Decisions of Firms (James Nisbet and Co., Ltd., Cambridge University Press, 1978).

Patrick, Hugh T., "Finance, Capital Markets and Economic Growth in Japan," in Financial Development and Economic Growth, Arnold W. Sametz, ed. (New York: New York University Press, 1972).

Rubinstein, Mark E., "A Mean Variance Synthesis of Corporate Financial Theory" in Issues in Management Finance, S.C. Myers, ed. (Cambridge, Massachusetts: M.I.T. Press, 1976) pp. 46-60.

Sakong, Il, "An Overview of Corporate Finance and the Long-Term Securities Market" in Planning Model and Macroeconomic Policy, Kim Chuk Kyo, ed. (Seoul: Korea Development Institute, 1977), pp. 228-262.

Shaw, Edward S., Financial Deepening in Economic Development (New York: Oxford University Press, 1973).

Sundararajan, V. and Subhash Thakur, "Public Investment, Crowding Out, and Growth: A Dynamic Model Applied to India and Korea," IMF, Staff Papers, Vol. 27, No. 4 (December 1980, pp. 814-855).

Taylor, Lance, Structuralist Macroeconomics (1983).

van Wijnbergen, Sweder, "Credit Policy, Inflation and Growth in a Financially Repressed Economy," Journal of Development Economics, (forthcoming, 1983).