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Capital Controls, Interest Rate Parity, and  
Exchange Rates: A Theoretical Approach <sup>1/</sup>

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Abstract

The main purpose of this paper is to analyze, from a theoretical point of view, the impact of capital control measures on the exchange rate. The introduction of capital controls increases transaction costs, and could also affect interest rates of financial assets that are traded at home and abroad. The impact of these controls on the transaction costs and the interest rates together determine a forward premium or discount for a given currency. The impact of control measures on the premium or discount, together with the speed of adjustment determines its impact on the spot and forward exchange rates. The control measures to encourage outflows or discourage inflows invariably depreciate the currency in both markets.

While the qualitative impact of the capital controls on exchange rates is straightforward, its quantitative impact depends upon many factors including the "speed of adjustment" referred to above, elasticities of demand for and supply of financial assets, and the extent of capital controls among other things. Furthermore, we must note that, since the introduction of controls results in welfare losses or declines in market efficiency, the authorities in charge of these controls must also weigh the importance of bringing about a desired exchange rate effect by introducing these capital controls against losses in welfare or economic efficiency, and must consider other options such as monetary policy as a possible means of influencing the exchange rate.

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## I. Introduction

Measures to influence the inflow and outflow of capital are no longer features typical of developing economies. Most countries, developing and developed alike, have imposed restrictions on the movement of capital in recent times. <sup>1/</sup> In many instances, such measures were introduced in response to the large amplitude of exchange rate fluctuations as witnessed during the decade of the 1970s and the early 1980s. There appears no first best justification for government intervention either in the foreign exchange market or the domestic securities market. When such an intervention is usefully employed, there are costs attached to it that must be taken into account.

Implementation of laws restricting the movement of capital in the foreign exchange and securities markets is frequently credited with having an important impact on (i) measured deviations from interest rate parity, (ii) the exchange rate, and (iii) the efficiency of the foreign exchange market. In our previous papers <sup>2/</sup> we have devised a quantitative indicator, consistent with foreign exchange and asset market equilibrium, which can capture the magnitude of such capital control measures. In those papers no detailed analysis was offered on the effect of uncertainty (regarding the future imposition of capital controls) on the interest rate parity relationship and the exchange rate. This paper develops a framework that is capable of analyzing the impact of capital controls on the spot and forward exchange rates and the domestic interest rate.

This paper is organized as follows: Section II discusses the theoretical and methodological issues involved in quantifying capital control measures under certainty and uncertainty. Section III develops a simple model to analyze the effect of capital control measures on the exchange rates. Section IV investigates the effect of controls on the domestic interest rate both at home and abroad and on the foreign exchange rates. Section V presents concluding comments. The appendix contains a technical note providing proofs of the relationship between uncertainty and the interest rate parity theory presented in Section II.

## II. Quantification of Capital Control Measures

In this section we discuss theoretical and methodological issues, and present the basic building blocks for quantifying capital control measures. According to the interest rate parity theory, in a world characterized by the absence of distortions and transaction costs, the current money market interest rate differential between interest bearing domestic and foreign assets is uniquely related to the forward premium or discount. This is formally expressed through the relationship:

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<sup>1/</sup> Saieh (1980) and International Monetary Fund (1977-83).

<sup>2/</sup> See Otani and Tiwari (1981), and Otani (1983).

$$\frac{(1 + r_d)}{(1 + r_f)} = \frac{F}{S} \quad (1)$$

where S and F are respectively the spot and the forward exchange rates, defined as the number of units of domestic currency per unit of foreign currency;  $r_f$  is the foreign interest rate for a security of a given maturity; and  $r_d$  is the domestic interest rate for a security of the same maturity. Equation (1) as expressed above, is an equilibrium condition and any deviations from it are an indication of unexploited profits. However, every transaction in securities and/or across currencies involves a positive transaction cost. Therefore, it is useful to consider how equation (1) should be modified for a world where transaction costs exist.

#### a. Issues in Transaction Costs

In the literature on the interest rate parity relationship, considerable attention has focussed on the significance of transaction costs and its identification with the discrepancy between the theoretical relationship expressed in equation (1) and empirically observed relationship. In two illuminating contributions, Frenkel and Levich (1975 and 1977) show that both over the tranquil and the turbulent periods, covered interest differentials were completely accounted for by pure transaction costs, thereby reflecting that no arbitrage opportunities existed. They propose using the "triangular arbitrage" method for estimating transaction costs in the foreign exchange market and follow Demsetz (1968) in estimating transaction costs in the securities market. Pure transaction costs, as the term indicates, are costs which an economic agent must incur while transacting across currencies or in financial assets and include not only brokerage fees but also search costs, etc. These costs are independent of whether the transaction is within the same nation or across national boundaries.

In an important contribution, Aliber (1973) argued that for assets issued in different political jurisdictions, political risk accounts for much of the observed covered interest rate differentials. Political risk, in turn, arises when agents transact across national boundaries. At a particular point in time, conditional on the information available at that point in time, the risk also reflects the probability that a given sum invested abroad will not return home as expected.

Dooley and Isard (1980) further refine the concept of transaction costs associated with capital controls. They have argued that the imposition of capital controls is an explicit or implicit imposition of taxes that economic agents must bear. These taxes are in addition to the costs associated with political risk that are analyzed by Aliber (1973).

From these analyses, it is clear that transaction costs consist of three components: (i) tp: pure transaction costs; (ii) tc: taxes and other costs that transactors must pay, explicitly or implicitly, in the face of capital controls; (iii) tr: costs associated with political

risk. A priori, it is not possible to specify as to what proportion of the total transaction costs are accounted for by  $t_p$ ,  $t_c$ , and  $t_r$ . However, some recent empirical estimates of total transaction costs clearly indicate that the size of these costs is too large to be attributed to pure transaction costs. 1/

Frenkel and Levich's procedure aggregates  $t_p$ ,  $t_c$ , and  $t_r$  together, and they call this aggregate transaction costs. Aliber calls this aggregate political risk. Otani and Tiwari (1981) succeeded in dividing the total transaction costs into two components: pure transaction costs on the one hand and an aggregate of costs associated with existing capital controls and political risks on the other. 2/

Given the existence of transaction costs in the foreign exchange and financial markets, it is essential that any discussion of the relationship between transaction costs and the interest rate parity condition must explicitly take into account the three components of total transaction costs. Bearing this in mind, we first recapitulate how the interest rate parity relationship expressed in equation (1) must be modified for a world where only pure transaction costs exist, and then analyze how this ought to be further adapted for a world where not only pure transaction costs but also costs associated with capital controls and political risk are present.

#### (1) Pure Transaction Costs and Interest Rate Parity

Assume that the initial position of an agent is in domestic securities. The agent can either continue to hold assets denominated in domestic currency and earn the domestic interest rate  $r_d$ , or transfer funds abroad at the prevailing spot rate  $S$  to hold foreign currency denominated assets earning the foreign interest rate  $r_f$ , and simultaneously buy a forward contract at the prevailing forward rate  $F$  so that when the foreign security matures he can switch back to the domestic security. The essence of the argument does not change if instead we assume that the initial position of the agent is in domestic currency. 3/ Defining  $t_{dp}$  as the percentage pure transaction costs in the domestic securities market;  $t_{sp}$  as the percentage pure transaction cost in the spot foreign exchange market;  $t_{fp}$  as the percentage pure transaction costs in the foreign securities market and  $t_{fp}$  as the percentage pure transaction cost in the forward foreign exchange market, it is readily seen that equation (2) must be satisfied if no capital is to flow across national boundaries in pursuit of unexploited profits:

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1/ Blejer (1982), Frenkel and Levich (1977).

2/ According to Dooley and Isard, who have examined the question of transaction costs from a slightly different point of view, the gap between the interest rate on DM denominated assets in Germany and that on DM denominated assets in the Eurocurrency market in Switzerland is mostly attributed to costs associated with the existing capital controls; only a small portion are attributed to political risk.

3/ For that formulation, see Frenkel and Levich (1977).

$$(1+t_{dp})(1+t_{fp})(1+t_{sp})(1+t_{fp}) \leq \frac{(1+r_d)S}{(1+r_f)F} \leq \frac{1}{(1+t_{dp})(1+t_{fp})(1+t_{sp})(1+t_{fp})} \quad (2)$$

approximating  $t_{dp} + t_{fp} + t_{sp} + t_{fp}$  by  $TC_p$  and ignoring terms of higher order, equation (2) can be rewritten to define the 'neutral-band' in Frenkel (1973) terminology, within which covered interest differentials must fall in order that no profit opportunities exist. This band can be defined as:  $\frac{1}{TC_p}$

$$\left| \frac{(1+r_d)S}{(1+r_f)F} - 1 \right| \leq TC_p \quad (3)$$

## (2) Capital Controls, Uncertainty, and Interest Rate Parity

As outlined earlier, the introduction of capital control measures will increase the cost of transaction in two different ways. First, transactors must incur costs due to existing controls, i.e., the direct and indirect costs associated with government regulations. These costs, however, have no elements of uncertainty. Second, risk averse transactors must implicitly incur costs due to uncertainty concerning future changes in existing controls. Therefore, in a world where capital controls exist, the total transaction costs consist of: (a) costs associated with certainty and (b) those associated with uncertainty. For the lack of better terminology, call the former "certainty costs," and the latter "uncertainty costs."

In discussing the effect of capital controls on transaction costs, define the gross certainty cost of borrowing one unit of currency at home as  $\bar{C}$  where  $\bar{C} = (1 + r_d)(1 + t_{dc})$ ;  $r_d$  is defined as before and  $t_{dc}$  is the "certainty cost" of transacting that asset in the presence of capital controls. Under certainty, define  $\bar{R}$  as the return from investing abroad with  $\bar{R} = F(1+r_f)/S(1+t_{sc})(1+t_{fc})(1+t_{fc})$  where  $F$ ,  $r_f$  and  $S$  are defined as before, and  $t_{sc}$ ,  $t_{fc}$ , and  $t_{fc}$  are the "certainty costs" associated with transactions in the spot and forward markets and in the foreign currency denominated financial asset. In equilibrium,  $\bar{C} = \bar{R}$  and the familiar expression of equation (3) is slightly modified to become equation (4):

$$\left| \frac{(1+r_d)S}{(1+r_f)F} - 1 \right| \leq \bar{TC} \quad (4)$$

where  $\bar{TC}$  is the total percentage transaction costs of arbitraging funds under certainty, and includes not only "pure" transaction costs ( $TC_p$ ) but

also costs associated with existing capital controls. Therefore,  $\bar{TC} > TC_p$ .

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<sup>1/</sup> McCormick (1979) has shown that nonsimultaneous observation of variables in the interest rate parity relationship biases the estimation of aggregate transaction cost.

For a meaningful discussion of transaction costs under uncertainty, we need to specify the utility function of the individual. If an individual engages in a sequence of transactions that generate the interest rate parity relationship, the observation that he buys a forward contract to eliminate the exchange risk is sufficient to rule out convex and linear utility functions. Therefore, the utility function of the individual is assumed to be strictly concave; it is increasing in the return  $R$  but decreasing in the cost  $C$ . We also assume it to be a continuous, twice differentiable real valued function; its first derivative with respect to  $R$  is positive and its second derivative negative, while the first derivative with respect to  $C$  is negative. <sup>1/</sup> (See the Appendix for a detailed discussion).

Although the analysis of the relationship mentioned above can be done in the  $R$  and  $C$  space, we will do it in the utility of income and income (generated by the return on financial assets) space. Under certainty, the gross return from investing abroad  $\bar{R}$  is equal to the cost of borrowing  $\bar{C}$ . Therefore, the utility associated with  $\bar{R}$  is exactly offset by the disutility with  $\bar{C}$  and there is no incentive for an agent to engage in arbitrage of funds. In this sense, an equilibrium condition is established.

Under uncertainty, the problem can be best analyzed by asking the following question: if the cost of borrowing funds at home,  $\bar{C}$  remains unaffected (even in the presence of capital controls), but the return from investing abroad is uncertain, (i.e., it is a random variable with a given mean and variance), then what is the expected return that will make a risk averse expected utility maximizing individual indifferent on the margin?

This question has been extensively analyzed in different contexts (see Tobin [1958], Markowitz [1958], Pratt [1964] and Arrow [1971] among others). In our context the answer can be expressed as:

$$\bar{C} = R_u - V(R_u) \quad (5)$$

where  $\bar{C}$  is defined as before;  $R_u$  is the expected gross return from investing abroad; and  $V(R_u)$  is the probability premium (depending upon risk aversion and the variance of return) that equalizes the utility from  $\bar{R}$  (the certainty rate of return) and the expected utility from receiving  $R_u$ . An equilibrium condition is established when the disutility,  $U(C)$  is just offset by the utility,  $U(R_u - V(R_u))$ . From the above discussion, it follows that for capital to flow out of the home country, it must be the case that  $C < R_u - V(R_u)$ , or alternatively

$$(1+r_d)(1+t_{dc}) < \frac{F(1+r_f)}{S(1+t_{fc})(1+t_{Fc})(1+t_{Sc})} - V(R_u) \quad (6)$$

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<sup>1/</sup> In this sense, the utility function measures the disutility of an individual who must borrow funds at home.

Thus, for no capital outflows seeking unexploited profit, it must be true that

$$\frac{S(1+r_d)}{F(1+r_f)} \geq \frac{1}{(1+TC)(1+\delta_1)} \quad (7)$$

where  $\delta_1 = V(R_u)/(1+r_d)(1+t_{dc})$

By similar reasoning, for no capital inflows seeking unexploited profit, the following must hold:

$$(1+TC)(1+\delta_2) \geq \frac{S(1+r_d)}{F(1+r_f)} \quad (8)$$

where  $\delta_2 = V(R_u)/(1+r_f)(1+t_{fc})$ .

Combining equations (7) and (8) above, the neutral band can now be defined as

$$(1+TC_{u2}) \geq \frac{S(1+r_d)}{F(1+r_f)} \geq \frac{1}{(1+TC_{u1})} \quad (9)$$

where

$$(1+TC_{u2}) = (1+\bar{TC})(1+\delta_i) \quad i = 1, 2$$

and

$$\left| \frac{S(1+r_d)}{F(1+r_f)} - 1 \right| \leq TC_{ui} \quad i = 1, 2 \quad (10)$$

and thus,  $TC_p \leq \bar{TC} \leq TC_u$

Notice that equations (9) and (10) provide a specification for the neutral band in the presence of pure transaction costs, control costs and "political" risk associated with uncertainty about future changes in existing capital control measures during the length of maturity of the asset. The introduction of political risk increases the neutral band, and thus the total percentage transaction costs to arbitraguers. Also, under uncertainty the neutral band is not symmetric around the zero line.

Suppose that only "pure" transaction costs exist, and that these costs remain constant. Furthermore, assume that there is no uncertainty. The deviations from the interest rate parity will in Figure 1 fall on the line  $TC_{pi}$  when capital flows into the home country, and on the line  $TC_{po}$  when capital flows out. The effect of the introduction of capital controls on

transaction costs can be analyzed in two steps. First, it increases the costs of transaction even without taking into account the element of uncertainty. The additional transaction cost shifts the original lines  $TC_{pi}$  and  $TC_{po}$  to  $\bar{TC}_i$  and  $\bar{TC}_o$ , respectively, so that the deviations from interest rate parity would fall on these two lines whenever capital flows take place under certainty. However, political risk or uncertainty arises whenever capital controls are introduced. As a result, it is unlikely that we will be able to observe all the deviations falling on  $\bar{TC}_i$  and  $\bar{TC}_o$ ; instead, these deviations will scatter around the two lines,  $TC_{ui}$  and  $TC_{uo}$ , and the expected value of these deviations will be equivalent to the distance OA or OB in Figure 1. <sup>1/</sup>

b. On the Measurement of Capital Controls

Our approach toward developing a meaningful measure of capital controls requires considering two alternative trading routes for the holders of domestic currency denominated assets. The first trading route lies across two different political jurisdictions--the home country H, which imposes controls on the movement of capital, and the foreign country A. To be consistent with earlier notation let  $r_d$  and  $r_f$  represent the interest rate on domestic currency and foreign currency denominated assets, respectively. Assets yielding  $r_d$  lies in jurisdiction H while that yielding  $r_f$  lies in jurisdiction A. These assets are, of course, of identical maturity. Similarly  $S_H$  and  $F_H$  represent the spot and the forward exchange rates in jurisdiction H. Define the percentage covered interest differentials in jurisdiction H as  $G_H$ .

When controls on the movement of capital between jurisdictions H and A are effective, the absolute value of  $G_H$  will be affected through the change in the transaction cost in various markets. For the moment, we can assume that this increased aggregate transaction costs will affect both the premium/discount on the domestic currency and the domestic interest rate such that the interest rate parity condition inclusive of transaction costs is maintained. <sup>2/</sup> We next need to consider the properties of the distribution of  $G_H$ . In particular, we are interested in determining whether  $G_H$

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<sup>1/</sup> While the above paradigms give a fairly reasonable depiction of the real world where arbitrage funds can move about, an additional element of reality must be introduced if we were to represent the real world more accurately. That is, even in the case where no political risk or uncertainty exists, still there exist elements of stochastic nature in observed deviations from the interest rate parity. These are due to data problems associated with the timing of observation, rounding of numbers, pure mistakes involved in reporting data, etc. These problems are dealt with in a later section.

<sup>2/</sup> An implicit assumption here is that the foreign interest rate,  $r_f$ , does not change.

is randomly distributed around a zero mean. <sup>1/</sup> If there is no change in the imposition of controls over the period under consideration, the condition of no unexploited profits will require that  $G_H$  be randomly distributed about a non-zero mean. However, when the intensity of controls changes we should expect  $G_H$  to be randomly distributed around a time trend. Finally, as it happens in reality, both the intensity and the direction of controls change over time. Therefore, in general,  $G_H$  is non-randomly distributed over the entire sample period, i.e.,

$$G_H \sim R(\lambda, \sigma_H^2); \lambda \neq 0 \quad (11)$$

where

$$G_H = \frac{(1+r_d)S_H}{(1+r_f)F_H} - 1$$

A naive interpretation of equation (11) would mean that if agents were to trade on every non-zero value of  $G_H$  they would on the average make positive profits. But strictly speaking, the non-zero mean that emerges in equation (11) is a measure of the expected cost that each agent will have to incur if he engages in trade and will, on the margin, be identical to the apparent profit opportunities suggested by equation (11). The sign of  $\lambda$ , of course, depends on the direction of capital flows.

Now consider the second trading route. We will require this trading route to lie within some narrowly defined political jurisdiction such that transactions within this jurisdiction are not subject to the controls imposed in jurisdiction H. In fact, we will define this jurisdiction to be A itself. Transactions within A are characterized by the absence of distortions; the physical location of A is irrelevant to the argument as long as A does not overlap with H in an economic sense. We require that the domestic currency denominated asset in jurisdiction A yielding interest  $r_d^*$  be denominated in the currency of jurisdiction H; further, the foreign currency denominated asset yielding interest  $r_f$  be the same as the previous foreign currency denominated asset. The specification that foreign assets be the same for both the trading routes and that the domestic asset in A be denominated in the currency of H is critical for the measurement of capital control since it shifts the focus onto that asset on which the effects of capital controls are felt. Let  $S_A$  and  $F_A$  represent the spot and the forward exchange rates in jurisdiction A. For notational convenience define the percentage covered interest differentials in A as  $G_A$ . As before, we are interested in the properties of the distribution of  $G_A$ . Since this jurisdiction is free of controls, we can hypothesize that  $G_A$  will be randomly distributed around a zero mean, i.e.,

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<sup>1/</sup> It may be more precise to say that deviations from interest rate parity are randomly distributed around  $\pm \lambda$  where  $\lambda$  is the expected transaction cost. Since the signs for  $\lambda$  can change randomly when capital movements are free, the presumption is that these deviations will be distributed randomly around the zero mean.

$$G_A \sim R(0, \sigma_A^2) \quad (12)$$

where

$$G_A = \frac{(1+r_d)S_A}{(1+r_f)F_A} - 1$$

The hypothesis that  $G_A$  is randomly distributed around a mean of zero would mean that if agents were to trade on every non-zero value of the mean, they would make zero profits in the long run.

Any indicator of capital controls must be capable of distinguishing between the effects of capital controls and those of transaction costs, financial market uncertainty, etc. on interest rate disparity.  $G_A$  is a measure of pure transaction costs and all other factors, except controls and the uncertainty associated with them affecting jurisdiction A. However,  $G_H$  is comprised of  $G_A$  and the certainty and uncertainty costs of controls. Therefore, a natural measure of controls,  $\emptyset$ , may be defined as:

$$\emptyset = E(|G_H| - |G_A|) \quad (13)$$

A meaningful interpretation of  $\emptyset$  is one of the increased costs that market participants have to incur if they attempt to evade existing controls or the cost that they have to pay in the form of explicit or implicit taxes under the existing controls as well as the risk (or uncertainty) premium associated with the probability that the government may change the control during the period of maturity of the assets. This increased cost is equal to the present discounted value of profits that can be made due to the existence of apparent disparity. Thus each expected utility maximizing risk-averse market participant, on the average, breaks even.

### III. Capital Controls and the Exchange Rate

Having analyzed various issues concerning the quantification of capital control measures, we are in a position to discuss the effect of capital controls on the exchange rates. For this purpose, we first present a simple model that can determine the exchange rate in the presence of capital controls. Next, depending on whether controls are applied to discourage inflows or outflows, we analyze the effect of capital controls on the exchange rate.

#### a. A Simple Model of Exchange Rate Determination in the Presence of Capital Controls

Our analysis of the effect of capital market distortions on the exchange rates will utilize the "asset view" of exchange rate determination and incorporate the "overshooting" hypothesis of exchange rate adjustment as developed by Dornbusch (1976). To develop this approach, consider the interest rate parity relationship in the presence of transaction costs as

represented by equation (9). When modified to focus especially on capital market distortions due to capital controls, this relationship (in the presence of capital controls) can be written as:

$$1 - \emptyset \leq \frac{S(1+r_d)}{F(1+r_f)} \leq 1 + \emptyset \quad (14)$$

where  $\emptyset$  is defined in equation (13) above and all other variables have their usual meaning. Equation (14) states that covered interest differentials, net of pure transaction costs, when bounded between two end points  $(1 - \emptyset)$  and  $(1 + \emptyset)$  imply the absence of unexploited profit opportunities. Ex post, on the margin, the following relationship holds:

$$\frac{S(1+r_d)}{F(1+r_f)} = 1 + \emptyset \quad (15)$$

We need to distinguish between the inflow and outflow of arbitrage funds before we can proceed further. <sup>1/</sup> The right hand side of equation (15) is  $(1 + \emptyset)$  when arbitrage funds flow into the home country and  $(1 - \emptyset)$  when arbitrage funds flow out of the home country. By taking the natural logarithm of equation (15), the following two equations are obtained:

$$\ln S - \ln F = \ln(1+r_f) - \ln(1+r_d) + \ln(1+\emptyset) \quad (16a)$$

$$\ln S - \ln F = \ln(1+r_f) - \ln(1+r_d) + \ln(1-\emptyset) \quad (16b)$$

Equations (16a) and (16b) are familiar expressions. Equation (16a) holds for an arbitrage inflow while (16b) holds for an arbitrage outflow. The difference between the spot and the forward exchange rate is also defined as the premium or the discount on the home currency depending on the sign of the difference. Since the exchange rates are defined as the number of units of domestic currency per unit of foreign currency, if the difference  $(\ln S - \ln F)$  is positive then it will be defined as a premium on the forward value of the domestic currency. Conversely, if the difference  $(\ln S - \ln F)$  is negative, it will be defined as a discount. Thus equations (16a) and (16b) can be respectively written as:

$$P_i = \ln S - \ln F = \ln(1+r_f) - \ln(1+r_d) + \ln(1+\emptyset) \quad (17a)$$

$$P_o = \ln S - \ln F = \ln(1+r_f) - \ln(1+r_d) + \ln(1-\emptyset) \quad (17b)$$

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<sup>1/</sup> See Figure 1 in the previous section.

Equations (17a) and (17b) above are a statement of "Fisher open" under capital market distortions. Alternatively, this difference can be defined as the expected appreciation/depreciation of the spot exchange rate over the length of maturity of the security, and can be written as:

$$\mu_i = E(\Delta \ln S) = \ln(1+r_f) - \ln(1+r_d) + \ln(1+\theta) \quad (18a)$$

$$\mu_o = E(\Delta \ln S) = \ln(1+r_f) - \ln(1+r_d) + \ln(1-\theta) \quad (18b)$$

where  $\mu_i$  = expected rate of change of the spot exchange rate when  
arbitrage funds flow into the home country.  
 $\mu_o$  = expected rate of change of the spot exchange rate when  
arbitrage funds flow out of the home country.

Under certainty (17a) and (17b) will be identical to (18a) and (18b), respectively. Under uncertainty, this identity is maintained under perfect foresight or rational expectations equilibrium. However, for our purposes the conditions under which this identity is maintained are not crucial and we will assume its maintenance.

We need to specify the formation of exchange rate expectations. In the Dornbush (1976) model, the exchange rate was expected to change in proportion to the discrepancy between the long run equilibrium exchange rate and the current exchange rate. The expected percentage change in the spot exchange rate is specified to depend on the gap between the expected long run equilibrium exchange rate and the current rate, plus the expected percentage change in the long run equilibrium rate, i.e.,

$$E(\Delta \ln S) = \theta [E(\ln \bar{S}) - \ln S] + E(\Delta \ln \bar{S}) \quad (19)$$

where  $\bar{S}$  = long-run equilibrium exchange rate

$\theta$  = speed of adjustment lying within the closed interval of zero and unity

Our specification is a modified version of Dornbusch's (1976) formulation which does not contain  $E(\Delta \ln \bar{S})$  on the right hand side of equation 19. <sup>1/</sup> Thus, in the long run, the expected change in the current spot rate will be identical to the expected change in the long-run equilibrium spot exchange rate. For the purpose of our analysis a further elaboration is not necessary.

Combine equations (18a) and (19), and equations (18b) and (19) to obtain the following:

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<sup>1/</sup>  $E(\Delta \ln \bar{S})$  can be assumed to depend upon the differential between foreign and domestic long-term expected rate of inflation, as in Frankel (1979).

$$\ln S = E(\ln \bar{S}) + \frac{1}{\theta} E(\ln S) + \frac{1}{\theta} \ln[(1+rf)/(1+rd)] + \ln(1+\phi) \quad (20a)$$

$$\ln S = E(\ln \bar{S}) + \frac{1}{\theta} E(\ln \bar{S}) + \frac{1}{\theta} \ln[(1+rf)/(1+rd)] + \ln(1-\phi) \quad (20b)$$

Equations (20a) and (20b) are expressions for the spot exchange rate for an inflow and an outflow of capital, respectively.

#### b. Qualitative Effects of Capital Controls on Exchange Rate

In order to analyze the effects of capital controls on the level of the spot and forward exchange rates, a few qualifying remarks are in order. When controls are imposed in response to large amplitudes in exchange rate fluctuations, they can operate in both directions. If so, the intensity of controls should change with the exchange rate cycle. For this reason, our analysis will be of the short run; consequently, we will assume that the equilibrium long-run expected exchange rate  $E\ln S$  is independent of capital control measures. If, however, a capital control measure is permanently introduced, there will be a new long run equilibrium exchange rate which will incorporate the effect of such a control. When this happens the effect on the spot exchange rate in equations (20a) and (20b) will be observed not through the term containing  $\phi$  but through a new long-run equilibrium exchange rate. The dynamic adjustment then closely follows the standard Dornbusch model. However, when the assumption of independence is maintained, implicit also is the assumption that capital control measures are transitory in nature.

We will assume that capital control measures influence the domestic interest rate but not the foreign interest rate. This assumption reflects the notion that the stock of domestic currency denominated assets are so small relative to the stock of foreign currency denominated assets, that the movement of arbitrage funds that are induced by capital control measures are a nontrivial proportion of the stock of domestic currency denominated financial assets but a trivial proportion of the stock of foreign currency denominated financial assets. Finally, we will also assume that a change in the transaction costs  $\phi$  is solely attributable to capital market distortions. In fact,  $\phi$  will henceforth be referred to as the control cost.

Suppose that the government introduces control measures intended to influence the flow of capital. In order to analyze the effect of these measures on the exchange rates, first note that these flows take place as a result of changes in the stock of financial assets held by residents or nonresidents; financial assets are denominated in either the domestic currency or the foreign currency, and the demander (supplier) of these assets are either nonresidents or residents. Therefore, potentially, four different cases of the currency-residency combination are possible; Case I: residents supply the domestic currency denominated assets; Case II: residents supply the foreign currency denominated assets; Case III: nonresidents supply the domestic currency denominated assets; and Case IV:

nonresidents supply the foreign currency denominated assets. In addition, controls can be applied on either the suppliers or the demanders in each of the four cases. Therefore, potentially, there are eight ways in which capital flows can be influenced.

As will be evident from the discussion below, the effect of capital controls on the exchange rate have elements of symmetry; namely, the exchange rate effect of measures to discourage inflows (encourage outflows) of capital will be opposite to that of measures to discourage outflows (encourage inflows). Therefore, a detailed analysis of the exchange rates effects of capital controls in all possible cases will not be offered. We will provide a detailed analysis of the exchange rate effects of capital controls to discourage inflows and other cases will be briefly touched upon.

(1) Exchange Rate Effects of Controls to Encourage Outflows When Inflows are Taking Place

As noted earlier, the exchange rate effects of the four possible control measures to discourage capital inflows can be studied by considering equations (16a), (17a), (18a), and (20a). Differentiating these equations with respect to  $\theta$  we obtain:

$$\begin{aligned} dP(\theta)/d\theta = (1/(1+\theta)) [1 - (\partial \ln(1+rd(\theta))/\partial \ln(1+\theta)) \\ + (\partial \ln(1+rf(\theta))/\partial \ln(1+\theta))] \end{aligned} \quad (21)$$

$$d\ln S(\theta)/d\theta = (1/\theta) \cdot (dP(\theta)/d\theta) \quad (22)$$

$$d\ln F(\theta)/d\theta = ((1-\theta)/\theta) \cdot (dP(\theta)/d\theta) \quad (23)$$

In order to interpret the results of the equation (21), (22), and (23), we must first examine the effects of capital controls on the interest rates that appear in these equations.

Case Ia

Consider the case whereby residents issue domestic currency denominated assets of the relevant maturity. The stock of assets supplied is

negatively related to the domestic interest rate,  $\frac{1}{r}$  and is represented by SS in Figure 2. Nonresidents (foreigners) demand domestic currency denominated assets, the demand schedule being represented by DD. <sup>2/</sup>

Notice that the foreigners' demand for domestic currency denominated assets is a function of the domestic interest rate ( $r_d$ ), the (given) foreign interest rate ( $r_f$ ) and the control costs ( $\phi$ ) that foreigners must pay to purchase domestic assets. Suppose that initially, there are no controls, so the DD in Figure 2 is drawn for an ideal market economy, with  $\phi = 0$ . Interest rate  $\ln(1 + r_d)$  clears the market. At  $[\ln(1 + r_d), \ln Q_0]$ , surplus accruing to the foreigners is given by area above segment AB and below the demand curve DD, while the surplus accruing to domestic residents is given by the area lying below the segment AB and above the supply curve SS.

One of the most common form of capital control measures to restrict the inflow of capital has been to restrict foreigners' access to domestic assets issued by residents. This is achieved through increasing the control cost that must be borne by foreigners to EF. This shifts the demand curve from DD to D'D' and reduces the stock of domestic currency denominated assets to  $\ln Q_1$ . Notice that when the control cost is explicitly introduced and collected by the government, area FCDE is a transfer payment from the foreigners to the government and the triangle ACD is the efficiency loss of imposing the controls. <sup>3/</sup> When quantity restrictions are imposed to limit the stock of domestic assets to  $\ln Q_1$ , then the government (in the absence of allocating/auctioning the quota) does not receive the rectangle FCDE. In that case FCDE is the amount of real resources that foreigners will have to spend to evade capital control measures (which the government does not collect) and triangle ACD continues to remain the welfare cost. <sup>4/</sup>

It is clear that as long as the elasticity of the supply schedule with respect to the domestic interest rate is not zero, the rise in the domestic interest rate (BF) is less than the increase in the control costs (EF). <sup>5/</sup> Therefore,  $0 \leq \frac{\partial \ln(1 + r_d(\phi))}{\partial (1 + \phi)} \leq 1$

From equation (21), we can thus infer that the premium rises less than the increase in transaction cost induced by the capital control. From equation (22) it follows that the effect of the capital controls on the current

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<sup>1/</sup> Since the vertical axis has the reciprocal of the natural logarithm of the domestic interest rate, it will be assumed that the nominal domestic interest rate is bounded away from zero.

<sup>2/</sup> In order to simplify the presentation, domestic residents demand for domestic currency denominated assets is ignored here. Such a simplification will not change the substance of the analysis.

<sup>3/</sup> For an excellent analysis of welfare losses see Harberger (1971).

<sup>4/</sup> Posner (1975) has in fact argued that area EDACF is the correct representation of the welfare cost of regulation.

<sup>5/</sup> By assumption  $\frac{\partial \ln(1 + r_f(\phi))}{\partial \ln(1 + \phi)} = 0$ .

spot exchange rate is  $1/\theta$  times the effect of controls on the premium. Since  $\theta$  lies in the closed interval  $[0, 1]$ , the effect of controls on the spot rate will always be greater than the effect of controls on the premium, except in the limiting case when  $\theta = 1$ , i.e., when adjustment is instantaneous. With instantaneous adjustment the entire effect of the change in the premium is completely passed on to the spot exchange rate. In general, the smaller the speed of adjustment the greater the effect on the spot rate. The effect of capital controls on the forward exchange rate is  $(1-\theta)/\theta$  times the effect on the forward premium. For values of  $\theta$  less than one-half, the effect on the forward exchange rate is greater than that on the premium; for values of  $\theta$  greater than one half, the effect on the forward exchange rate is less than that on the premium. Also, for every value of  $\theta$ , the effect on the spot rate is larger than the effect on the forward rate.

#### Case Ib

In the case where capital control measures are intended to restrict the issuance of domestic currency denominated assets by the residents, the supply curve SS (in Figure 3) shifts to S'S' due to the increased control cost of  $\theta$ . An increase in the control cost by BC leads to a decline of the domestic interest rate from  $r_d$  to  $r_{d1}$ . Therefore,  $-1 \leq \partial \ln(1+r_d(\theta))/\partial \ln(1+\theta) \leq 0$ . This implies that  $(1 - \partial \ln(1+r_d(\theta))/\partial \ln(1+\theta))$  is positive and lies between unity and two. From equations (21), (22), and (23) it is clear that the premium rises less than the increase in the control cost, and that the domestic currency depreciates both in the spot and the forward market.

#### Case IIa

Consider the case where residents supply the foreign currency denominated assets while the nonresidents demand them. Since the domestic residents are price takers in the market for foreign currency denominated assets, they face a perfectly elastic demand schedule while their supply schedule of the foreign currency denominated assets is upward sloping. These are depicted in Figure 4.

When the government introduces measures to restrict the issuance of the foreign currency denominated assets by domestic residents, the supply schedule SS upward by  $\Delta \theta$  to S'S'; the interest rate received by the demanders remains the same while the implicit interest rate paid by the suppliers rises from  $r_f$  to  $r_{f1}$  with the differential  $\Delta \theta$  accruing to the government. <sup>1/</sup> Thus the stock of foreign currency denominated assets (at home) will decline. As a result, funds repatriate from the home country to the foreign country, resulting in a capital outflow, which has the same impact on the capital account as a reduction in the net inflow of capital. Therefore,  $dP(\theta)/d\theta = 1/(1+\theta)$  since  $\partial \ln(1+r_d(\theta))/\partial \ln(1+\theta) = 0$  and  $\partial \ln(1+r_f(\theta))/\partial \ln(1+\theta) = 0$ . Accordingly,

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<sup>1/</sup> If the suppliers have free access to the world market, no foreign currency denominated asset will be offered in the home country.

the measures to restrict the issuance of the foreign currency denominated assets by residents have no impact on interest rate of either the domestic or the foreign currency denominated asset. However, these measures increase the premium or reduce the discount on the domestic currency in the forward market, and the currency depreciates in both the spot and the forward market.

### Case IIb

Finally, consider the case where the government attempts to restrict foreigners' demand for the foreign currency denominated assets issued by residents. Any controls that increase transaction costs to be borne by nonresidents would entirely eliminate their demand for these assets, since they can obtain such assets in the foreign country. If this takes place, there will be a massive capital outflow from the home country with the resident suppliers being unable to find customers. Therefore, the market ceases to exist in the domestic country. Under these conditions, such measures would obviously have impact on the exchange rate (i.e., depreciate the domestic currency), but the model developed in this paper is not capable of finding the exact solution as to the impact on the interest rate or the premium.

In short, the effect of capital controls to discourage the inflow of arbitrage funds will lead to an increase in the domestic rate of interest when the controls are used to restrict demand. Conversely, it will lead to a decline in the rate when these are used to restrict supply. Nevertheless, the forward premium on the domestic currency rises, and the currency will depreciate in both the spot and forward markets.

### (2) Exchange Rate Effects of Capital Controls to Encourage Inflows When Outflows are Taking Place

Capital control measures to encourage the inflow of arbitrage funds are aimed to counteract potential and actual outflows of capital. Thus the relevant equations to consider are (16b), (17b), (18b), and (20b). Differentiate equations (17b) and (20b), respectively, with respect to the control costs and manipulate to obtain:

$$dP(\theta)/d\theta = (-1/(1-\theta))\{1-(\partial \ln(1 + rd(\theta))/\partial \ln(1-\theta)) + (\partial \ln(1 + rf(\theta))/\partial \ln(1 - \theta))\} \quad (24)$$

$$d\ln S/d\theta = (1/\theta) dP(\theta)/d\theta \quad (25)$$

$$d\ln F/d\theta = ((1 - \theta)/\theta) dP(\theta)/d\theta \quad (26)$$

In interpreting equations (24), (25), and (26) above, potentially four different cases must be examined, as in the previous section where we

analyzed exchange rate effects of capital controls to encourage outflows. Since the essential arguments are symmetrical to those presented in the previous section, our discussions here will be brief.

When nonresidents supply domestic currency denominated assets to the home country and residents demand them, one way to encourage inflows of capital is to implement measures to restrict the domestic residents' demand for domestic currency denominated assets (Case IIIa below) and the other is to impose controls to restrict the foreigner's issuance of these assets (Case IIIb below).

#### Case IIIa

In Figure 5, the demand scheduled DD, is a function of the domestic interest rate when the foreign interest and the control costs are held constant. <sup>1/</sup> The control costs  $\theta$  acts as a shift parameter on the demand curve. SS schedule is the nonresidents' supply schedule of the domestic currency denominated asset. <sup>2/</sup>

When the government imposes controls to restrict resident's demand for the domestic currency denominated assets, the demand schedule shifts downward to D'D', the interest rate rises from  $\ln(1+r_{do})$  to  $\ln(1+r_{dl})$ , the domestic residents' holding of these assets (issued by foreigners) declines from  $\ln Q_0$  to  $\ln Q_1$ , and capital flows into the home country. Under these conditions,  $\partial \ln(1+rd(\theta)) / \partial \ln(1-\theta)$  is negative, and thus  $(1 - \partial \ln(1+rd(\theta)) / \partial \ln(1-\theta))$  is positive, and greater than unity. Hence  $dP(\theta)/d\theta$  is negative. Thus measures to encourage capital inflows in this case result in a reduction of the premium or an increase in the discount of the domestic currency in the forward market. It follows, therefore, that the domestic currency appreciates in both the spot and the forward market, as equations (25) and (26) indicate.

#### Case IIIb

On the other hand when the government restricts the issuance of domestic currency denominated assets by foreigners, the supply schedule (in Figure 6) will shift upward by the increase in the transaction cost, with the interest rate  $\ln(1+r_{do})$  decreasing to  $\ln(1+r_{dl})$  and the residents' holding of these assets falls to  $\ln Q_1$ . As long as the demand schedule is downward sloping,  $\partial \ln(1+rd(\theta)) / \partial \ln(1-\theta)$  is positive, but less than unity. Therefore,  $(1 - \partial \ln(1+rd(\theta)) / \partial \ln(1-\theta))$  is positive. Hence,  $dP(\theta)/d\theta$  is negative. Again, measures to encourage capital inflows by restricting the foreigners' issuance of domestic currency denominated assets result in a decline in the premium (or an increase in the discount), and in an appreciation of the domestic currency in the spot and forward markets.

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<sup>1/</sup> The assumption of the foreign interest rate being constant is adopted for the simplification of diagrammatic presentation. The relaxation of this assumption will not change the essence of the analysis.

<sup>2/</sup> We continue to assume that the nominal interest rate is bounded away from zero.

Now consider the case when domestic residents demand foreign currency denominated assets while nonresidents supply them. Since the domestic residents are price takers in the market for foreign currency denominated assets, they face a perfectly elastic supply schedule (from the rest of the world) while their demand schedule for foreign currency denominated assets is downward sloping. This is depicted in Figure 7.

#### Case IVa

When the government implements measures to restrict the residents' demand for the foreign currency denominated assets, the demand curve (in Figure 7) shifts from DD to D'D'. The interest rate on these assets remains constant because supply is perfectly elastic at the given interest rate. The residents' holding of these assets declines from  $\ln Q_0$  to  $\ln Q_1$ , resulting in an inflow of capital as domestic residents repatriate funds from abroad. Since  $\partial \ln(1+rf(\theta)) / \partial \ln(1-\theta)$  is zero and  $\partial \ln(1+rd(\theta)) / \partial \ln(1-\theta)$  can be assumed to be zero in this particular case,  $dP(\theta)/d\theta = -1/(1-\theta) < 0$ . As in cases IIIa and IIIb, these capital control measures lead to a reduction in the premium (or an increase in the discount), and the spot and forward values of the domestic currency appreciate.

#### Case IVb

Potentially, the government can introduce measures to restrict the issuance of these assets by foreign residents, by imposing, say, transaction tax on them. Such measures, however, would make foreign suppliers of these assets find markets in the foreign country. As a result, there could be a massive inflow of capital (i.e., repatriation of funds to the home country). Under these circumstances, the domestic currency would appreciate in both the spot and the forward market, but the model presented in this paper cannot provide a precise answer to what happens to the premium, except for noting that it would decline.

In short, the measures to encourage capital inflows invariably decrease the premium (or increases the discount) and appreciate the domestic currency. Their impact on the interest rates differs depending on whom the control measures are imposed.

Table 1 below summarizes effects of capital controls on interest rates, capital movements, premium and exchange rates.

#### IV. Capital Controls, and Implications for "Domestic Interest Rates," Exchange Rates, and "Speed of Adjustment" at Home and Abroad

Analyses of capital controls so far have been focused on their effect on the domestic interest rate, and exchange rates at home. In this section, we examine how capital controls introduced at home can affect the

Table 1. Effects of Capital Controls on Interest Rates ( $r_d$  and  $r_f$ ), Capital Movements ( $\Delta K$ ), Premium ( $P$ ), and Exchange Rates ( $S$  and  $F$ )

Domestic currency -  
Denominated Assets

Foreign currency -  
Denominated Assets

D: Non-residents and S: Residents

Restriction on demand

$r_d$ : increase  
 $\Delta K$ : outflow  
 $P$ ,  $S$  and  $F$ : increase

Restriction on demand

market ceases  
to exist as demand  
disappears completely

Restriction on supply

$r_d$ : decrease  
 $\Delta K$ : outflow  
 $P$ ,  $S$  and  $F$ : increase

Restriction on supply

$r_f$ : unchanged for nonresident  
increase for residents by  
 $\Delta K$ : outflow  
 $P$ ,  $S$  and  $F$ : increase

D: Residents and S: Nonresidents

Restriction on demand

$r_f$ : decrease  
 $\Delta K$ : inflow  
 $P$ ,  $S$  and  $F$ : decrease

Restriction on demand

$r_f$ : same for nonresidents  
increase for residents by  
 $\Delta K$ : inflow  
 $P$ ,  $S$  and  $F$ : decrease

Restriction on supply

$r_d$ : increase  
 $\Delta K$ : inflow  
 $P$ ,  $S$  and  $F$ : decrease

Restriction on supply

market ceases to exist as  
supply disappears completely

interest rate on the domestic currency denominated assets and exchange rates abroad, and what implications we can draw for the speed of adjustment ( $\theta$ ) at home and abroad.

For this purpose, it would be useful to consider the following example. Suppose that initially no capital controls exist and the equilibrium conditions in the foreign exchange and assets markets are established in jurisdictions H (home country) and A (abroad). Perfect capital mobility ensures that the interest rate on the domestic currency denominated assets at home ( $r_d$ ) is identical to the rate abroad ( $r^*d$ ). So are the spot ( $S_H$ ) and the forward rate ( $F_H$ ) at home identical to the spot ( $S_A$ ) and the forward rate ( $F_A$ ) abroad. The interest rate on the foreign currency denominated assets ( $r_f$ ) is common to the market in each jurisdiction.

Now, suppose that capital controls are imposed to restrict, say, the foreigners demand for the domestic currency denominated assets issued by residents. This would induce capital outflows from the home country; all or some of this outflow of capital will be invested in the domestic currency denominated assets abroad. As these flows take place, the domestic interest rate at home would rise while the rate abroad would decline. That is, the impact of the capital control on the domestic interest rate at home ( $\partial \ln(1+r_d) / \partial \ln(1+\theta)$ ) is positive (as argued in Section II), and that on the rate abroad ( $\partial \ln(1+r^*d) / \partial \ln(1+\theta)$ ) is negative.

This is demonstrated in Figure 8, where the demand and supply curves for the domestic currency denominated assets in jurisdiction H and A are plotted. Notice that the demand and supply curves for jurisdiction H are the same as before, but the demand curve for jurisdiction A is drawn with  $\ln(1-f(\theta))$ , indicating that the demand curve shifts out when capital controls are introduced at home.

In the absence of controls, equality of equations (11) and (12) requires that  $r_d = r^*d$ . We start from this situation, and then introduce control costs to encourage outflows in jurisdiction H. The demand curve DD shifts to D'D', raising the interest rate to  $rd_1$  and reducing the quantity to  $\ln Q_1$ . Notice that the line segment  $AB = \theta$ , so that the effective rate of return becomes  $rd_1$ . However, in jurisdiction A, the introduction of controls in jurisdiction H shifts the demand curve from  $D^*D^*$  to  $D^*'D^*$ . A new equilibrium is achieved at  $[1/\ln(1+r^*d_1), \ln Q^*1]$ . Notice that  $rd^*$  has fallen to  $r^*d_1$ . In equilibrium  $rd_1$  and  $rd^*1$  will differ by  $\theta$ .

The above analysis is based on the assumption that capital controls are intended to encourage outflows from the home country or to encourage inflows to the foreign country. A similar analysis can be made on the interest rate effect of capital controls intended to encourage inflows at home. The formal aspects of the analysis are identical, except for the fact that the direction of the interest rate effect is reversed, i.e., the controls to encourage inflows to the home country or outflows

from the foreign country reduces the domestic interest rate at home and increases it abroad. Once again, the difference between the rate at home and abroad is identically equal to the control costs.

We are now in a position to consider the impact of the controls on the premium and the exchange rates. We have shown that for the home country  $dPH/d\theta = (1/(1+\theta)) [1 - \partial \ln(1+rd)/\partial \ln(1+\theta)]$  when controls are intended to encourage outflows (from the home country) and  $dPH/d\theta = -(1/(1-\theta)) [(1 - \partial \ln(1+rd)/\partial \ln(1-\theta))]$  when they are intended to encourage inflows. For the foreign country, it can be shown that  $dPA/d\theta = -\partial \ln(1+r^*d)/\partial \ln(1+\theta) > 0$  when controls are used to encourage outflows (from the home country) and  $dPA/d\theta = \partial \ln(1+r^*d)/\partial \ln(1-\theta) < 0$  when they are used to encourage outflows from the foreign country. If the interest rate effect at home and abroad is the same in absolute value, then the impact on the premium is also the same,  $1/\theta$  and therefore, by implication, the effect on the spot and the forward exchange rate is also the same. Consequently, the "adjustment speed" at home and abroad takes the same value, i.e.,  $\theta = \theta^*$ .

In addition to the special cases discussed above, it would be interesting to note two other extreme cases: one is when the (gross) domestic interest rate at home,  $(1+rd)$ , increases by the same proportion as the increase in the (gross) transaction cost  $(1+\theta)$  following the introduction of capital controls. This can happen when the supply schedule is completely inelastic. As a result, no capital outflows are induced, and thus the demand schedule for the domestic currency denominated assets abroad does not shift. In this case,  $dPH/d\theta = 0$  since  $\partial \ln(1+rd)/\partial \ln(1+\theta) = 1$  <sup>2/</sup> and  $dPA/d\theta = 0$  since  $\partial \ln(1+r^*d)/\partial \ln(1+\theta) = 0$ . Accordingly, the exchange rate remains unchanged both at home and abroad.

The other case is when the interest rate impact of the capital controls is zero at home; this can happen when the supply schedule is perfectly elastic. If so, the demand schedule abroad shifts "right" and the interest rate declines. In this case,  $dPH/d\theta = 1$  since  $\partial \ln(1+rd)/\partial \ln(1+\theta) = 0$ , and  $dPA/d\theta > 0$  since  $\partial \ln(1+r^*d)/\partial \ln(1+\theta) < 0$ . Since the exchange rate effects in the spot market at home (abroad) is obtained as a product of  $1/\theta$  and the premium effect while that in the forward market at home (abroad) is obtained as a product of  $(1-\theta)/\theta [(1-\theta^*)/\theta^*]$  and the premium effect, it is not clear whether the domestic currency can depreciate more or less at home than abroad. Without knowing the precise magnitude of the adjustment coefficient and the interest rate effect in each country, it is impossible to determine the magnitude of the exchange rate effect of capital controls measures.

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1/ Provided that initially  $\theta = 0$  and  $d \ln(1 + \theta) \approx 0$ .

2/ Recall equation (21).

## V. Concluding remarks

The major purpose of this paper has been to analyze, from a theoretical point of view, the impact of capital control measures on the exchange rate. Even though both developed and developing countries have employed such measures as one of the instruments for exchange rate policies, there are (several) divergent opinions about their effect on the exchange rate. While the analysis has been theoretical, it has been formulated with a view to empirical investigation.

We have established that capital control measures increase the transaction costs. In the presence of capital controls, the costs consist of (a) "pure" transaction costs including brokerage fees and cost of gathering information, (b) implicit and explicit taxes or costs that transactors must incur due to the existing controls, and (c) political risk due to uncertainty concerning the possible future changes in the laws governing these controls. In their absence, these costs consist of pure transaction costs only. It has been shown that, since these costs are reflected in deviations from the interest rate parity condition, the extent of capital controls can be measured by comparing interest rate parity differentials.

The introduction of capital controls, apart from increasing the transaction cost, could also affect the interest rate on financial assets that are traded at home and abroad. The impact of these controls on the transaction costs and the interest rates together determine a forward premium or discount for a given currency. The impact of control measures on the premium or discount, together with the speed of adjustment determines its impact on the spot and forward exchange rates.

We have shown that the control measures to encourage outflows or discourage inflows invariably depreciate the currency in both the spot and the forward exchange rate; conversely, control measures to encourage inflows and discourage outflows appreciate the currency in both markets.

While qualitative impact of the capital controls on exchange rates are straightforward, its quantitative impact depends upon many different factors including the "speed of adjustment" referred to above, elasticities of demand for and supply of financial assets, and the extent of capital controls among other things. Furthermore, we must note that, since the introduction of controls result in welfare losses or declines in market efficiency, the authorities in charge of these controls must also weight the importance of bringing about a desired exchange rate effect by introducing capital controls against losses in welfare or economic efficiency, and must consider other options such as monetary policy as a possible means of influencing the exchange rate.

## APPENDIX

### Uncertainty and Interest Rate Parity

This appendix provides a detailed discussion of the relationship between uncertainty and interest rate parity.

Let the costs of borrowing one unit of currency at home be defined as  $C$  where  $C = (1 + r_d)(1 + t_d)$ , where  $r_d$  is the interest rate on the domestic currency denominated assets and  $t_d$  is the percentage transaction costs of transacting these assets. Under certainty, define  $R_c$  as the return from investing abroad with  $R_c = F(1 + r_f)/S(1 + TC_f)$ . Under certainty,  $C = R_c$  and the familiar expression of equation (3) is established.

For a meaningful discussion of uncertainty, we need to specify the utility function of the individual. If an individual engages in a sequence of transactions that generate the interest rate parity relationship, the observation that he buys a forward contract to get rid of the exchange risk is sufficient to rule out convex and linear utility functions. The utility function  $U$  is positively related to  $R$ , negatively related to  $C$ , and strictly concave. It is a continuous, twice differentiable real valued function with  $U_R > 0$  and  $U_{RR} < 0$ , and  $U_C < 0$ , and  $U_{CC} > 0$ . Although this analysis can be done in the  $R$  and  $C$  space we will do it in terms of income and utility derived from.

Under certainty, Point A in Figure 9 is an equilibrium point at which  $C = R_c \Leftrightarrow U(C) = U(R_c)$ . Uncertainty in this model is introduced through the draw of a random variable  $e$  from a fixed c.d.f.  $F(e)$ . Take  $F$  to be strictly increasing over the interval  $I = (e, \bar{e})$  where  $\underline{e} = \inf\{F(e)\}$  and  $\bar{e} = \sup\{F(e)\}$ . Before we specify the distribution of  $e$ , we need to specify how the random variable  $e$  affects the model structure.

Under uncertainty the problem will be posed as follows: if the cost of borrowing funds at home  $C$  remains unaffected but the return from going abroad is random with a given mean and variance, then what is the expected return that will make a risk averse individual break even on the margin. We will assume that the shock  $e$  appears as an additive component in the return function with

$$\int_{\underline{e}}^{\bar{e}} e F_e de = 0 \text{ and } \int_{\underline{e}}^{\bar{e}} [e - \int_{\underline{e}}^{\bar{e}} e F_e de]^2 F_e de = \sigma_e^2.$$

Let the return from investing abroad under uncertainty be denoted by  $R_u(e)$  with  $\int_{\underline{e}}^{\bar{e}} R_u(e) F_e de = R_u$ . Notice that  $R_u$  in Figure 9 is greater than  $R_c$ . The expected utility maximizing risk averse individual will equate the loss in utility  $U(C)$  associated with the costs of borrowing funds with the expected utility of the return. In equilibrium these must be equal. Thus:

$$U(C) = \int_{\underline{e}}^{\bar{e}} U[R_U + e] F_e de \quad (a)$$

The right hand side of the above expression is segment BE in Figure 9. Notice that the way Figure 9 was constructed segment BE equals segment AD. The distance AE can thus be interpreted as the risk premium. Define  $V(R_U, e)$  to be the certainty equivalent. It is evident that

$$U(C) = U\left[R_U + \int_{\underline{e}}^{\bar{e}} e F_e de - V(R_U, e)\right] = \int_{\underline{e}}^{\bar{e}} U(R_U + e) F_e de$$

or 
$$U(C) = U(R_U - V(R_U, e))$$

where 
$$V(R_U, e) = (-1/2)(U''(R)/U'(R))\sigma_e^2 + O(\sigma_e^2) \quad (b)$$

Since the utility function is strictly increasing, equation (b) can now be expressed as:

$$C = R_U - V(R_U, e) \quad (c)$$

Equation (C) above is an equilibrium condition. For capital to flow out of the home country it must be the case the  $C < (R_U - V(R_U, e))$ . or alternatively

$$(1+r_d)(1+t_d) < [F(1+r_f)/S(1+t_f)(1+t_p)(1+t_s)] - V(R_U, e) \quad \text{or}$$

$$(S/F(1+r_d)(1+TC)/(1+r_f)) \leq 1 - (S/F)[(1+t_f)(1+t_p)(1-t_s) V(R_U, e)/(1+r_f)] \quad (d)$$

Thus, for no capital outflows seeking unexploited profits it must be true that

$$(S/F)[(1+r_d)/(1+r_f)] \geq 1/(1+TC)[1+V(R_U, e)/(1+r_d)(1+t_d)] \quad (e)$$

By similar reasoning as above, for no capital inflows seeking unexploited profits the following must hold:

$$(1+TC)[1+V(R_U, e)/(1+r_f)(1+t_f)] \geq (S/F)[(1+r_d)/(1+r_f)] \quad (f)$$

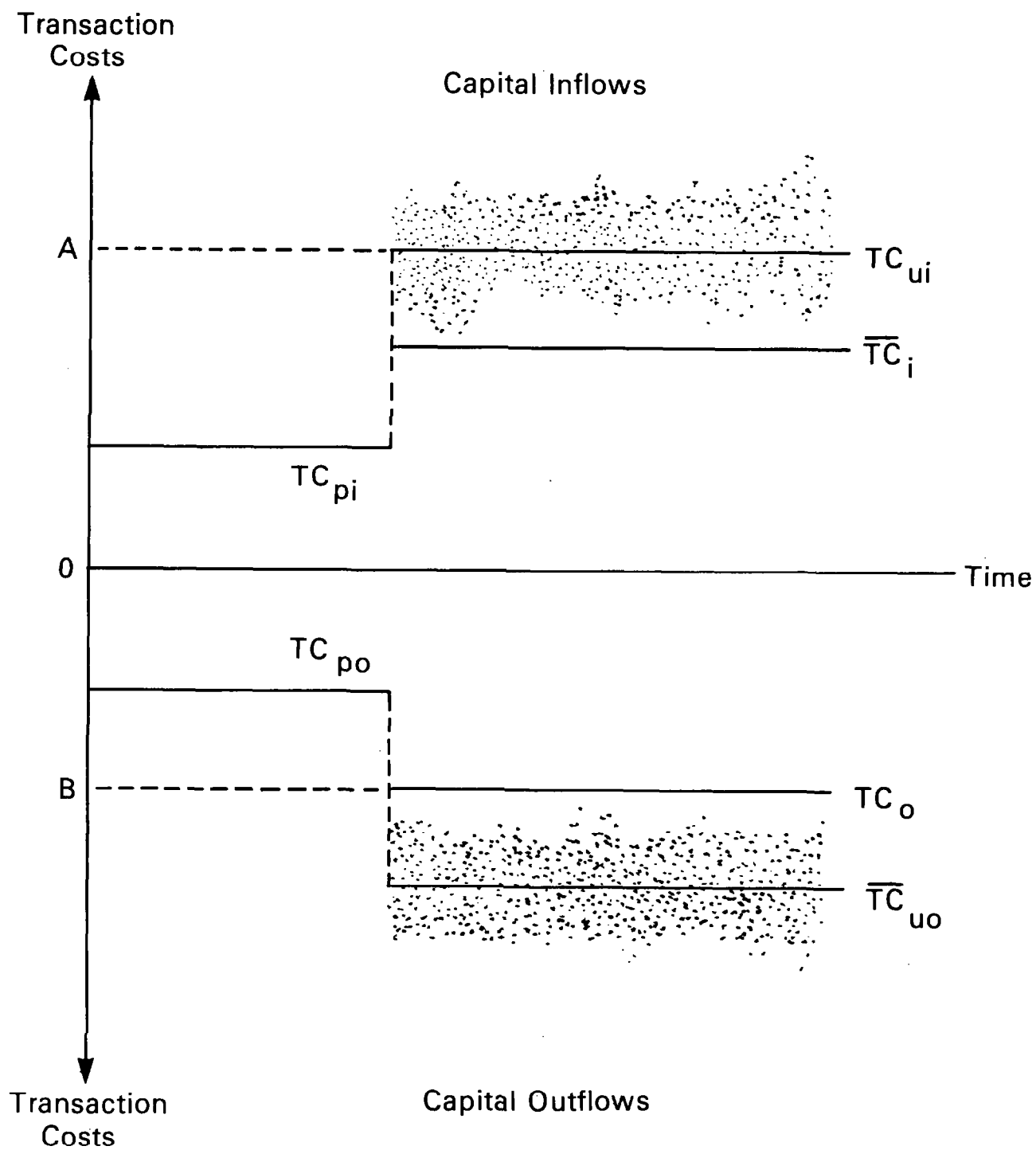
Combining equations (e) and (f) above, the neutral band can now be defined as:

$$\begin{aligned} (1+TC)[1+V(R_U, e)/(1+r_f)(1+t_f)] &\geq S(1+r_d)/F(1+r_f) \geq \\ (1/(1+TC))[1+V(R_U, e)/(1+r_d)(1+t_d)] \end{aligned}$$

Notice that equation (g) above gives a specification for the neutral band both when pure transaction costs, control costs and political risk are present. The introduction of political risk increases the neutral band within covered interest differentials must lie. The terms in the square brackets above should be interpreted as the certainty equivalent that agents, on the margin, must be compensated by.



FIGURE 1  
TRANSACTION COSTS AND UNCERTAINTY



100

100

100

100

FIGURE 2

# DOMESTIC CURRENCY DENOMINATED ASSETS MARKET

(DD: nonresidents and SS: residents)

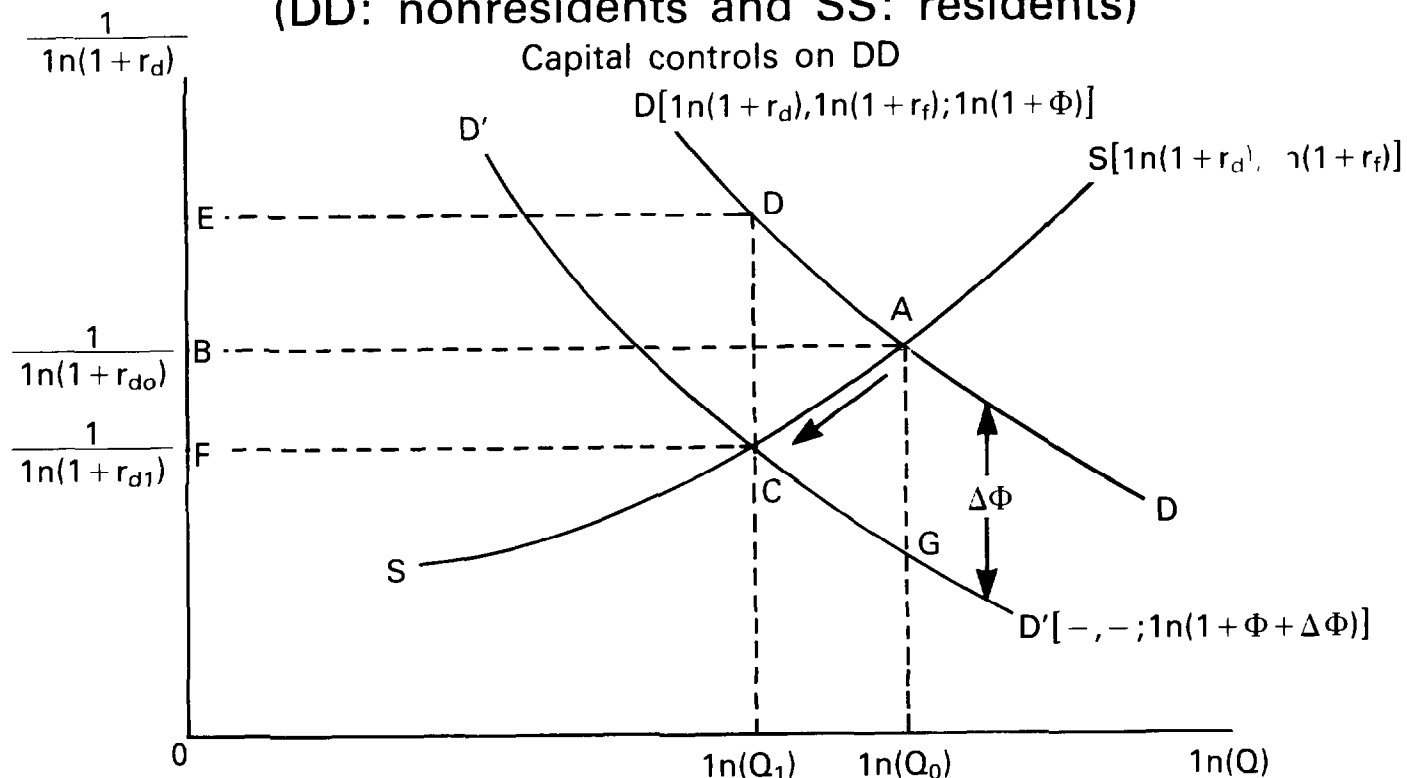
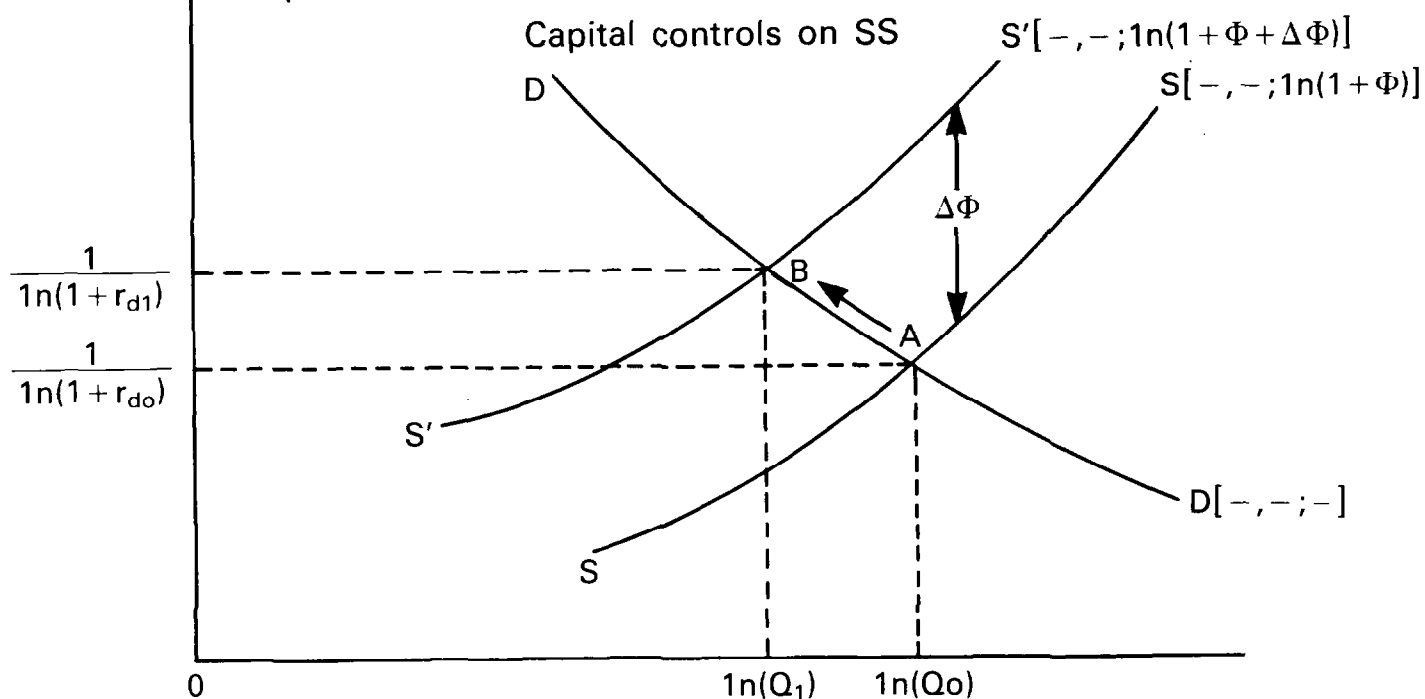


FIGURE 3

# DOMESTIC CURRENCY DENOMINATED ASSETS MARKET

(DD: nonresidents and SS: residents)

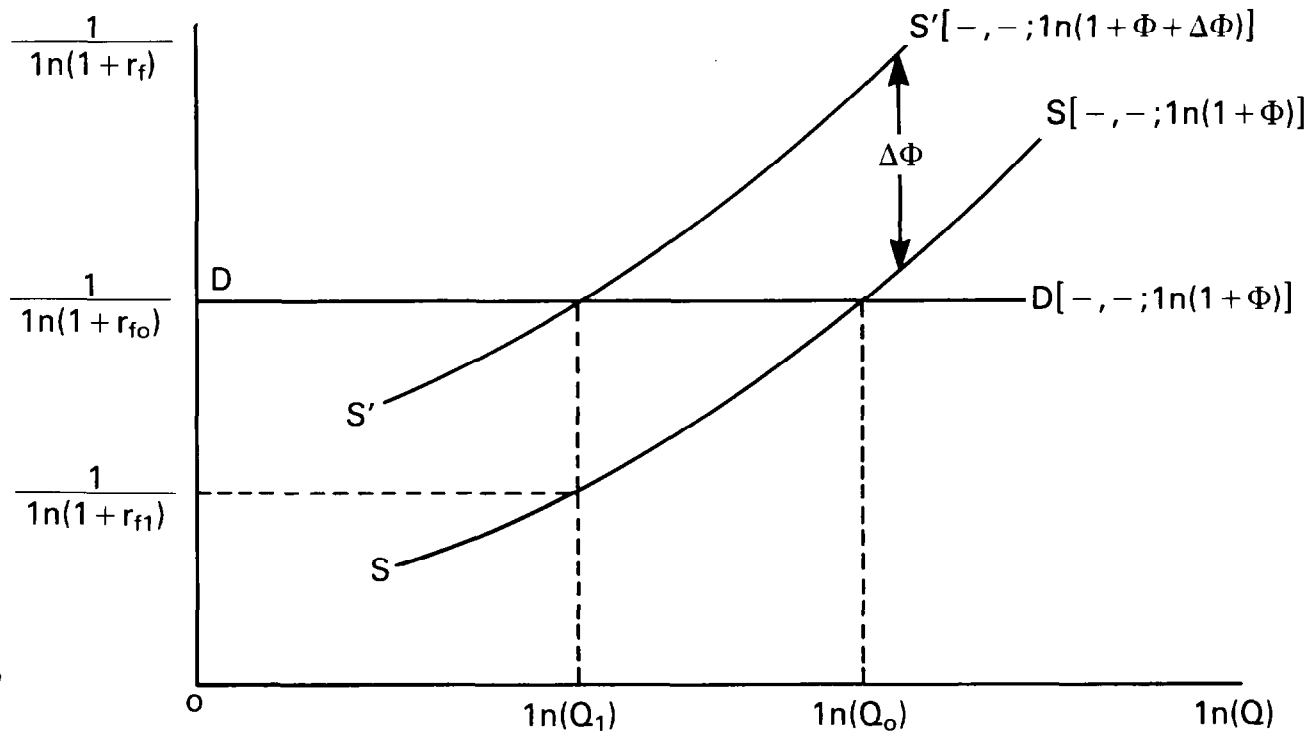




# FIGURE 4 FOREIGN CURRENCY DENOMINATED ASSETS MARKET

(DD: nonresidents and SS: residents)

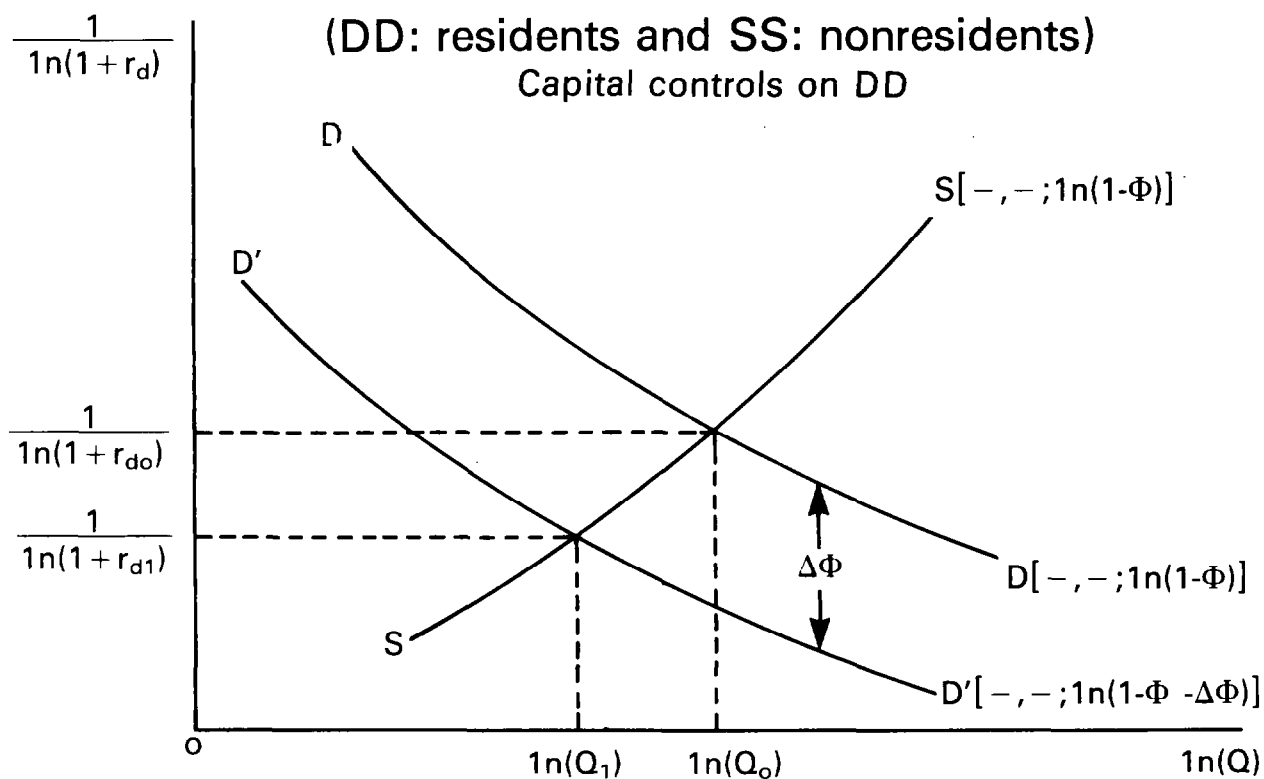
Capital controls on SS



# FIGURE 5 DOMESTIC CURRENCY DENOMINATED ASSETS MARKET

(DD: residents and SS: nonresidents)

Capital controls on DD



2



FIGURE 6

# DOMESTIC CURRENCY DENOMINATED ASSETS MARKET

(DD: residents and SS: nonresidents)

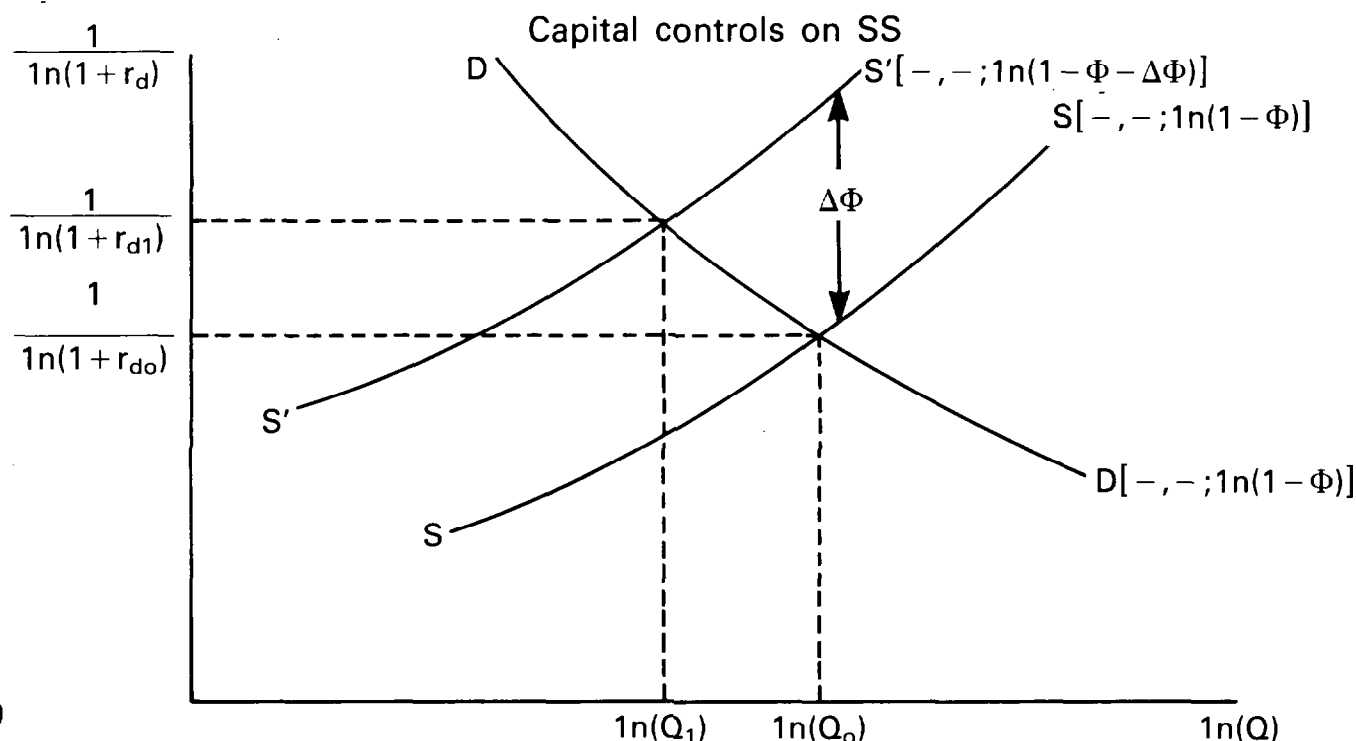


FIGURE 7

# FOREIGN CURRENCY DENOMINATED ASSETS MARKET

(DD: residents and SS: nonresidents)

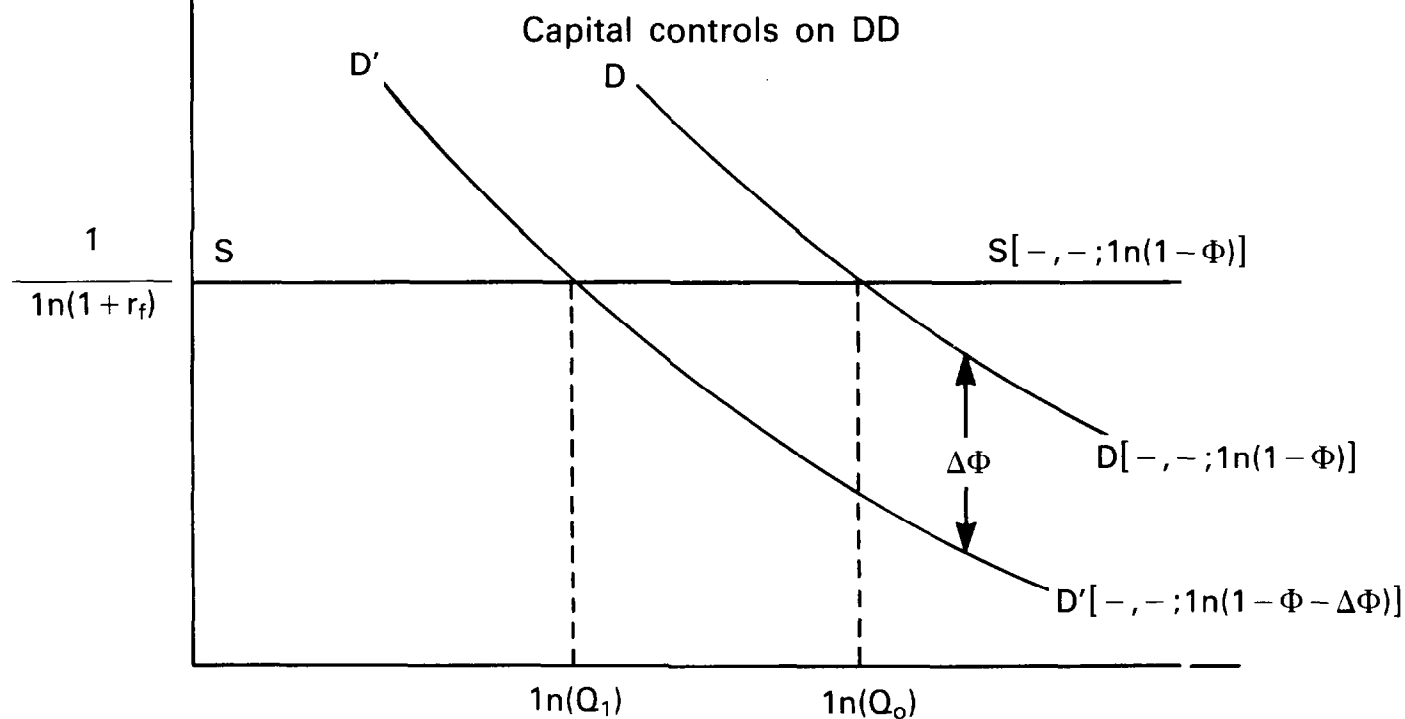




FIGURE 8

# EFFECTS OF CAPITAL CONTROLS ON INTEREST RATES AT HOME AND ABROAD

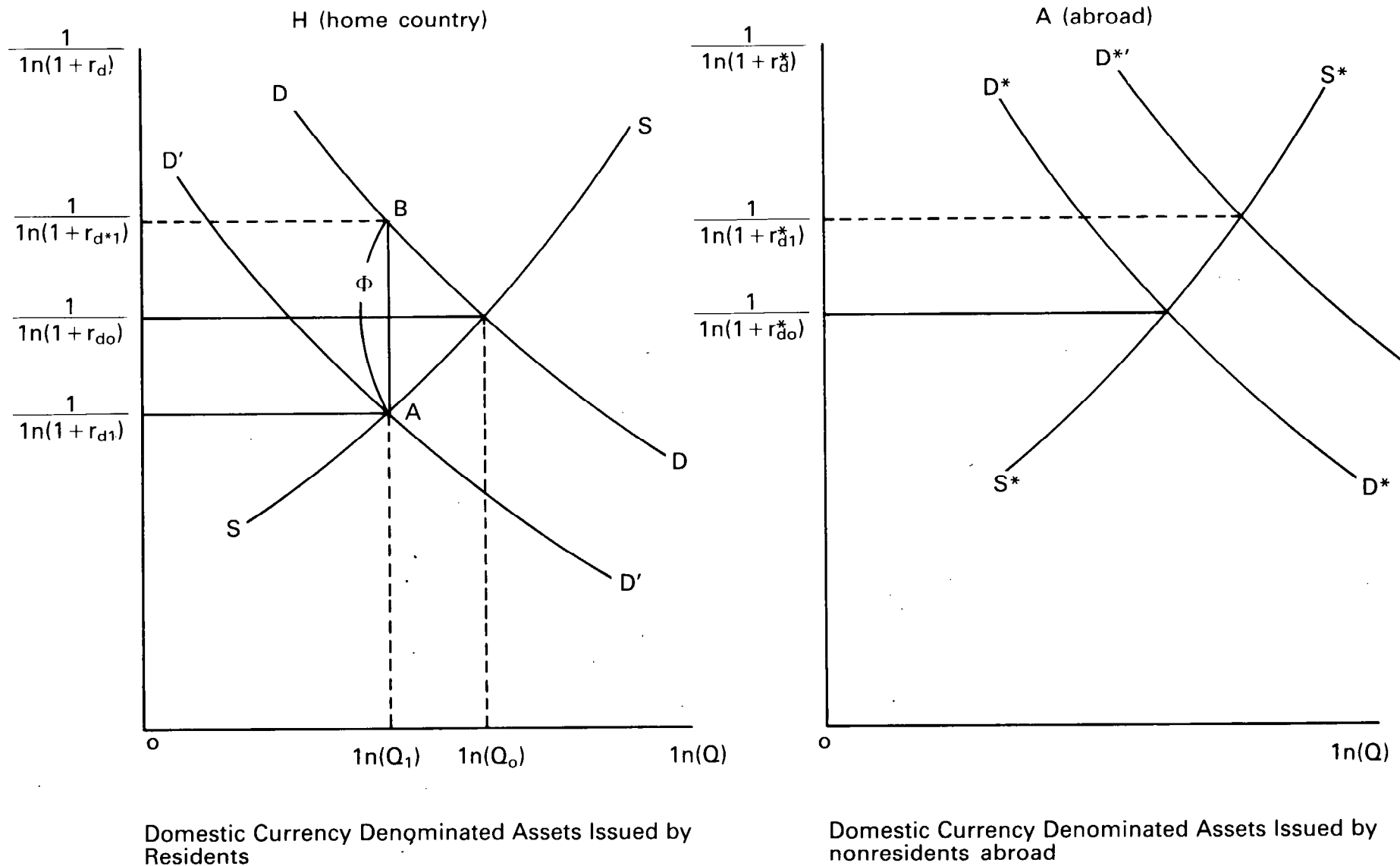
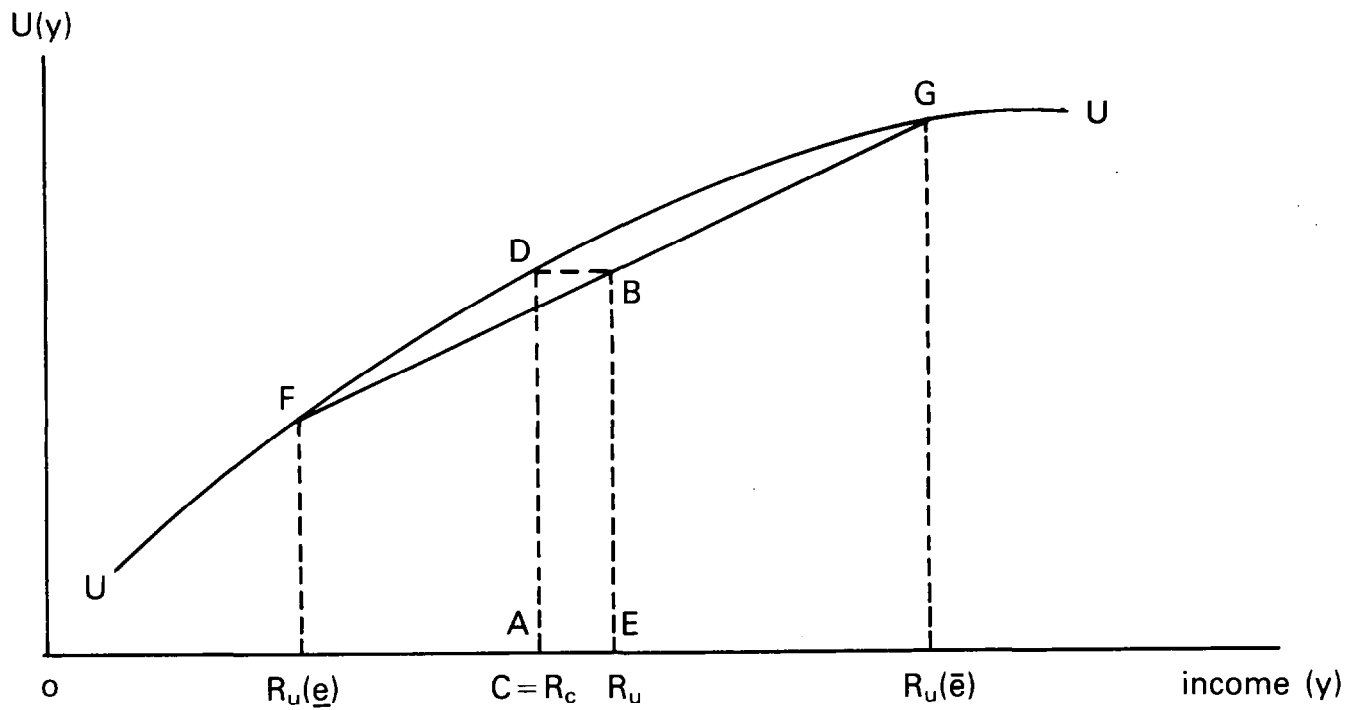


FIGURE 9

# UNCERTAINTY, EXPECTED RETURN, AND UTILITY





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