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Maximizing Export Taxes in the Presence of Illegal Trade

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I. Introduction

In a recent publication, Franco (1981) addresses the question of the optimal producer price for a primary commodity. 1/ In the accompanying empirical work, he analyzes the case of cocoa in Ghana. Two separate optimality criteria are considered--maximization of export receipts and maximization of government export taxes. The result he derives is that in the presence of smuggling, the producer price that maximizes tax revenue is greater than the producer price that maximizes export receipts, and that, as smuggling diminishes to zero, the discrepancy between the two prices also diminishes to zero.

Franco considers a constant-rate export duty. However, the government revenue yield in Ghana (and many other countries) is the difference between export revenues and the allowed costs of the Marketing Board (inclusive of payments to producers), which implies that the tax rate is not independent of producer prices and export revenues. This note re-examines optimal producer prices in the context of a variable tax rate. 2/

With one exception, only direct government revenues from exports will be considered; the effect of trade policies on income tax revenues is, for example, not included in the analysis. The one exception

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1/ Franco, G.R., "The Optimal Producer Price of Cocoa in Ghana," Journal of Development Economics (Amsterdam), Vol. 8 (No. 1, February 1981), pp. 77-92.

2/ It should be noted that the "optimality" criteria focused on here (those used by Franco (1981)) do not necessarily produce optimal policies for the total economy. First, policies oriented toward the maximization of export revenues can clearly result in excessive subsidies to exports. Second, there is no reason why government should maximize its revenues from any particular source. For example, a more reasonable target might be to achieve revenue targets with minimum deadweight loss. Third, the model pays no attention to issues such as exchange rate management and black-market exchange premiums, which are frequently the primary incentive to engage in illegal trade.

relates to the effect of export policies, in an exchange-constrained economy, on government receipts from taxes on imports; increased export earnings allow a rise in imports and import taxes in such an economy.

It is easily demonstrated that Franco's result does not hold for a variable tax rate. Government receipts are the product of the tax rate (t_x) and export revenues (E). The derivative of government revenues (R) with respect to the producer price (P^P) is

$$(1) \quad \frac{\delta R}{\delta P^P} = \frac{\delta t_x}{\delta P^P} E + t_x \frac{\delta E}{\delta P^P}$$

If this is evaluated at the export receipts-maximizing price, it is clear that the second term is zero. Hence, at this price, a reduction in the producer price will increase government revenues, since the tax rate is inversely related to the producer price. 1/ Hence, without smuggling, the government revenue-maximizing producer price is less than the producer price that maximizes export receipts. With a fixed tax rate, as in Franco's model, the two producer prices are the same in the absence of smuggling.

The next section of the paper will outline the structure of the model to be used. In Section III, the optimal producer prices that maximize export receipts and government revenue will be derived analytically and their relative sizes compared. It is shown that in an economy which is not exchange-constrained, the producer price that maximizes government revenue is greater than the producer price that maximizes export receipts only if the marginal propensity to smuggle with respect to smuggling incentives is sufficiently high and if the latter producer price does not involve an export subsidy. 2/ In Section IV, these analytic results will be illustrated with some numerical examples. Finally, Section V will contain conclusions.

1/ For this not to be the case would require quite special circumstances. Specifically, the increase in the producer price would have to lead to an increase in the world price, and this increase in the world price would have to be greater in percentage terms than the increase in domestic prices. A necessary, but not sufficient, condition is that the supply curve be backward-bending. In addition, since the export receipts-maximizing price is being considered, the percentage decrease in export volume must be at least as great as the percentage increase in world prices, implying that the absolute value of supply elasticity with respect to the producer price is at least one.

2/ In an exchange-constrained economy, this condition is less strong because an increase in exports allows increased imports, thereby increasing import tax revenues.

II. Structure of the Model

Consider a primary commodity exporting country where the primary product is the country's only foreign exchange earner. Production, Q, is assumed to be a positive linear function of the producer price, 1/

$$(2) \quad Q = \alpha_0 + \alpha_1 P^P$$

where P^P denotes the producer price. It is assumed that the exporter faces a downward-sloping demand schedule

$$(3) \quad D = \sigma_0 - \sigma_1 P^*$$

where P^* denotes the world price for the country's exports.

Using equations (2) and (3), the equilibrium world price can be expressed as a function of the producer price

$$(4) \quad P^* = \frac{\sigma_0 - \alpha_0}{\sigma_1} - \frac{\alpha_1 P^P}{\sigma_1}$$

Exporters can export their output through the official market at the official exchange rate (s) or through illegal channels at the parallel exchange market rate (e). The price for illegally exported output (P^I) is written as follows:

$$(5) \quad P^I = (k+\phi P^*)e$$

This specification can encompass the whole range between a constant producer price in a neighboring country and a price which moves proportionally with the world market price (P^*). It is assumed that illegal

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1/ Franco assumes a different supply function, separating the decisions to produce for the official and illegal markets. Here it is assumed that the official producer price is the marginal output price and that the incentive to smuggle affects the distribution of output between official and illegal exports. In general, legal and illegal trade coexist, implying that, at the margin, the benefits of exporting through the two channels have been equalized. Thus, at the margin, the net additional benefits of exporting illegally (the price discrepancy) are offset by the net additional costs (costs of trying to avoid detection and the expected value of penalties if detected). This difference in the assumed supply functions does not qualitatively affect the comparison of the results from the two models being made here.

exports are a positive function of the discrepancy between P^I and P^P--that is 1/

(6) $Q^I = \beta_0 + \beta_1(P^I - P^P)$

The rest of production (Q^0) is exported through the official market, or

(7) $Q^0 = (\alpha_0 - \beta_0) + (\alpha_1 + \beta_1)P^P - \beta_1(k + \phi P^*)e$

In general, a government's revenue yield from a primary product is the difference between the export revenue (net of marketing costs) and payments to producers. Here it is assumed, without affecting the qualitative nature of the results, that marketing costs are zero. The government also gains revenue from taxing imports. If the economy is foreign exchange-constrained, then the import tax base is in part determined by the value of official market exports. Incorporating these direct and indirect revenue effects, government revenue can be expressed as 2/

$$R = (P^*s - P^P)Q^0 + P^*Q^0 st_m$$

where t_m denotes the import tax rate. (If it is desired to consider only direct government export revenues, t_m can be set equal to zero.) This can be rewritten as

(8) $R = P^*Q^0(1+t_m)s - P^PQ^0$

III. Derivation of Optimal Prices

1. Export receipts

Total export receipts (in foreign-currency terms) are written as follows: 3/

1/ It is assumed that the marginal costs of smuggling rise with the volume smuggled.

2/ The specification here assumes all exports get reflected in increased imports. However, by interpreting t_m as the product of the import tax rate and the propensity to import out of exports, the specification becomes more general. It is also assumed here that illegal foreign exchange earnings are not used for official market imports. While this may seem to be a restrictive assumption, it does not affect the qualitative nature of the results.

3/ Including both legal and illegal export receipts.

$E = QP^*$

Differentiating with respect to P^P, one obtains

(9) $$\frac{\delta E}{\delta P^P} = Q\,\frac{\delta P^*}{\delta P^P} + P^*\,\frac{\delta Q}{\delta P^P}$$

Using equations (2) and (4), equation (9) can be rewritten as

(10) $$\frac{\delta E}{\delta P^P} = -(\alpha_0+\alpha_1 P^P)\,\frac{\alpha_1}{\sigma_1} + \left[\frac{(\sigma_0-\alpha_0)}{\sigma_1} - \frac{(\alpha_1 P^P)}{\sigma_1}\right]\alpha_1$$

Setting equation (10) equal to zero, one obtains

(11) $$P^P_E = \frac{\sigma_0 - 2\alpha_0}{2\alpha_1}$$

Inserting equation (11) into equation (4) yields

(12) $$P^*_E = \frac{\sigma_0}{2\sigma_1}$$

where P^P_E denotes the producer price which maximizes export receipts, and P^*_E is the corresponding world price. Not surprisingly, P^*_E can easily be shown to be the point where the demand schedule is unit elastic.

2. <u>Government revenue maximization</u>

Taking the derivative of equation (8) with respect to P^P, one obtains

(13) $$\frac{\delta R}{\delta P^P} = Q^0(1+t_m)s\,\frac{\delta P^*}{\delta P^P} - Q^0 + \left[P^*(1+t_m)s - P^P\right]\frac{\delta Q^0}{\delta P^P}$$

The producer price that maximizes revenue is obtained by setting equation (13) equal to 0 and solving for P^P. The complete expression for P^P is given in the Appendix.

3. Comparison of optimal prices without smuggling ($i.e., \beta_1 = \beta_0 = 0$)

First, consider the case with $t_m = 0$--that is, considering only direct government revenue from exports--and with s set equal to 1

$$(14)\quad P_R^P = \frac{\alpha_1\sigma_0 - 2\alpha_1\alpha_0 - \alpha_0\sigma_1}{2(\sigma_1+\alpha_1)\alpha_1}$$

where P_R^P denotes the producer price which maximizes government revenue.

Using equations (11) and (14), it is easily shown that

$$P_E^P - P_R^P = \frac{\sigma_1(\sigma_0-\alpha_0)}{2(\sigma_1+\alpha_1)\alpha_1} > 0$$

This illustrates the point made in the introduction that without illegal trade, the producer price that maximizes government revenue is always less than the producer price that maximizes export receipts.

Next, expanding consideration to $t_m > 0$,

$$(15)\quad P_{RM}^P = \frac{[\alpha_1(\sigma_0-\alpha_0)(1+t_m)] - [(1+t_m)\alpha_1 + \sigma_1]\alpha_0}{2[(1+t_m)\alpha_1 + \sigma_1]\alpha_1}$$

where P_{RM}^P denotes the producer price that maximizes government revenue when indirect revenue effects through the import tax base are considered.

Using equations (11), (14), and (15), one obtains

$$(16)\quad P_{RM}^P - P_R^P = \frac{\sigma_1(\sigma_0-\alpha_0)t_m}{2[(1+t_m)\alpha_1 + \sigma_1](\alpha_1+\sigma_1)} > 0$$

and

$$(17)\quad P_E^P - P_{RM}^P = \frac{\sigma_1(\sigma_0-\alpha_0)}{2[(1+t_m)\alpha_1 + \sigma_1]\alpha_1} > 0$$

Thus, it can be seen that incorporating indirect government-revenue effects increases the producer price that maximizes government revenue. The reasoning is straightforward. Including the tax-base effect, government revenues are

$$R = (t_x+t_m)P^*Q_s^0$$

Taking the derivative with respect to P^P, one obtains

$$(18)\quad \frac{\delta R}{\delta P^P} = \frac{\delta t_x P^* Q_s^0}{\delta P^P} + t_m \frac{\delta P^* Q_s^0}{\delta P^P} + t_x \frac{\delta P^* Q_s^0}{\delta P^P}$$

If import-tax-base effects (the second component of equation (18)) are ignored, then equation (18), evaluated at P_R^P, equals zero. At this producer price, the tax-rate-reducing effect of an increase in P^P (the first component of equation (18)) is exactly offset by the tax-base-increasing effect (the third component of equation (18)). Introducing the import-tax-base effect gives additional weight to the tax-base-increasing effect. Hence, P_{RM}^P is greater than P_R^P. However, it is still less than the producer price that maximizes export revenue. In the absence of smuggling, the tax base is maximized at the producer price that maximizes export receipts. Evaluated at this latter price, the second and third components of equation (18) are zero. The first component is negative, and hence P_{RM}^P must be less than P_E^P.

4. <u>Comparison of optimal prices in presence of smuggling</u>

Extending consideration to this more general case, it can easily be shown that

$$P_{RS}^P > P_{RM}^P > P_R^P$$

where P_{RS}^P denotes the producer price that maximizes government revenue in the presence of smuggling and takes account of indirect-revenue-raising effects through the import tax base. However, a definite ranking of P_E^P and P_{RS}^P is not possible. 1/ To illustrate this ranking ambiguity, consider equation (13), which is the expression for the derivative of export revenues with respect to the producer price. Evaluated at P_E^P--that is the producer price that maximizes export revenue--equation (13) can be rewritten as

$$(19)\quad \left.\frac{\delta R}{\delta P^P}\right|_{P_E^P} = \overbrace{\left. Q^0\right|_{P_E^P}}^{A} \cdot \overbrace{\left[-1 - (1+t_m)\frac{\alpha_1}{\sigma_1}\right]}^{B} +$$

$$\overbrace{\left[\alpha_1 + \beta_1 + \beta_1 \phi e \frac{\alpha_1}{\sigma_1}\right]}^{C} \cdot \overbrace{\left[(1+t_m)\frac{\sigma_0}{2\sigma_1} - \frac{\sigma_0 - 2\alpha_0}{2\alpha_1}\right]}^{D}$$

1/ This is also the case if P_{RS}^P excludes indirect import tax effects.

If equation (19) is positive, then P^P_{RS} is greater than P^P_E, and the reverse is true if equation (19) is negative. The first part of equation (19) (the product of A and B) is negative--for a given value of Q, increasing the producer price reduces the tax rate directly (i.e., for a given world price) and also indirectly (owing to the effects of increasing supply on the world price). The rest of equation (19) shows the effect of increasing the volume of official market exports--these coming from increased production and reduced illegal exports. This term is not necessarily positive. A sufficient condition for it to be positive is that the world price in domestic currency terms ($s\sigma_0/2\sigma_1$) be greater than the producer price (the algebraic expression is normalized on $s = 1$). This means that (evaluated at official rates) the government is not subsidizing exports. The necessary condition for it to be positive is that the sum of t_x and t_m be positive--that is, if exports are being subsidized, they must yield, at the margin, more in additional import tax revenues than they cost in direct export subsidies.

If it is assumed that there is no smuggling ($\beta_0 = \beta_1 = 0$), it is easy to show that equation (19) has a negative sign, which confirms the result already obtained that, in such cases, the producer price that maximizes government revenue is less than the producer price that maximizes export revenue. The effects of smuggling on the sign of equation (19) will now be examined. Let the right-hand side of equation (19) be denoted as Z. It is easily seen that the derivative of Z with respect to the marginal propensity to smuggle (β_1) is

$$(20)\quad \frac{\delta Z}{\delta\beta_1} = -\overbrace{\left[1 + (1+t_m)\frac{\alpha_1}{\sigma_1}\right]}^{E}\overbrace{\left[\frac{\sigma_0-2\alpha_0}{2\alpha_1} - \frac{\phi\sigma_0 e}{2\sigma_1} - ke\right]}^{F}$$

$$+\overbrace{\left[(1+t_m)\frac{\sigma_0}{2\sigma_1} - \frac{(\sigma_0-2\alpha_0)}{2\alpha_1}\right]}^{G}\overbrace{\left[1 + \phi e\frac{\alpha_1}{\sigma_1}\right]}^{H}$$

Provided there is an incentive to export illegally (i.e., $P^I > P^P$), the first part of this expression (the product of E and F) is positive. This is explained as follows: for given prices, the higher is β_1, the greater is the percentage of output exported illegally. This means that the tax-rate-reducing effects of increasing the producer price work on a smaller base and, therefore, have a smaller adverse impact on government revenues. The second part of equation (20) is positive, provided that, ceteris paribus, increasing official market exports increases government revenues. 1/ In such cases, measures which draw exports back into the

1/ For an export subsidy, this requires that the import tax effects offset the cost of the subsidy.

official market will have positive revenue effects. The larger β_1 is, the greater the volume of exports that will be drawn back into the official market for a given increase in producer prices.

If equation (20) is positive, it can definitely be stated that for a sufficiently high β_1, Z is positive and the producer price that maximizes government revenue is greater than the producer price that maximizes export receipts. Thus, provided there is a positive export tax, or that if there is a subsidy, it is offset by import-tax effects, such a β_1 does exist. However, for this to make sense within the model, we have to constrain β_1 to a reasonable size. Clearly, for a given producer price and a given incentive to export illegally, a high enough β_1 will, according to the specification, yield more than 100 percent of exports going through the illegal market. This does not make sense. Thus, at a minimum, β_1 must be constrained so as to allow Q^0 to be positive. It is easily shown that this restriction does not affect the result stated in the first two sentences of this paragraph. This can be verified by looking back to equation (19). The term A is Q^0, official market exports. As Q^0 diminishes to zero, the first part of equation (19) (the product of A and B) diminishes to zero while the second part remains positive. Thus, there exists a β_1 consistent with positive official market exports at P^P_E, for which equation (19) is positive. 1/ But, clearly, the presence of smuggling is not sufficient to make $\bar{P}^P_{RM} > P^P_E$.

Furthermore, there exist cases where the producer price that maximizes government revenue is lower than the producer price that maximizes export receipts, irrespective of the size of the propensity to smuggle (β_1). If the producer price that maximizes export revenue involves a subsidy and this subsidy is large enough to outweigh the indirect revenue (import tax) benefits of increasing official market exports, then equation (19) is negative, irrespective of the size of β_1. This may not be apparent from equation (20), where it is possible for $\delta Z/\delta\beta_1$ to be positive even if E in equation (20) is negative. However, when a negative Q^0 is ruled out, equation (19) cannot be positive.

Finally, a number of points seem worth noting in the context of the above-mentioned results. First, it is theoretically possible that the producer price that maximizes government revenue could conceivably involve an export subsidy. This is because of the indirect revenue

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1/ If Q^0 is zero, then the tax-rate-lowering effect of increasing the producer price will have no effect on revenue, but revenue will be enhanced due to an increase in Q^0 (stemming from increased production and reduced illegal exports).

benefits derived from expanding the import-tax base. Second, little has been said in the analysis about exchange rate policy. But, clearly, the setting of the exchange rate is not irrelevant to the government revenue yield which results from a given producer price. At a given producer price, an increase in the exchange rate automatically increases both the direct export-revenue yield and the indirect import-tax yield, through valuation effects on the import- and export-tax bases. It also increases revenue by drawing exports back into the official market. (The black-market premium will be reduced, both by the depreciation and, probably, also by an appreciation of the black-market rate.) 1/

IV. Some Numerical Examples

In Table 1, there are some numerical examples of how the producer price that maximizes government revenue (P^P_{RS}) varies with β_1, t_m, and δ_1. The values assumed for other parameters are not varied and are noted in the footnote to Table 1. It is assumed that the price for illegally exported output is the world price converted at the black-market rate. The supply parameter is set at $\alpha_1 = 2$, the black-market exchange rate premium at 50 percent, and the smuggling parameter β_1 is varied between 0 and 1. In all cases, it can be seen that P^P_{RS} is positively related to both t_m and β_1. In case (a), there is a large gap between the producer price that maximizes export receipts (P^P_E) and the corresponding world price, converted to domestic currency using the official exchange rate $P^*_E s$. In this case, relatively small values of β_1 and t_m ($\beta_1 = 0.1$, $t_m = 0.1$) give a government-revenue-maximizing producer price greater than the export-receipts-maximizing producer price. In case (b), the difference between $P^*_E s$ and P^P_E is much smaller. For any given value of t_m, a much higher value of β_1 is required to make P^P_{RM} greater than P^P_E. The reason is as follows: the smaller gap between $P^*_E s$ and P^P_E means that at P^P_E the erosion of the tax base, for a given value of β_1, is much smaller, and this increases the weight of the tax-rate-reducing effects of a rise in producer prices relative to the tax-base-increasing effects; in addition, the smaller gap means a lower revenue yield per unit of official market exports and, hence, a lower revenue gain for each unit of exports encouraged back to the official market. In case (c), P^P_E is the same as $P^*_E s$, which means that at P^P_E there is a zero tax rate on exports. In this case, if P^P_{RS} is to be greater than P^P_E, both the import tax rate and β_1 must be relatively

1/ Of course, a more complete evaluation of the exchange rate issue would also have to consider how exchange rate policies affected domestic prices and how these affected producer incentives. But, even if the government raised producer prices by the same percentage as the devaluation, there would be a revenue gain (assuming there was already a positive revenue yield from taxation of the export commodity).

Table 1. Producer Prices that Maximize Government Revenue Under Different Assumptions About the Values of β_1, t_m, δ_1 1/

(a) $\sigma_1 = 0.4$, $P_E^* s = 125$, $P_E^P = 50$

t_m	β_1: 0	0.10	0.25	0.50	0.75	1.00
0	43.8	48.9	54.4	57.8	59.5	60.5
0.10	44.2	50.4	54.9	58.3	60.0	61.0
0.25	44.8	51.0	55.5	58.9	60.6	61.6
0.40	45.3	51.5	56.0	59.4	61.1	62.1

(b) $\sigma_1 = 0.8$, $P_E^* s = 62.5$, $P_E^P = 50$

t_m	β_1: 0	0.10	0.25	0.50	0.75	1.00
0	39.3	42.6	45.7	48.7	50.3	51.4
0.10	40.0	43.3	46.4	49.4	51.1	52.1
0.25	40.9	44.2	47.3	50.3	52.0	53.1
0.40	41.7	45.0	48.1	51.0	52.7	53.8

Table 1 (concluded). Producer Prices that Maximize Government Revenue Under Different Assumptions About the Values of β_1, t_m, δ_1 1/

(c) $\sigma_1 = 1.0$, $P_E^* s = 50$, $P_E^P = 50$

t_m	β_1: 0	0.10	0.25	0.50	0.75	1.00
0	37.5	40.1	42.8	45.4	47.0	48.0
0.10	38.3	40.9	43.6	46.2	47.8	48.8
0.25	39.3	41.9	44.6	47.2	48.8	49.8
0.40	40.1	42.8	45.4	48.0	49.6	50.7

(d) $\sigma_1 = 1.2$, $P_E^* s = 41.67$, $P_E^P = 50$

t_m	β_1: 0	0.10	0.25	0.50	0.75	1.00
0	35.9	38.1	40.4	42.7	44.2	45.2
0.10	36.8	38.9	41.2	43.5	45.0	46.0
0.25	37.8	40.0	42.3	44.6	46.1	47.1
0.40	38.8	40.9	43.2	45.5	47.0	48.0

1/ Values for unvarying parameters are $\alpha_0 = -50$, $\alpha_1 = 2$, $\delta_0 = 100$, $s = 1$, and $e = 1.5$.

high. If $\beta_1 = 1$ and $t_m = 0.4$, then P^P_{RS} is greater than P^P_E. Finally, in case (d), the producer price that maximizes export receipts involves an export subsidy of 16.7 percent. For the range of parameter values used in the Table, there is no case where $P^P_{RM} > P^P_E$. It is known, however, from the analysis in the text that for any $t_m > 0.167$, there exists a β_1 which makes $P^P_{RM} > 50$.

V. Conclusion

In this note, the question of "optimal" producer prices has been addressed. As pointed out earlier, the notions of optimality used are not ones derived by the author but rather ones used elsewhere in the literature. They clearly may not be optimal in an economy-wide context. Furthermore, not enough attention has been paid to closely associated policy tools, such as the exchange rate. It is shown that if there is no smuggling, the producer price that maximizes government revenue is always less than the producer price that maximizes export revenue, even if one takes into consideration the indirect revenue effects deriving from changes in the import-tax base. In the presence of smuggling, the producer price that maximizes government revenue is greater than the producer price that maximizes export revenue, provided that the marginal propensity to smuggle is sufficiently high and that the latter producer price either does not involve an export subsidy or, if an export subsidy is involved, the import-tax-revenue effects outweigh the cost of the subsidy. However, the existence of smuggling does not ensure $P^P_{RS} > P^P_E$, as in the Franco (1981) result. Finally, it should be noted that for a small exporter with no effect on world prices, exports can always be increased by raising subsidies. Hence, the producer price that maximizes export receipts is always greater than the producer price that maximizes government revenue.

Complete Expression for the Revenue Maximizing Producer Price

The complete expression for the producer price that maximizes revenue P^P_{RS} is as follows:

$$P^P_{RS}\ \left[(1+t_m)\alpha_1 s + \sigma_1\right]\left[\alpha_1 + \beta_1 + \beta_1\phi(\alpha_1/\sigma_1)e\right]$$

$$= \left[(1+t_m)\alpha_1 s + \sigma_1\right]\left[\beta_1 ke + \beta_1\phi\ (\sigma_0-\alpha_0)e/\sigma_1 - (\alpha_0-\beta_0)\right] \cdot 1/2$$

$$+ \left[(\sigma_0-\alpha_0)(1+t_m)\ s\ (\alpha_1+\beta_1+\beta_1\phi e\alpha_1/\sigma_1)\right] \cdot 1/2$$