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Exchange Rate Differential and Balance of Payments Under Dual Exchange Markets

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I. Introduction

Several countries use dual exchange market systems comprising an official market in which the exchange rate is determined by central bank intervention and a non-official market in which the exchange rate is determined by market forces. The official market usually handles the external transactions of the public sector, as well as "priority" imports and "traditional" exports of the private sector, while the free market handles the remaining current transactions and the capital transactions of the private sector. The two exchange markets have different clearing mechanisms. While an excess demand for foreign exchange in the official market causes a balance of payments deficit, an excess demand for foreign exchange in the free market causes a depreciation of the free exchange rate, which in turn may be reflected in a widening differential between the free and the official exchange rates.

Since the use of dual exchange markets constitutes a multiple currency practice, the Fund often encourages countries using dual systems to adopt policies leading to a unification of the exchange markets. In general, this unification involves a gradual reduction in the differential between the free and the official exchange rate. Consequently, Fund stabilization programs in countries operating dual systems sometimes include specific targets regarding reductions in the differential between the exchange rates, in addition to the usual targets on the balance of payments.

This raises the issue of what effect various types of adjustment policies have on the balance of payments and on the differential between the free and the official exchange rates. Previous models on dual exchange markets are in general not suited for examining many of these issues. For example, most previous models cannot be used to discuss transfers of current transactions between the official and free markets,

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because all current transactions are assumed to be carried out in the official market. 1/ Earlier analyses that did allow for current transactions in both markets are of a partial equilibrium nature or are inadequate for other reasons. 2/ This paper develops a model that is not subject to these limitations. Current transactions are assumed to be carried out in both the official market and the free market, while capital transactions are restricted to the free market. The official exchange rate is assumed to be either fixed or crawling at a constant rate, while the free exchange rate is assumed to be determined by market forces. The private sector allocates its wealth between two alternative assets, domestic money and foreign money, depending on the expected rate of depreciation of the free exchange rate. The evolution of the stocks of both assets is derived from the model rather than being exogenously postulated.

The model in this paper is used to discuss the effects of various policies, with the following implications. A once-and-for-all devaluation of the official exchange rate, without any accompanying policy changes, has only a transitory effect on the differential between the free and the official exchange rate and on the balance of payments. An increase in the rate of crawl of the official exchange rate, however, has permanent effects on both the differential between the exchange rates and the balance of payments, and the direction of these effects depends on the elasticity of the demand for money with respect to the expected rate of depreciation of the free exchange rate. Transfer of some export receipts from the official to the free market reduces the steady state differential between the exchange rates and worsens the balance of payments, while transfers of some import payments from the official to the free market produce the opposite results. A once-and-for-all official intervention in the free market has only transitory effects on both the differential between the exchange rates, and the balance of payments. An increase in taxes, or a reduction in public sector expenditures, improves the balance of payments and reduces the steady state differential between the exchange rates.

1/ See, for example, Argy and Porter (1972), Swoboda (1974), Flood (1978), Marion (1981), Flood and Marion (1982), Cumby (1983), Aizenman (1983), and Gardner (1984).

2/ For example, in Dornbusch (1976) the free rate has no effect in the real system and, therefore, the free rate is indeterminate in the long run. Dornbusch, et al. (1983) use a partial equilibrium model. Blejer (1978) develops a model more suited for empirical applications than for theoretical discussions. Macedo (1982) arbitrarily imposes balance of payments equilibrium in steady state and forces domestic credit creation to be consistent with that steady state solution. Nowak (1983) assumes that the authorities restrict the level of imports in the official market so as to equal the proceeds from exports surrendered in that market; thus, the balance of payments is always in equilibrium.

The rest of the paper is organized as follows. Section II develops the model of an economy with a dual exchange market system under the assumption that all goods are traded. Section III discusses the effects of various policies on the steady state differential between the free and the official exchange rates, and on the steady state balance of payments. Section IV incorporates non-traded goods and examines how the implications of the model are affected. Section V contains some concluding remarks. Appendices I and II examine some aspects of the dynamics of the model.

II. The Model

Consider a small economy that operates a dual exchange market system. The official exchange rate, e, is determined by central bank intervention and applies to some commercial transactions; and the free exchange rate, s, is determined by market forces and applies to the rest of commercial transactions and to capital transactions. 1/ The central bank decides the rule of intervention in the official market by selecting a constant rate of depreciation, π, of the official exchange rate. 2/

The central bank also decides the goods that must be exported or imported through each of the markets. It is assumed that there are only traded goods and their foreign currency prices are normalized at unity. 3/ The economy produces a continuum of goods indexed by u, $0 \leq u \leq 1$, and consumes a continuum of goods indexed by v, $0 \leq v \leq 1$. The central bank determines the coverage of the markets by selecting $\bar{u}$, which is the boundary between goods that must be exported through the free market, $0 \leq u \leq \bar{u}$, and through the official market, $\bar{u} < u \leq 1$; and by selecting $\bar{v}$, which is the boundary between goods that must be imported through the free market, $0 \leq v \leq \bar{v}$, and through the official market, $\bar{v} < v \leq 1$. It is assumed that there is no domestic expenditure on domestic output, that total domestic output is fixed at level y and evenly distributed across the goods u, $0 \leq u \leq 1$; and that total expenditure of the private sector is evenly distributed across the goods v, $0 \leq v \leq 1$. 4/ Under these assumptions, $\bar{u}$ is the proportion of total output exported through the free market, and $\bar{v}$ is the proportion of total expenditure of the private sector allocated to goods imported through the free market. 5/

1/ Exchange rates are defined in units of domestic currency per unit of foreign currency.

2/ This allows for the case of constant official exchange rate by setting $\pi = 0$.

3/ Non-traded goods are incorporated into the model in Section IV.

4/ Most of these assumptions simplify the presentation without affecting the qualitative results of the model.

5/ If the free exchange rate is substantially higher than the official exchange rate, there are incentives for overinvoicing imports and underinvoicing exports in the official market, and hence actual imports and exports in each of the markets may be different from reported imports and exports. Overinvoicing and underinvoicing, however, are not considered in this paper. Fleming (1971) and Lanyi (1975) discuss the difficulties of enforcing segmentation between the markets.

Nominal financial wealth of the private sector, W, is comprised of stocks of two non-interest bearing assets, domestic money, M, and foreign money, F. Foreign residents do not hold domestic money, so the entire stock of domestic money is held by the domestic private sector. Capital transactions are carried out in the free market, thus domestic money is exchanged for foreign money at the free exchange rate. Nominal financial wealth is equal to

$$(1) \quad W = M + sF$$

Private domestic residents have to allocate their wealth between the two available assets. Since those assets are exchanged at the free exchange rate, the fraction of wealth that domestic residents want to hold in domestic money is a decreasing function of the expected rate of depreciation of the free exchange rate. Assuming that private domestic residents possess perfect foresight, wealth allocation depends on the actual rate of depreciation of the free exchange rate.

$$(2) \quad M^d = \lambda(\dot{s}/s)W \qquad 0 < \lambda < 1 \qquad \lambda' < 0$$

Assuming that private domestic residents can achieve their desired portfolio composition instantaneously, i.e., $M^d = M$, we obtain

$$(3) \quad M = \frac{\lambda(\dot{s}/s)}{1 - \lambda(\dot{s}/s)} sF$$

Equation (3) is a portfolio relationship that holds at every point in time. It indicates that the higher the rate of depreciation of the free exchange rate, the lower is the ratio of domestic money to the domestic currency value of foreign money.

The evolution of the two assets depends on the transactions carried out in both exchange markets and on the public sector budget. In the absence of central bank intervention in the free market, the change in the stock of foreign money held by the private sector, $\dot{F}$, is equal to the balance of the commercial transactions carried out in the free market. 1/ Exports in the free market are equal to a fraction $\bar{u}$ of total output, and

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1/ Most of the previous models on dual exchange markets asume that no current transactions are carried out in the free market. Under that assumption, $\dot{F} = 0$, the stock of foreign assets held by the private sector remains constant at the level prevailing at the time the dual system was adopted.

imports in the free market are equivalent to a fraction $\bar{v}$ of total expenditure of the private sector. Assuming that total nominal expenditure of the private sector is a fixed proportion, a, of nominal financial wealth, we obtain

$$(4) \quad \dot{F} = \bar{u}y - \frac{\bar{v}a(M+sF)}{s}$$

Abstracting from the banking system, the change in the money stock, $\dot{M}$, is equal to the change in central bank domestic credit, $\dot{D}$, plus the change in the domestic currency value of international reserves, generated by central bank intervention in the official exchange market, $\dot{R}$.

$$(5) \quad \dot{M} = \dot{R} + \dot{D}$$

Changes in domestic credit are assumed to be the result of public sector deficits or surpluses. It is assumed that the public sector buys g units of goods, all of which are imported through the official market, and that real taxes in terms of goods channeled through the official market are constant at level t.

$$(6) \quad \dot{D} = e(g - t)$$

The change in international reserves is equal to the balance of the transactions in the official market. 1/ Since a fraction $(1 - \bar{u})$ of output is exported through the official market and a fraction $(1 - \bar{v})$ of total expenditure of the private sector is allocated to goods imported through the official market, we obtain

$$(7) \quad \dot{R} = e(1 - \bar{u})y - (1 - \bar{v})a(M + sF) - eg$$

From (5), (6), and (7),

$$(8) \quad \dot{M} = e(1 - \bar{u})y - (1 - \bar{v})a(M + sF) - et$$

1/ It is assumed that the central bank does not monetize changes in the domestic currency value of international reserves arising from changes in the official exchange rate.

Equations (3), (4), and (8) determine the dynamics of the three endogenous variables, s, F, and M. Since the official exchange rate is changing continuously for $\pi \neq 0$, all the variables expressed in terms of domestic currency will change at the rate π in steady state. Therefore, it is useful to rewrite equations (3), (4), and (8) in terms of $m = (M/e)$ and $d = (s/e)$, to obtain

$$(9) \quad m = \frac{\lambda(\frac{\dot{d}}{d} + \pi)}{1 - \lambda(\frac{\dot{d}}{d} + \pi)} dF$$

$$(10) \quad \dot{F} = \bar{u}y - \bar{v}a(\frac{m}{d} + F)$$

$$(11) \quad \dot{m} = (1 - \bar{u})y - (1 - \bar{v})a(m + dF) - t - m\pi$$

where m is the real stock of money in terms of goods channeled through the official market and d measures the differential between the free and the official exchange rates. Equations (9), (10), and (11) determine the dynamics of d, F, and m. As is usual in models that assume perfect foresight, the system exhibits saddle-point stability. 1/ The steady state values of the endogenous variables, denoted by m*, F*, and d*, are the following

$$(12) \quad m^* = \frac{[(1 - \bar{u})y - t]\lambda(\pi)}{a(1 - \bar{v}) + \pi\lambda(\pi)}$$

$$(13) \quad F^* = \frac{\bar{u}y(1 - \lambda(\pi))}{a\bar{v}}$$

$$(14) \quad d^* = \frac{a\bar{v}[(1 - \bar{u})y - t]}{[a(1 - \bar{v}) + \pi\lambda(\pi)]\bar{u}y}$$

In steady state, the real stock of domestic money, the stock of foreign money and the differential between the exchange rates are constant. 2/ The nominal stock of domestic money and the free exchange rate increase at the rate π. Prices of all goods also increase at the same rate.

1/ This is shown in Appendix I.

2/ From equation (14) it is clear that d* could be lower than one; that is, the free exchange rate could be lower than the official exchange rate. Although the model allows for this possibility, we discuss the case of d* higher than one because it is empirically more relevant.

The steady state balance of payments, in terms of foreign currency, is equal to

$$(15) \quad (\dot{R}/e)^* = t - g + m^*\pi$$

Thus, the steady state balance of payments surplus or deficit is equal to the public sector surplus or deficit plus the steady state inflation tax. Since a country possesses only a finite stock of international reserves, the dual system is not sustainable indefinitely if there is a public sector deficit which is higher than the steady state inflation tax. If a country in this situation does not implement policies that improve its balance of payments, its international reserves will eventually be depleted and the central bank will not be able to continue intervening in the official foreign exchange market. 1/ In addition, equation (15) implies that policy changes, other than changes in public sector expenditure or taxes, have a permanent effect on the balance of payments only if they affect the steady state inflation tax.

III. Effects of Policy Changes

This section discusses the effects of various policy changes on the steady state differential between the free and the official exchange rates and on the steady state balance of payments. 2/ We start with devaluation policy.

It is necessary to distinguish between a once-and-for-all devaluation and an increase in the rate of devaluation of the official exchange rate, whether the initial situation is a fixed official exchange rate ($\pi = 0$) or not ($\pi > 0$). A once-and-for-all devaluation has no permanent effect

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1/ In this sense it is not possible to have a "steady-state" balance of payments deficit. However, we will continue using this term for expositional convenience, and we will continue using equation (15) even for balance of payments deficits because it is a useful point of reference for discussing the effects of various policies.

2/ The dynamics of the variables from one steady state to another cannot be determined unambiguously. Since an analytical expression for the characteristic roots of the system could not be obtained, it is not possible to know whether all the roots are real for every admissible value of the parameters of the model, and therefore, whether the solutions always converge monotonically to the new steady state. Numerical solutions of the system, however, were obtained for a wide variety of parameter values and in every case the three roots were real. Appendix II illustrates the dynamics of the balance of payments and the differential between the exchange rates, for some particular values of the parameters, when the economy is subject to the policy changes discussed in this section.

on either the differential between the free and the official exchange rate or the balance of payments. From equations (14) and (15), the steady state values of d and $(\dot{R}/e)$ are independent of the level of the official exchange rate. However, a once-and-for-all devaluation may initially reduce the differential between the exchange rates and improve the balance of payments. Since a once-and-for-all devaluation immediately reduces the real stock of domestic money and does not affect the stock of foreign money, from equation (9), there would be a portfolio disequilibrium if the differential between the exchange rates were to remain at the original level. The share of foreign currency in private sector wealth would be higher than desired. Portfolio equilibrium is maintained through two changes. First, the differential between the exchange rates falls immediately, reducing the actual share of foreign money in private sector wealth. Second, since the differential between the exchange rates starts rising from its new lower level, the desired share of foreign money in private sector wealth increases. 1/ The initial fall in m and d causes a decline of real imports in the official market and therefore improves the balance of payments. However, all these changes are transitory, they last until the real stock of domestic money, which increases due to the improvement in the balance of payments and the reduction in the base for the inflation tax, returns to its steady state level. 2/ In the new steady state the differential betwen the free and the official exchange rates and the balance of payments are the same as in the original steady state.

An increase in the rate of devaluation of the official exchange rate, on the other hand, has permanent effects on both the differential between the exchange rates and the balance of payments. These effects arise from the change in portfolio composition that is induced by the increase in the rate of devaluation of the official exchange rate. Since an increase in π implies that the steady state rate of depreciation of the free exchange rate also increases, the private sector reallocates its portfolio out of domestic money and into foreign money. This portfolio reallocation affects the steady state stock of both assets and their relative price. The direction of the effects of an increase in π on the steady state differential between the exchange rates and on the balance of payments depend on the elasticity of the demand for money with respect to the expected rate of depreciation of the free exchange rate. 2/ From equation (14), the effect on the steady state differential between the exchange rates is equal to

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1/ Since both changes work in the direction of maintaining portfolio equilibrium, the effect of the first change alone falls short of offsetting the initial fall in m, thus, the initial fall in d is proportionally lower than the initial fall in m. This implies that at the time of the devaluation the free exchange rate increases, although proportionally less than the official exchange rate.

2/ See equation (11).

3/ The elasticity of the demand for money, $M = \lambda(\dot{s}/s)W$, is in steady state equal to $\eta = -\pi\lambda'(\pi)/\lambda(\pi)$.

$$(16) \quad \Delta d^* = \frac{d^*\lambda(\pi)}{a(1-\bar{\nu}) + \pi\lambda(\pi)}(\eta-1) \quad \Delta\pi \quad \gtrless 0$$

where the symbol Δ before a variable denotes the change in that variable. If the elasticity of the demand for money is lower than one the differential between the exchange rate declines, and if the elasticity of the demand for money is higher than one the differential increases. From equation (15), the effect of an increase in π on the steady state balance of payments is equal to its effect on the inflation tax, which is given by

$$(17) \quad \Delta(\dot{R}/e)^* = \frac{m^*a(1-\bar{\nu})}{a(1-\bar{\nu}) + \pi\lambda(\pi)}(1-\eta) \quad \Delta\pi \quad \gtrless 0$$

If the elasticity of the demand for money is lower than one the steady state inflation tax increases and the steady state balance of payments improves, and if the elasticity of the demand for money is higher than one the steady state inflation tax declines and the steady state balance of payments worsens. Equations (16) and (17) imply that there is a limit until which an increase in the rate of devaluation of the official exchange rate improves the steady state balance of payments and reduces the steady state differential between the free and the official exchange rates. Once the demand for money becomes elastic, an increase in the rate of devaluation of the official exchange rate worsens the steady state balance of payments increases the steady state differential between the exchange rates.

Changes in the coverage of the markets also have permanent effects on the differential between the free and the official exchange rates and on the balance of payments. These effects arise because the transfer of exports or imports from one of the exchange markets to the other changes the process of accumulation of the two assets, favoring the accumulation of foreign money if exports are increased or imports are reduced in the free market, and vice versa. This changes the steady state stock of both assets and their relative price. A transfer of exports from the official to the free market, an increase in $\bar{u}$, reduces the steady state differential between the exchange rates and worsens the steady state balance of payments. The effect on the steady state differential between exchange rates is equal to

$$(18) \quad \Delta d^* = -\frac{d^*(y-t)}{\bar{u}[(1-\bar{u})y - t]} \quad \Delta\bar{u} \quad < 0$$

The effect on the steady state balance of payments, obtained by differentiating $m^*\pi$ with respect to $\bar{u}$, is equal to

$$(19) \quad \Delta(R/e)^* = -\frac{\lambda(\pi)\pi}{a(1-\bar{\nu}) + \lambda(\pi)\pi} \quad \Delta\bar{u}y \quad \leq 0$$

The worsening of the balance of payments is due to a reduction in the steady state real stock of domestic money and, hence, a reduction in the steady state inflation tax. Since when $\bar{u}$ is increased by $\Delta\bar{u}$ the amount of exports transferred from the official to the free market is $\Delta\bar{u}y$, (19) implies that the worsening of the steady state balance of payments is smaller than the amount of exports transferred from the official to the free market. Furthermore, if the official exchange rate is constant ($\pi = 0$), there is no effect on the steady state balance of payments, because in that case the inflation tax is always equal to zero. 1/

A transfer of imports from the official to the free market, an increase in $\bar{v}$, increases the steady state differential between the exchange rates and improves the steady state balance of payments. The effect on the steady state differential between the exchange rates is equal to

$$(20) \quad \Delta d^* = d^* \frac{a + \pi\lambda(\pi)}{[a(1 - \bar{v}) + \pi\lambda(\pi)]\bar{v}} \Delta\bar{v} \qquad > 0$$

The effect on the steady state balance of payments, obtained by differentiating $m^*\pi$ with respect to $\bar{v}$, is equal to

$$(21) \quad \Delta(\dot{R}/e)^* = \frac{\lambda(\pi)\pi}{a(1 - \bar{v}) + \lambda(\pi)\pi} \Delta\bar{v}a(m^* + d^*F^*) \qquad > 0$$

where m^*, d^*, and F^* are the steady state values of the variables at the time of the transfer of imports. The improvement in the balance of payments is due to an increase in the steady state real stock of money and, hence, an increase in the steady state inflation tax. Since when $\bar{v}$ is increased by $\Delta\bar{v}$ the amount of imports originally transferred from the official to the free market is equal to $\Delta\bar{v}a(m^* + d^*F^*)$, (21) implies that the improvement in the steady state balance of payments is smaller than the amount of imports originally transferred from the official to the free market. Furthermore, if the official exchange rate is constant there is no effect on the steady state balance of payments. 1/

A once-and-for-all official intervention in the free market has no permanent effect on either the differential between the free and the official exchange rate or the balance of payments. 2/ Since official intervention in the free market initially changes the composition of the

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1/ If foreign inflation were positive there would be an effect on the steady state balance of payments even under fixed official exchange rate.

2/ Official intervention in the free market refers here to a once-and-for-all official purchase or sale of foreign exchange in the free market. It is obvious that continuous official intervention in the free market would have permanent effects, but such policy implies an exchange system different from the one analyzed in this paper.

private sector portfolio but does not affect the determinants of the steady state portfolio composition, there is no permanent effect in any of the variables. Official intervention in the free market, however, may temporarily reduce the differential between the exchange rates. Official sales of foreign money in exchange for domestic money in the free market increases F and reduces m immediately. From equation (9), at the initial differential between the exchange rates there is an excess of foreign money with respect to domestic money. Therefore, the differential between the exchange rates falls immediately and starts rising from its new lower level; these two changes restore portfolio equilibrium. However, this sets into motion a process of adjustment in the stock of the two assets until the initial steady state equilibrium is restored.

Regarding the steady state effect of fiscal policy, while changes in public sector expenditures affect only the balance of payments, changes in taxes affect both the balance of payments and the differential between the exchange rates. Since in this model an increase of Δg in public sector expenditure only means Δg additional imports through the official market, it worsens the balance of payments by Δg and has no effect on any other variable. 1/ An increase in taxes by Δt, on the other hand, improves the balance of payments and reduces the differential between the free and the official exchange rates. The effect on the steady state differential between the exchange rates is equal to

$$(22) \quad \Delta d^* = -\frac{d^*}{(1-\bar{u})y - t}\,\Delta t \qquad < 0$$

The effect on the steady state balance of payments, obtained by differentiating equation (15), is given by

$$(23) \quad \Delta(\dot{R}/e)^* = \left(1 - \frac{\lambda(\pi)\pi}{a(1-\bar{v}) + \lambda(\pi)\pi}\right)\Delta t \qquad > 0$$

The net effect on the balance of payments comprises a direct improvement due to a reduction in the public sector deficit, and an indirect worsening due to a reduction in the proceeds from the inflation tax caused by a reduction in the real stock of domestic money. Since this indirect worsening is smaller than the direct improvement, the steady state balance of payments improves.

1/ This result has to be modified in the presence of non-traded goods, as discussed in the next section.

IV. Non-traded Goods

This section incorporates non-traded goods into the model, and shows that the conclusions regarding the qualitative effects of the various policies discussed in the previous section remain valid, with the exception of the effects of an increase in public sector expenditure. This section also examines the determinants of the relative price between traded and non-traded goods, which sometimes is referred to as the "real" exchange rate.

Non-traded goods are incorporated with a minimum of modifications in the structure of the model. The supply of non-traded goods is assumed to be fixed at a level denoted by y_N. 1/ The demand for non-traded goods originates in both the private sector and the public sector. Private sector expenditure on non-traded goods is a constant fraction, α, of total expenditure. The remaining fraction, $(1-\alpha)$, is evenly distributed across the importable goods. As before, total nominal expenditure of the private sector is a fixed proportion, a, of nominal financial wealth. Therefore, the private sector demand for non-traded goods is equal to $\alpha a(m + dF)r$ where $r = (e/p_N)$ is the real exchange rate and p_N is the domestic currency price of non-traded goods. 2/

Public sector expenditure on non-traded goods is a constant fraction, β, of total expenditure. As before, total public sector expenditure and taxes are assumed to be fixed in terms of goods channeled through the official market, and the public sector deficit is assumed to be financed by domestic credit creation. Hence,

$$(24) \quad \dot{D} = e(g-t)$$

$$(25) \quad g_T = (1-\beta)g$$

$$(26) \quad g_N = \beta g r$$

where g is total public sector expenditure, g_T is the public sector demand for traded goods, and g_N is the public sector demand for non-traded goods. Equilibrium in the non-traded goods market requires

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1/ In order to simplify the analysis, relative price effects on the supplies of traded and non-traded goods are ignored.

2/ Under dual exchange markets there are two relative prices between traded and non-traded goods, (e/p_N) and (s/p_N). The real exchange rate, r, in this paper refers to the relative price of traded goods channeled through the official market. The relative price of traded goods channeled through the free market is equal to rd.

$$(27) \quad y_N = [\alpha a(m + dF) + \beta g]r$$

It is assumed that the non-traded goods market always clears; thus, equation (27) holds continuously.

Under the new assumptions, the dynamics of the system are determined by

$$(28) \quad m = \frac{\lambda(\frac{\dot{d}}{d} + \pi)}{1 - \lambda(\frac{\dot{d}}{d} + \pi)} dF$$

$$(29) \quad \dot{F} = \bar{u}y_T - (1 - \alpha)\bar{v}a(\frac{m}{d} + F)$$

$$(30) \quad \dot{m} = (1 - \bar{u})y_T - (1 - \alpha)(1 - \bar{v})a(m + dF) + \beta g - t - m\pi$$

where y_T denotes the total production of exportable goods.

The steady state values of the endogenous variables are

$$(31) \quad m^* = \frac{[(1 - \bar{u})y_T + \beta g - t]\lambda(\pi)}{(1 - \alpha)a(1 - \bar{v}) + \pi\lambda(\pi)}$$

$$(32) \quad F^* = \frac{\bar{u}y_T(1 - \lambda(\pi))}{(1 - \alpha)a\bar{v}}$$

$$(33) \quad d^* = \frac{(1 - \alpha)a\bar{v}[(1 - \bar{u})y_T + \beta g - t]}{[(1 - \alpha)a(1 - \bar{v}) + \pi\lambda(\pi)]\bar{u}y_T}$$

$$(34) \quad (\dot{R}/e)^* = t - g + m^*\pi$$

$$(35) \quad r^* = \frac{y_N}{\frac{\alpha a[(1 - \bar{u})y_T + \beta g - t]}{(1 - \alpha)a(1 - \bar{v}) + \pi\lambda(\pi)} + \beta g}$$

From comparing (31), (32), (33), and (34) with (12), (13), (14), and (15) it is clear that the conclusions regarding the qualitative effects on m^*, F^*, d^* and $(\dot{R}/e)^*$ of the various policies that were examined in the previous section remain unchanged, with one exception. Under the new assumptions, an increase of Δg in public sector expenditure increases the differential between the exchange rates and worsens the balance of payments by less than Δg. Under the previous assumptions, an increase of Δg in public sector expenditure did not affect the differential between the exchange rates and worsened the balance of payments by Δg. The effects are now different because an increase in public sector expenditure that falls at least partially on non-traded goods increases the steady state stock of domestic money. As a result, d^* increases in order to maintain portfolio equilibrium, and the inflation tax increases, partially offsetting the direct worsening of the steady state balance of payments.

The effects of various policies on the steady state real exchange rate can be obtained from (35). While a once-and-for-all devaluation of the official exchange rate has no effect on r^*, an increase in the rate of crawl of the official exchange rate depreciates the real exchange rate if the elasticity of the demand for money is lower than one and appreciates the real exchange rate if the elasticity of the demand for money is higher than one. Regarding fiscal policy, an increase in public sector expenditures appreciates the real exchange rate, while an increase in taxes depreciates it.

With respect to the dynamics of the system, it is clear from (28), (29), and (30) that r does not intervene in the determination of $\dot{d}$, $\dot{m}$, and $\dot{F}$. Hence the qualitative results of the previous discussion regarding the dynamics of d, m, F, and $(\dot{R}/e)$ still hold, with the exception of the effects of a change in public sector expenditure, as mentioned above. The dynamics of the real exchange rate can be obtained from equation (27) for any of the policy changes discussed before. For example, since a once-and-for-all devaluation of the official exchange rate reduces m and causes a drop in d, the demand for non-traded goods declines, and the real exchange rate depreciates immediately in order to maintain equilibrium in the non-traded goods market. This effect, however, is only transitory. As m and d increase and return to their original steady state levels, the real exchange rate appreciates and also returns to its original level.

V. Concluding Remarks

The various policies examined in this paper can be classified according to the effectiveness in attaining the two external objectives that are usually included in stabilization programs in countries under dual exchange markets: an improvement in the balance of payments and a reduction in the differential between the free and the official exchange rate. First, a less expansionary fiscal policy, either through higher taxes or lower public sector expenditure, attains both objectives

simultaneously. Second, transfers of current transactions between markets attain one of the objectives but have adverse consequences with respect to the other. For example, a transfer of imports from the official to the free market will improve the balance of payments but it will also increase the differential between the exchange rates. In addition, the permanent effect on the balance of payments is smaller than the initial amount of transactions transferred between the markets. Third, an increase in the rate of crawl of the official exchange rate has desirable (undesirable) effects with respect to both objectives simultaneously if the elasticity of the demand for money with respect to the expected rate of depreciation of the free exchange rate is lower (higher) than one. This also implies that there may be a limit to the reduction in the differential between the exchange rates and the improvement in the balance of payments that can be obtained by changing the rate of crawl. Once the rate of crawl is such that the elasticity of the demand for money is equal to one, a change in the rate of crawl in either direction will produce undesirable results. Finally, a once-and-for-all devaluation of the official exchange rate and a once-and-for-all official intervention in the free market can help to attain both objectives only temporarily. These policies will have no permanent effects unless they are accompanied by other, appropriate, policies.

Appendix I

This Appendix shows that the system formed by equations (9), (10), and (11) exhibits saddle-point stability. In order to simplify the notation, the argument of the function $\lambda(\pi)$ is omitted. Linearizing the system around the steady state, $d = d^*$, $F = F^*$ and $m = m^*$, we obtain

$$\begin{vmatrix} \dot{d} \\ \dot{F} \\ \dot{m} \end{vmatrix} = \begin{vmatrix} E & G & -H \\ I & -J & -K \\ -L & -N & -P \end{vmatrix} \begin{vmatrix} d - d^* \\ F - F^* \\ m - m^* \end{vmatrix}$$

where E, G, H, I, J, K, L, N, and P are positive constants defined as follows:

$$E = \frac{-\lambda(1-\lambda)}{\lambda'} \qquad G = \frac{-\lambda(1-\lambda)d^*}{\lambda' F^*} \qquad H = \frac{-(1-\lambda)^2}{\lambda' F^*}$$

$$I = \frac{a\bar{v}m^*}{d^{*2}} \qquad J = a\bar{v} \qquad K = \frac{a\bar{v}}{d^*}$$

$$L = a(1-\bar{v})F^* \qquad N = a(1-\bar{v})d^* \qquad P = a(1-\bar{v}) + \pi$$

The characteristic equation of the system is given by

$$(A1) \quad x^3 + Bx^2 + Cx + D = 0$$

where

$$(A2) \quad B = J + P - E \gtrless 0$$

$$(A3) \quad C = PJ - EP - EJ - LH - NK - GI \gtrless 0$$

$$(A4) \quad D = NKE - EJP - LHJ - GIP - GKL - HIN < 0$$

To obtain a unique perfect foresight path, the dimension of the convergent subspace must be equal to the number of predetermined variables. 1/

1/ See Begg (1982).

Since we have two predetermined variables, F and m, one characteristic root must be real and positive and the other two must have negative real parts.

The signs of the real parts of the characteristic roots can be obtained from the following three conditions:

$$(A5) \quad x_1x_2x_3 = -D \qquad (> 0 \text{ from (A4)})$$

$$(A6) \quad x_1 + x_2 + x_3 = -B \qquad (\gtrless 0 \text{ from (A2)})$$

$$(A7) \quad x_1x_2 + x_2x_3 + x_1x_3 = C \qquad (\gtrless 0 \text{ from (A3)})$$

where x_1, x_2, and x_3 are the roots of equation (A1). 1/

From (A5) it follows that there is at least one positive real root and that the real parts of the other two additional roots must have the same sign. If $B > 0$, it follows from (A6) that the sign of the real parts of the two additional roots cannot be positive. If $B < 0$, it can be shown that $C < 0$. In that case, it follows from (A7) that the sign of the real part of the two additional roots cannot be positive. Therefore, there is one positive real root and the other two roots have negative real parts; the system exhibits saddle-point stability.

1/ See Allen (1960).

Appendix II

This appendix illustrates the dynamics of the balance of payments and the differential between the exchange rates when the economy is subject to various unanticipated policy changes. The examples presented here, which result from numerical solutions of the system formed by equations (9), (10), and (11) for particular values of the parameters, should not be interpreted as an estimation of the quantitative effects of those policies but as an indication of their qualitative effects.

In each of the examples, the initial steady state assumes the following parameters: $y = 100$, $t = 10$, $g = 25$, $\bar{u} = 0.4$, $\bar{v} = 0.6$, $a = 4$, $\pi = 0.4$, $\lambda = 0.9$, and $\lambda' = -0.5$. The initial steady state values of the endogenous variables are: $m^* = 22.96$, $F^* = 1.67$, $d^* = 1.53$ and $(\dot{R}/e)^* = -5.82$.

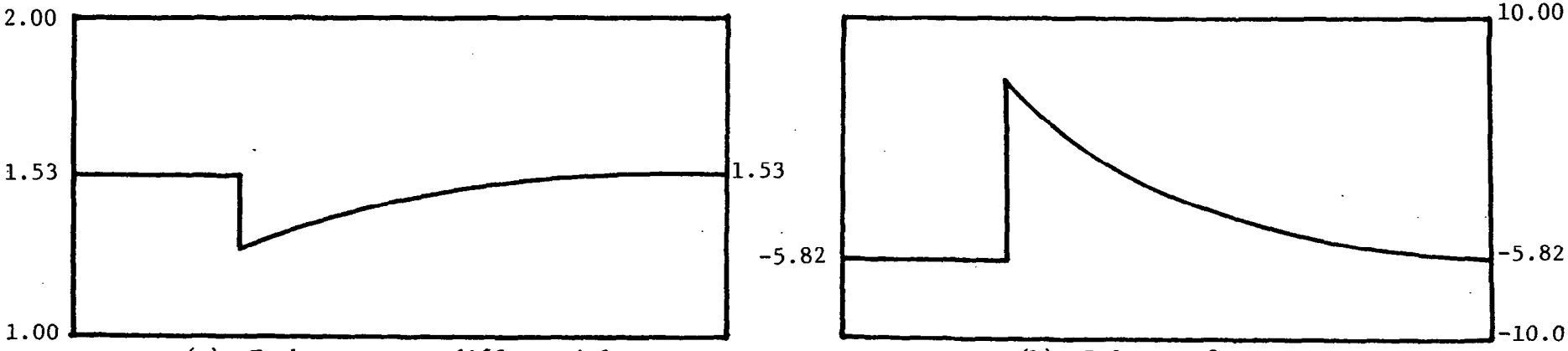
Figure 1 shows the effects of a once-and-for-all 33 percent devaluation of the official exchange rate. Although initially the balance of payments improves and the differential between the exchange rates declines, there is no permanent effect on either of those two variables.

Figure 2 describes the effects of incresing by 10 percent the rate of devaluation of the official exchange rate, π increases from 0.4 to 0.5. The steady state balance of payments improves and the steady state differential between the exchange rates declines because the demand for money is inelastic. Since at the initial steady state $\lambda = 0.9$, $\lambda' = -0.5$, and $\pi = 0.4$, the elasticity of the demand for money is $\eta = 0.22$.

Figure 3 shows the effects of transferring 5 percent of total exports from the official to the free market, $\bar{u}$ increases from 0.4 to 0.45. The steady state differential between the exchange rates declines and the steady state balance of payments worsens. However, the steady state balance of payments worsening, 0.91, is lower than the amount of exports transferred to the free market, 5.00.

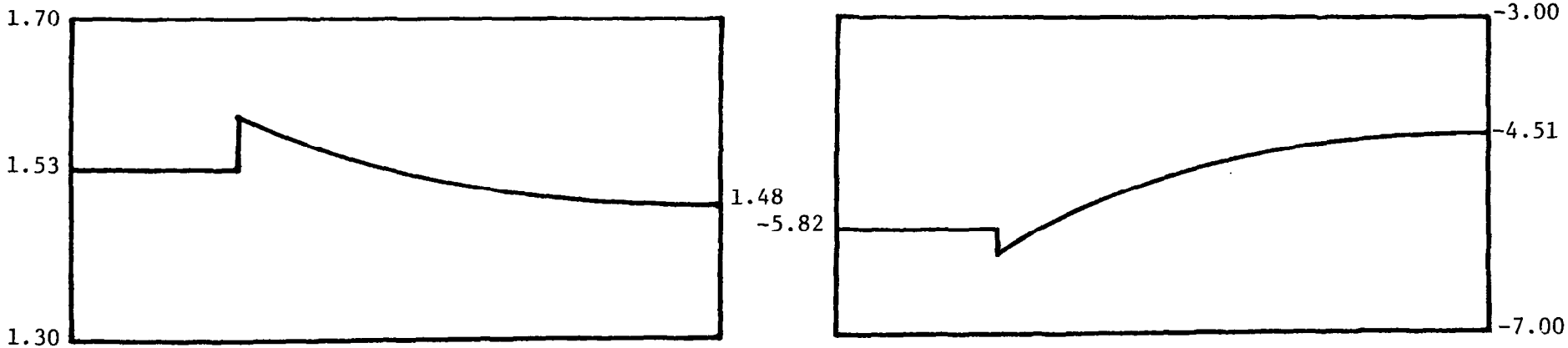
Figure 4 shows the effects of transferring 5 percent of the original private expenditure in imports from the official to the free market, $\bar{v}$ increases from 0.6 to 0.65. The steady state differential between the exchange rates increases and the steady state balance of payments improves. However, the steady state balance of payments improvement, 1.05, is lower than the amount of imports initially transferred to the free market, 5.10.

Figure 5 describes the effects of an official sale of foreign money in the free market by an amount equivalent to 20 percent of the steady state stock of foreign money, F increases exogenously from 1.67 to 2.00 and m declines exogenously from 22.96 to 22.45. Although initially the balance of payments improves and the differential between the exchange rates declines, there is no permanent effect on either of those two variables. It also has to be mentioned that Figure 5 does not capture the initial decline in international reserves of the Central Bank caused by the official sale of foreign money in the free market.



(a) Exchange rates differential (b) Balance of payments

Figure 1. A once-and-for-all devaluation of the official exchange rate



(a) Exchange rates differential (b) Balance of payments

Figure 2. An increase in the rate of devaluation of the official exchange rate

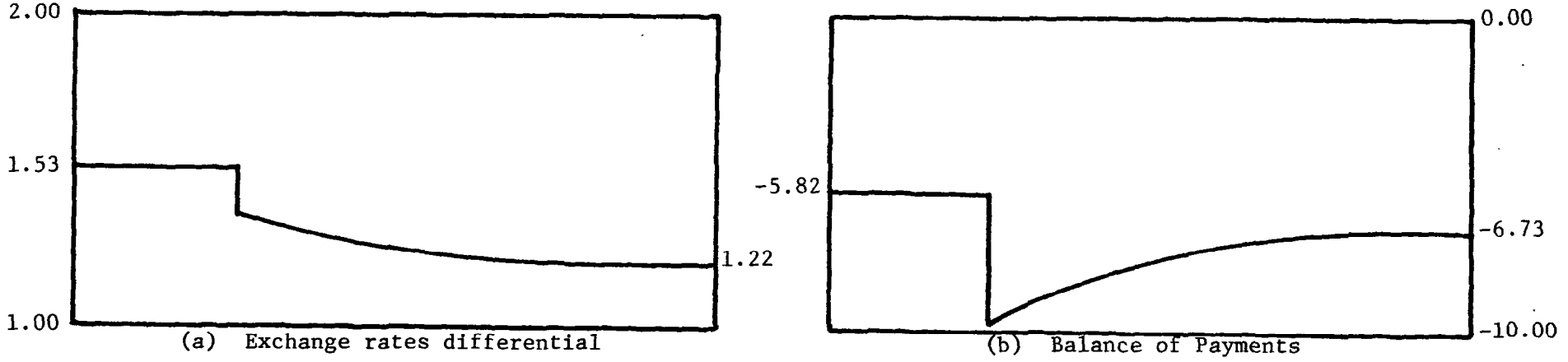


Figure 3. A transfer of exports from the official to the free market

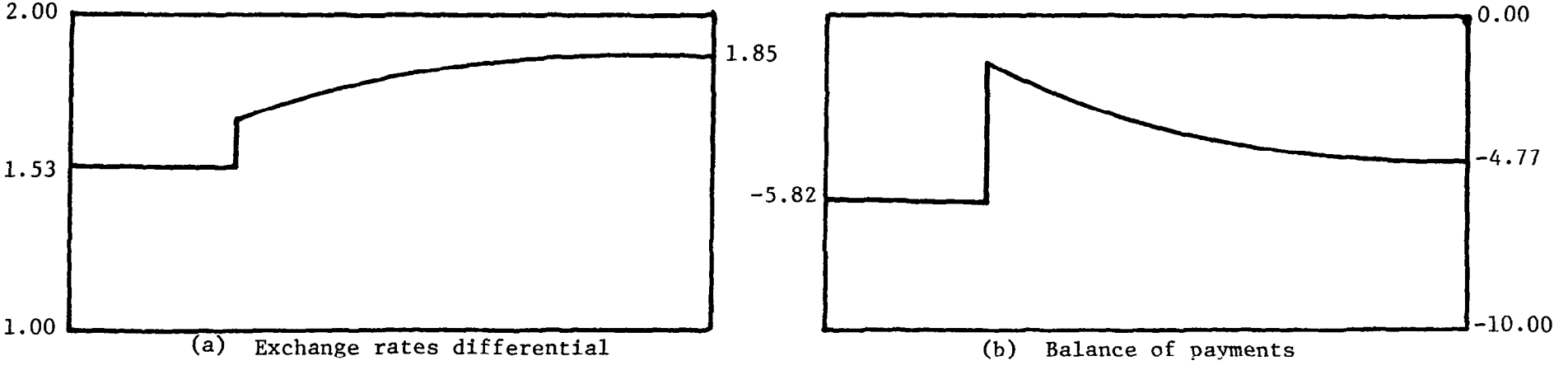
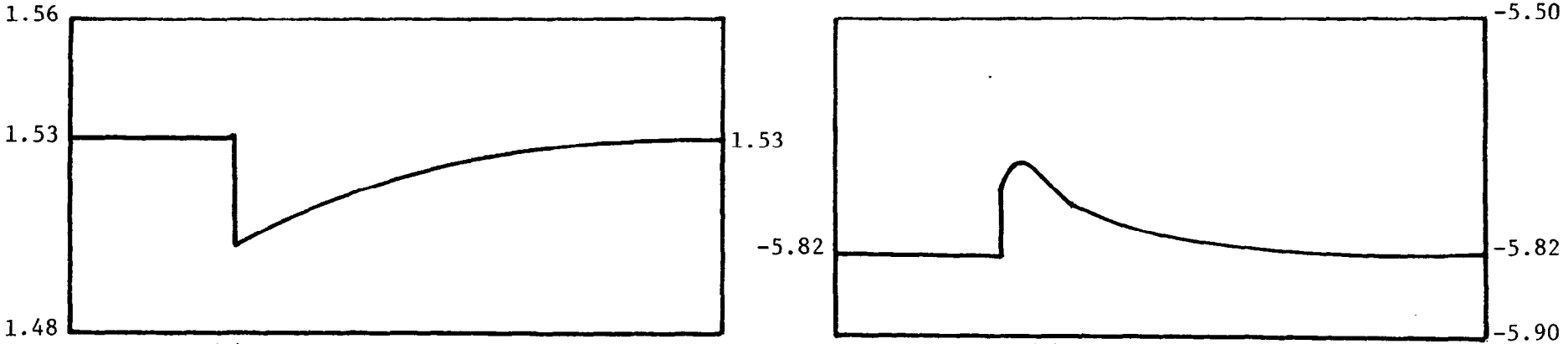
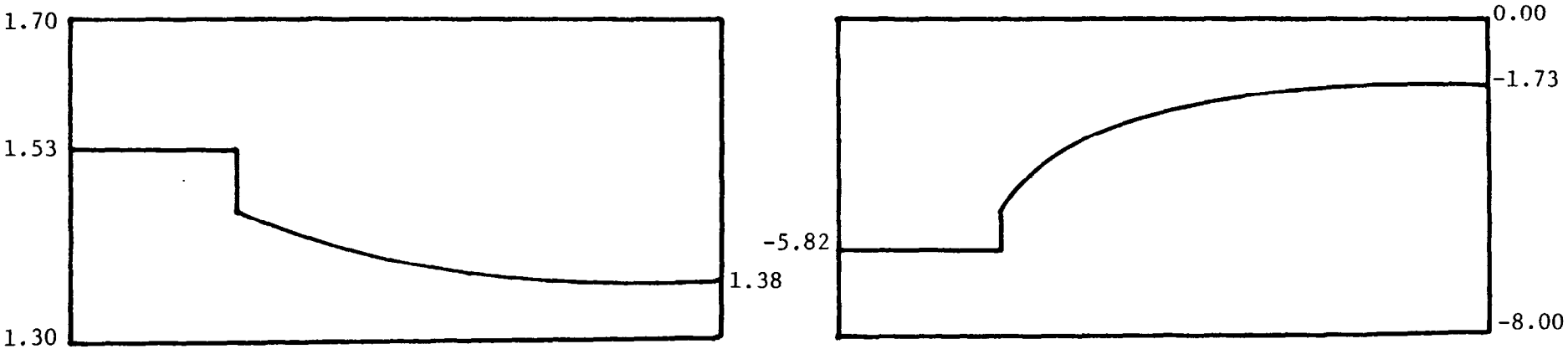


Figure 4. A transfer of imports from the official to the free market



(a) Exchange rates differential (b) Balance of payments

Figure 5. An official sale of foreign exchange in the free market



(a) Exchange rates differential (b) Balance of payments

Figure 6. An increase in real taxes

Figure 6 shows the effects of increasing real taxes by an amount equivalent to 5 percent of real income, t increases from 10 to 15. The steady state differential between the exchange rates declines and the steady state balance of payments improves. However, the steady state balance of payments improvement, 4.09, is lower than the increase in taxes, 5.00.

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