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Quantitative Controls and Unofficial Markets in Foreign Exchange: A Theoretical Framework

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I. Introduction and Summary

The purpose of this paper is to examine the effects of monetary, fiscal, and exchange rate policy in a dual exchange rate regime which has arisen in response to the imposition of quantitative controls on current transactions. A number of results are obtained which suggest that the standard open economy model provides a misleading guide for policy under such an exchange regime.

Multiple exchange rate regimes have been a common feature of the international monetary system since the 1950s. While the characteristics of these regimes have varied widely across countries and over time, attention in the literature has generally been focused on the dual arrangements adopted by a number of developed countries as a means of providing insulation from large and potentially volatile capital flows. 1/ Typically, this type of arrangement is characterized by an official market in which current transactions are conducted at a fixed or managed exchange rate, and a financial market for capital transactions in which the exchange rate is freely determined. While exchange controls are necessary in order to segregate the two markets, quantitative restrictions on current payments in the official market are usually limited, if not altogether absent. Consequently, the official market in this type of regime clears through some combination of reserve intervention and exchange rate adjustment.

This paper focuses on a second broad type of multiple exchange arrangement which has attracted rather less interest, despite its prevalence, particularly among less developed economies. It is distinguished from the first by the presence of quantitative trade and exchange controls in the official market which are designed to protect depleted reserve positions and avoid unpalatable official exchange rate adjustments in the face of balance of payments pressures. Under such a regime, there exists no mechanism, such as reserve movements, which ensures that all prevailing demand for foreign exchange at the official exchange rate is met and that the official market clears. If the costs of engaging in illegal transactions are not prohibitive, the excess demand for foreign exchange in the official market will be satisfied at a premium price in a secondary or parallel market. Thus, for example, the same imported good may be purchased simultaneously at two different exchange rates.

A number of writers have recognized that the standard model of a restriction-free economy may not be applicable to an economy in which there is rationing in the official foreign exchange market. 2/ However, little progress has been made in incorporating quantitative restrictions

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1/ For example, the United Kingdom (1947-79), Belgium (since 1951), France (1971-74), Italy (1973-74), and the Rand Monetary Area (1974-1983).

2/ For example, Krueger has stated that "...exchange rate changes under QR's [quantitative restrictions] cannot be analyzed in exactly the same manner as would a devaluation undertaken from an initial position of currency convertibility" (p. 55, 1978). See also Bhagwati (1978).

into a general-equilibrium open economy framework. 1/ In developing such a framework, this paper presents a number of results which differ significantly from those associated with the standard model. Consequently, reliance on the standard model may give rise to erroneous policy prescriptions. 2/

For example, it is demonstrated that, under a reasonably plausible set of assumptions, an official devaluation of the domestic currency will have no inflationary impact; indeed, under certain conditions, the price level may actually fall. Furthermore, for a given level of reserve intervention by the central bank, it is shown that an official devaluation will lead to an appreciation of the domestic currency in the parallel market and that changes in domestic credit and the official exchange rate may have permanent real effects.

The remainder of this paper is structured as follows: Section II presents a partial-equilibrium pure-flow model of the foreign exchange market which incorporates smuggling; Section III extends the analysis to a general-equilibrium framework which features currency substitution and rationally formed exchange rate expectations; Section IV examines

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1/ Dickie and Noursi (1975) estimate a model of the Syrian parallel foreign exchange market but, while acknowledging the existence of formal trade controls, do not explicitly take them into account. Bhagwati (1967) builds a simple flow model of the parallel market but restricts attention to an examination of the impact of illegal transactions on the officially recorded and actual balance of payments. Dornbusch et al (1983) have constructed a portfolio balance/currency substitution model of the black market for foreign exchange in Brazil, but the partial-equilibrium approach adopted precludes an examination of the economy-wide impact of policy shifts in the presence of quantitative controls. See also Lorie (1983) for a partial-equilibrium analysis of the black market under conditions of uncertainty.

2/ This conclusion is applicable not only to unified exchange rate models but to dual market models in which the official foreign exchange market clears through movements in central bank reserves. For example, Argy and Porter (1972) and Swoboda (1974) use a pure-flow model, and Cumby (1983) employs a portfolio balance framework to examine the implications of segregating current and capital transactions into fixed and floating rate markets. The short-run insulation properties of this type of dual system are dealt with in taxonomic fashion by Marion (1981). See also Flemming (1974), Flood (1977), Blejer (1978), and Flood and Marion (1981). With regard to the effectiveness of monetary policy under conditions of high capital mobility, this class of model essentially generates results which correspond to the basic flexible unified exchange rate model in the short run and the fixed rate model in the long run. The long-run effect of a devaluation in all these models is a once-and-for-all increase in reserves and an equi-proportionate rise in all prices. Leakages between the two markets as a result of illegal transactions weaken, but do not qualitatively alter the basic properties of the dual market model; see for example Lanyi (1975) and Macedo (1982).

the implications of the model for exchange rate policy; and Section V summarizes the paper's major conclusions.

II. The Foreign Exchange Market in a Partial-Equilibrium Setting

The economy in question is small in the sense that it cannot influence its external terms of trade (in foreign currency terms). It consists of a private sector and an official sector. The private sector produces a nontraded good and an export good, with a variable factor, labor, which is geographically mobile and flexible in price. It also consumes two goods, the nontraded good and an imported good; all three goods are nonstoreable.

The official sector comprises a government and a central bank. The central bank operates an official foreign exchange market in which all transactions are conducted at a fixed price. Both the government and the private sector participate in this market. Producers of the export good are legally obliged to sell their output to the government, which in turn surrenders the foreign exchange proceeds to the central bank. The central bank also sells foreign exchange for the purchase of the imported good. However, exchange control regulations are in force which restrict the availability of foreign exchange for the purchase of the imported good. The excess demand thus created is satisfied in a secondary or parallel market. The source of foreign exchange to this market consists of proceeds from smuggling the export good. The illegality of such transactions involves a cost which is reflected in a premium of the parallel market price over the official price.

It is assumed that all imports purchased in the official market are resold. Hence, while the imported good may be purchased at two different exchange rates, the price at which it is consumed is uniform, being determined by its (marginal) cost on the parallel market. For simplicity, it is further assumed that there are no risks or penalty costs associated with the purchase of foreign currency on the parallel market or the resale of imported goods.

1. Prohibitive smuggling costs

The consequence of quantitative import controls when prohibitive costs rule out smuggling are illustrated in Figure 1A. Demand for the imported good, II_o, and supply of the export good to the official market, XX_o, are depicted as functions of the exchange rate e (defined as the price of foreign currency in terms of domestic currency). At the official exchange rate, e_1, the prevailing demand for imports will be fully satisfied if official reserves, equal to AB, are sold each period. 1/ If the quantity of imports is restricted to OA through, say, a licensing system, then no

1/ This description corresponds to the fixed unified exchange rate case and to the official market segment of a dual rate regime without import controls.

reserve sales are necessary at $\bar{e}_1$. Consequently, at this rate there is an excess demand for imports and the foreign exchange market does not clear. However, imports purchased at $\bar{e}_1$ will be resold at the premium price of e_1, the profits from this activity being given by the product of $(e_1-\bar{e}_1)$ and OA. Thus, although no foreign exchange is transacted outside the official market, e_1 may be interpreted as a "shadow" parallel market exchange rate. A discriminatory arrangement which charged importers e_1, but offered exporters $\bar{e}_1$, would have equivalent price and quantity effects, but the exchange profits would accrue to the central bank rather than the private sector. 1/ It is also clear from Figure 1A that, for a given level of official reserves, successive devaluations of the official rate will produce appreciations of the shadow parallel rate and, therefore, a decline in the price of imports until the foreign exchange market clears at e_2. In this sense, a devaluation will have a deflationary impact.

2. Nonprohibitive smuggling costs 2/

We now turn to the more interesting case where the costs of smuggling are not prohibitive. The left-hand quadrant in Figure 1B depicts, as a function of the parallel market exchange rate e, the foreign currency value of sales of the export good to the official market, X, while the left-hand quadrant traces sales of the export good to the parallel market, also as a function of e. The schedules are drawn for given values of all other prices, including the official exchange rate $\bar{e}$.

For illustrative purposes, schedule OA is drawn to show supply of the export good to the parallel market when the official exchange rate is set equal to zero. Its slope shows the expansion in production of the export good which occurs as its own price rises relative to that of the nontraded good. 3/ As the official rate increases, it becomes

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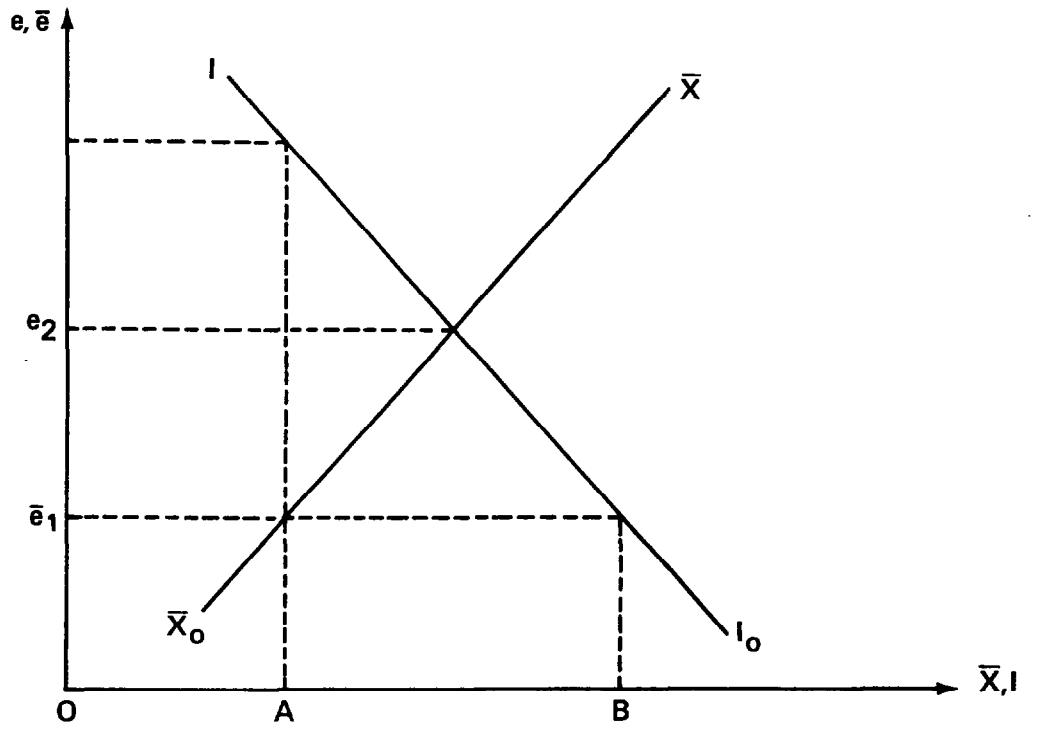
1/ This may be recognized as a case of the Marshallian equivalence proposition. Bhagwati (1978) has argued that, in practice, this equivalence will break down because the two prices cannot be adjusted parametrically in accordance with changes in the supply and demand schedules. This problem can, however, be overcome simply by introducing a scheme for auctioning import licenses. The price paid for these licenses will correspond to the shadow parallel exchange rate and will fluctuate in response to variations in the supply and demand for foreign exchange.

2/ The rest of this section is in the spirit of a model by Michaely (1954) which presents a geometric exposition of the determination of black market prices. For an application of this model to black markets in foreign exchange, see Sheikh (1976). The presentation here is somewhat simpler than in these two papers since it avoids construction of an exceedingly complicated black market demand curve. It also allows the path of the black market exchange rate to be traced as it responds to changes in the official rate.

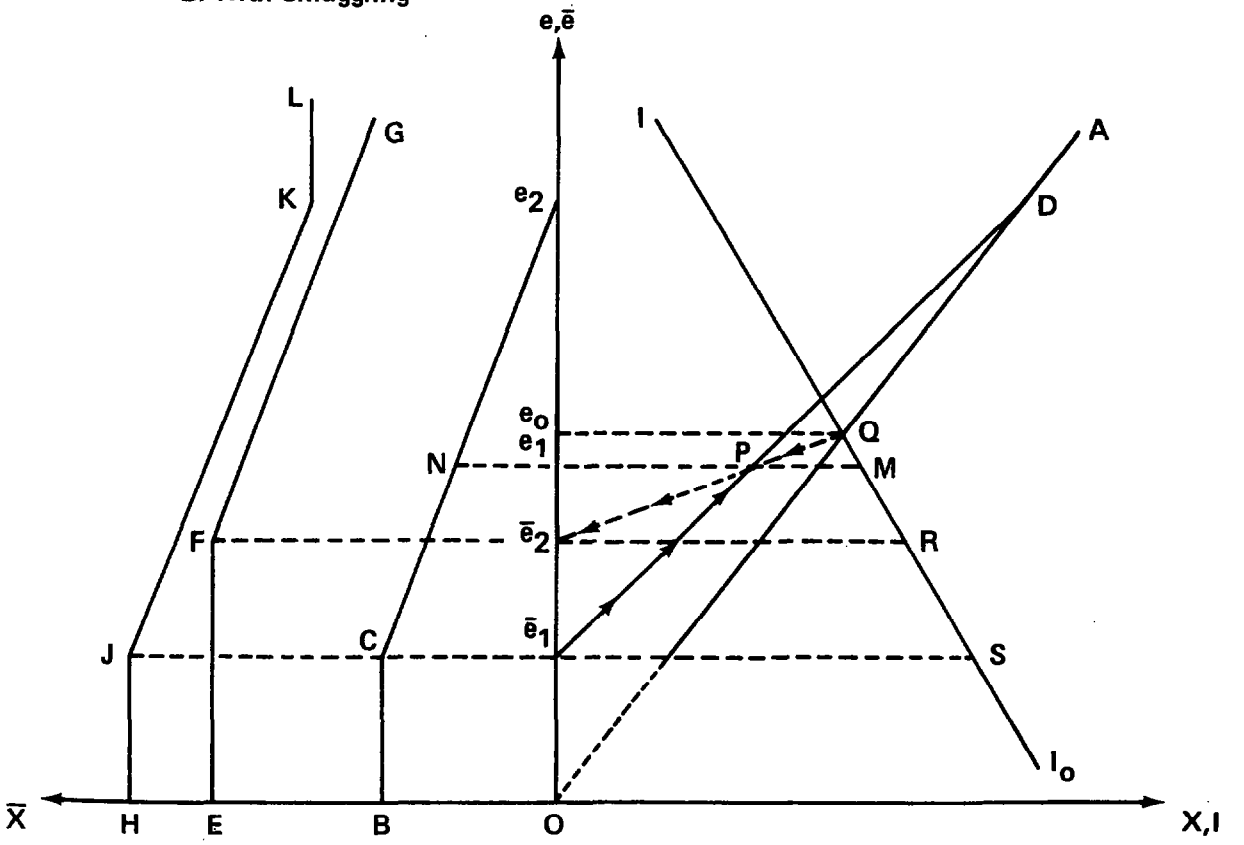
3/ It is assumed throughout this section that the economy is on a given production possibility frontier, so that aggregate production, appropriately measured, is fixed. The assumptions underlying the curves drawn in figure 1B are set out fully in the Appendix.

Figure 1. Determination of Parallel Market Exchange Rate

A. Without Smuggling



B. With Smuggling



profitable for those producers whose costs of smuggling are greatest to switch supply to the official market. 1/ Thus, if the official rate is, say, e_1 and there is no parallel market, OB units will be sold on the official market. At e_2, the parallel market premium (e_2/e_1) is sufficiently great to divert all sales from the official to the parallel market. The official and parallel market supply curves corresponding to e_1 are, therefore, BCe_2 and e_1DA, respectively. e_1D is flatter than OD since, as the parallel rate rises, sales of the export good are diverted from the official to the parallel market. An increase in the official rate from e_1 to, for example, e_2 produces an outward shift of the official market supply curve to EFG.

A rundown in official foreign exchange reserves is depicted by an outward shift in the official market supply schedule; thus, HJKL traces the total supply of foreign exchange in the official market when the price in this market is e_1 and reserves are being sold at a rate of BH per period.

To see how the parallel market rate is determined, consider an initial situation where all of the export good is sold on the parallel market because the official rate is "too low" (in this example zero) and where there are no reserve sales by the central bank. Under such conditions, the equilibrium parallel rate will be e_o, which corresponds to the intersection of the supply schedule OA and the demand schedule II_o. If the official rate is raised to e_1, then the equilibrium parallel rate will be e_1, since, at this price, total demand (e_1M) equals supply from the official market (e_1N = PM, by construction) plus supply from the parallel market (e_1P). Successive increases in the official rate will lead to a decline in the parallel rate along a "unification path" QPe_2, until the two rates merge at e_2 and the parallel market ceases to function. Note that at e_2 the quantity of foreign exchange demanded (e_2R) is equal to the quantity supplied in the official market (Fe_2). Both e_1 (the parallel rate consistent with an official rate of e_1) and e_2 are equilibrium exchange rates in the sense that, at these prices, the demand for foreign exchange is equal to supply. However, only e_2 is consistent with the absence of binding restrictions on the availability of foreign exchange in the official market. In this regard, it may be referred to as a "restriction-free" equilibrium.

The reason why the official and parallel market exchange rates are inversely related to each other is straightforward. 2/ By diverting resources from production of the nontraded good to the export good, a devaluation of the official rate will, ceteris paribus, generate an expansion in the total supply of the export good even though sales to the parallel market contract. Since the parallel rate equilibrates the total demand and supply of foreign exchange in the economy, it must

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1/ In this deterministic model, the (known) marginal cost of smuggling varies according to distance from the frontier. On an elaboration of this, see the Appendix.

2/ This result conforms with that obtained by Michaely (1954) and Sheikh (1976).

appreciate not only relative to the official rate but also in absolute terms. 1/

By similar reasoning, an increase in official reserve sales will also lead to an appreciation of the parallel rte. This is also illustrated in Figure 1B where, at the official rate $\bar{e}_1$, reserves of $\bar{BH}$ are sold each period. Since, by construction, demand at this price ($\bar{e}_1S$) equals the amount supplied in the official market ($\bar{e}_1J$), the two markets are effectively unified. However, $\bar{e}_1$ can only be regarded as a temporary "restriction-free" equilibrium since the capacity of the central bank to sell foreign currency reserves over time is limited. For reserve sales of less than BH at $\bar{e}_1$, the parallel market rate will lie on the unification path $\bar{e}_1P$. It is entirely possible, therefore, that the parallel rate is lower (the value of the domestic currency higher) than the "restriction-free" equilibrium rate, $\bar{e}_2$. The implication here is that use of the parallel rate as an indicator of the "restriction-free" equilibrium exchange rate must take into account movements in net official reserves or, equivalently, the size of the officially recorded overall balance of payments position.

III. General-Equilibrium Framework

This section extends the framework of analysis from a partial- to a general-equilibrium framework, thereby permitting an examination of the macroeconomic impact of major policy shifts when there is rationing in the official exchange market. The analysis is confined to a descriptive exposition of the workings of the model; a mathematical treatment is provided in the Appendix.

1/ The conclusions of this section, however, are obviously sensitive to the assumptions relating to the penalty costs of transacting in the parallel market. In this regard, the existence of smuggling costs is a crucial assumption underlying the negative slope of the unification path. In the absence of such costs, all producers would sell on the parallel market whenever the premium in this market was positive. Thus, the supply curve OA in Figure 1B would be flatter, passing through the point R. Without official reserve intervention, the unification path would, therefore, be a horizontal line passing through the points $\bar{e}_2$ and R, with the parallel rate equal at all times to the restriction-free equilibrium rate. Penalty costs on the demand side, if sufficiently large, may introduce an element of indeterminacy into the analysis. A rise in the parallel rate relative to the official rate may cause an increase or decrease in the quantity demanded on the parallel market. Hence the demand curve for imports may shift upwards or downwards if demand-side penalty costs are present. The assumptions adopted above which give the unambiguous result that an official devaluation results in an appreciation of the domestic currency in the parallel market, should, however, be regarded as empirically plausible.

1. Structure of the economy

a. Real sector.

Domestic production consists of the output of the nontraded good and the exported good. By the small open economy assumption, the demand for the export good is perfectly elastic at an exogenous foreign currency price, so that its output is supply-determined. Also, it is assumed initially that the only factor of production, labor, is fixed in supply. Aggregate output is therefore exogenous and equal in equilibrium to the sum of the demand for the nontraded good by the private sector and the government, $N(\ )$ and G, respectively, and the supply of the export good to the official market and the parallel market, $\bar{X}(\ )$ and $X(\ )$, respectively: 1/

$$(1) \quad Y = N(e/p_n, Y) + \bar{X}(\bar{e}/e, \bar{e}/p_n)\bar{e} + X(e/\bar{e}, e/p_n) + G$$

$$\text{where} \quad N_1,\ N_2,\ \bar{X}_1,\ \bar{X}_2,\ X_1,\ X_2 \ > \ 0$$

and where $\bar{e}$ and e denote the official and parallel market exchange rates, respectively, and p_n the price of the nontraded good. It should be noted that e represents not only the price obtained by selling the export good on the parallel market but also the local currency price of the imported good.

With reference to Figure 2, the IS curves show combinations of e and p_n which satisfy the condition that the demand for the economy's output be equal to its supply; they are upward sloping because an increase in p_n produces an excess supply of domestic output which must be accompanied by a rise in e for equilibrium to be maintained. Point Z on IS_o represents the stage reached when the parallel market premium $e/\bar{e}$ has increased to a level at which all production of the export good is sold on the parallel market. 2/ Thus, IS has an elasticity of greater than one to the southwest of point Z and equal to one to the northeast of Z. 3/

1/ The following is a national income accounting treatment of aggregate output. All base period prices including the parallel exchange rate e have been set equal to one, with the exception of the official exchange rate $\bar{e}$, which must be less than e. Measurement errors arise when relative prices in the current period differ from those in the base period. However, for small changes in relative prices over time, this error may be safely ignored.

2/ Points on IS_o to the northeast of Z satisfy (1) with the condition that the partial derivative X_1 and the function $\bar{X}(\ )$ are set equal to zero.

3/ The greater than unitary elasticity of IS below point Z reflects the fact that equal proportionate changes in p_n and e but not $\bar{e}$ will result in a reallocation of resources from production of the export good to the production of nontraded good.

b. Financial sector

The private sector holds two financial assets, domestic currency and foreign currency. Neither is interest-bearing. The demand for domestic currency in real terms is related positively to the level of output, negatively to the expected rate of return on holding foreign currency, but is independent of wealth. 1/ It is assumed that the sale of foreign exchange for capital transactions in the official market is prohibited; the return on foreign currency therefore corresponds to the rate of depreciation of the domestic currency in the parallel market. The supply of domestic currency is defined as the sum of domestic credit and the local currency value of the net foreign exchange reserves of the official sector.

Due to the presence of quantitative controls on both imports and capital payments, the stock of reserves in foreign currency terms may be treated as exogenous or, equivalently, as a target variable. This is a relatively strong assumption since it implies that any parametric shift in the supply function for foreign exchange in the official market will be accompanied by an intensification or relaxation of controls so as to maintain the reserve or balance of payments target.

These considerations imply the following equilibrium condition for domestic money:

$$(2) \quad R + D = L(Y, e^*_{+1}/e)p \qquad L_1 > 0;\ L_2 < 0$$

where R is the level of official reserves in local currency terms, D domestic credit, and e^*_{+1} the expected value of e one period hence.

The aggregate price level, p, is defined as a geometrically weighted average of the price of the nontraded good and the imported good:

$$(3) \quad p = p_n^{\alpha} e^{(1-\alpha)} \qquad 0 < \alpha < 1$$

LM_o in Figure 2 depicts the locus of points along which the equilibrium condition for domestic money is satisfied. It is downward-sloping since, for equilibrium, an excess demand for money caused by an increase in p_n must be matched by a fall in e. While LM is stationary, the price level remains unchanged; rightward shifts in the curve arising, for example, from an increase in the money supply, imply an increase in the price level.

1/ This implies of course that all wealth effects are compressed into the demand function for foreign currency.

Figure 2. Effects of an Increase in Domestic Credit and a Devaluation

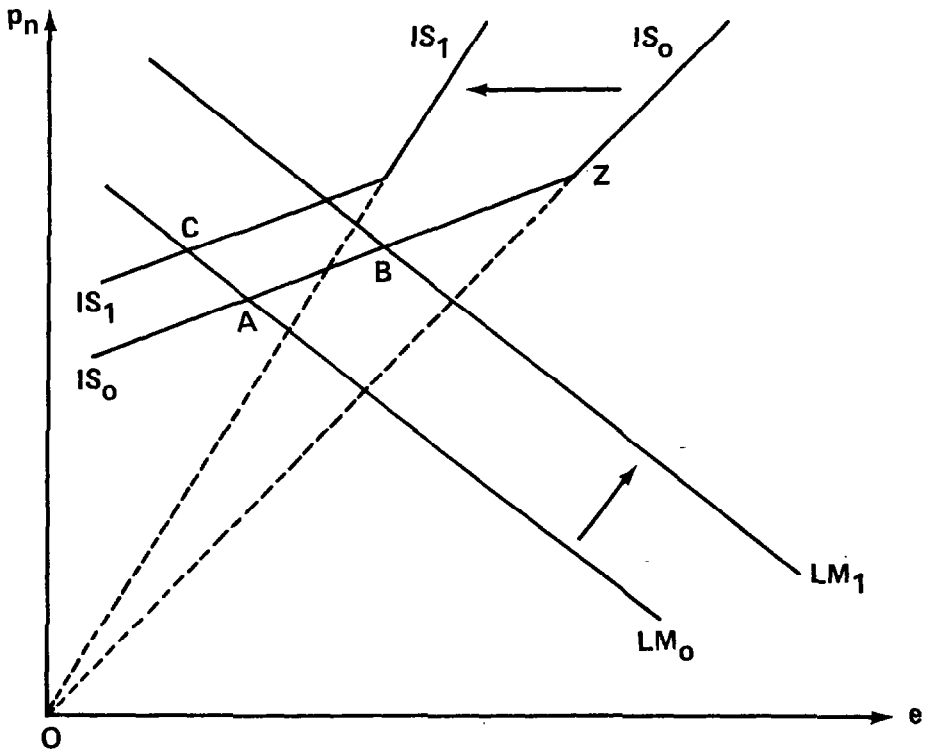
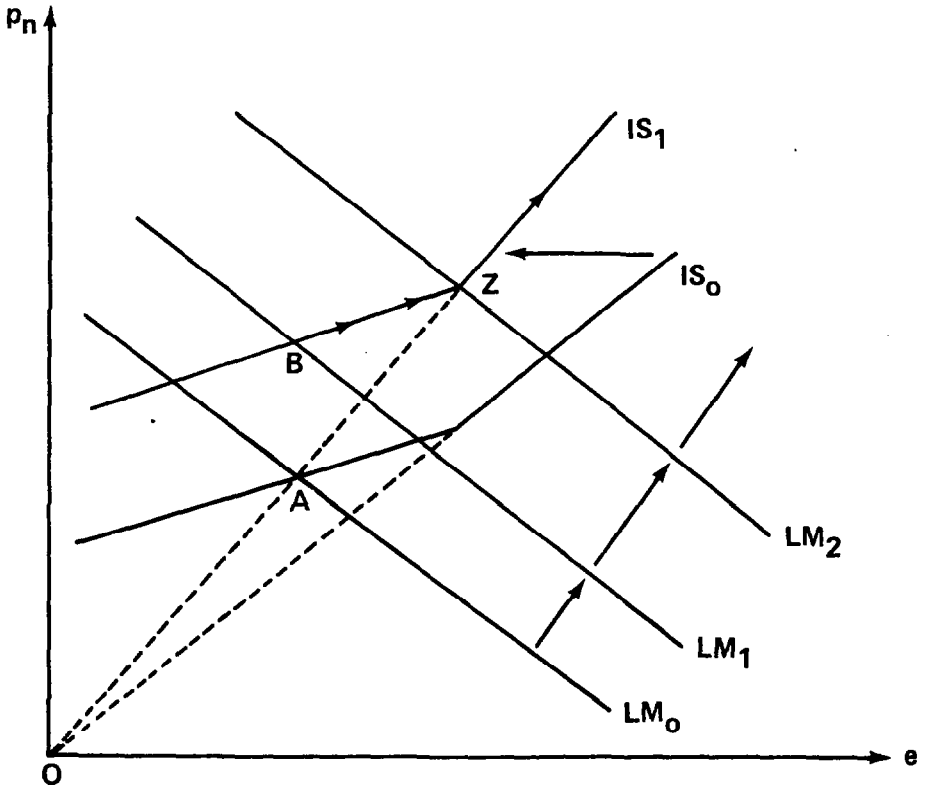


Figure 3. Effects of an Increase in Money—financed Government Spending



Assuming for the moment static exchange rate expectations, 1/ equations (1) - (3) can be used to solve for the endogenous variables e, p_n, and p. The remainder of this section examines the effects of various policy instruments on these variables.

2. Impact of policy shocks

a. Change in domestic credit

Consider first the impact of an increase in domestic credit. 2/ With the net stock of foreign reserves given (and stationary), this results in an expansion of the money supply and a rightward shift of the LM curve in Figure 2 from LM_0 to LM_1. As a result, from an initial equilibrium at point A, the economy moves to point B, where e, p_n, and p are higher. 3/ Since the IS curve has an elasticity greater than one over this range, the depreciation of the parallel rate is proportionately greater than the increase in the price of the nontraded good. This shift in the structure of relative prices demonstrates the non-neutrality of money with respect to real variables when the availability of foreign exchange through official channels is restricted. It simply reflects the fact that e, the price received for selling the export good in the official market, is fixed while all other prices and costs are free to vary.

This result stands in sharp contrast to those derived from the standard fixed/flexible exchange rate models (or their dual market variant). In the fixed rate model, an expansion in domestic credit is fully offset through the balance of payments, while in the flexible rate model it produces a proportionate and uniform rise in all prices, leaving their relative levels unchanged.

b. *Money-financed government spending*

The consequences of a sustained increase in money-financed government expenditure on the nontraded good are shown in Figure 3. In the current period there is a leftward shift of the IS curve and a rightward shift of the LM curve, such that the economy moves from point A to point B. Thus, while the overall impact effect is inflationary, the direction of change in the parallel rate is ambiguous. However, in subsequent periods, continued deficit-financing generates successive outward shifts of the

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1/ Static expectations mean that next period's expected value is equal to its current (known) value, i.e., $e^*_{+1} = e$. The expected return on holding foreign currency is then always zero.

2/ Since the analysis throughout abstracts from the existence of interest-bearing assets, it is assumed that increases in domestic credit which are not associated with the financing of government expenditures arise out of "helicopter" operations.

3/ An inspection of equations (1) and (2) reveals that point B is both a short- and long-run equilibrium position. This result is independent of the assumption made regarding wealth effects.

LM curve along IS_1, raising all prices including the parallel rate. 1/ Eventually, the point Z is reached on the intersection of LM_2 and IS_1 when supply of the export good to the official market is exhausted. Beyond this point, the economy will be in a steady state equilibrium with D, e, p_n, and p all increasing at the same constant rate.

c. Devaluation (static expectations/fixed output)

In Section II, it was demonstrated that, in a partial-equilibrium setting, a devaluation of the official rate leads to an appreciation of the parallel rate for a given stock of reserves. This result continues to hold within a general-equilibrium framework. With reference again to Figure 2, a devaluation results in a leftward shift of the IS curve from IS_0 to IS_1. Equilibrium in the economy, therefore, shifts from point A to point C where the parallel rate is lower and the price of the nontraded good higher. However, because of the assumptions of static exchange rate expectations and fixed aggregate output, the LM curve remains stationary and the price level is therefore unchanged. The absence of any effect on the price level may appear a somewhat counter-intuitive result, but it is clear from the domestic money equilibrium condition (2) that, with Y and e^*_{+1}/e fixed by assumption, the price level is determined solely by domestic credit and the stock of foreign exchange reserves, both of which are exogenously given. Only if the central bank decides, contemporaneously with the devaluation, to accumulate foreign exchange reserves on a net basis will the price level rise.

This example highlights a fundamental assumption of the model, namely, the exogeneity of reserves. It implies that any shift in the supply of foreign exchange to the official market as a result, for example, of a devaluation must be accompanied by an offsetting shift in the incidence of controls in order to maintain an unchanged net reserve position. In fact, since often a devaluation of the official rate will be supported by official sector borrowing, there will be a tendency for net reserves to decline. In practice, therefore, a devaluation may be associated with a deflationary rather than inflationary price impact.

1/ The indeterminate effect on the parallel rate in the impact period reflects the adoption of an end-of-period asset equilibrium specification. In a beginning-of-period formulation, asset supplies (including the stock of money) are held stationary in the impact period. Hence, an increase in government spending initially involves only a leftward shift of the IS curve and leads unambiguously to an appreciation of the parallel rate. However, as in the end-of-period formulation, money financing in subsequent periods results in a rightward shift of the LM curve and produces a progressive depreciation of the parallel rate. A discussion of the issues involved in the alternative specifications of asset equilibrium is contained in Karni (1979). For an example of an end-of-period model which incorporates a government budget constraint, see Turnovsky (1975).

d. Devaluation (rational expectations/flexible output)

We now turn to a further interesting implication of the model that, when either the assumption of static expectations or that of fixed aggregate output is relaxed, a devaluation may have deflationary consequences. Consider first the role played by exchange rate expectations. If these are formed rationally, a devaluation of the official rate in the current period which is unanticipated will not affect the expected rate of change of the parallel rate, e^*_{+1}/e; both the numerator and the denominator of this term decline proportionally. Consequently, as in the case of a devaluation under the assumption of static expectations, the price level will again be unaffected.

In contrast, an anticipated devaluation will bring about a temporary decline in the price level. 1/ This may be seen as follows. Under the assumption of rational expectations, the time paths for the parallel market rate and the domestic price level associated with a preannounced devaluation of the official rate will be as shown in Figure 4. 2/ At time t, the central bank announces that it will devalue the exchange rate at time $(t+\theta)$ from $\bar{e}_0$ to $\bar{e}_1$. Since all market participants are presumed to know at time t that this will involve an appreciation of the parallel rate from e_0 to e_1, the actual return on holding foreign currency, e^*_{+1}/e, declines immediately. The resulting contraction in the demand for foreign currency holdings is matched by a rise in the demand for domestic currency, and, as a consequence, both the parallel rate and the price level fall to maintain money-market equilibrium. The parallel exchange rate continues to appreciate at an increasing rate (i.e., e^*_{+1}/e declines further) until the devaluation actually takes place at time $(t+\theta)$. 3/ At this point, no further change in the parallel rate is anticipated and the rate of return on holding foreign currency rises abruptly. The resulting substitution from domestic back into foreign currency thereby causes the price level to jump at time $(t+\theta)$ back to its original level, p_0.

While the price level is therefore independent of the official exchange rate in the long run when output is fixed, an observer measuring the movement in prices between time $(t+\theta-1)$ and time $(t+\theta)$ may erroneously conclude that the impact of a devaluation is inflationary. Moreover, the devaluation may incorrectly appear to have only a minimal effect on the parallel rate. This example thus illustrates the importance of assumptions about expectations in any empirical analysis of price and exchange rate movements .

1/ The analytical techniques used in this section are discussed in detail in Sargent and Wallace (1973).

2/ These time paths are derived mathematically in the Appendix.

3/ The decline in e^*_{+1}/e prior to $t+\theta$ is illustrated in Figure 4 by the concave (to the origin) shape of the time path for e. This concavity is necessary for stability since the demand for money must rise, and therefore e^*_{+1}/e must fall, if e is to decline to its long-run equilibrium level.

Once the assumption relating to the exogeneity of aggregate output is relaxed (i.e., labor is no longer in fixed supply), the domestic price level ceases to be independent of changes in the official exchange rate in the long run. An unanticipated devaluation leads to an increase in total output which, by raising the demand for domestic money, exerts downward pressure on the domestic price level. The deflationary impact of the devaluation is reinforced by a fall in the price of the imported good as a result of the appreciation of the parallel market rate. A corollary that follows from this result is that the imposition of controls result in a permanent once-and-for-all reduction in output and an increase in the price level. Again, these predictions stand in sharp contrast to those obtained from the standard unified or dual market model which imply that a devaluation will raise the money supply and all prices proportionately, thereby leaving relative prices and all real variables unchanged.

3. Exchange rate unification

It should be clear from the above analysis that, once a parallel market emerges, no automatic forces operate to bring about a reunification of the exchange system. Indeed, in the case of an increase in money-financed govenment expenditures or in an inflationary environment, the official exchange rate will become increasingly overvalued. This will be reflected in an increase in the parallel market premium over time and a contraction in the volume of transactions conducted in the official market. A merging of the two markets can only be achieved through a reduction in domestic credit, a devaluation, the sale of official foreign exchange reserves, 1/ and/or a relaxation of controls.

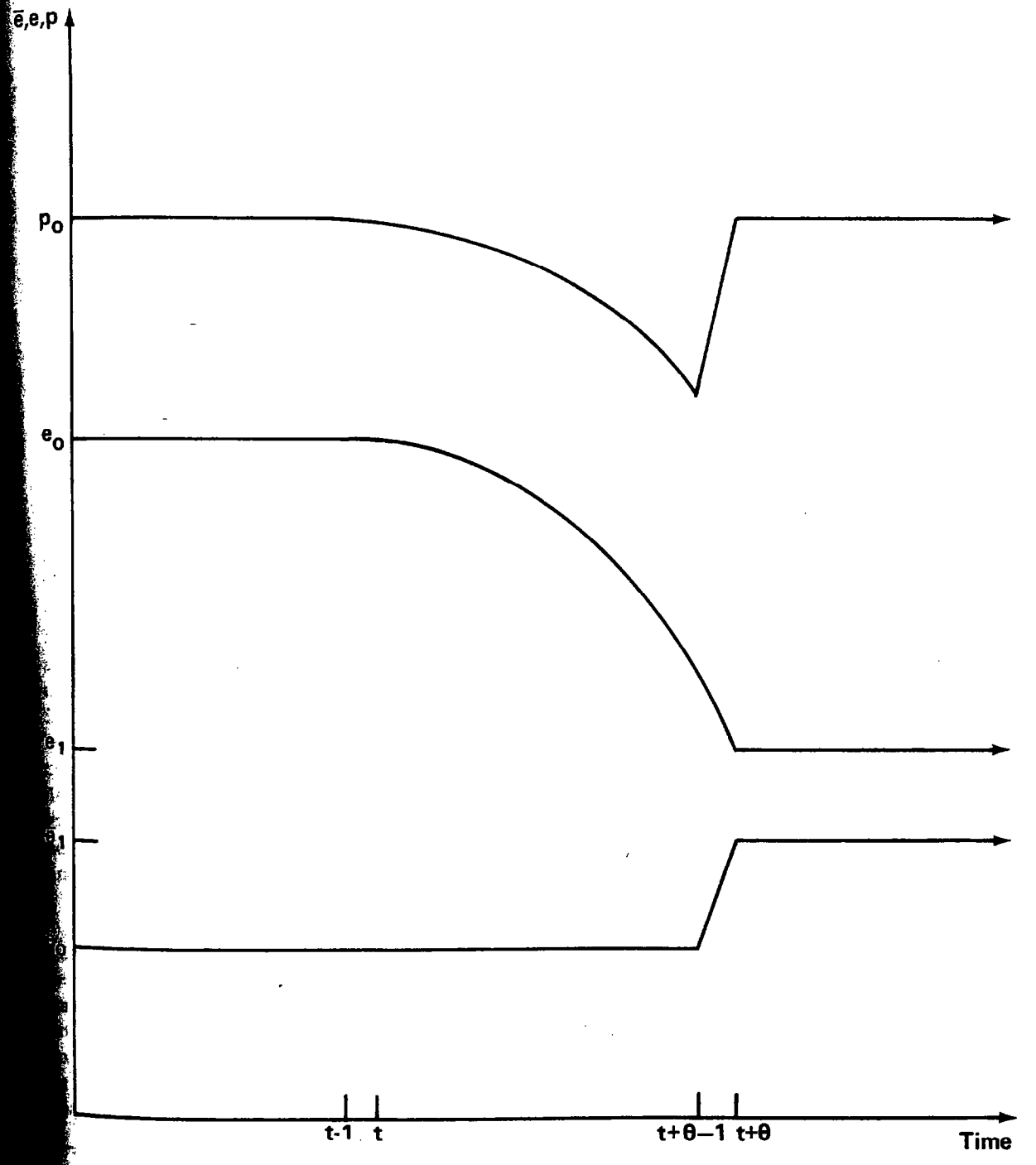
IV Some Implications for Exchange Rate Policy

An important implication of the model presented above is that the imposition of quantitative controls on current payments as a substitute for a formal devaluation does not avoid the adverse repercussions on the rate of inflation or real wages of a devaluation. The emergence of a parallel market in response to such controls and the depreciation of the exchange rate in this market have similar inflationary consequences to those of an official devaluation. Furthermore, the illegality of transacting in the parallel market gives rise to real resource costs that are absent in a unified exchange system. Such a dual market also provides an environment conducive to corrupt practices and permits economic rent to accrue to certain groups who are in a position to exploit the exchange rate differential between the two markets. 2/

1/ Net sales of foreign exchange by the central bank produce a leftward shift in the LM curve and are thus identical in their effect to a contraction in domestic credit.

2/ In a perfectly competitive environment, however, the rent will be fully offset through a bidding up of the cost of obtaining access to the official market.

Figure 4. Effects of a Pre-announced Devaluation[1]



[1]/The assumption in the text that p_o and e_o are set equal to one is relaxed here for illustrative purposes.

Ordinarily, under a fixed rate regime, the price impact of expan-
ionary policies will be mitigated by a loss of foreign exchange reserves.
his brake on inflationary pressures is absent if quantitative controls
re imposed as a means of averting reserve loss. Consequently, if and
hen the official exchange rate is devalued, there will be no direct
nflationary effect since the increase in prices and the reduction in
eal wages normally associated with a devaluation will have already tken
lace through a depreciation of the currency in the parallel market.
his point is worth reiterating by way of the following simple example.

Assume first that the domestic economy and the rest of the world
re in stationary equilibrium, with no inflation and no growth in real
monetary variables. Then, suppose that there is an increase in credit
the domestic economy. In the absence of foreign exchange restrictions,
his credit expansion will be fully offset by a loss in reserves and the
omestic money supply and price level will remain unaffected. A devalua-
ion aimed at restoring the stock of reserves to the level prevailing
rior to the credit expansion will produce a once-and-for-all increase
the money supply with corresponding increases in the price of imports
d the aggregate price level. 1/

Now consider, for the same economy, a similar increase in domestic
edit but one accompanied by quantitative controls aimed at avoiding
oss in reserves. In such a situation, a parallel foreign exchange
ket will develop and the exchange rate in this market will depreciate.
rate of inflation will be equal to the rate of credit expansion, but
price of imports will rise at a more rapid rate. A reunification of
exchange system brought about by a devaluation of the official rate
the removal of all controls (such that the stock of reserves remains
hanged) will cause the parallel rate to appreciate and the price of
rts to decline. However, the devaluation will have no inflationary
ct in the current or subsequent periods since, with reserves constant,
money supply will be determined only by movements in domestic credit.
e, while the time paths for prices will be different under the two
ange regimes, the rate of inflation over the period as a whole will
e same. The movement in prices and reserves under the restrictive
estriction-free regimes is illustrated in Figure 5.

The preceding argument is also applicable to movements in the
wage rate under the two trade regimes. Although the model
ribed in this paper assumes a flexible money wage rate, the
cations for exchange rate policy remain qualitatively the same if
age rate is sticky and lies above its market-clearing level.
a restrictive trade regime, the real wage rate will fall as the
lel market rate depreciates, but be unaffected by a devaluation of
fficial rate. 2/ This is in contrast to the situation under a

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It is assumed that the devaluation is not anticipated.

In fact, there will be a tendency for the real wage to rise with a
nation since, as discussed above, the price level will decline with
ncrease in output.

restriction-free regime where a devaluation results in a decline in the real wage.

V. Conclusions

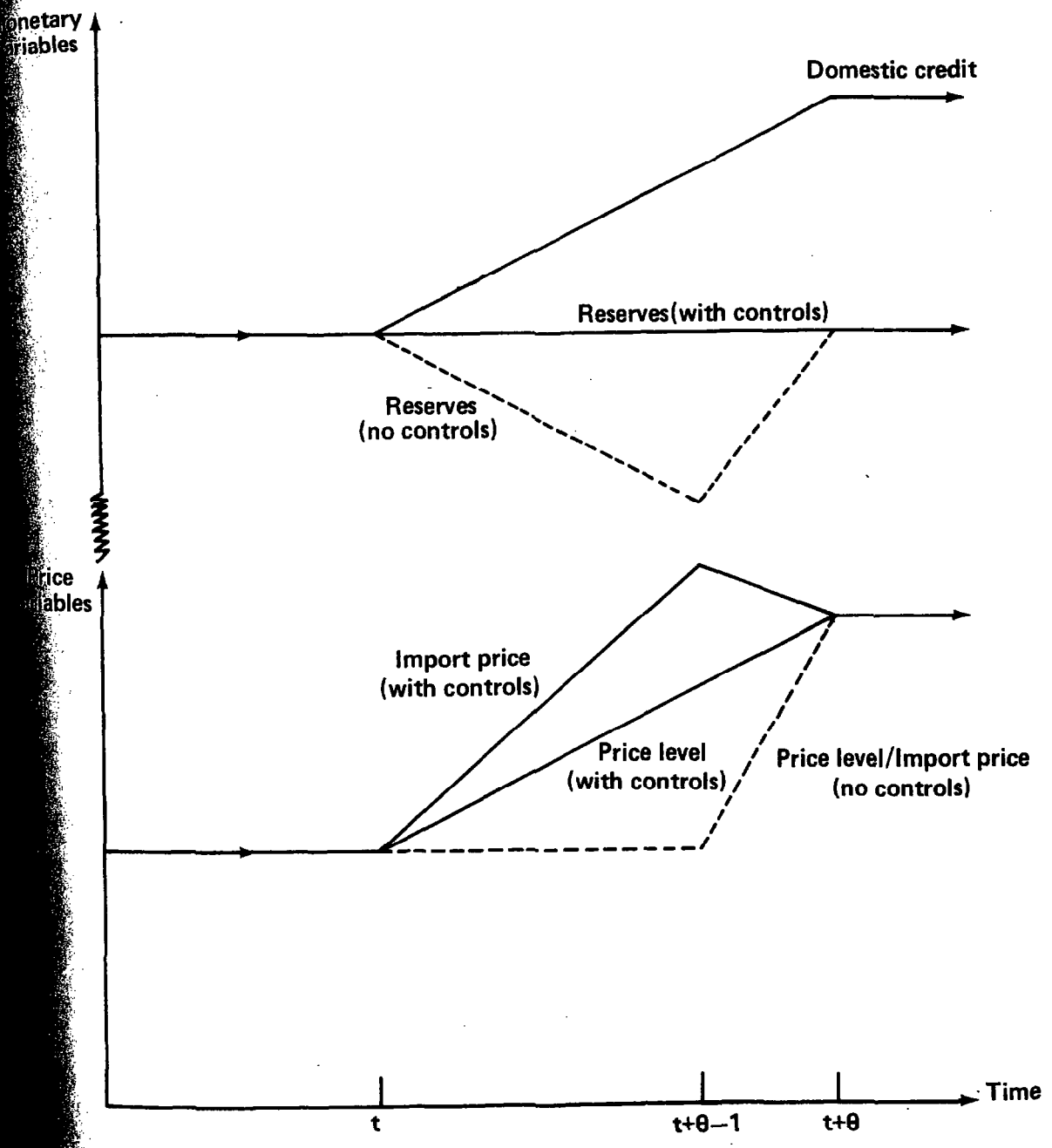
This paper has developed a simple macroeconomic model in which the official exchange rate is fixed and the availability of foreign exchange through official channels is restricted. Thus, unlike conventional fixed exchange rate models (including the dual market variants) which abstract from the existence of quantitative controls, the stock of official foreign exchange reserves is autonomously given. The foreign exchange premium offered in the parallel market as a result of inconvertibility in the official market provides an incentive for exporters to sell their output illegally. Thus, while the parallel rate equilibrates supply and demand in the parallel market, it responds directly to shocks in the official market, such as a change in the exchange rate or an intensification of import controls.

The analysis has demonstrated within both general- and partial-equilibrium frameworks that the official and parallel market exchange rates are inversely related to each other. Furthermore, in the absence of foreign exchange sales by the central bank, the parallel rate will be more depreciated than the "restriction-free" equilibrium rate. The relationship between these two rates will, however, be reversed beyond a given level of reserve sales by the central bank. Some caution should, therefore, be exercised in using the parallel market rate as an indicator of the appropriate magnitude of exchange rate adjustment necessary in order to attain balance of payments equilibrium.

The model presented in this paper generates a number of results that differ strikingly from those derived from conventional fixed/flexible exchange rate models and their dual market variant. Because not all prices are flexible, specifically that received by exporters who sell on the official market, and because the stock of reserves is, in effect, fixed through the imposition of controls, changes in domestic credit and the official exchange rate have real effects in the long run. In addition, the model gives rise to some interesting conclusions regarding the price effects of a devaluation of the official rate. Since the presence of controls prevents a deterioration in the balance of payments when inflationary financial policies are being pursued, the domestic currency will undergo an effective depreciation in the parallel market. If the official exchange rate is devalued, there will be no subsequent inflationary impact since the increase in prices and the reduction in real wages normally associated with a devaluation will have already taken place. Indeed, the analysis shows that deflationary forces are set in motion if the devaluation is preannounced or widely anticipated, or if it produces an expansion in domestic output.

The analysis also underscores the importance of expectations in any empirical study of price and exchange rate movements. In particular, it shows that failure to take expectations of official exchange rate

Figure 5. A Comparison of Price and Reserve Movements Under Alternative Trade Regimes[1]



1/ In this example, domestic credit expansion begins at t and an unanticipated devaluation of the official rate takes place at t+θ. In the absence of controls, the rise in the price level occurs the time of the devaluation t+θ, while under the restrictive regime , it takes place between t+θ and t. The actual inflation rate between t and t+θ is, however, identical under both types of regime.

anges adequately into account may erroneously lead to the conclusion at a devaluation of the official rate in the presence of quantitative strictions is inflationary, and that its influence on the parallel rket rate is minor.

Finally, it is shown that suppression of parallel market activities d a reunification of the exchange rate system can only be brought about rough a tightening in domestic credit, a devaluation of the official te, or a relaxation of exchange and trade controls. No automatic rces operate to realign the two exchange rates. Reunification also has portant welfare implications since it raises the surpluses enjoyed by nsumers of the imported good and producers of the export good, while t eliminates the economic rent accruing to traders who can obtain cess to the official foreign exchange market.

APPENDIX

Formal Description of the Model

1. Partial-equilibrium analysis

This section of the Appendix provides a more thorough description of the model's real sector than that contained in the text and presents a formal derivation of the partial-equilibrium results obtained graphically in Section II.

Production of both the nontraded and export good is assumed to take place in a competitive environment in which each producer takes the product price as given. All producers of the export good face identical (variable) costs of production but not of distribution. Specifically, it is assumed that, while it is costless for producers to sell the export good through official channels, the illicit nature of sales to the parallel market involves a penalty cost. This cost, which is known with certainty, 1/ is strictly proportional to (a) the distance between the production site and the frontier and (b) the actual volume sold. 2/ Hence, the marginal cost of distribution is constant for each producer engaged in parallel market trade but varies across producers according to distance from the frontier. The marginal distribution costs facing producers operating at the frontier are taken to be nonzero, but negligible.

It follows that the individual producer's decision of whether to produce the nontraded or the export good, and in what quantities, will depend on relative output prices and wage costs. Wage costs will be positively related to the price of the private sector's consumption bundle since the supply of labor is assumed to vary with the real wage rate. Any decision to produce the export good will involve a secondary decision as to which market to supply. The producer will not sell through official channels if the premium he can obtain on the parallel market exceeds his per-unit distribution (penalty) costs.

These considerations imply a set of supply functions of the following form:

$$(A1)\quad \overline{X} = \overline{X}\,(\overline{e}/e,\ \overline{e}/p,\ \overline{e}/p_n), \qquad \overline{X}_1,\ \overline{X}_2,\ \overline{X}_3 > 0$$

$$(A2)\quad X = X\,(e/\overline{e},\ e/p,\ e/p_n), \qquad X_1,\ X_2,\ X_3 > 0;\ X_1 = \overline{X}_1$$

$$(A3)\quad S = S\,(p_n/\overline{e},\ p_n/e,\ p_n/p), \qquad S_1,\ S_2,\ S_3 > 0$$

1/ The assumption that penalty costs are known with certainty is only a simplification; uncertainty combined with risk-neutrality on the part of producers could generate equivalent results.

2/ The purpose of these assumptions is to ensure that for a range of parallel market premia the exported good is sold simultaneously through both official and illegal channels.

where $\bar{X}$ and X represent the quantities of the export good sold on the official and parallel markets, respectively; S production of the nontraded good; $\bar{e}$ and e the prices of foreign currency on the official and parallel markets, respectively; p_n the price of the nontraded good; and p the price of the private sector's consumption bundle (or aggregate price level). 1/

Thus, with regard to (A1), for example, an increase in e relative to $\bar{e}$ induces a switch of sales of the export good from the official to the parallel market, while an increase in p_n results in an increase in production of the export good and a decline in production of the nontraded good. A rise in p produces a reduction in the supply of labor (through a decline in the real wage) and hence a decline in the output of both goods.

The supply functions (A1) and (A2) will be subject to certain discontinuities. For example, when the parallel market premium is sufficiently large (zero), none (all) of the export good will be sold on the official market. Additional parameter restrictions depend on the elasticity of the supply of labor with respect to the real wage. If, at one extreme, this elasticity is zero, the aggregate volume of production of domestically produced goods will be fixed in the sense that the economy will be on, at all times, a given production possibility frontier. This implies $\bar{X}_2$, X_2, $N_2 = 0$. Production decisions will, therefore, be independent of the real wage. If, at the other extreme, the supply of labor is perfectly elastic, then the aggregate volume of output will be variable and production decisions independent of relative output prices. This implies $\bar{X}_3$, X_3, $N_3 = 0$.

The private sector's consumption of the nontraded and import good, will depend on the relative prices of these goods and income:

$$(A4) \quad N = N(e/p_n, Y) \qquad N_1, N_2 > 0$$

$$(A5) \quad I = I(e/p_n, Y) \qquad I_1 < 0; \; I_2 > 0$$

where N and I are the demands for the nontraded and imported good, respectively, and Y is the aggregate level of output measured as the sum of the production of the exported and nontraded good, both valued in terms of constant base period prices.

Equilibrium in the parallel market is given by the condition that the demand for foreign exchange be equal to supply:

$$(A6) \quad I - \bar{I} + \Delta F = X$$

1/ The foreign currency price of the export and imported good is assumed fixed and has been set equal to one.

where $\bar{I}$ denotes import purchases in the official market, F resident holdings of foreign currency, and Δ a first difference operator. The feature which distinguishes this model from the standard dual market approach is the presence of binding quantitative controls in the official market which renders the stock of reserves exogenous. The amount of official market imports is therefore determined by the available supply in that market.

$$(A7) \quad \bar{I} = \bar{X} - \Delta R/\bar{e}$$

where R is the stock of official reserves in terms of domestic currency. (A7) is the officially recorded balance of payments. Adding (A6) and (A7) yields:

$$(A8) \quad I + \Delta F = \bar{X} + X - \Delta R/\bar{e}$$

Ex ante, (A8) may be used to determine the parallel market rate for given values of all other variables; ex post, it is the balance of payments identity.

Some of the results obtained in the text may now be readily verified algebraically. Holding F constant, substituting (A1), (A2), and (A5) into (A8), and differentiating with respect to time, yields a solution for changes in the parallel market rate in terms of changes in the official rate and reserves: 1/

$$\frac{\Delta e}{\Delta \bar{e}} = \frac{\bar{X}_2 + \bar{X}_3}{I_1 - X_2 - X_3} < 0$$

$$\frac{\Delta e}{\Delta R} = \frac{-1}{I_1 - X_2 - X_3} > 0$$

The result is therefore confirmed, that in a partial-equilibrium setting, the parallel rate appreciates when the official rate is devalued and when the central bank sells reserves.

1/ Reserves have been held constant in the base period, such that $\Delta^2 \bar{R} = 0$.

2. General-equilibrium analysis

(a) Fixed output/static expectations

The impact multipliers for e and p in terms of the policy variables D, $\bar{e}$, and G may be derived from equations (1)-(3) together with a government budget constraint: $\Delta D = p_n G$. Letting $\hat{}$ denote a proportionate rate of change and ' an elasticity, the solutions are:

$$\begin{bmatrix} \hat{e} \\ \\ \hat{p} \end{bmatrix} = \begin{bmatrix} \phi_1/\phi_2 & -eX.X_3'/Y & (\phi_1 - \alpha G)/Y \\ \\ \frac{R}{R+D} & 0 & \frac{R}{R+D} \end{bmatrix} \begin{bmatrix} \hat{D} \\ \hat{\bar{e}} \\ \hat{G} \end{bmatrix}$$

where $\phi_1 = (N.N_1' + \bar{e}\,\bar{X}.\bar{X}_3')D/Y(R + D) \quad >0$

$\phi_2 = [N.N_1' + \bar{e}\,\bar{X}.\bar{X}_3'(1 - \alpha) + X.X_3']/Y \quad >0$

Therefore:

$\hat{e}/\hat{D} > \hat{p}/\hat{D} > \hat{p}_n/\hat{D} > 0$

$\hat{e}/\hat{\bar{e}} < 0;\ \hat{p}/\hat{\bar{e}} = 0;\ \hat{p}_n/\hat{\bar{e}} > 0$

$\hat{e}/\hat{G} \gtrless 0;\ \hat{p}_n/\hat{G} > \hat{p}/\hat{G} > 0$

(b) Flexible output/static expectations

Under the assumption that the supply of labor is perfectly elastic, the partial derivatives $\bar{X}_3, X_3, N_3$, in equations (A1), (A2), and (A3), are set equal to zero. This renders output endogenous. Y, e, p_n and p may then be solved using (2) and (3) together with the modified clearing condition for domestic output:

$$(A9) \quad Y = N(e/p_n, Y) + \bar{X}(\bar{e}/e, \bar{e}/p)\bar{e} + X(e/\bar{e}, e/p) + G$$

and the clearing condition for the nontraded good:

$$(A10) \quad S(p_n/p) = N(e/p_n, Y)$$

The impact of a devaluation of the official exchange rate on the price level is:

$$(A11) \quad \hat{p}/\hat{\bar{e}} = -\bar{e}\,\bar{X}.\bar{X}_2'/Y - L_1'[N_1' + S_1'(1-\alpha)]/\emptyset_3 \quad <0$$

where $\emptyset_3 = L_1'[\bar{e}\,\bar{X}.N_1'/Y - (1-\alpha)X_2'S_1'] - N_1' - S_1'(1-\alpha)$

$$- N_2'[X(1+\alpha X_2') + (1-\alpha)(N.S_1' + \bar{e}\,\bar{X})]/Y$$

The numerator in (A11) is unambiguously negative while the denominator, $\emptyset_3$, must be negative if the tatônnement process is to be stable.

3. Rational expectations/fixed output

We now assume that exchange rate expectations are formed rationally, but return to the fixed output assumption. As in the text, attention is confined to the effect over time of a devaluation on the parallel rate and the aggregate price level. For convenience, the weighting of the price level is changed from a geometric to an arithmetic average:

$$(3') \quad p = \alpha p_n + (1-\alpha)e \qquad 0< \alpha <1$$

The reduced solutions for e^*_{+1} and p in terms of e and $\bar{e}$ are derived from (1), (2), and (3'). Linearising around long-run equilibrium yields:

$$(A12) \quad e^*_{t+1} = \beta_1 e_t + \beta_2 \bar{e}_t + \beta_3 Z$$

$$(A13) \quad p_t = \delta_1 e_t + \delta_2 \bar{e}_t + \delta_3 Z$$

where Z denotes a vector of exogenous variables (including the long-run value of $\bar{e}$), which are constant over time, and where:

$$\beta_1 = 1/L_2\gamma - L[(1-\alpha)(\bar{X}_2 + \bar{X})\bar{e} + N_1 + X_2 + X]/L_2\gamma \quad >1$$

$$\beta_2 = -\alpha L(\overline{eX}_2 + X)/L_2\gamma \quad >0$$

$$\delta_1 = [(1-\alpha)(\bar{X}_2 + \bar{X})\bar{e} + N_1 + X_2 + X]/\gamma \quad >0$$

$$\delta_2 = (\overline{eX}_2 + X)/\gamma \quad >0$$

$$\gamma = \overline{eX}_2 + N_1 + X_2 + \overline{eX} + X \quad >0$$

It may be noted that $\delta_1 \beta_2/(\beta_1 - 1) = \delta_2$. Since the model is nonstochastic, $e^*_{+1} = e_{+1}$, and (A12) represents a first order difference equation in e. However, because $\beta_1 > 1$, it is necessary to solve (A12) in a forward direction and set the arbitrary constant equal to zero in order to ensure a stable time path for e, and, therefore, p.

The forward-looking solution for e is:

$$(A14) \quad e_t = (\beta_3/1-\beta_1)Z - (\beta_2/\beta_1)\sum_{i=o}^{\infty}(1/\beta_1)^i\bar{e}_{t+i}$$

Letting $\bar{t}$ be the current time period, t be the time of the announcement of the devaluation, $t + \theta$ the time it actually takes place, and $\bar{e}_o$ the value of the official exchange rate prior to $t + \theta$, (A14) may be written as:

$$(A15) \quad \bar{e}_t = (\beta_3/1-\beta_1)Z - (\beta_2/\beta_1-1)\bar{e}_o - (\beta_2/\beta_1)\sum_{t+\theta-\bar{t}}^{\infty}(1/\beta_1)^i(\bar{e}-\bar{e}_o)_{\bar{t}+i}$$

From (A12) the time path for p is given by:

$$(A16) \quad \bar{p}_t = [\delta_1\beta_3/(1-\beta_1) + \delta_3]Z + \delta_2(\bar{e}-\bar{e}_o) - (\delta_1\beta_2/\beta_1)\sum_{t+\theta-\bar{t}}^{\infty}(1/\beta_1)^i(\bar{e}-\bar{e}_o)_{\bar{t}+i}$$

APPENDIX

The time paths, (A15) and (A16), are illustrated in Figure 4 of the text. It may be confirmed that, when

(i) $\bar{t} = t-1$ (and, therefore, $\bar{e} = \bar{e}_o$)

and (ii) $\bar{t} = t+\theta$,

$$p_{\bar{t}} = [\delta_1\beta_3/(1-\beta_1) + \delta_3]Z$$

Hence, in the long run, the aggregate price level is independent of the official exchange rate when output is fixed.

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