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Stocks, Flows and Some Exchange Rate Dynamics for the Currency Substitution Model

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I. Introduction

In recent years the asset market or portfolio balance approach to the determination of exchange rates, viewed here as the relative price of monies, has gained dominance. 1/ The theory appeals because it focuses on a salient consequence of the move to generalized floating (accompanied by the progressive dismantling of capital controls), that involves transactors routinely holding assets denominated in different currencies. With the lowering in transaction costs occasioned by various market innovations, shifts between such assets can occur with ease. This in turn can cause a floating exchange rate to fluctuate substantially. However, as currency flows generated by trade, say, over a small interval of time, are likely to be insignificant relative to possible movements in currency stocks, it might seem appropriate, as in the standard asset market approach, to emphasize the role of foreign exchange stocks in determining the evolution of the exchange rate, and to ignore flows. 2/

1/ There is by now a vast literature which has been variously surveyed. See Isard (1978), Frankel (1983), and Krueger (1983). The earlier stimulus is provided in Mundell (1968) and the narrower monetary approach.

2/ Niehans remarks, "One of its features [of the monetary tradition in foreign exchange theory] is the tendency to downplay, by suitable simplifications, the flow aspects of the problem, just as the elasticity theorists often neglected the stock aspects." (1977, p. 1246).

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Nevertheless, such an approach does not lead to a satisfactory analysis of exchange rate dynamics as has been recognized in a number of contributions that have attempted to assess the effect of a current account surplus (deficit) on the exchange rate path. 1/ Unfortunately, the resolutions proposed, while representing an advance over the assumptions of the standard asset market approach, do not appear to take adequate account of the stock-flow problem. Failure to satisfactorily resolve this problem can be the source of several difficulties. 2/ In the argument of the paper, this failure hampers both a rigorous analysis of exchange rate dynamics and the appropriate incorporation of exchange rate phenomena in a more general macroeconomic framework.

In order to develop the argument, some key conceptual issues relating to the stock-flow distinction are examined, and their implications for exchange rate determination are noted. For this purpose it is convenient to employ the standard currency substitution (CS) model as this constitutes the simplest example of the portfolio balance approach, and has been extensively studied 3/ In the following section alternative stock-flow approaches are briefly evaluated, in the context of the CS model, and a preferred approach is selected. In Section III the stock-flow resolution that is proposed is incorporated into the CS model. This enables solutions for the spot rate of depreciation--a crucial first step in describing the evolution of exchange rates--to be derived under different expectational hypotheses, ranging from perfect foresight to adaptive expectations.

The more rigorous analysis permitted by the proposed stock-flow resolution is shown to clarify the dynamic properties ascribed to the model in the literature and to correct some erroneous conclusions, while generalizing certain other findings. Of particular interest is the generalization of the so-called accelerationist hypothesis, 4/ whereby a rise in the current account deficit always leads to a transitional depreciation of the exchange rate. This hypothesis helps explain the depreciation of the U.S. dollar rate vis-à-vis other major currencies in 1978-79, unlike the standard monetary or asset market approach, which

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1/ Branson (1974), Dornbusch and Fischer (1980), Kouri (1976, 1983), Niehans (1977) and Rodriguez (1980) have explicitly addressed the issue, which has also been noted by several other authors.

2/ For example, Meese and Rogoff (1983) provide vivid demonstrations of the general failure of structural models of the exchange rate to outperform the random walk model in out-of-sample forecasts. See also Frenkel (1983). One explanation for this surprising finding could be the mis-specification of the structural models, from not attending adequately to the stock-flow issues.

3/ See especially Kouri (1976). The major long-run results of the CS model correspond formally, mutatis mutandis, to those of the two-asset, closed-economy, monetary growth models developed by Tobin (1965) and Sidrauski (1967).

4/ See Kouri (1983) and Rodriguez (1980), in particular.

on the basis of markedly lower U.S. monetary growth would have predicted an appreciation. 1/ However, the accelerationist hypothesis is unable to account for the appreciation of the U.S. dollar in the recent period of rising current account deficits. 2/ The generalization provided shows how the latter association is possible, even in the simple confines of the CS model. These and other results are presented in Section IV in the course of assessing some effects of monetary and fiscal policies on the time paths of the exchange rate. Concluding comments are presented in Section V, while a number of mathematical results that are needed for the analysis are set out in the Appendix.

II. The Currency Substitution Model and Some Stock-Flow Issues

1. A critique of alternative proposals

In the simplest version of the model, transactors hold stocks of domestic currency, M, and non-interest bearing foreign assets, F, in their portfolios. 3/ Normalizing the import price at unity and assuming continuous purchasing power parity (PPP) renders the domestic price level equal to the exchange rate, E. Real financial wealth, A, can therefore be expressed as

$$(1) \quad A = M/E + F = m + F, \quad \text{where } m = M/E.$$

The model is in temporary equilibrium when the stock demand for foreign assets, F(real), equals the assumed given stock supply at any moment in time, 4/

$$(2) \quad F^d = F(\lambda^e, Y, A) = F, \quad \text{and } F_1 > 0, \; F_2 > 0, \; F_3 > 0.$$

1/ See Frankel (1983) for a discussion.

2/ U.S. monetary growth significantly exceeded that of Canada, Germany and Japan in 1982, so that according to the asset market hypothesis, the rate should have depreciated vis-à-vis the currencies of these countries. Notice that the theory neglects terms-of-trade effects, which provides a further channel for the appreciation to worsen the current account deficit.

3/ Although the model is set out here in terms of non-interest bearing assets, this can be readily generalized in the manner set out in Dornbusch and Fischer (1980), for example. However, some have argued that to the extent government debt implies future tax liabilities to pay the interest, only non-interest bearing government debt should be allowed. See especially Barro (1978).

4/ This is a simplifying assumption that is made by several authors, e.g., Kouri (1976), Dornbusch and Fischer (1980) and Rodriguez (1980), who assume that domestic assets are not held by foreigners so that the only way of acquiring foreign assets for the non-reserve country in question is by running a current account surplus.

Here λ^e, the expected spot rate of depreciation, equals $(\dot{E}/E)^e$; Y, the full employment level of output, is assumed to be fixed throughout.

On the face of it, and in an assumed perfect foresight context, equation (2) can be used to solve for the spot exchange rate level, E, that ensures instantaneous portfolio balance. 1/ Alternatively, the equation could be used to solve for the expected spot rate of depreciation, λ^e. Several authors, for example Dooley and Isard (1983), contend that from a portfolio balance condition such as equation (2), either the spot level of the exchange rate, E, or its expected rate of depreciation, λ^e, can be determined, but not both, and in order to analyze exchange rate movements, choose to solve for λ^e.

However, as Sargent and Wallace (1973) demonstrate, using the portfolio balance condition to solve only for the expected rate of change in the asset price is a source of dynamic instability. This is because the solution that results is, in their terminology, a "backward looking" solution, whereby the current spot exchange rate is historically determined, rather than by reference to the expected future. As a consequence, an excess demand for the foreign asset is eliminated, on the demand side, by an appropriate adjustment in the opportunity cost term. Thus, an expected appreciation of the domestic currency will reduce the demand for foreign assets. However, given the perfect foresight assumption of the model, the realized appreciation of the domestic currency reduces the supply of foreign assets in domestic currency units, thereby leading to a further excess demand for foreign assets. The solution proposed by Sargent and Wallace involves the direct elimination of the excess demand for the foreign asset by a depreciation, instead, of the domestic currency. Assuming forward looking rational expectations, Sargent and Wallace demonstrate that the solution to equation (2) not only yields the current spot price, E, but that the latter is derived from solving into the future the expected time path of E, so that both λ^e and λ are known. 2/ At the same time, more acceptable dynamic stability properties result. Subsequently, we shall revert to the Sargent and Wallace argument and show its compatibility to the more general approach to be adopted below.

Given portfolio balance, the next step in the analysis is to determine the rate of change in foreign assets. This is derived using the national income accounting identity for stationary economies (investment equals zero)

$$(3) \quad B = Y - C - G,$$

1/ Given the wealth constraint, the spot exchange rate could equally well have been solved from equating the stock demand for real balances to its supply. In equation (2) E is present as the denominator of real balances that enter into A.

2/ See also the solution in Rodriguez (1980), which is of the forward looking variety.

where B is the balance of trade (current account), G is government purchases of goods and services, and C is private consumption.

The following balance of payments flow identity results

(4) Current account + capital account $= B - \dot{F} = 0.$

The issue to be posed at this stage is one of whether or not there is an underlying equilibrating process that ensures that the rate of accumulation of foreign assets, $\dot{F}$, is an equilibrium rate, and if so, how it relates to the stock equilibrium condition expressed by equation (2), where the given stock of F at a point in time is determined as an equilibrium amount.

Before attempting a resolution, it is useful to consider some of the major positions taken in the literature with regard to the stock-flow issue. Branson (1974), in his extension of the continuous equilibrium portfolio balance model to an open economy, draws the appropriate sharp distinction between stock-shift phenomena and continuing flow effects that is enforced by a continuous-time dynamic analysis. However, this separation is not maintained in the determination of the values taken by the market adjustment variables over the economy's time path. For Branson, these variables are determined essentially through the simultaneous interaction of stock and flow transactions, as in the traditional IS/LM model. In particular, the instantaneous value of the exchange rate is determined from the requirement that the balance of payments be in equilibrium. 1/

Thus, for Branson, the flow equation (4) above, on substituting behavioral expressions for B and $\dot{F}$, is an equilibrium condition, which he uses to solve for the instantaneous equilibrium level of the exchange rate. 2/ The problem with this procedure is that it takes no account of the possible role of stock-shift or portfolio balancing factors stated in the stock equilibrium condition (2) for influencing the exchange rate, but relies entirely on flows. The model provides for a menu of three assets that residents can hold--domestic nontraded equities, domestic money, and foreign securities. Because of the standard, simplifying assumption that external liabilities cannot be created, transactions in foreign securities lead to opposite movements in foreign reserves or,

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1/ See equation (15) in Branson (1974, p. 47).

2/ The trade balance, B, is expressed as a function of the exchange rate level, while the rate of change in the domestic demand for foreign securities, $\dot{F}$, is a (variable) function of the rate of growth of wealth. Although the latter function can include a stock adjustment element, this is not essential and the issue remains as to the status of the portfolio balance condition.

when the exchange rate is freed, to exchange rate fluctuations. However, for a satisfactory theory of exchange rate determination, these portfolio balance or stock-shift effects must be reconciled with the continuing flows or balance of payments effects. 1/

Niehans (1977) pursues a different track by not allowing for instantaneous portfolio balance, thereby abandoning the stock equilibrium equation (2). Instead, any disequilibrium in stock holdings of the two assets that he provides--domestic and foreign non-interest bearing monetary assets--is eliminated over time through a stock adjustment scheme. Ignoring instantaneous stock-shift phenomena in this way results in a pure flow model. The balance of payments flow equilibrium condition (4), that equates the flow demand for the foreign asset with its supply, as given by the current account of the balance of payments, is used just as in Branson to determine the instantaneous exchange rate level. 2/ While Niehans' formulation avoids the problem of what to do with the stock equilibrium condition, his procedure of using the flow equilibrium condition to solve for the exchange rate level rather than its rate of change is problematic, because of the assumption that the model is one of continual market clearing for flows. Unless the model is discontinuous, so that the market clears only periodically, ruling out instantaneous portfolio balance in a context of continual market clearing precludes jumps in asset prices. Instead, these prices become historically determined and can only adjust as a continuous function of time. In such a framework the flow equilibrium condition should be used to solve for the instantaneous rate of change in the asset price and not its level. 3/

Suppose, however, we wish to model stock-shift phenomena with continual market clearing and, in particular, to allow for the equilibration of asset preferences through discontinuous jumps in asset prices. It will then be necessary to employ the stock equilibrium condition (2) for this purpose, as was undertaken in Kouri (1976). The issue now arises as to how equilibrium in flows is provided.

1/ Branson recognizes the problem and states at the end of his paper, "But the introduction of capital movements, with the value of the exchange rate being determined by both continuing flows and discontinuous stock shifts, raises some analytical problems that are yet to be solved." (1974, p. 48). Fair (1979), who quotes the same sentence, asserts that he has solved the stock-flow problem in his paper. Unfortunately, this is not a valid claim. Fair appears to take the first part of the quotation from Branson literally and sets out stock conditions and flow conditions that he asserts jointly determine the exchange rate. In his discrete time framework, the stock conditons and flow conditions are additive, with the consequence that asserting them jointly simply has the effect of pushing the stated stock conditions forward one period.

2/ Thus see equation (11), op. cit, p. 1249.

3/ See Purvis (1973) for the specification of a pure flow, closed economy, monetary growth model that preserves these distinctions.

Unlike with Branson and Niehans, Kouri (1976) states emphatically that no meaning as an ex ante equilibrium condition should be attached to the ex post balance of payments (flow) identity expressed in equation (4) above. In his words, "the reason is that when using the assumption of a continuous portfolio equilibrium, the flow demand equation for individual assets cannot be defined" (Kouri (1976), p. 286). In a subsequent paper, Kouri (1983) abandons this position, but before considering his proposed alternative the present postulate should be examined more closely, as it has been influential in a number of models. 1/ Essentially, the postulate derives from the notion that at any point in time flows are insignificant compared to stocks so that allowing for stock equilibrium eliminates any equilibrating role for flows. In this approach flows are left hanging. Although some of these flows are important in determining the evolution of the economy, for example F or B, they are not the outcome of any equilibrating process but merely accounting statements. In such a context, any time paths that are derived cannot generally be described as equilibrium time paths. Nor can it be maintained that stock equilibrium, as expressed by equation (2), holds continuously over time, unless it is shown that the flows that change stock positions are themselves being continuously equilibrated. The latter property may follow implicitly in a perfect foresight context, which is one of the expectational alternatives considered in Kouri (1976). However, he does not clarify the nature of the solution.

A substantial clarification of the perfect foresight solution in Kouri's basic CS model is presented in Rodriguez (1980) who, responding to Niehans' (1977) challenge, demonstrates that flows are of fundamental importance to the evolution of exchange rates, even when instantaneous portfolio balancing is permitted. The demonstration takes account of the nature of the perfect foresight assumption, whereby transactors "know" the future evolution of the currently available stock of foreign assets, and, hence, the underlying stream of future current account surpluses (flows). In this model, an increase in the current account surplus generates an instantaneous appreciation of the spot rate. At the same time, the expected rate of appreciation of the exchange rate is revised upwards, for the duration of the higher surplus.

The solution that Rodriguez presents is virtually identical to Sargent and Wallace's (1973) forward looking solution. While correct as far as it goes, the model and its solution do not generalize easily. In particular, it is not obvious how the current account surplus (deficit) will affect the evolution of exchange rates in a context that does not allow for perfect foresight. Nor, even under the perfect foresight assumption, is it clear how the current account surplus relates to domestic saving, or more generally to the market for goods and services.

1/ See Foley (1975) and Mussa (1976) for an elaboration of this standpoint.

Some further elaboration is provided in Dornbusch and Fischer (1980), who specify a link between the current account surplus of the balance of payments and savings behavior. They also generalize the Kouri CS model to allow for the endogenous determination of the price of domestic output. Unfortunately, despite the gain in realism, the explicit elaboration of savings behavior and its role in affecting exchange rates is problematic. This is because in their presentation Dornbusch and Fischer treat the net savings flow (domestic investment is zero) as being simultaneously equal to both the flow supply of foreign assets (the current account surplus) and its demand. As will be evident from the subsequent discussion, such a procedure does not facilitate the determination of the time path of the exchange rate.

Finally, we consider the so-called accelerationist model in Kouri (1983). The analysis is confined to the foreign exchange market and, as Kouri emphasized, is one of dynamic partial equilibrium. Nevertheless, the model represents an important advance in its explicit recognition of the role of flow demands for foreign exchange in determining movements in the exchange rate. In a full reversal of his earlier position (see the quotation from Kouri (1976) above), Kouri argues, "Given the short-run equilibrium value, the exchange rate must change <u>per unit of time</u> in such a way as to equilibrate flow demands for and supplies of foreign exchange derived from capital flows on the one hand and current account transactions on the other" ((1983), p. 117). However, the implications of this new approach are not fully drawn out by Kouri. In particular, an integration of the flow demand for foreign assets with savings behavior is lacking; as are linkages between the flow market for foreign assets and other flow markets (e.g., the market for goods and services) and specifications of the underlying equilibrating mechanisms. Each of these aspects will need to be resolved in moving to a more general setting such as that provided in Kouri's (1976) CS model.

2. A stock-flow resolution

An essential feature of the approach presented here for handling stocks and flows in a dynamic general equilibrium setting is to treat flows from the viewpoint of maintaining stock equilibrium continuously over time. 1/ This is undertaken as follows.

Suppose that at any point in time, transactors can achieve their desired portfolio composition instantaneously, subject to the overall wealth constraint. Over time, flows of foreign assets supplied through the balance of payments and money flows generated via the government budget constraint will, respectively, affect stocks of foreign assets and cash in the economy. Assume next that savers wish to accumulate the two assets at varying rates, depending on their perception of opportunity costs but subject to the overall savings constraint. Given an equilibrating scheme for flows, an initial state of portfolio balance can be

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1/ See Chand (1981) for an elaboration.

continously maintained over time through the elimination of any excess flow demands for each of the assets.

Of course, if at any point in time a disruptive event occurs, such as changes in the perceptions of exchange risks or in policies and other "news", and wealth owners desire a new portfolio composition, they can engage in an instantaneous stock shift, provided that transaction costs are not prohibitive. The time path of the exchange rate will thus exhibit periodic jumps as a result of stock-shift behavior, and stretches of relative smoothness, reflecting predominantly flow adjustments. In the suggested approach equation (2) features as a stock equilibrium condition, but equation (4) is employed as a flow equilibrium condition. The former solves for the level of the exchange rate, while the latter solves for the spot rate of depreciation (viewed as the <u>right</u>-hand time derivative of E).

It can be readily shown that the suggested approach is consistent with the perfect foresight, forward looking, procedure employed by Sargent and Wallace (1973) for determining the current spot asset price. The consistency with the Sargent and Wallace procedure follows from the fact that the flow equilibrium condition employed here for solving for the rate of change in the asset price is, as will be evident subsequently, the same operation as time differentiating the stock equilibrium condition (but only for the perfect foresight case) and finding the equilibrium rate of change in the asset price. Where the assumed environment is one of perfect foresight, the flow equilibrium condition constitutes the first link in a chain stretching from now to the future that solves for the entire forward time path of the price level. It is especially important to recognize the role played by this condition in setting out the basic structure that is employed to determine the time path. However, if expectations are governed by some other, not necessarily forward looking, process only the instantaneous rate of change in the asset price generated by the flow condition is needed for the analysis of exchange rate dynamics, as is demonstrated further below.

III. A Reformulation of the Basic Model

In this section the currency substitution model is reformulated with respect to its treatment of stocks and flows. The model is then solved for both the spot level and the spot rate of depreciation under alternative expectations hypotheses. As the basic model is well known, 1/ its structure is presented here in abbreviated form.

1. Structure

The country is small and produces output that is a perfect substitute for imports. Output is fixed throughout at its full employment level. It

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1/ See especially Kouri (1976).

is assumed that residents hold both the non-interest bearing domestic and foreign assets, but that foreigners do not hold domestic money. Furthermore, residents as a group can only acquire foreign assets through running a current account surplus. Portfolio positions are described in equations (1) and (2) above.

Consumption function

(5) $C = C(Y_d, A)$, where Y_d is disposable income and $C_1 > 0$, $C_2 > 0$.

Disposable income 1/

(6) $Y_d = Y - T - \lambda^e m$.

Here Y is the full employment level of output, T is revenue, from lump sum taxes (so as to ensure neutrality), m is real balances (M/E) as before, and λ^e is the expected rate of depreciation, ($\lambda^e = (\dot{E}/E)^e$). The inclusion of the expected rate of erosion in real balances in the income concept is because the savings and consumption functions are assumed to be influenced by this broader concept of disposable income. 2/

Savings and its disposition 3/

(7) $S = (Y - T) - C(Y_d, A) = \dot{M}^d/E + \dot{F}^d$.

Ex ante saving is defined here in physical terms as output that is not consumed, which is appropriate for the consideration of sectoral constraints that is undertaken subsequently (see equation (14)). In this treatment the capital gain term $\lambda^e m$ is treated as a balance sheet item rather than as part of the income-expenditure account, even though it influences consumption behavior. However, when considering the net change in wealth, this item must be netted against S.

1/ As the domestic price level and the exchange rate are identical given continuous PPP, the expected erosion in real balances is determined by the expected rate of depreciation. Disposable income is defined here in ex ante terms. Ex post the expected term would be replaced by its realized magnitude.

2/ This treatment is similar to that of closed economy, money and growth models, for example, Sidrauski (1967), and owes its lineage to the Haig-Simons argument for the extension of the disposable income concept to include expected capital gains or losses. Some authors, Kouri (1976), for example, employ the narrower concept.

3/ Savers here are concerned not only with adding to their wealth, but also in the composition of the wealth addition, i.e., the shares to be assigned to the net accumulation of real balances and foreign assets, respectively.

Government

$$(8) \quad G - T = \dot{M}^s/E = \theta m,$$

where $\theta = \dot{M}^s/M$ and G is government expenditure. Alternative model structures are generated depending on whether T or G is taken as residually determined. Here it will be assumed that both T and M^s are policy determined, thus forcing G to be a residual. It is assumed that $\theta > 0$.

Balance of Payments

$$(9) \quad B = (X - I) = Y - C(Y_d, A) - G = \dot{F}^s,$$

where X is exports, I is imports, and $\dot{F}^s$ represents the flow supply of foreign assets (positive or negative).

Demand for real balances 1/

$$(10) \quad M^d/E = m^d = L(\lambda^e)F, \quad L' < 0.$$

2. Conditions for momentary stock-flow equilibrium

Drawing on the stock-flow resolution proposed in Section II, the momentary stock and flow conditions for determining the exchange rate level and its rate of change, respectively, are derived next.

Instantaneous stock equilibrium

Stock demands and supplies must always satisfy the wealth constraint

$$(11) \quad A = M^d/E + F^d = M^s/E + F^s.$$

Hence, by Walras' law, stock market equilibrium will prevail when

$$(12) \quad \text{either } M^d - M^s = 0 \text{ or } F^d - F^s = 0.$$

Employing one of these conditions, the equilibrium exchange rate level at a point in time, t, is determined for given expectations, λ^e, as

1/ This analytically appealing formulation of the demand function for real balances assumes homogeneity in wealth holdings. It is derived as a reduced form from a conventional specification, such as L(Y,i,A). The separate role of output or income, Y, in determining the transactions demand, has been suppressed, as this is assumed constant.

(13) $E = M^s/L(\lambda e)F.$

Continuing stock or flow equilibrium

Sectoral budget constraints (equations (7), (8) and (9)) are consolidated, and, on placing real flows on the left-hand side (LHS), and financial flows on the right-hand side (RHS), give rise to

(14) $Y - C - G - B \equiv 0 \equiv (\dot{F}^d - \dot{F}^s) + (\dot{M}^d/E - \theta m).$

As is apparent from the right-hand side of equation (14), the excess flow demands for foreign assets or for real balances are related to the stock conditions set out in equation (12) through the latter's time derivatives. While in a formal sense the mandatory time derivative relationship between stocks and flows is preserved, there are important issues of interpretation concerning, in particular, the relationship between the financial flows and the goods market, which are taken up next.

For the model at hand, the LHS of equation (14) states that the national product market is identically in equilibrium, as any excess supply of domestic output is always eliminated through a corresponding balance of trade surplus (given that domestic output is indistinguishable from the foreign good and that the country is small). Hence, the sum of excess demands for financial flows is also identically equal to zero, although ex ante non-zero flow excess demands for each of the financial assets can prevail. The flow demands for financial assets are defined by reference to the time rate of change in stock demands. Ex ante, they are based on expectations concerning the market adjustment variable, E, and its rate of change, λ. For the system to be in flow equilibrium

(15) either $(\dot{M}^d/E - \theta m) = 0$ or $(\dot{F}^d - \dot{F}^s) = 0.$

In what follows we focus on the condition for domestic monetary flows. This may be conveniently re-expressed by the following condition for ex post equilibrium

(16) $D(M^d/E) - (\theta - \lambda)m = 0,$

where D is the time derivative operator. 1/

1/ The substitution employed is $D(M^d/E) = \dot{M}^d/E - \lambda m.$

An expression for the flow demand for real balances is obtained by time differentiating the stock demand function: 1/

$$(17) \quad D(M^d/E) = FL'(\lambda^e)\dot{\lambda}^e + L(\lambda^e)\dot{F}.$$

The flow demand for real balances depends both on the expectation hypothesis governing λ^e, which in turn determines the value of $\dot{\lambda}^e$, and on the rate of change in foreign asset holdings, $\dot{F}$. In the event of financial flow disequilibrium, the solution technique here requires the actual spot rate of depreciation to take that value that will eliminate the <u>ex ante</u> excess flow demand for real balances (or equivalently, foreign assets). In other words,

$$(18) \quad D(M^d/E) - (\theta - \lambda^e)m = m(\lambda^e - \lambda),$$

with λ diverging from λ^e if the <u>ex ante</u> excess flow demand for real balances shown on the LHS of equation (18) is not equal to zero.

A general solution for the spot rate of depreciation follows from equation (18). Substituting for $D(M^d/E)$ from equation (17) and using the money demand function in equation (10)

$$(19) \quad \lambda = \lambda^e + \rho(\dot{\lambda}^e/\lambda^e) - \dot{F}/F - (\lambda^e - \theta),$$

where $\rho = -(L'/L)\lambda^e$ is the demand elasticity for real balances with respect to λ^e.

The momentary solution of the model is completed by using equations (8), (9), (14) and (15) to solve for the rate of change in foreign assets, which is now an equilibrium rate.

$$(20) \quad \dot{F} = Y - C(Y - T - \lambda^e m, A) - T - \theta m.$$

3. The spot rate of depreciation under <u>alternative expectations hypotheses</u>

Three alternative expectations hypotheses are employed here to cover some expectational extremes. The first is the rational expectations or, in the present deterministic context, the perfect foresight hypothesis,

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1/ The flow demand for foreign assets, $\dot{F}^d$, can be obtained from equation (10).

whereby transactors anticipate correctly the entire time path of the exchange rate. The second is static long-run expectations, which assumes that transactors can solve for the steady state rate of depreciation (the terminal point for the perfect foresight solution), but not for the intervening path. Being uncertain as to when the long-run solution will prevail, they are assumed to employ the long-run expectation to predict the spot rate at each point in time. Under the third hypothesis of adaptive expectations, transactors are assumed to lack knowledge both of the long-run solution and the intervening path, but adjust their expectations through a simple error learning mechanism. Most other expectational hypotheses can be formulated as combinations of these three. 1/

The solutions for the spot rate of depreciation, λ, at any time t outside the steady state are as follows:

Rational expectations

(21) $\lambda e = \lambda$ and $D^n(\lambda e) = D^n(\lambda)$ 2/ for $n = 1 \ldots \infty$.

Hence, using equation (21) in equation (19),

(22) $\lambda = \rho(\dot{\lambda}/\lambda) + (\theta - \dot{F}/F)$.

Static long-run expectations

(23) $\lambda e = \theta$ and $\dot{\lambda} e = 0$.

Hence, from equations (19) and (23)

(24) $\lambda = \theta - \dot{F}/F$.

Adaptive expectations

(25) $\dot{\lambda} e = b(\lambda - \lambda e)$,

1/ Thus a particular case of perfect myopic foresight would be $\lambda e = \lambda$ but without imposing equality for higher-order time derivatives. It might be noted that although individual transactors are presumed rational, their behavior in the aggregate can exhibit non-rational patterns depending on the restrictions imposed on different transactors' access to information and their varying stages in acquiring knowledge of the environment.

2/ D^n symbolizes the operation of taking the time derivative at time n.

where b is a speed of adjustment coefficient, assumed constant, and $0 < b < \infty$.

Hence, from equations (19) and (25)

$$(26) \quad \lambda = (1 - 1/Z)\lambda^e + (\theta - \dot{F}/F)/Z,$$

where $Z \equiv 1 + (L'/L)b$ or $1 - \rho(b/\lambda^e)$.

Each of the solutions for the spot rate of depreciation has as a common element $\theta - \dot{F}/F$. This is because λ adjusts in part to ensure that the desired proportional rate of growth in real balances, which under the assumptions made here is a unit elastic function of the proportional rate of growth in foreign assets, $\dot{F}/F$, is being met for a given rate of growth of the money supply, θ. It can readily be shown that in steady state each of the λ's reduces to θ.

IV. Alternative Financial Policies and the Solution of the Model

The existence and uniqueness of the steady state solutions for the model comprising equation (20) and either one of the three expectational hypotheses is readily demonstrated, and will be assumed here. Given a rate of growth in the money supply, θ, and a specific fiscal policy regime, there exists a long-run equilibrium solution with the following characteristics

$$(27) \quad \lambda^{e*} = \theta, \quad F = F^*, \quad \dot{\lambda}^e = 0, \quad \dot{F} = 0.$$

where * refers to steady state values.

The stability of the system under the alternative expectational hypotheses is examined in the appendix. With static long-run expectations, the model is globally stable, provided the marginal propensity to consume is less than one. Under adaptive expectations, the condition for local stability requires, in addition, that the speed with which the expected rate of inflation is revised, b, not exceed a critical level that is determined in part by the elasticity of substitution between assets (see Appendix). In the perfect foresight case, expectations are in effect revised instantaneously so that the long-run equilibrium solution has the characteristics of a saddlepoint. Hence, there is only one path to which the state variables can be displaced, if the economy is to return to its steady state solution. In what follows, the relevant stability conditions will be assumed.

1. The long-run solution

In order to examine the effects of financial policies on exchange rate behavior, their implications for the steady state solution should first be considered, as these are invariant under the alternative expectational hypotheses.

The pursuit of a more expansionary monetary policy, represented by a higher rate of growth in the nominal supply, θ, will cause the long-run expected and actual rate of depreciation to rise by the same amount. 1/ In keeping with the standard two-asset model (e.g., Sidrauski, 1967), substitution away from real domestic balances will occur in portfolios, owing to their higher opportunity costs. However, whether or not the long-run amount of the alternative asset increases will depend on ρ, the demand elasticity of domestic real balances with respect to λ^e. The condition governing the long-run behavior of foreign assets is derived from setting $\dot{F}$ in equation (20) equal to zero, substituting (m+F) for A and differentiating with respect to θ:

$$(28) \quad dF^*/d\theta = m[\theta(\rho-1)(C_1-1)C_2\rho]/\theta D \gtrless 1/(1+a),$$

where $a = -C_2/[\theta(C_1-1)]$ and $D = \theta L(C_1-1) - C_2(1+L) < 0$. 2/

The associated implication for long-run real balances is given from the stock demand for domestic real balances (equation (10) as

$$(29) \quad dm^*/d\theta = LF^*(\delta-\rho)/\theta.$$

where δ is the response elasticity of the demand for foreign assets with respect to θ. Clearly, long-run real balances will fall, although there is the possibility that if the increase in foreign assets is sufficiently sizable, there could be an increase in real balances as well.

Turning next to a more expansionary fiscal policy, but keeping the money financed budget deficit constant, it is easily shown that foreign asset holdings in the long run unambiguously decline. The government budget constraint was earlier assumed to have expenditure determined as a residual. Hence, an expansionary fiscal policy, with deficit financing constant, requires that lump sum taxes be increased to support an increase

---

1/ Note that the assumption that θ is maintained constant assumes complete sterilization of the foreign exchange market.

2/ In the absence of a wealth effect in the consumption function ($C_2 = 0$), the above condition reduces to that of the demand elasticity ρ, of real balances with respect to λ^e, being greater or smaller than unity.

in expenditure. The effect of expansionary fiscal policy is thus the same as that predicted by the balanced budget multiplier, whereby the tax increase reduces consumption expenditure by less than the increased government expenditure. The current account surplus is reduced, while the long-run implications for holdings of foreign assets are as follows:

$$(30) \quad dF^*/dT = -(C_1-1)/D < 0.$$

Hence, long-run holdings of domestic real balances always decline as well.

$$(31) \quad dm^*/dT = L \cdot dF^*/dT < 0.$$

Finally, as the pure fiscal policy case does not involve a change in the rate of growth of the money supply, the spot rate of depreciation is unaffected.

2. Some transitional dynamics

With the above information on the long-run outcomes, the transitional dynamics under the three expectational hypotheses can now be analyzed for expansionary monetary and fiscal policies.

Static long-run expectations

As the expected rate of depreciation, λ^e, remains static at the given rate of growth in the money supply, the analysis will focus on the movements of the actual rate of depreciation, λ, and net foreign assets. The movements in these variables over time are described by the two equations noted above that are reproduced here for convenience

$$(24) \quad \lambda = \theta - \dot{F}/F.$$

$$(20) \quad \dot{F} = Y - C(Y - T - \theta m, A) - T - \theta m,$$

where

$$(10) \quad m = M/E = L(\theta)F.$$

The preceding three equations together indicate that the actual rate of depreciation, λ, is determined only by net foreign assets, F, for given θ and T. That is,

$$(32) \quad \lambda = f(F, \theta, T).$$

A diagrammatic representation of the system is presented in Figure 1. The $\dot{F} = 0$ line is based on equation (20) and is vertical in the diagram as the long-run solution, F^*, does not depend on λ or λ^e but only on θ. However, deviations from the $\dot{F} = 0$ line trigger movements in the spot rate of depreciation along the λf line, which has a positive slope, as is readily seen from differentiating equation (24) with respect to F and using the stability condition in the Appendix.

$$(33) \quad \partial\lambda/\partial F = -\partial(\dot{F}/F)/\partial F > 0.$$

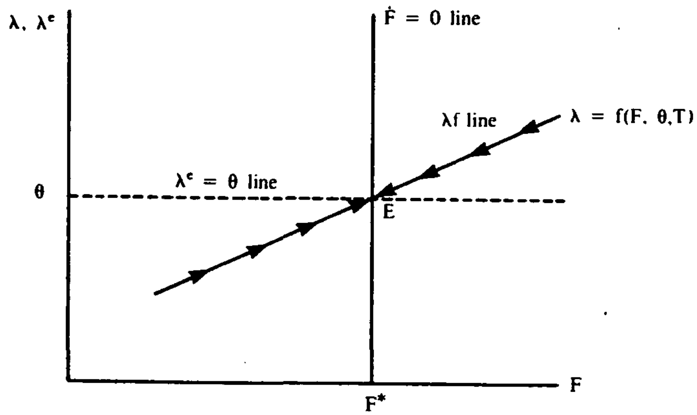
The adjustment dynamics of this model are straightforward. When actual foreign asset holdings are below the long-run equilibrium level, current account surpluses will be generated in the transition to F^*, while the opposite occurs when holdings are excessive. The λf line represents the adjustment path and indicates the spot rate of depreciation required to equilibrate the flow supply of foreign assets and their flow demand at each point on the path.

An expansionary monetary policy, unlike an expansionary (pure) fiscal policy, involves an increase in the deficit on account of the government budget constraint and the absence of domestic government debt in this paper. In order to analyze the effects of such an expansionary monetary policy, it is useful to distinguish between the cases of high and low asset substitutability as the associated adjustment paths are quite different. In part this is because the higher expected opportunity costs to holding domestic real balances will cause the initial depreciation (jump) in the exchange rate level to vary by different amounts depending on the degree of asset substitutability. Figure 1b provides the information needed to assess exchange rate dynamics when asset substitutability is high. On implementing the expansionary monetary policy the $\dot{F} = 0$ line shifts to the right in accordance with the earlier demonstration that the new long-run equilibrium level of F^* will be higher. Regarding the shifts in the λf line, two possibilities are present shown in Figures 1b(i) and 1b(ii) respectively, depending on the strength of high asset substitutability effects. In both cases the spot rate of depreciation will, during the transition, lie below the new equilibrium rate θ_1, set by the expansionary monetary policy, indicating that after the initial jump in the exchange rate level E, the transitional adjustment path will be one where E depreciates less rapidly than at its long-run rate of depreciation. In the case shown as Figure 1b(i) there could even be some appreciation of the exchange rate level following the initial increase in the money supply.

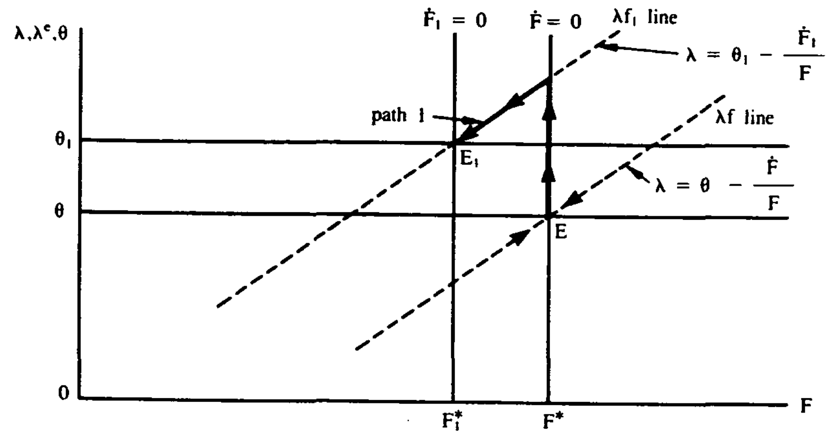
It is through examining the movements in the spot rate of depreciation that the nature of the initial exchange rate jump can be inferred. For the cases shown in Figure 1b, the depreciation in the exchange rate level must have overshot its trend value, which is undone by the subsequent movements in the exchange rate. During the transition, the current account surpluses generated are not, contrary to the accelerationist hypothesis, associated with an appreciating exchange rate, but instead with a

Figure 1. THE STATIC LONG-RUN EXPECTATIONS CASE

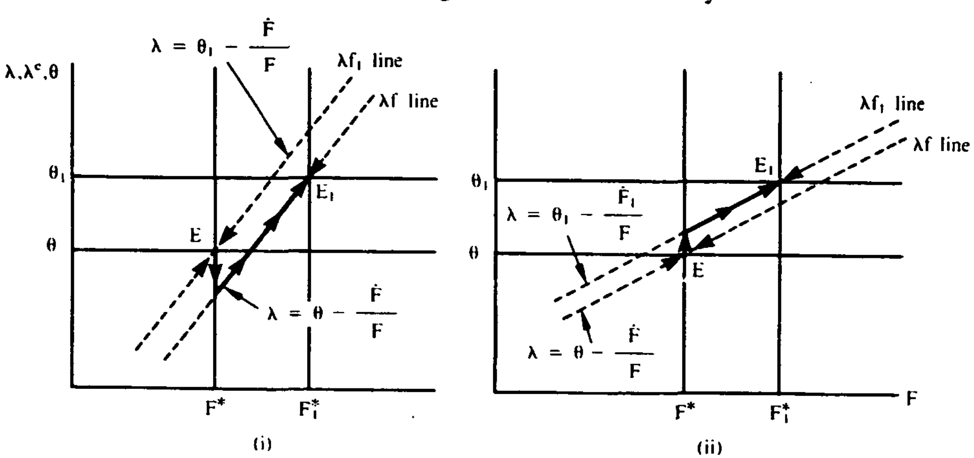
(a) The Basic Case



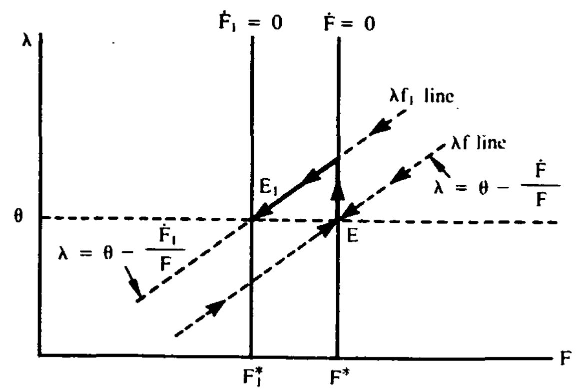
(c) Monetary Expansion and Low Asset Substitutability



(b) Monetary Expansion and High Asset Substitutability



(d) Fiscal Expansion



continuing depreciation. This seemingly counter-intuitive result follows from the flow supply of foreign exchange being less than its flow demand.

Results that are converse to the preceding are obtained in the case of monetary expansion when asset substitutability is low. The dynamics are represented in Figure 1c and, as can be readily inferred, imply an initial undershooting in the exchange rate level that is followed by some appreciation, in the transition to the new long-run equilibrium trend path. The latter appreciation is associated with current account deficits that enable the economy to decumulate its foreign assets holdings. 1/

The effects of a more expansionary fiscal policy are illustrated in Figure 1d. As exchange rate expectations are not affected, there is no initial jump in the exchange rate level. However, during the transition, the balance of payments is in deficit because of excess aggregate demand. The resulting decumulation of foreign assets reduces the flow demand for real balances. Hence, given an unchanged rate of growth in the money supply, a higher rate of depreciation results to ensure continuing portfolio equilibrium in the transition to lower final stocks of both m and F.

Rational expectations

Certain similarities between the perfect foresight case and static long-run expectations are present, as both are forward looking, even though under static expectations only the long-run equilibrium rate of depreciation is anticipated. Of course, in the perfect foresight case no exchange losses or gains are incurred during the transitions.

Based on the mathematical results in the appendix, Figure 2a shows the dynamics of the system under a more expansionary monetary policy and high asset substitutability. There is only one trajectory, depicted as the SS line, that the economy can pursue if it is to reach its new long-run equilibrium. At the time the new policy is implemented, transactors recompute the future time path of the exchange rate. 2/ The new situation will necessitate a rebalancing of portfolios at a lower level of real balances. In order to establish whether this causes the exchange rate jump to overshoot its new long-term trend, once again we consider the behavior of the spot rate of depreciation. There is an initial discontinuous jump in this rate to reach the trajectory SS, following which, and in consequence of, the undershooting, the rate of depreciation rises

1/ It should be emphasized that it is not the appreciation that brings about the current account deficit as no terms of trade effects are present in the model and the stimulative real balance effect on domestic private expenditure is limited. Instead the current account deficit results from the higher level of government dissaving associated with the more expansionary monetary policy.

2/ Announcement effects from knowing the policy change before it is actually implemented can be introduced as in Boyer and Hodrick (1982).

until it equals the new money supply growth rate, θ_1. Thus overshooting must have occurred in the initial exchange rate level jump, with the excessive initial rise in E compensated by below-equilibrium rates of depreciation during the subsequent transition period.

In the case of a monetary expansion when the substitutability between assets is low, the reverse of the preceding occurs, with the jump in the exchange rate level undershooting its new long-term trend. The dynamics of this case are illustrated in Figure 2b. A somewhat similar situation is obtained if an expansionary fiscal policy is applied. However, just as with static expectations, the result here does not depend on the degree of asset substitutability (see Figure 2c). The spot rate of depreciation exceeds θ during the transition, while the balance of payments is in deficit. Unlike with static expectations, however, there is an initial exchange rate level jump in order to rebalance portfolios, as a consequence of new expectations.

<u>Adaptive expectations</u>

In this scheme exchange rate expectations adjust gradually. Since there is no initial impact on stock demands, the exchange rate level does not exhibit jumps, and the entire adjustment relies instead on movements in the spot rate of depreciation.

Drawing on results in the Appendix, phase diagrams are presented in Figure 3 that exhibit the dynamics following an expansionary monetary policy. As before, a distinction is drawn concerning the degree of asset substitutability when considering monetary expansion. There is only one path that the economy will follow in the transition from E to E_1 in the high asset substitutability case (see Figure 3a). However, in the alternative case of Figure 3b, three possible paths can be distinguished. The characteristics of certain of these transition paths are shown in more detail in Figure 4. In the two cases of monetary expansion that are illustrated, the adjustment process entails an excessive initial decumulation of foreign assets. As a consequence, the balance of payments path undergoes a reversal during the transition, generating surpluses in order to reconstitute foreign asset holdings. Related fluctuations are induced in the time path of the spot rate of depreciation that depend on the variation in the flow demand for real balances as foreign asset holdings evolve.

The dynamics of an expansionary fiscal policy are illustrated in the phase diagram of Figure 3c. There is some similarity in results with the rational expectations case, which is also evident from the time paths detailed in Figure 4c. Initially, the spot rate of depreciation jumps, but thereafter declines continuously until it reaches its earlier steady state level. In the manner of adaptive, first-order error learning schemes, the expected rate of depreciation consistently falls behind actual developments. In effect, a semi-circular path is pursued.

Figure 2. THE PERFECT FORESIGHT CASE

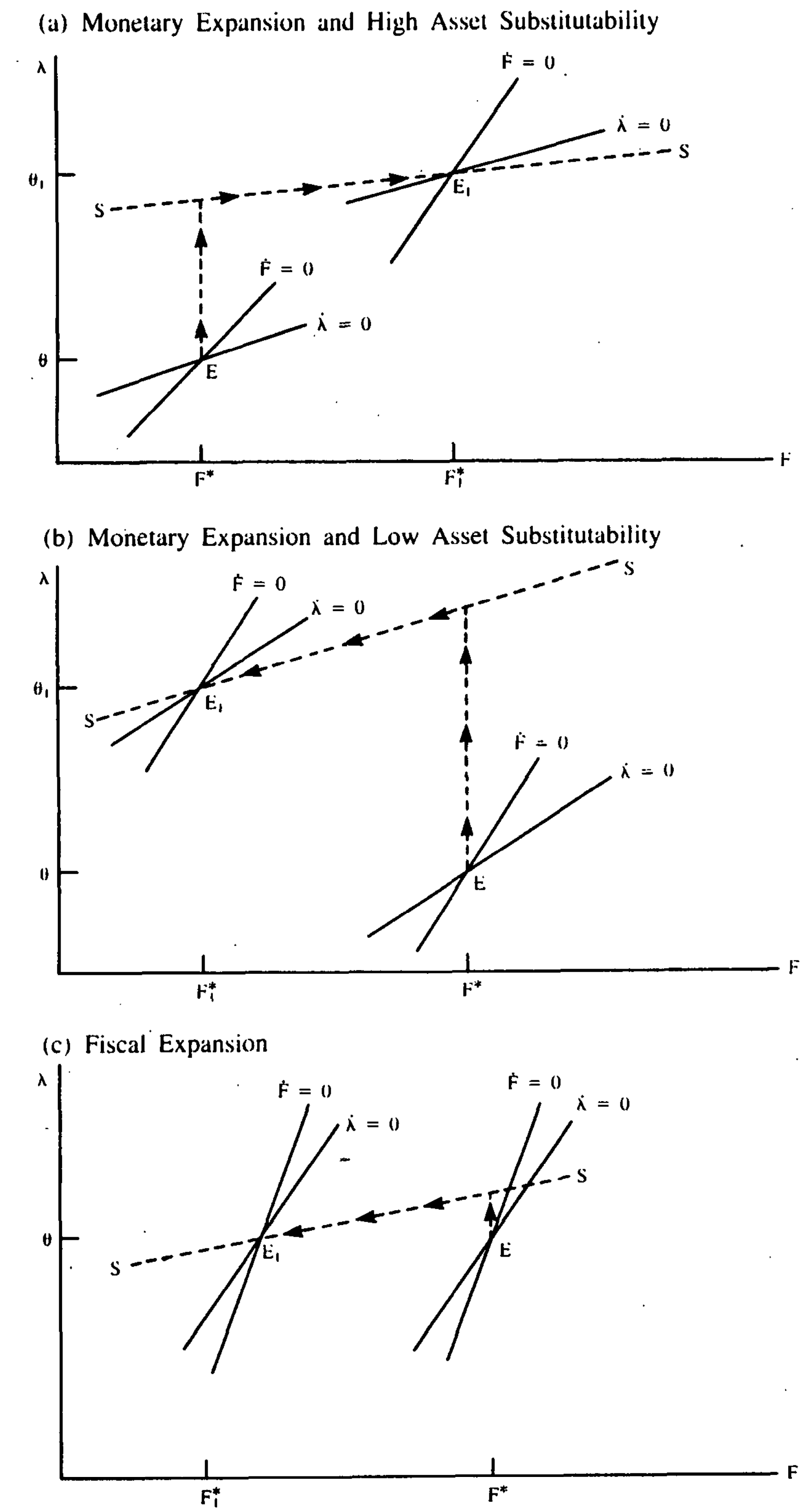


Figure 3. THE ADAPTIVE EXPECTATIONS CASE

(a) Monetary Expansion and High Asset Substitutability

(b) Monetary Expansion and Low Asset Substitutability

(c) Fiscal Expansion

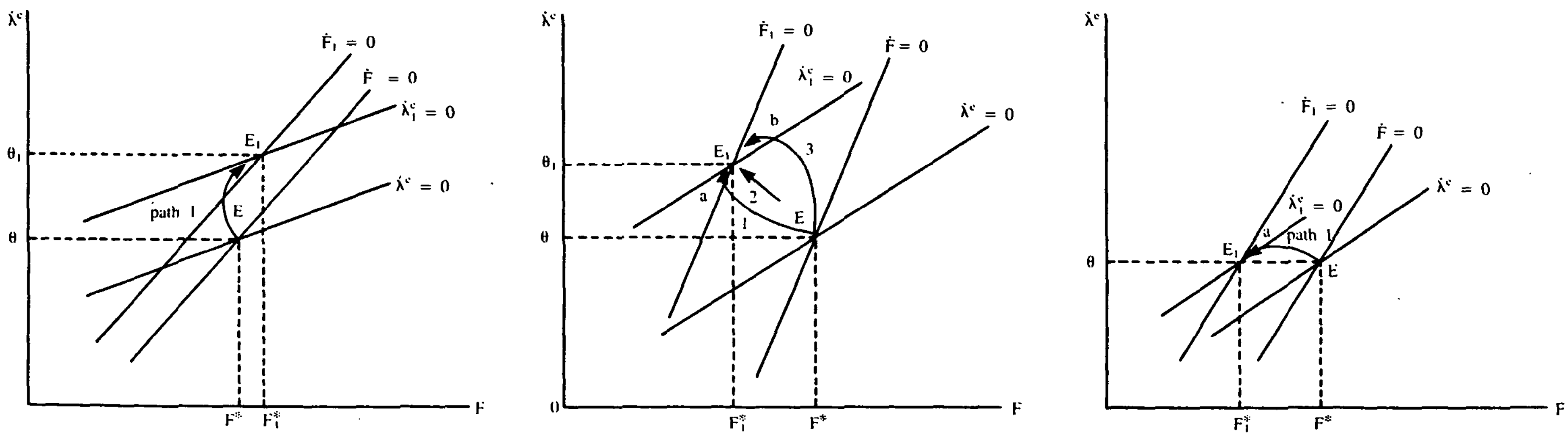
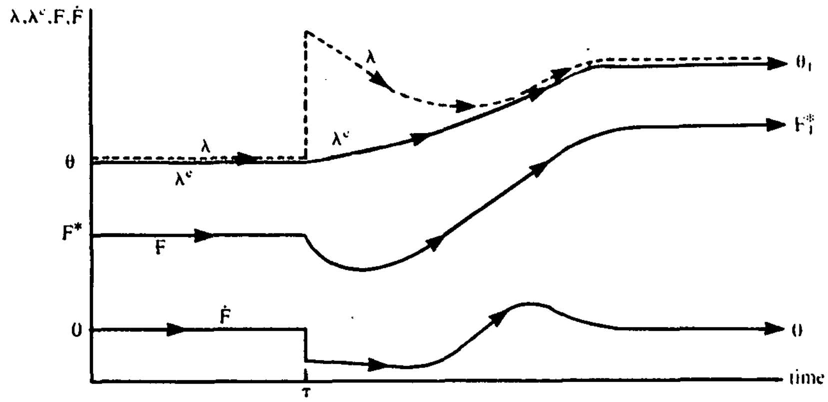
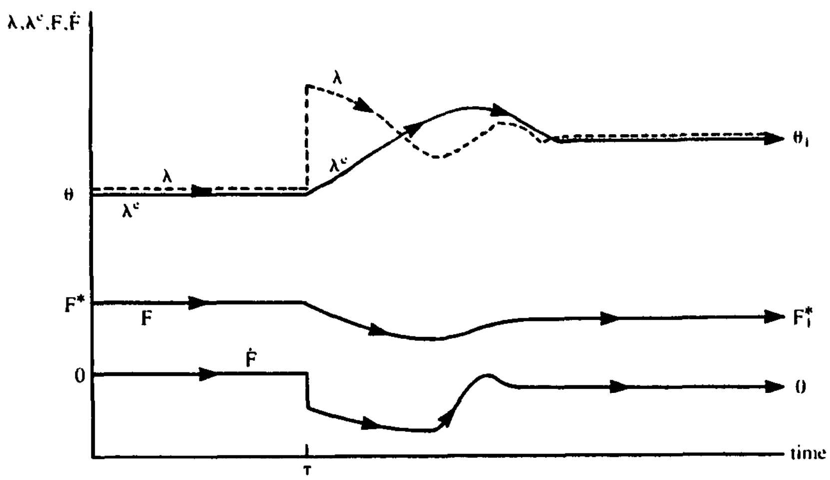


Figure 4. SOME TRAVERSES UNDER ADAPTIVE EXPECTATIONS

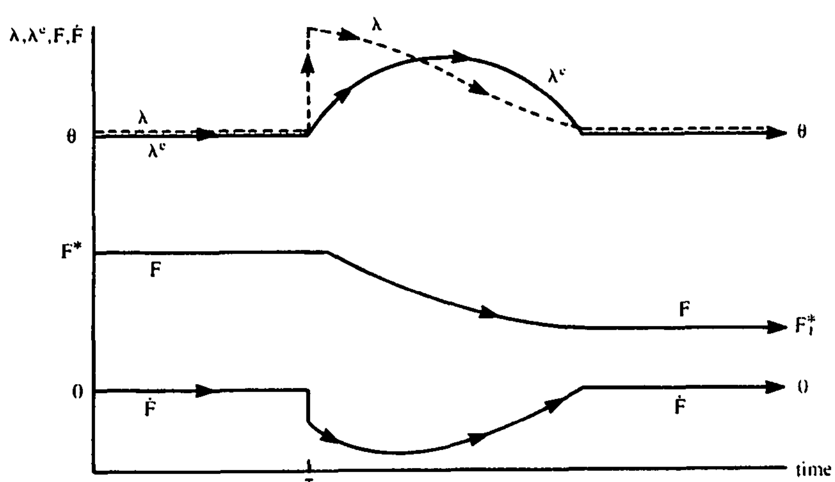
(a) Monetary Expansion and High Asset Substitutability



(b) Monetary Expansion and Low Asset Substitutability



(c) Fiscal Expansion



V. Concluding Comments

The reworking above of possibly the simplest portfolio balance model of exchange rate determination was undertaken in an attempt to render more transparent the underlying stock-flow transactions and the dynamics of the asset market approach. For this purpose a consistent theory of equilibrium in stocks and flows was employed. The analysis based on this theory emphasized the role of stock-shift factors in explaining exchange rate jumps, while relating flows to the evolution of the exchange rate over time.

Despite its simplicity, the reformulated currency substitution model was capable of generating a sufficient variety of exchange rate movements to account for several observed patterns. One finding of interest was the demonstration that the relationship between the current account of the balance of payments and exchange rate movements is not as simple as generally believed. The current account balance in the model set out above decomposes into the sum of private and public saving. The public sector was assumed to dissave by issuing monetary liabilities, which may not accord with the private sector's desired rate of accumulating these liabilities. Given the budgetary constraints, such an imbalance would carry over to the flow supply of foreign assets and their flow demand. A resulting excess flow demand for foreign assets will cause the exchange rate to depreciate, even though the current account is in surplus. This property was most clearly exhibited assuming forward looking expectations and high asset substitutability, but it can arise when expectations are adaptive.

Turning to some of the other results, it was shown that a meaningful analysis of static expectations is possible, provided the stock-flow resolution is appropriate. 1/ Because of the latter, it was possible to obtain additional results concerning the exchange rate dynamics of the adaptive expectations case. Although no new results were obtained for the perfect foresight case--as there can be only one stable path or trajectory for the CS model--it was shown how these results can be obtained using an interpretation that fully integrates real and financial flows and readily generalizes to other expectational hypotheses, unlike with other treatments in the literature.

Using the model, the exchange rate implications of financial policies were examined. Regarding fiscal policy, the analysis showed that neither the degree of asset substitutability nor alternative assumptions about the process by which expectations are formed markedly changed the qualitative nature of the results. Generally, the expected result, of an expansionary fiscal policy leading to deterioration in the current account of the balance of payments and a faster transitional depreciation in the exchange rate, is upheld.

1/ This is contrary to Kouri's surprising assertion (when analyzing the effects of an increase in the rate of monetary expansion), ". . .It is meaningless to assume static expectations in this case. . ." (1976, p. 299).

However, monetary policy effects on the exchange rates are more sensitive to assumptions concerning expectations and asset substitutability. With forward looking expectations, an expansionary monetary policy will initially result in a depreciation in the exchange rate level. The more substitutable the assets, the more likely that the depreciation in the exchange rate level will overshoot its long-run path. Under certain expectational hypotheses, it is possible that the initial overshooting is so large that the exchange rate appreciates subsequently. Generally, whenever there is an initial overshooting in the exchange rate level, the analysis of the model showed that the transitional path to long-run equilibrium would be characterized by the exchange rate depreciating at a slower rate than the new rate of growth in the domestic money supply. Results converse to the preceding hold when assets are not close substitutes, with the exchange rate now undershooting its long-run path. 1/

Where expectations adjust sluggishly, as under adaptive expectations, no jump in the exchange rate level will be discernible at the time the monetary policy change occurs. Hence, during the transition, the exchange rate level must depreciate faster than its new long-run rate. In practice, however, exchange rate expectations are likely to involve both forward looking elements and an adaptive component. As the relative emphasis on these components can vary, a wide variety of exchange rate adjustment paths are possible.

Consequently, it might appear redundant to complicate the model by relaxing some of the simplifying assumptions, particularly those of continuous purchasing power parity and of rudimentary financial markets. Generalizations in these directions will, nevertheless, be needed in order to better comprehend the underlying causes of observed exchange rate phenomena and to take account of their effects in a broader macroeconomic context. For these purposes the reformulated currency substitution model provides a convenient stepping stone.

1/ It might be noted that the possibility of overshooting in the model does not depend on the presence of sticky prices or other market frictions, as has often been contended in the literature. See, for example, Frankel (1983).

Mathematical Solutions of the Model

1. Static long-run expectations

The system is comprised of the following:

$$\lambda = \theta - \dot{F}/F, \tag{34}$$

$$\dot{F} = Y - C(Y - T - \theta m, A) - T - \theta m. \tag{35}$$

The condition for stability around an equilibrium node, F^*, is

$$\partial\dot{F}/\partial F^* = \theta L(C_1 - 1) - C_2(1 + L) < 0.$$

This is assured provided $C_1 < 1$.

The slope of the $\lambda = f(F, \theta, T)$ line in the phase diagram is

$$d\lambda/dF \Big|_{\lambda=f(F)} = -[\partial(\dot{F}/F)]/\partial F > 0,$$

as $[\partial(\dot{F}/F)]/\partial F = (1/F^*)/[\theta L(C_1 - 1) - C_2(1 + L)] < 0$ in the neighborhood of F^*.

Policy impact effects on λ are:

$$\partial\lambda/\partial\theta = 1 + [\rho(1 - C_1 + C_2/\theta) - (1 - C_1)] > 1,$$

provided $\rho > 1/\{[1 + C_2/[\theta(1 - C_1)]\}$.

$$\partial\lambda/\partial T \cdot \Delta T = (1/F)/(1 - C_1)\Delta T > 0.$$

2. Rational expectations

In order to find the effect of changes in expectations on $\lambda(t)$, derive from the first-order structural differential equation for λ stated in the text (see equation (22)) and estimated at $\lambda^e = \theta$,

$$\lambda = (L/L')(\theta - \lambda - \dot{F}/F). \tag{36}$$

Using the operator notation (Sargent (1979), Chapter 1) a forward looking solution is derived from equation (36) as:

$$\lambda(t) = -1/\alpha \int_t^{\infty} e^{(s-t)/\alpha}\{\theta(s) - [1/F(s)][Y(s) - C[Y(s) - T(s)]$$

$$- \lambda(s)[M(s), A(s)] - T(s) - \theta(s)M(s)\}ds,$$

where α is a constant which equals $-\theta/\rho$, with ρ defined at θ, F^*, and the appropriate terminal condition ensures that the integration constant is zero. The effects of more expansionary monetary and fiscal policies are readily seen as involving a discontinuous jump in λ. Thereafter, λ adjusts continuously and its motion is described by the following system:

$$\dot{F}/F = \{Y - C[Y - T - \theta LF, (1 + L)F] - T - \theta LF\}/F, \quad (37)$$

$$\dot{\lambda} = (L/L')(\theta - \lambda - \dot{F}/F). \quad (38)$$

Linearizing around λ^* (= θ) and F^*,

$$\begin{vmatrix} \dot{F}/F \\ \\ \dot{\lambda} \end{vmatrix} = \begin{vmatrix} [(\partial\dot{F}/F)/\partial F^*] & [\partial(\dot{F}/F)/\partial\lambda^*] \\ \\ -\frac{L}{L'}[\frac{\partial(\dot{F}/F^*)}{\partial F^*}] & -\frac{L}{L'}[1 + \frac{\partial\dot{F}/F)}{\partial\lambda^*}] \end{vmatrix} \begin{vmatrix} F - F^* \\ \\ \lambda - \lambda^* \end{vmatrix}.$$

The elements of the Jacobean are as follows:

$$a_{11} = (\partial\dot{F}/F^*)/\partial F^* = \theta L(C_1 - 1) - C_2(1 + L) < 0,$$

$$a_{12} = (\partial\dot{F}/F^*)/\partial\lambda^* = C_1L + \theta L'(C_1 - 1) - CL'_2 > 0,$$

$$a_{21} = \partial\dot{\lambda}/\partial F^* = -(L/L')[\partial(\dot{F}/F^*)\partial F^*] < 0,$$

$$a_{22} = \theta\dot{\lambda}/\partial\lambda^* = -(L/L')[1 + \partial(\dot{F}/F^*)/\partial F^*] < 0,$$

The system is a saddle point, as is indicated by the sign of the Jacobean determinant.

$$|J| = -(L/L')[\partial(\dot{F}/F^*)]/\partial F^* = \lambda_1\lambda_2 < 0,$$

where $\lambda_1 < 0$ is the assumed stable root. For the stable root $\lambda_1 < 0$, the eigenvector, x, is determined as:

$$x_1/x_2 = a_{12}/\lambda_1 - a_{11} \quad \text{or} \quad \lambda_1 - a_{22}/a_{21}.$$

The solution along the stable trajectory is:

$$\begin{vmatrix} F - F^* \\ \lambda - \lambda^* \end{vmatrix} = K_1 \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} e^{\lambda_1}.$$

Here K_1 is an initial condition constant determined by reference to the $\dot{F} = 0$ line at the point of policy impact.

The slope of the stable trajectory is

$$x_1/x_2 = \lambda_1 - a_{22}/a_{21} > 0.$$

The constant K_1 is determined as follows:

$$F - F^*_1\Big|_{t=0} = F_0^* - F_1^* = [(a_{12} - m)/a_{11}]d\theta.$$

Equating the solution $(F - F^*) = K_1 a_{12} e^{\lambda_1 t}$ to the preceding yields

$$K_1 = [(a_{12} - m)/(a_{11}a_{12})]d\theta.$$

In order to construct the phase diagram, find the slopes of the $\dot{F} = 0$ and $\dot{\lambda} = 0$ lines as follows:

$$d\lambda/dF\Big|_{\dot{F}=0} = -a_{11}/a_{12} > 0,$$

$$d\lambda/dF\Big|_{\dot{\lambda}=0} = -a_{11}/(1 + a_{12}) > 0.$$

Clearly from the above, the slope of the $\dot{F} = 0$ line exceeds that of the $\dot{\lambda} = 0$ line. Deviations from each of these lines results in the following movements:

$$\partial\dot{F}/\partial F\Big|_{\dot{F}=0} = Fa_{11} < 0 \quad \text{and} \quad \partial\dot{\lambda}/\partial\lambda\Big|_{\dot{\lambda}=0} = -(LF/L')(1 + a_{12}) > 0.$$

Using the above results, the basic phase diagram is constructed in Figure 5a.

3. Adaptive expectations

The dynamic system consists of

$$\dot{\lambda}^e = (b/Z)(\theta - \lambda^e - \dot{F}/F), \tag{39}$$

$$\dot{F}/F = Y/F - (C/F)(Y - T - \theta L(\lambda^e)F, (1 + L(\lambda^e))F) - T/F - \theta L(\lambda^e), \tag{40}$$

where $Z = 1 + (L'/L)b$.

Elements of the Jacobian of this system evaluated at $\lambda^e = \theta$ and $F = F^*$, are formally similar to those for the perfect foresight case. Thus:

$$(\partial\dot{F}/F^*)/\partial F^* = a_{11} < 0,$$

$$(\partial\dot{F}/F^*)/\partial\lambda^{e*} = a_{12} > 0,$$

$$(\partial\dot{\lambda}^e)/\partial F^* = -(b/Z)a_{11} > 0,$$

$$(\partial\dot{\lambda}^e)/\partial\lambda^{e*} = -(b/Z)(1 + a_{12}) < 0.$$

Stability, which is assumed here, requires that $b/Z < 0$. This implies the restriction on the speed of the adjustment term $b < \theta/\rho$.

Hence, the trace $= a_{11} + a_{22} < 0$ and the Jacobian determinant $|J| = \lambda_1\lambda_2 > 0$. Thus provided $C_1 < 1$ and $b < \theta/\rho$, the system is locally stable.

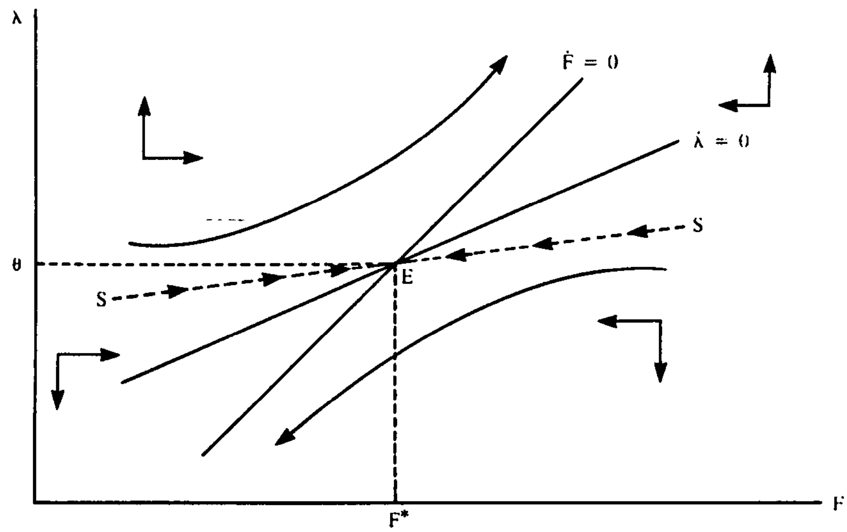
For the phase diagram, the slopes of the $\dot{\lambda}^e = 0$ and $\dot{F}^* = 0$ line are, respectively

$$d\lambda^e/dF\Big|_{\dot{\lambda}^e=0} = -a_{11}/(1 + a_{12}) > 0,$$

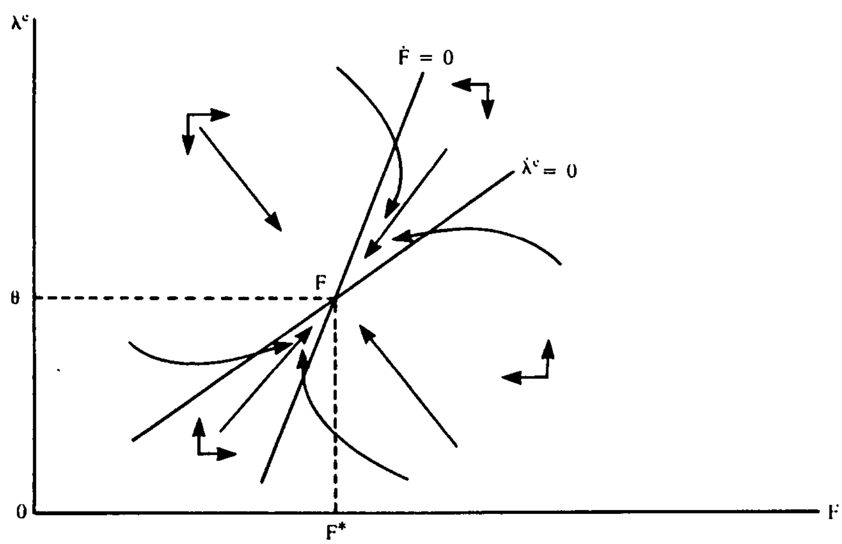
$$d\lambda^e/dF\Big|_{\dot{F}=0} = -a_{11}/a_{12} > 0.$$

Figure 5. PHASE DIAGRAMS

(a) The Perfect Foresight Case



(b) The Adaptive Expectations Case



The slope of the $\dot{F} = 0$ line clearly exceeds that of the $\dot{\lambda}^e = 0$ line. In the event of displacements from these lines

$$\partial\dot{\lambda}^e/\partial\lambda\Big|_{\dot{\lambda}^e=0} = a_{22} < 0,$$

and

$$\partial\dot{F}/\partial F\Big|_{\dot{F}=0} = Fa_{11} < 0.$$

Using the above result, the basic phase diagram is constructed as Figure 5b.

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