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Cooperative Monetary Policies with Flexible Exchange Rates

Prepared by Kenneth Rogoff

Approved by Donald J. Mathieson

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I. Introduction

Schemes to improve the macroeconomic performance of the present managed-floating exchange rate regime frequently involve increased coordination among central banks of employment and inflation-rate stabilization policies. Unfortunately, these schemes often focus entirely on how central banks might cooperate to offset unanticipated disturbances, and pay little or no attention to the problem of maintaining low expected rates of inflation. The present paper is an effort to provide a simple macroeconomic framework in which to examine both issues. The exchange rate regime is a managed float, but unlike the majority of recent studies, the emphasis of the analysis is game-theoretic and not information-theoretic. 1/ Within the context of a two-country macroeconomic model, we simultaneously analyze the strategic interactions of sovereign monetary authorities across countries, and the strategic interactions of private agents and the monetary authorities within a given country. 2/

1/ This is not to suggest that the information-theoretic aspects of exchange rate management are not important. Indeed, our empirical knowledge of the determinants of exchange rate movements is, as yet, rather limited; see Meese and Rogoff (1983).

2/ The analysis of the strategic interactions between the two central banks is based, in part, on Canzoneri and Gray (1983). Their paper contains a number of interesting results as well as references to other game-theoretic analyses of international monetary policy. These earlier analyses, including the one by Canzoneri and Gray, do not incorporate rational expectations and do not model the strategic relationships between central banks and wage setters. That component of the present analysis is based on Rogoff (1983b), which is in turn based, in part, on Kydland and Prescott (1977) and on Barro and Gordon (1983).

The analysis suggests that monetary policy cooperation should be viewed as a two-stage process. 1/ The first stage consists of choosing among alternative intermediate monetary targets (the money supply, nominal GNP, the price level, etc.) The need for intermediate targeting arises if the central banks are otherwise unable to guarantee to the private sector that they will refrain from systematic inflation, and only use discretionary monetary policy to offset disturbances. At this first stage, the central banks must also decide on the degree to which they will credibly precommit themselves to achieving their targets. (In other words, the weight they will promise to place on their intermediate monetary target relative to ultimate social objectives.) Provided that the weights on the intermediate targets are less than infinite, the central banks will retain some scope to respond to unanticipated disturbances. The credibility of the central banks' promise to place weight on their intermediate monetary targets presumably derives from the incentive structure of the central banks (through proper institutional design) and/or from the reputation of the individuals who govern them. The second stage of monetary policy cooperation consists of jointly responding to unanticipated disturbances within the constraints of the "flexible" intermediate targeting regime. Cooperation at this stage definitely produces superior responses to unanticipated supply disturbances, and may produce superior responses to other types of disturbances depending on the choice of intermediate monetary target (if any).

It would be useful for central banks to coordinate their selection of intermediate monetary targets even if they are unable to coordinate their efforts to offset unanticipated disturbances. The reason is that some types of intermediate targets produce more favorable <u>non</u>cooperative responses to unanticipated disturbances than do other types. For example, precommitment to nominal GNP or price level targets is preferable to precommitment to money supply targets if money demand shocks are important. We demonstrate that in the absence of money supply targeting, the noncooperative response to country-specific money demand shocks is optimal and equivalent to the cooperative response. 2/

While cooperation in the selection of intermediate monetary targets is useful even if cooperation in offsetting disturbances is infeasible, the reverse is not necessarily true. Without intermediate monetary targeting, a fully discretionary noncooperative regime may produce higher

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1/ Whether or not it is really possible to implement a cooperative monetary system is, regrettably, an issue which is beyond the scope of this paper. von Furstenberg (1983) appraises the prospects for bringing greater international influence to bear on national money supply processes. Hamada (1976) stresses that the problem may best be thought of as one in which the central banks cooperate to construct a regime with the best possible (self-enforcing) noncooperative equilibrium.

2/ Henderson (1984) derives a similar result. The present paper contains only a limited discussion of the issues involved in target selection. Rogoff (1983b) provides a more detailed analysis.

welfare in one or both countries than would a fully discretionary cooperative regime. Although the fully discretionary cooperative regime does lead to better responses to unanticipated supply disturbances, it also leads to higher expected rates of inflation: private sector wage setters will choose higher rates of nominal wage increase if they rationally fear that the central banks will coordinate their efforts to systematically lower real wages and raise employment. Underlying this tension is the assumption that there is some type of labor market distortion which causes the market-determined levels of real wages and unemployment to be too high.

The results described above are based on a stochastic, two-country, two-good, time-consistent, rational expections cum wage contracting, macroeconomic model of a managed floating exchange rate regime. Sterilized intervention is ineffective because there is perfect capital mobility and because private agents regard bonds denominated in different currencies as perfect substitutes. 1/ But domestic monetary policy can be used to temporarily offset the real effects of unanticipated disturbances because (partially-indexed) wage contracts are negotiated a period in advance. Unanticipated foreign monetary policy can also have real effects on the domestic economy through the real exchange rate and real interest rates. The equations and solution of the underlying macroeconomic model are relegated to the Appendix so as to focus on the more novel game-theoretic aspects of the analysis in the text.

Section II of the text describes the home and foreign welfare functions, which depend on own employment and CPI inflation. Section III details the objectives of the wage setters, as well as the nature of a time-consistent equilibrium. Section IV describes the cross-effects of home and foreign monetary policy. Section V derives the stochastic equilibrium path of the world economy in a regime in which central banks conduct stabilization policy unilaterally without adopting intermediate monetary targeting. Section VI contrasts the results of Section V with social welfare under a cooperative, fixed real exchange rate regime, in which again there is no targeting. Section VII extends the results to a simple case of cooperation cum intermediate monetary targeting.

1/ There seems to be little harm in abstracting from the macroeconomic effects of sterilized intervention, since those effects appear to be extremely limited. See, for example, Rogoff (1983a).

II. Domestic and Foreign Social Objective Functions

Each central bank attempts to minimize a social loss function which depends on deviations of own-country employment and inflation from their optimal (socially-desired) values: 1/

(1a) $\Lambda_t = (n_t - \tilde{n})^2 + \chi(\pi_{It} - \tilde{\pi}_I)^2,$

(1b) $\Lambda_t^* = (n_t^* - \tilde{n}^*)^2 + \chi(\pi_{It}^* - \tilde{\pi}_I^*)^2,$

where Λ (Λ^*) is the home (foreign) social loss function. Star superscripts denote foreign country variables, t subscripts denote time, and lower case letters are used to represent logarithms. (Henceforth, we will discuss only domestic variables and equations in circumstances where discussion of their foreign counterparts is superfluous.) Employment is given by n, and π_{It} is the rate of consumer price level inflation; e.g., $\pi_{It} = (p_I)_t - (p_I)_{t-1}$, where p_I is the consumer price level. 2/ The socially-preferred values of n and π_I are denoted by $\tilde{n}$ and $\tilde{\pi}_I$. χ is the relative weight which society places on inflation stabilization versus employment stabilization. As with most of the technological and behavioral parameters of the model of the Appendix, χ is assumed to be the same for both countries (though $\tilde{\pi}_I$ and $\tilde{\pi}_I^*$ may differ). 3/ The objective functions (1) are static, but the results below would be unchanged even we replaced equations (1) by objective functions which also depended on expected future values of inflation and employment. The reasons are that the model of the Appendix has no dynamics (except for the inflation rate) and wage contracts are for only one period.

1/ A similar specification of the social loss function is used by Barro and Gordon (1983), and by Kydland and Prescott (1977).

2/ See equation (18) of the Appendix. Including the rate of inflation of the home-currency price of the home good in the social objective function would have no qualitative impact on the results below.

3/ Most rational expectations *cum* wage contracting analyses include only output or employment deviations directly into the social loss function Λ, and do not include the level of the inflation rate. It is indeed difficult to rigorously justify including the anticipated component of the inflation rate into Λ. Some costs of perfectly anticipated inflation include the administrative costs of posting new prices, the costs of adjusting the tax system to be fully neutral with respect to inflation, and the costs incurred because high rates of inflation force private agents to economize on their holdings of non-interest bearing money. The optimal rate of inflation may nevertheless be non-zero; it may be optimal to make some use of the seignorage tax when other available methods of taxation are also distortionary. See Phelps (1973).

III. The Conflict Between Wage Setters and the Central Banks

The tension between wage setters and the central bank within each country derives from the assumption that $\tilde{n}$, society's target employment rate, is greater than $\bar{n}$, wage setters' target employment rate. Possible factors which might distort the labor-leisure decision, thereby causing wage setters to target levels of employment which are too low and levels of real wages which are too high, include income taxation and unemployment insurance. 1/ Monopolistic unions might also cause the average real wage to be too high to sustain society's target employment rate $\tilde{n}$.

Despite the fact that their targets are socially suboptimal, wage setters are able to frustrate any effort by their home central bank to systematically raise the level of employment or lower real wages. They can do so by setting base nominal wage increases at a sufficiently high level so that, in the absence of disturbances, the central bank will not choose to inflate the money supply beyond the point consistent with wage setters' desired real wage. 2/ At this sufficiently high level of inflation, the central bank finds that the marginal utility gain from inflating (further) to raise employment above wage setters' desired level is fully offset by the marginal disutility from the added inflation. Thus equilibrim is characterized by "stagflation." 3/

It is, of course, possible to consider a cooperative equilibrium between wage setters and their home central bank, in which the central bank promises not to systematically inflate for the sake of raising the average level of employment. The problem is how to design a system to enforce such a promise without constraining the ability of the central bank to offset to unanticipated shocks. If it were possible to anticipate every type of disturbance, one could write a law specifying a path of the money supply contingent on each possible disturbance. Later, in Section VII, we shall consider intermediate monetary targeting as an alternative (second-best) solution. 4/

1/ See Barro and Gordon (1983).

2/ Each individual group of wage setters is indeed concerned with the inflation rate, just as society is. But because the impact of an individual firm's contract on the aggregate inflation rate is small, they have little incentive to temper their nominal wage increases.

3/ Kydland and Prescott (1977) were the first to demonstrate why a time-consistent macroeconomic equilibrium might be characterized by stagflation. While we focus here on labor market distortions, there are other factors which may cause the time-consistent rate of inflation to be too high. Examples include seignorage and the existence of nominal government debt; see Barro and Gordon (1983).

4/ Other possible schemes involve lagged feedback rules; see Canzoneri, Henderson and Rogoff (1982).

IV. Mutual Elements in the Home and Foreign Central Bank Objective Functions and the Potential for Cooperative Stabilization Policy

Using equations (18), (19) and (20) of the Appendix, we can rewrite the home and foreign social objective functions as

$$(2a)\quad \Lambda_t = [z_t/\alpha + \gamma(p_t - \bar{w}_t) - \tau q_t - (\tilde{n} - \bar{n})]^2$$

$$+ \chi[p_t - p_{t-1} + .5(q_t - q_{t-1}) - \tilde{\pi}_I]^2$$

$$(2b)\quad \Lambda_t^* = [z_t/\alpha + \gamma(p_t^* - \bar{w}_t^*) + \tau q_t - (\tilde{n}^* - \bar{n}^*)]^2$$

$$+ \chi[p_t^* - p_{t-1}^* - .5(q_t - q_{t-1}) - \tilde{\pi}_I^*]^2,$$

where p is the home-currency price of the home good, p* is the foreign-currency price of the foreign good, and q is the real exchange rate: $q \equiv e + p^* - p$ (e is the home-currency price of foreign currency). $\bar{w}$ ($\bar{w}^*$) is the home (foreign) base nominal wage rate, and z is a supply shock common to both countries.

The first term in the home social objective function Λ represents squared deviations of home employment from its socially desired value; it consists of four components: The first component depends on the supply shock, z, and on the coefficient of the Cobb-Douglas production function, α. The second component depends on the difference between the actual period t price level, p_t, and the base wage rate $\bar{w}$. By construction of the model of the Appendix, the base wage rate is equal to the expected value of the consumer price index where expectations are based on period t-1 information. (Recall that the base wage rate is set a period in advance.) The coefficient γ is equal to $(1-\beta)/\alpha$, where β is the degree to which wages are indexed to unanticipated changes in the consumer price level. It is assumed that $0 \leq \beta < 1$. 1/ The third component involves the

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1/ It would, of course, be attractive to extend the present model to allow for an endogenous determination of β. For the full information setup of the text, one can show that individual groups of wage setters would be likely to choose $\beta = 0$ if β is bounded between zero and one. Wage setters have no need to worry about aggregate goods market demand and money demand disturbances, because these are offset by the central banks. But because the central banks are concerned with price level stability, they do not allow prices (and therefore real wages) to move as much as would be required to achieve employment stability. (See the Appendix.) From the point of view of wage setters at an individual firm, indexation would only serve to further damp desired movements in the real wage.

real exchange rate, q, and arises because wages are indexed to a price index which includes the foreign good. By construction of the model of the Appendix, the expected value of q is zero. A similar component in q would arise if, as in the model of Daniel (1981), the foreign good entered as an intermediate good in the domestic production function. The two goods have equal weights in the consumer price index, and $\tau \equiv .5\beta/\alpha$. The fourth and final component depends on the difference between society's and wage setters' target levels of employment, $\tilde{n} - \bar{n}$.

The second term in the social objective function is the squared difference between the actual rate of CPI inflation and society's target rate. Again, the real exchange rate enters. Obviously, it is the presence of the real exchange rate in both objective functions which creates opportunities for cooperation or conflict. We shall begin by considering the noncooperative or "Nash" equilibrium.

V. Equilibrium When Central Banks Do Not Cooperate to Achieve Their Stabilization Objectives

Each central bank has only one instrument at its disposal, the money supply. 1/ As demonstrated in the Appendix, any unilateral effort by either central bank to influence its own country's price level will also have an effect on the real exchange rate. 2/ But if the central banks inflate jointly, they can raise prices at home and abroad without altering the real exchange rate.

Here we will examine the noncooperative (Nash) equilibrium which obtains when each central bank perceives that it cannot improve its own objective function through unilateral action. To focus on the game-

1/ There will obviously be no tradeoffs and no conflicts if the central banks have as many independent instruments as objectives.

2/ If the central bank could make fully credible announcements about future monetary policy, then it could also affect real interest rates and the real exchange rate by making unanticipated announcements of future money supply changes. In such an environment, the central bank could offset disturbances by (temporarily) changing the real interest rate without affecting the real exchange rate. In the setup of the text, however, "prospective" feedback rules are not, in general, time consistent (credible); see Rogoff (1983b). If time consistency were not a problem, lagged feedback rules would still be inferior to contemporaneous feedback rules in the model of the text. The reason is that both inflation rate stabilization and employment stabilization enter the social objective function. Lagged feedback rules succeed in decreasing the variance of employment only by increasing the unconditional variance of inflation; see Canzoneri, Henderson and Rogoff (1982). A similar problem with lagged feedback rules arises in the presence of multiperiod overlapping contracts.

theoretic aspects of the model, we assume that central banks and private investors have full current-period information. The first-order conditions for the Nash equilibrium are given by 1/

$$(3a) \quad (\partial\Lambda_t/\partial p_t)^N = 2[\gamma - \tau\Psi][z_t/\alpha + \gamma(p_t - \bar{w}_t) - \tau q_t - (\tilde{n} - \bar{n})]$$

$$+ 2\chi(1 + .5\psi)[p_t - p_{t-1} + .5(q_t - q_{t-1}) - \tilde{\pi}_I] = 0,$$

$$(3b) \quad (\partial\Lambda_t^*/\partial p_t^*)^N = 2(\gamma + \tau\Psi^*)[z_t/\alpha + \gamma(p_t^* - \bar{w}_t^*) + \tau q_t - (\tilde{n}^* - \bar{n}^*)]$$

$$+ 2\chi(1 - .5\Psi^*)[p_t^* - p_{t-1}^* - .5(q_t - q_{t-1}) - \tilde{\pi}_I^*] = 0,$$

where N superscripts stand for "Nash" equilibrium, and $\Psi \equiv (\partial q/\partial dm)/(\partial p/\partial dm) > 0$ (m is the home money supply, $dm = m_t - E_{t-1}(m_t)$). Due to the symmetry of the model of the Appendix, $\Psi^* \equiv (\partial q/\partial dm^*)/(\partial p^*/\partial dm^*) = -\Psi$; that is, holding base wage rates constant, a unilateral foreign money supply increase has exactly the opposite effect on the real exchange rage q as a unilateral home money supply increase. One can also prove, using the model of the Appendix, that $(\gamma - \tau\Psi) > 0$; i.e., an unanticipated increase in the home money supply raises home employment.

Wage setters are assumed to correctly anticipate whether a cooperative or Nash regime will be in place in the ensuing period. 2/ By examining the first-order conditions (3), wage setters choose base wage rates so that the expected real wage equals their target real wage. The model of the Appendix is constructed so that wage setters' target (logarithm of the) real wage is zero, and so that $E_{t-1}(q_t) = 0$. Taking t-1 expectations across (3), and setting expected real wages and the expected real exchange rate equal to zero yields 3/

$$(4a) \quad (\bar{w}_t)^N = E_{t-1}(p_t)^N = p_{t-1} + .5q_{t-1} + \bar{\pi}_I^N,$$

$$(4b) \quad (\bar{w}_t^*)^N = E_{t-1}(p_t^*)^N = p_{t-1}^* - .5q_{t-1} + (\bar{\pi}_I^*)^N,$$

1/ The second order conditions for a local minimum are met; because of the quadratic forms of (1), the minimum is global.

2/ Central banks could try to fool wage setters by indicating that they have no plans to cooperate, and then turning around and doing so. They could not systematically gain by randomizing policy, however.

3/ Here is the first of many times where we make use of the fact that certainty equivalence obtains because the objective functions are quadratic; see Sargent (1979).

where $\bar{\pi}_I^N \equiv \tilde{\pi}_I + (\gamma - \tau\Psi)(\tilde{n} - \bar{n})/\chi(1 + .5\Psi)$, and

$$(\bar{\pi}_I^*)^N \equiv \tilde{\pi}_I^* + (\gamma - \tau\Psi)(\tilde{n}^* - \bar{n}^*)/\chi(1 + .5\psi).$$

By choosing $\bar{w}$ and $\bar{w}^*$ according to equations (4), wage setters assure themselves that the noncooperating central banks will, in the absence of disturbances, produce price levels consistent with wage setters' target real wages. We see, by inspection of equations (4), that the rate of nominal wage growth is an increasing function of the difference between the central banks' target employment rate, $\tilde{n}$, and wage setters' target employment rate, $\bar{n}$. The rate of nominal wage growth is a decreasing function of the relative weight which society places on stabilizing inflation versus stabilizing employment.

We may deduce from equations (4) that the expected rate of change of the nominal exchange rate, e, may be non-zero. 1/ First, the two countries may have different average inflation rates if either $(\tilde{n} - \bar{n})$ or $\tilde{\pi}_I$ differ from $(\tilde{n}^* - \bar{n}^*)$ and $\tilde{\pi}_I^*$, and second because real exchange rate movements are expected to be temporary.

Using equations (3) and (4), we can analyze the responses of the noncooperative system to unanticipated disturbances. Because the two countries are completely identical except for their labor market distortions and target inflation rates, and because the goods market demand and aggregate supply (but not money demand) shocks are perfectly correlated, the real exchange rate q always turns out to be zero in Nash equilibrium, regardless of the realization of the disturbances. 2/ To confirm that q = 0 is indeed always a Nash equilibrium, first substitute (4) into (3), and calculate $(p_t)^N$ and $(p_t^*)^N$ under the assumption that q = 0. Subtract $E_{t-1}(p_t)^N$ and $E_{t-1}(p_t^*)^N$ from the resulting expressions to obtain: 3/

1/ By inspection of equations (4), the reader will observe that the expected rate of inflation of the domestic price of the domestic good fluctuates. But the expected rate of change of the CPI, $(p_t + .5q_t) - (p_{t-1} + .5q_{t-1})$ is constant and equal to $\bar{\pi}_I^N$, since $E_{t-1}(q_t - q_{t-1}) = - q_{t-1}$. We use this fact later in the text and again in solving the model of the Appendix.

2/ Although the real exchange rate does not turn out to move in the Nash equilibrium, the fact that it can move (out of Nash equilibrium) plays a major role in everyone's decisions.

3/ A simpler method of obtaining equations (5) is to solve (3) for p_t and p_t^* holding q = 0. Then take t-1 expectations across the resulting expressions and form $(p_t)^N - E_{t-1}(p_t)^N$ and $(p_t^*)^N - E_{t-1}(p_t^*)^N$.

(5a) $(p_t)^N - E_{t-1}(p_t)^N \equiv (dp_t)^N$

$$= -z_t/\alpha[\gamma + \chi(1 + .5\Psi)/(\gamma - \tau\Psi)],$$

(5b) $(p_t^*)^N - E_{t-1}(p_t^*)^N \equiv (dp_t^*)^N$

$$= -z_t/\alpha[\gamma + \chi(1 + .5\Psi)/(\gamma - \tau\Psi)].$$

Although the two countries may have different expected inflation rates, the unanticipated component of home and foreign price movements is the same. Feasibility of the q = 0 Nash equilibrium is confirmed by examining equations (28) of the Appendix; q = 0 and $dp_t = dp_t^*$ if and only if (dm - v) = (dm* - v*), where v (v*) is the home (foreign) money demand disturbance. 1/ One can also prove that the Nash equilibrium is unique. 2/

Since we will later want to compare social welfare under the Nash regime with social welfare under a cooperative regime, it is necessary to compute $E_{t-1}(\Lambda_t)^N$, and $E_{t-1}(\Lambda_t^*)^N$. (We want to compare regimes on the basis of expected performance, not on the basis of performance for any given realization of the disturbances.) Using the fact that in a Nash equilibrium q equals zero for all realizations of the supply shock z, we can decompose the social loss functions into three components:

(6a) $E_{t-1}(\Lambda_t)^N = (\tilde{n} - \bar{n})^2 + \chi\Pi^N + \Gamma^N,$

(6b) $E_{t-1}(\Lambda_t^*)^N = (\tilde{n}^* - \bar{n}^*)^2 + \chi(\Pi^*)^N + (\Gamma^*)^N,$

1/ If q varied, we would need to substitute into the social objective functions for q, p, and p* in terms of $\bar{w}$, $\bar{w}^*$, dm, dm*, v and v* in order to solve for the Nash equilibrium. Aside from providing enormous algebraic simplification, the symmetry between the two countries is convenient later when we analyze a cooperative solution which leads to equal welfare gains in both countries.

2/ To prove uniqueness of the Nash equilibrium, take t-1 expectations across equations (3), and subtract the resulting expressions from their corresponding equations in (3) to obtain two equations in dp_t, dp_t^*, and dq_t. Use equations (28) of the Appendix to substitute out for these three variables in terms of (dm - v) and (dm* - v*). The result is two linear, independent equations in (dm - v) and (dm* - v*), and the solution is unique.

where $\Pi^N \equiv [(\bar{\pi}_I)^N - \tilde{\pi}_I]^2$, $(\Pi^*)^N \equiv [(\bar{\pi}_I^*)^N - \tilde{\pi}_I^*]^2$,

$$\Gamma^N \equiv E_{t-1}\{[z_t/\alpha + \gamma(dp_t)^N]^2 + \chi[(dp_t)^N]^2\},$$

and $(\Gamma^*)^N = \Gamma^N$, since $(dp)^N = (dp^*)^N$.

The first element of $E_{t-1}(\Lambda_t)^N$ is nonstochastic and depends on the difference between the socially optimal level of employment, $\tilde{n}$, and wage setters' target level of employment, $\bar{n}$. Because monetary policy cannot systematically raise the employment rate, this term can be reduced only by directly addressing the underlying cause of the distortion. (This issue is beyond the scope of the present paper.) The second term, $\chi\Pi^N$, measures the extent to which the average rate of CPI inflation in a Nash equilibrium exceeds society's target rate. ($\bar{\pi}_I^N$ is defined in equation (4); as pointed out in an earlier footnote, $\bar{\pi}_I^N$ would be constant even if q were not.) This second term is also independent of the disturbances, but as we shall later confirm, it is a function of the policy regime. The final term, Γ^N, measures the extent to which the central bank succeeds in stabilizing the employment rate and the CPI inflation rate around their <u>market-determined</u> values. Note that although the central banks actually attempt to stabilize inflation and employment around their socially-preferred values, in a time-consistent equilibrium the central banks respond to disturbances as if they were trying to stabilize inflation and employment around their average (mean) market-determined values. 1/ To evaluate Γ^N, substitute in for $(dp_t)^N$ using equation (5a):

$$(7) \quad \Gamma^N = (\sigma_z^2/\alpha^2)[(\chi')^2 + \gamma^2\chi]/(\gamma^2 + \chi')^2 = (\Gamma^*)^N,$$

where σ_z^2 is the variance of the zero mean supply shock, $E_{t-1}(z_t^2)$, and $\chi' \equiv \gamma\chi(1 + .5\Psi)/(\gamma - \tau\Psi)$.

There has been a great deal of attention devoted to the transmission of monetary disturbances under managed floating exchange rates, and the reader may be puzzled why these disturbances do not appear in equation (7). Money demand disturbances do appear in the underlying model of the Appendix, and this model contains the customary transmission mechanisms: the real exchange rate and real interest rates. The reason that money demand shocks do not appear in equation (7) is that, to the extent the shocks are known (here information is perfect 2/), they can be completely

1/ This result is due to the quadratic form of the objective function.
2/ Rogoff (1983b) demonstrates how to extend a framework such as the present one to the case where central banks have incomplete contemporaneous information.

neutralized. For example, consider a temporary disturbance to home money demand. The home central bank can temporarily raise the home money supply and, as one may confirm from the model of the Appendix, prevent any effect from being transmitted abroad through exchange rates or interest rates. Furthermore, the resulting stabilization of the domestic price level is consistent with both home price level stability and home employment stability. Thus the home central bank will react to the disturbance in the same fashion whether or not it takes the utility function of the foreign central bank into account. Clearly, if we relax the assumption that investors and central banks have complete contemporaneous information, then the home monetary authority may no longer have sufficient information to fully offset home money demand disturbances. Consequently, these disturbances will have some effects both at home and abroad. If the only problem is incomplete information, however, there is no reason to presume that a cooperative monetary policy regime will lead to better outcomes in response to money demand disturbances than would a fully discretionary noncooperative regime. (As we shall discuss below, money supply targeting distorts responses to money demand shocks under both cooperative and noncooperative regimes, in which case the noncooperative response to money demand disturbances may be inferior.)

While our result that money demand disturbances do not create a conflict between ultimate home and foreign objectives is fairly general, our result that home and foreign goods market demand shocks create no conflict is not general. Goods market demand shocks do not disturb the Nash equilibrium only because we have made the simplifying assumption that home and foreign goods market demand shocks are perfectly correlated. Provided that the two shocks are perfectly correlated, and that both countries alter their money supplies equally to offset the mutual disturbance, there will be no effect on home and foreign prices and employment. The level of world real interest rates will move to offset the disturbance, but the real exchange rate will remain fixed. The resulting equilibrium is Nash because neither side will have an incentive to unilaterally alter its money supply further.

On the other hand, aggregate supply disturbances do affect the price levels which arise in Nash equilibrium in spite of our simplifying assumption that the home and foreign supply shocks are perfectly correlated. The reason is that an aggregate supply shock creates a tradeoff between price level stability and employment stability. As we discuss more fully in the Appendix, an aggregate supply shock alters the market-determined full information real wage; that is, the real wage which wage setters would agree on after observing the productivity disturbance. Because the base nominal wage is set a period in advance, and because wage indexation is incomplete, the central bank can effect wage setters' desired movements in the real wage by allowing the price level to adjust in response to supply shocks. But while such price level movements are consistent with one social objective, employment stability, they are inconsistent with another social objective, price level stability. In the next section we demonstrate that, holding base wage rates constant, the cooperative response to supply shocks is superior to the noncooperative response.

VI. Equilibrium When Central Banks Cooperate to Achieve Their Stabilization Objectives

Examining the first-order conditions for the Nash equilibrium (equations (3)), we see that the incentives of the central banks to unilaterally inflate are reduced by their fears of the concommitant effects on the real exchange rate. The real exchange rate depreciation caused by unilateral home money growth raises consumer price level inflation. Aside from the direct disutility of the higher inflation rate, the home country enjoys less of an increase in employment from a given price level increase then it would if the real exchange rate were to remain constant. The reason is that nominal wages are indexed to the CPI, which includes the domestic price of the foreign good. (We note again that the presence of intermediate imports in the production function would imply a similar result.) 1/

From the above discussion, it would appear self-evident that each central bank would prefer to increase its money supply beyond the level consistent with Nash equilibrium, provided it could count on the other central bank to increase its money supply. In this section we will assume that the two central banks agree to fix the real exchange rate at the same level which would arise if all the disturbances were zero, and to confer with each other on the most mutually advantageous set of price levels consistent with that real exchange rate. 2/ We will demonstrate below that such an agreement is mutually beneficial if the variance of supply shocks is large.

Under the cooperative fixed real exchange rate ($q = 0$) regime, the first-order conditions for minimization of the home and foreign social welfare functions (equation (2)) are given by equations (8) below:

1/ Holding the foreign money supply constant, the foreign country may benefit or suffer when the home country unilaterally inflates from a position of Nash equilibrium. The outcome depends, in part, on whether the expenditure-switching effects of the real depreciation of the foreign-country currency outweigh the expenditure-increasing effects of the lower level of world real interest rates. (See Appendix.)

2/ There are other, asymmetric, cooperative schemes which, holding wages constant, lead to Pareto improvements over the Nash equilibrium. Qualitatively, all of these schemes are similar in that they involve higher money growth at home and abroad than in the Nash equilibrium. The cooperative scheme considered in the text is a logical one to consider since the two countries are identical in almost every respect. Canzoneri and Gray (1983) have emphasized that fixed exchange rate regimes may be viewed as noncooperative in that either country may unilaterally decide to fix the exchange rate. If country A firmly believes that country B is committed to not allowing its exchange rate to float, country A's behavior will be affected. However, country B will generally have an incentive to cheat on any unilateral commitment it makes to fix the exchange rate.

$$\text{(8a)}\quad \gamma[z_t/\alpha + \gamma(p_t - \bar{w}_t) - (\tilde{n} - \bar{n})] + \chi(p_t - p_{t-1} - \tilde{\pi}_I) = 0,$$

$$\text{(8b)}\quad \gamma[z_t/\alpha + \gamma(p_t^* - \bar{w}_t^*) - (\tilde{n}^* - \bar{n}^*)] + \chi(p_t^* - p_{t-1}^* - \tilde{\pi}_I^*) = 0.$$

In deriving equations (8), we have made use of the fact that $\partial q/\partial dm$ is zero in the cooperative equilibrium, since each central bank can count on the other to match any further unanticipated change in its money supply. Equations (9), (10), and (11) are derived using the same algorithm as we used to derive equations (4), (5), and (7) of the previous section; "C" superscripts stand for "cooperative regime".

$$\text{(9a)}\quad (\bar{w}_t)^C = E_{t-1}(p_t)^C = p_{t-1} + (\bar{\pi}_I)^C,$$

where $(\bar{\pi}_I)^C \equiv \gamma(\tilde{n} - \bar{n})/\chi + \tilde{\pi}_I$,

$$\text{(9b)}\quad (\bar{w}_t^*)^C = E_{t-1}(p_t^*)^C = p_{t-1}^* + (\bar{\pi}_I^*)^C,$$

where $(\bar{\pi}_I^*)^C \equiv \gamma(\tilde{n}^* - \bar{n}^*)/\chi + \tilde{\pi}_I^*$,

$$\text{(10)}\quad (p_t)^C - E_{t-1}(p_t)^C \equiv (dp_t)^C = (dp_t^*)^C = -z_t/\alpha(\gamma + \chi/\gamma),$$

$$\text{(11)}\quad \Gamma^C = (\sigma_z^2/\alpha^2)[\chi/(\gamma^2 + \chi)] = (\Gamma^*)^C.$$

Comparing equations (9) with their Nash counterparts, equations (4), reveals that the mean rate of CPI inflation is higher under the cooperative regime than under the noncooperative regime. The intuitive explanation is simple: wage setters recognize that the central banks will have stronger incentives to inflate in a regime where they can count on each others' cooperation, so that the benefits of inflation are not reduced by real exchange rate depreciation. Anticipating that the central banks are going to "gang up" on them, wage setters respond by setting higher rates of base nominal wage increase.

A comparison of equations (10) and (5) reveals that $|(dp_t)^C| > |(dp_t)^N|$. In the Nash equilibrium, the central banks allow the supply shock to affect employment more, and inflation less, than in the cooperative equilibrium. 1/

1/ Canzoneri and Gray (1983) analyze a one-time supply disturbance and find that the cooperative response to this disturbance may call for either smaller or larger changes in the money supply than in the noncooperative equilibrium. In their framework, the authorities try to stabilize employment and money growth. The present framework may also yield the result that $|dp_t|^C < |dp_t^N|$ when the central banks adopt the money supply as an intermediate monetary target, along the lines discussed in the next section.

It is easy to prove that the cooperative response to disturbances is superior and produces a lower weighted average of employment and inflation around their market-determined values; i.e., $\Gamma^N > \Gamma^C$: 1/

Proof: Note that expressions (11) and (7) can be written in the same general form since $\chi/(\gamma^2 + \chi) = (\chi^2 + \gamma^2\chi)/(\gamma^2 + \chi)^2$. Differentiating the expression $(y^2 + \gamma^2\chi)/(\gamma^2 + y)^2$ with respect to y yields $2\gamma^2(y - \chi)/(\gamma^2 + y)^{-3}$. Note that this derivative is strictly positive for $y > \chi$ (since $\chi > 0$), and note that $\chi' > \chi$. Q.E.D.

So the cooperative equilibrium produces a better response to unanticipated supply disturbances, but a higher average inflation rate. Whether or not social welfare is higher under the cooperative or noncooperative regimes thus depends, in part, on the variance of the supply shock and on the size of the labor market distortion. Note that it is perfectly possible for the country with the smaller labor market distortion to prefer the cooperative regime, while the other country would be better off under a noncooperative regime. (If the home country had no labor market distortion, it would always prefer the cooperative regime regardless of the variance of the disturbances.)

VII. Cooperation in Conjunction with Intermediate Monetary Targeting

The possibility that the regime we have labeled "cooperative" might be inferior to the "noncooperative" regime appears to violate the basic tenets of game theory. The reason our result is possible, of course, is that the "cooperative regime is not truly cooperative: the central banks cooperate with each other, but not with private sector wage setters. If the central banks were able to credibly guarantee to wage setters that they would not inflate to try to systematically raise employment, and would only use monetary policy to offset disturbances, it would be possible to achieve a superior and truly cooperative outcome. Such a guarantee would eliminate the inflationary bias of monetary policy, and consequently both countries would definitely be better off if the central banks were able to coordinate their stabilization policies. 2/

1/ An alternative proof is possible along the following lines: Define $\rho z_t = dp_t$, and find the value of ρ which minimizes Γ as defined in equations (6). This value turns out to be the same as for $(dp_t)^C$.

2/ A still better cooperative equilibrium would be available if the two countries could eliminate their labor market distortions at low cost.

There is fortunately an alternative and perhaps more feasible method to ensure that a world in which central banks coordinate their stabilization policies is always at least as attractive as one in which they do not cooperate. The alternative is one which, in some respects, resembles the intermediate monetary targeting regimes of the past decade. In these regimes, individual central banks credibly commit themselves to placing great weight (relative to ultimate social objectives) on intermediate monetary targets, without totally abandoning the use of discretionary policy. Here we shall present a simple example: 1/

Suppose the two central banks again agree to cooperate as in the previous section, guaranteeing to each other that they will coordinate money supply changes so as to fix the real exchange rate. But this time the central banks also (credibly) guarantee to wage setters that they will attempt to minimize the following objective functions instead of the social loss functions:

$$\text{(12a)} \quad I = (n_t - \tilde{n})^2 + \chi(\pi_{It} - \tilde{\pi}_I)^2 + \varepsilon(\pi_{It} - \tilde{\pi}_I)^2,$$

$$\text{(12b)} \quad I^* = (n_t^* - \tilde{n}^*)^2 + \chi(\pi_{It}^* - \tilde{\pi}_I^*) + \varepsilon(\pi_{It}^* - \tilde{\pi}_I^*)^2.$$

The objective functions (12) differ from the social loss functions (equations (1)) due to the presence of the additional third terms. These terms represent each central bank's commitment to achieving an intermediate monetary target (here the inflation rate, which is also a final target). Credibility presumably derives not from announcements, but from concrete changes in the institutional structure and incentive structure of the central banks. 2/

$$\text{(13a)} \quad \gamma[z_t/\alpha + \gamma(p_t - \bar{w}_t) - (\tilde{n} - \bar{n})] + (\chi + \varepsilon)(p_t - p_{t-1} - \tilde{\pi}_I) = 0,$$

$$\text{(13b)} \quad \gamma[z_t/\alpha + \gamma(p_t^* - \bar{w}_t^*) - (\tilde{n}^* - \bar{n}^*)]$$
$$+ (\chi + \varepsilon)(p_t^* - p_{t-1}^* - \tilde{\pi}_I^*) = 0.$$

1/ Rogoff (1983b) analyzes alternative targets and demonstrates that the optimal degree of commitment to an intermediate monetary target is, except in certain special cases, nonzero but noninfinite.

2/ In other words, the term in ε represents the central bank's commitment to place an even greater weight on the inflation rate than the social objective function does. One way to implement such a regime is to appoint an agent to head the independent central bank who places a greater weight on inflation stablization relative to employment stabilization than does society as a whole.

Now compare equations (13) with the first-order conditions for the noncooperative regime, equations (3), and recall that in the Nash equilibrium $q = 0$. Observe that when $q = 0$, the two sets of first-order conditions are equivalent if $\chi + \varepsilon = \gamma\chi(1 + .5\Psi)/(\gamma - \tau\Psi)$. It is easy to see that if two regimes yield identical first-order conditions, then the two regimes also yield identical social welfare when evaluated by the social loss functions (1). Thus, by committing themselves to intermediate inflation-rate targets, the cooperating central banks can do at least as well as in the fully discretionary noncooperative regime. In fact, they can generally do better since the optimal degree of commitment to an intermediate inflation-rate target may be greater than or less than the one implicit in the noncooperative regime. Also, the inflation rate may not be the intermediate monetary target with the best stabilization properties; the central banks may be able to do better still by committing themselves to noninflationary money supply or nominal GNP targets. 1/

Indeed, in the model we develop here, central banks will find it attractive to unilaterally adopt intermediate monetary targeting to reduce inflation even if they are unable to cooperate in responding to unanticipated disturbances. The nature of the intermediate target selected by the home country may affect the welfare of the foreign country, and vice versa. If, for example, the home country chooses the money supply as an intermediate target, then home money demand shocks will produce a conflict between the home and foreign central bank objective functions. Such a conflict does not arise in the absence of intermediate monetary targeting, or if the intermediate monetary target is the price level or nominal GNP. Thus there is scope for central bank cooperation in target selection even if the central banks are unable to cooperatively respond to unanticipated disturbances.

VIII. Conclusions

The present analysis extends the recent game-theoretic literature on two-country monetary policy by incorporating rational expectations, and by examining paths along which wages and price expectations are determined in a time-consistent fashion. We confirm that monetary policy cooperation produces superior outcomes in response to unanticipated supply shocks, and that cooperation should not be necessary to produce the optimal response to unanticipated country-specific money demand shocks (unless one or both of the countries is targeting the money supply). We also demonstrate that a fully discretionary cooperative monetary policy regime may produce higher expected rates of inflation in both countries than would a fully discretionary noncooperative regime. Intermediate monetary targeting improves welfare in both the cooperative and noncooperative regimes by reducing expected inflation rates. Provided that intermediate monetary

1/ See Rogoff (1983b).

targeting is feasible, the optimal cooperative regime is never inferior to any noncooperative regime.

In conclusion, we note that the type of analysis developed here focuses on institutional design, and not simply on policy rules within a given institutional framework.

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Appendix: The Underlying Two-Country Macroeconomic Model

Here we describe the two-country, two-good, rational expectations cum wage contracting model on which the results of the text are based. 1/ Monetary policy can have short-run real effects here because nominal wage contracts are negotiated a period in advance; these contracts are only partially indexed to the current-period consumer price level. 2/ To facilitate algebraic manipulation, the technological and behavioral parameters in the two countries are constrained to be equal.

1. Aggregate Supply

The good produced by home-country firms differs from the good produced by foreign-country firms. But within each country, all firms have identical Cobb-Douglas production functions. Using lower case letters to denote logarithms, the aggregate production functions can be written as

$$(14a) \quad y = c_0 + \alpha\bar{k} + (1 - \alpha)n + z,$$

$$(14b) \quad y^* = c_0^* + \alpha\bar{k}^* + (1 - \alpha)n^* + z^*,$$

where star (*) superscripts indicate the foreign country, 3/ y is output, $\bar{k}$ is the fixed capital stock, n is labor, c_0 is the constant term, and z is a serially uncorrelated aggregate productivity disturbance; $z \sim N(0, \sigma_z^2)$. z and z* are assumed to be perfectly correlated so that z = z*. Time subscripts are omitted where the meaning is obvious; throughout, all parameters are non-negative.

Firms hire labor until the marginal value product of labor equals the nominal wage rate, w:

$$(15a) \quad c_0 + \log(1 - \alpha) + \alpha\bar{k} - \alpha n_d + z = w - p,$$

$$(15b) \quad c_0^* + \log(1 - \alpha) + \alpha\bar{k}^* - \alpha n_d^* + z = w^* - p^*,$$

1/ Apart from its game-theoretic aspects, the model is quite similar to ones employed by Henderson and Waldo (1981), Daniel (1981), and Canzoneri and Gray (1983). Daniel's two-country model includes intermediate imports instead of wage indexation. Canzoneri and Gray's model does not incorporate rational expectations but like the present one, is highly symmetric.

2/ As Gray (1976) demonstrates, full price-level indexation is suboptimal in the presence of supply (productivity) shocks.

3/ In the discussion below, we sometimes refer only to domestic variables and equations if discussion of their foreign counterparts is redundant.

where p is the nominal price of the domestically produced good, and n_d is aggregate labor demand. The notional labor supply curve is assumed inelastic: 1/

(16a) $n_s = \bar{n}$,

(16b) $n_s^* = \bar{n}^*$.

To simplify algebra, $\bar{n}$ is set equal to $\bar{k} + (1/\alpha)[\log(1 - \alpha) + c_0]$, $\bar{n} = \bar{n}^*$, $\bar{k} = \bar{k}^*$, and $c_0 = c_0^*$.

CPI-indexed wage contracts for period t are negotiated at the end of period t-1. The base wage rate is $\bar{w}$ ($\bar{w}^*$), and the indexation parameter is β:

(17a) $w = \bar{w} + \beta(p_I - \bar{w})$,

(17b) $w^* = \bar{w}^* + \beta(p_I^* - \bar{w}^*)$, $0 < \beta < 1$,

where

(18a) $p_I = .5p + .5(p^* + e)$,

(18b) $p_I^* = .5p^* + .5(p - e)$;

e is the (logarithm of the) exchange rate (the domestic currency price of foreign currency). The nature of the employment contract is that laborers agree to supply (ex-post) whatever amount of labor is demanded by firms in period t, provided firms pay the negotiated wage. The actual levels of employment in period t are thus found by substituting the wage equations (17) into the labor demand equations (15):

(19a) $n = \bar{n} + \gamma(p - \bar{w}) - \tau q + z/\alpha$,

(19b) $n^* = \bar{n} + \gamma(p^* - \bar{w}^*) + \tau q + z/\alpha$,

1/ Making notional labor supply depend on the real wage, as in Rogoff (1983b), would not qualitatively affect the results here.

where $\gamma \equiv (1 - \beta)/\alpha$, $\tau \equiv .5\beta/\alpha$, and

(20) $q = e + p^* - p.$

As described in the text, wage setters choose $\bar{w}$ to minimize $E_{t-1}(n_t - \bar{n})^2$, where E_{t-1} denotes rational expectations based on period t-1 information. This implies that $\bar{w} = E_{t-1}[p_t - .5\beta q_t/(1-\beta)]$, and $\bar{w}^*_t = E_{t-1}[p^*_t + .5\beta q_t/(1-\beta)]$.

Equations (14) and (19), together with the assumption that $-c_0 = \alpha\bar{k} + (1 - \alpha)\bar{n}$, imply that the aggregate supply equations can be written as

(21a) $y_s = \theta(p - \bar{w}) - \kappa q + z/\alpha,$

(21b) $y^*_s = \theta(p^* - \bar{w}^*) + \kappa q + z/\alpha,$

where $\theta \equiv (1-\alpha)(1-\beta)/\alpha$ and $\kappa \equiv .5(1-\alpha)\beta/\alpha$.

2. Money and bond markets

Only domestic residents hold the domestic money and only foreign residents hold the foreign money. However, residents of both countries hold both domestic- and foreign-currency denominated bonds. The demand for real money balances in each country is a decreasing function of the nominal interest rate and an increasing function of real income:

(22a) $m - p_I = -\lambda r + \phi(p + y - p_I) + v,$

(22b) $m^* - p^*_I = -\lambda r^* + \phi(p^* + y^* - p^*_I) + v^*,$

where m is the logarithm of the nominal money supply and v is the money market disturbance term; $v \sim N(0, \sigma^2_v)$, $v^* \sim N(0, \sigma^2_{v^*})$, and v and v* are independent.

Domestic- and foreign-currency denominated bonds are perfect substitutes so that uncovered interest parity holds:

(23) $E_t(e_{t+1}) - e_t = r_t - r^*_t.$

Agents are assumed to have full period t information (including knowledge of the period t disturbances) in making their portfolio and investment decisions.

3. Goods market demand

Demand for the good produced in each country is a decreasing function of its relative price, an increasing function of real income at home and abroad, and a decreasing function of the real interest rate:

$$(24a) \quad y_d = \eta q - \delta'\{r - E_t[p_{I(t+1)}] + p_{It}\} + \Delta(p + y - p_I) + \Delta(p^* + y^* - p_I^*) + u',$$

$$(24b) \quad y_d^* = -\eta q - \delta'\{r^* - E_t[p_{I(t+1)}^*] + p_{It}^*\} + \Delta(p + y - p_I) + \Delta(p^* + y^* - p_I^*) + u',$$

where $u' \sim N(0, \sigma_{u'}^2)$ and $\Delta < .5$. 1/ Note that we are assuming that home and foreign goods market demand shocks are perfectly correlated. Using equations (18), equations (24) can be rewritten as

$$(24a') \quad y_d = \eta q - \delta\{r - E_t[p_{I(t+1)}] + p_{It}\} + u,$$

$$(24b') \quad y_d^* = -\eta q - \delta\{r^* - E_t[p_{I(t+1)}^*] + p_{It}^*\} + u,$$

where $\delta = \delta'/(1-2\Delta)$, and $u = u'/(1-2\Delta)$. In deriving equations (24'), we have made use of the fact that, because the CPI weights are the same in both countries and because home and foreign bonds are perfect substitutes, real interest rates are always equal even though there may be short-run deviations from PPP. Otherwise, the different real interest rates faced by both home and foreign consumers would enter separately in (24').

4. Solution of the model

To close the model, it is necessary to specify how wage setters and investors form expectations of future prices. The procedure for deriving time-consistent, rational, price expectations is discussed in the text.

1/ The assumption that $\Delta < .5$ implies that, in each country, the marginal propensity to consume out of real income is less than one. (Δ is the marginal propensity to consume either one of the two goods.)

There we demonstrated that $E_t[p_{I(t+1)}] - p_{It}$ is constant and equal to $\bar{\pi}_I^N$ or $\bar{\pi}_I^C$ depending on whether the equilibrium between central banks is noncooperative or cooperative. (See equations (4) and (9) of the text.) Using equations (18) and (23), it is easy to deduce that $E_t(e_{t+1}) - e_t = r - r^* = \bar{\pi}_I - \bar{\pi}_I^*$. In the time-consistent equilibrium, rational expectations of exchange rate appreciation are static. Exchange rate appreciation and CPI inflation expectations would change, of course, if the governments' objective functions changed. We are now prepared to solve for the expected values of the real exchange rate, outputs, and real interest rates. Taking t-1 expectations across equations (21) and (24) and recalling that wage setters set $\bar{w} = E_{t-1}[p_t - .5\beta q_t/(1-\beta)]$ and $\bar{w}^* = E_{t-1}[p_t^* + .5\beta q_t/(1-\beta)]$ one can solve for

(25) $$E_{t-1}(q_t) = 0 = \bar{w} - E_{t-1}(p_t) = \bar{w}^* - E_{t-1}(p_t^*),$$

(26a) $$E_{t-1}(r - \bar{\pi}_I) = E_{t-1}(y_t) = 0, \text{ and}$$

(26b) $$E_{t-1}(r^* - \bar{\pi}_I^*) = E_{t-1}(y_t^*) = 0.$$

We can use the above equations together with equations (22) to obtain

(27a) $$E_{t-1}(m_t) = \bar{w} - \lambda\bar{\pi}_I,$$

(27b) $$E_{t-1}(m_t^*) = \bar{w}^* - \lambda\bar{\pi}_I^*.$$

We are now prepared to solve for q, p and p* as functions of $\bar{w}$, $\bar{w}^*$, z, u, v, v*, dm_t and dm_t^*, where $dm_t \equiv m_t - E_{t-1}(m_t)$. These solutions are given below:

(28a) $$q = \upsilon[(dm - dm^*) - (v - v^*)],$$

(28b) $$p = \bar{w} + \xi(dm - v) + (H - \xi)(dm^* - v^*) + \lambda Hu/\delta - (\phi + \lambda/\delta)z/\alpha,$$

(28c) $$p^* = \bar{w}^* + \xi(dm^* - v^*) + (H - \xi)(dm - v) + \lambda Hu/\delta - (\phi + \lambda/\delta)z/\alpha,$$

where $\upsilon \equiv [2(\eta + \kappa)/\theta + 2\phi\eta + (1 - \phi)]^{-1}$,

$H \equiv (1 + \theta\lambda/\delta + \phi\theta)^{-1}$, and

$\xi \equiv \{1 + \upsilon[\kappa\phi - (1 - \phi)/2 + \lambda(\kappa + \eta)/\delta]\}H$.

In the text we require knowledge of $\Psi \equiv (\partial q/\partial dm)/(\partial p/\partial dm)$, which by inspection of equations (28) equals υ/ξ. We assume that the income elasticity of money demand, ϕ, is less than or equal to one; thus υ is definitely greater than zero. One can also demonstrate that $\xi > 0$. 1/

The coefficient on unanticipated foreign monetary disturbances in equation (28b), $H - \xi$, may be positive or negative. When there is no wage indexation, for example, an unanticipated foreign money supply expansion (or an unanticipated negative disturbance to foreign money demand) may raise or lower the domestic price level, depending on whether the expenditure-switching effect of the foreign real exchange rate depreciation outweighs the expenditure-increasing effect of lower world real interest rates. 2/

1/ One can prove that $\xi > 0$ directly from equations (28). A simpler way to prove that $\partial p/\partial dm > 0$ is to sum and difference like equations in the model, and show that $\partial(p - p^*)/\partial(dm - dm^*) > 0$, and $\partial(p + p^*)/\partial(dm + dm^*) > 0$. This is similarly the simplest way to prove that $\partial y/\partial dm > 0$, a fact we use in the text.

2/ Daniel (1981) discusses these transmission channels, as do Canzoneri and Gray (1983).