DM/83/43

INTERNATIONAL MONETARY FUND

# Expectations of Inflation and Interest Rate Determination

Research Department

Prepared by Rüşdü Saracoglu

Approved by Anthony Lanyi

June 9, 1983

## Summary

In recent years there has been considerable dispute concerning the underlying causes of both the high levels and the variability of interest rates. In this paper the determination of interest rates in five industrial countries (the United States, France, Germany, Japan, and the United Kingdom) is studied, with particular attention paid to the roles of monetary factors and of expectations of inflation.

The theoretical discussion centers on Fisher's hypothesis that relates interest rates to expected rates of inflation. It is argued that much of the earlier research on this issue is flawed, because it is based on microeconomic partial equilibrium analysis and assumes an implausible expectation formation mechanism. This paper, by contrast, follows recent developments in economic theory in suggesting that the formulation of expectations--and, therefore, the determination of interest rates--is influenced not only by past rates of inflation but also by a host of other variables, such as changes in the money supply or in the level of economic activity. Moreover, variables that influence the expected rates of inflation are themselves likely to be influenced by these expectations. Consequently, the methodology developed in this paper is intended to reflect the simultaneities and dynamic patterns of feedback that characterize the macroeconomic data, as well as accounting for the fact that market participants utilize data other than past rates of inflation in forming their expectations of inflation. Market participants' subjective expectations of inflation are proxied by forecasts of inflation that are optimal in the sense of minimizing the variance of forecast errors conditional on information that is available to market participants. These forecasts are then used to compute ex-ante real interest rate series for each country and to test several hypotheses regarding interest rates.

The empirical results of the paper indicate that twice during the 1968-82 period real interest rates underwent substantial changes. These changes had a substantial macroeconomic impact, particularly with respect to inflation. It is argued that the expansionary impact of negative real interest rates following the first oil shock led to the intensification

of the inflationary process, whereas the emergence of positive real interest rates following the second oil shock was instrumental in bringing inflation rapidly under control.

Other results suggest that systematic, and thus predictable, changes in the money supply that lead to systematic shifts in the time path of inflation leave real interest rates unchanged, thus supporting a variant of Fisher's hypothesis. Regarding the role of monetary factors in the variability of real interest rates, it is found that there is little evidence, except in Japan, that unanticipated changes in the money supply contributed significantly to the variation in real interest rates.

## I. Introduction

The relationship between nominal interest rates and expectations of inflation has received widespread attention over the years. 1/ Although the theoretical issue is not a new one, the relatively high rates of inflation and nominal interest rates experienced in recent years have brought the question into sharp focus. The concern of policymakers has been directed not only at the large fluctuations in nominal and real interest rates, but also at real interest rates that are very high relative to historical levels and have risen rapidly to those levels.

The concern over the behavior of real interest rates stems from the prominent role they play in macroeconomic analysis and policy formulation. First and foremost, it is widely believed that high real interest rates discourage productive investment and thereby lead to economic slowdown, which hinders growth. 2/ Secondly, the movements in real interest rates constitute the main link in the economic transmission mechanism whereby shifts in monetary policy induce changes in the level of economic activity--assuming that the monetary authority is indeed able to affect changes in real interest rates. Thirdly, it is often argued that policies that result in higher real interest rates tend to increase the external value of the domestic currency due both to the induced capital inflows and to the strengthening of the current account position resulting from the dampening effect of higher real rates of interest on the level of economic activity.

---

1/ Some examples are Begg (1977), Fama (1975, 1976). Makin and Tanzi (1982), Mishkin (1980), Sargent (1973, 1973a, 1973b, 1976).

2/ In formulations of this view, investment demand depends upon the ratio of the value of existing stocks of captial to their replacement cost. This ratio, known as Tobin's q, is equal to the ratio of the net marginal product of capital to the real rate of interest.

Although the importance of real interest rates in economic analysis is widely recognized, not much research has been undertaken on their determination, and the theory developed by Irving Fisher over half a century ago remains as the foundation of most of the current investigations of the subject. Nevertheless, Fisher's theory is a controversial one, and particularly when it is interpreted as suggesting a constant real rate. The controversy is due, in part, to the fact that real interest rates are not observed directly, but inferred from the behavior of the rates of nominal interest and inflation. Because the concept of real interest rate involves expectations of future rates of inflation, any analysis of real rates requires assumptions regarding the formation of expectations. Consequently, hypotheses tested, and conclusions arrived at, regarding real interest rates depend upon the validity of the assumed mechanisms of expectation formation.

In this paper, we propose to adopt a statistical approach to the determination of expectations. One advantage of this approach is that it minimizes the amount of economic theorizing one must do in order to obtain a meaningful series on expected rates of inflation which can then be used to generate time series on real rates of interest. An estimated system of equations is then used to derive a representation of the unobserved ex ante real rates of interest as a linear function of the "information" set presumed to be known by economic agents. The methodology developed here is a general one and could be used with alternative specifications of the information vector. In this sense, the present paper is preliminary, developing the methodological framework and providing a simple application.

This paper is organized as follows. In Section II there is a selective review of the literature, with discussion of some theoretical issues pertaining to the behavior of the real rates of interest. Section III deals with the appropriate econometric methodology to be used in testing the theory. Section IV presents the empirical results, which are relevant not only for the messages they convey regarding the real rates of interest but also for the information they give about the dynamic structure of the economies examined. Section V summarizes the conclusions of the study.

## II. Theoretical Issues

### 1. The role of real interest rates

According to neo-Keynesian theory, a change in the real rate of interest alters the demand for investment by changing the present value of the future stream of returns to capital goods. A rise in the real rate of interest lowers the value of the existing stock of capital, because the future stream of returns is discounted more heavily. As the value of existing capital falls, relative to its replacement cost, the demand for investment is reduced, leading to a fall in the aggregate demand and employment.

The link between the real rate of interest and the level of investment demand is also a crucial aspect of the mechanism through which monetary policy affects the real side of the economy. Within the Keynesian framework, changes in the rate of monetary expansion primarily induce changes in nominal interest rates, through the portfolio allocation behavior of asset holders. To the extent that changes in nominal interest rates are translated into changes in real interest rates, and to the extent that investment demand responds to these changes, monetary policy is able to affect the level of economic activity.

An understanding of the behavior, and the determinants of real rates of interest is also crucial for our understanding of foreign exchange markets, particularly in terms of the impact of alternative policies upon the exchange rates. Higher real interest rates cause an appreciation of the domestic currency both through increased capital inflows and through the current account impact of the economic slowdown. It is thus frequently argued that excessive reliance on monetary measures to combat inflation induce large fluctuations in exchange rates owing to the response of real interest rates to restrictive monetary policy.

The foregoing analysis is essentially Keynesian in nature in that it is based, at least implicitly, on the assumption that the elasticity of real interest rate with respect to money is non-zero which, in turn, implies that nominal interest rates and expectations of inflation respond differently to changes in policy. Although one can give several justifications for this differential response, the most common one is that expectations adjust slowly to a changing economic environment. It is also assumed that nominal wages are fixed in the short run. With the sluggish response of expectations, changes in monetary policies will indeed alter the real interest rates, thereby allowing the monetary authority to attenuate cyclical fluctuations in the level of economic activity through the impact of variations in the real interest rate upon aggregate demand. An implication of the sluggish response of price expectations and other rigidities is that policymakers face a tradeoff between unemployment and inflation.

The two crucial elements of the mechanism through which the policy actions of the monetary authorities affect the level of employment, non-zero elasticities of real interest rates and real wages with respect to the money supply, have come under substantial criticism by the monetarists. Earlier critics emphasized that any trade-off between inflation and unemployment is only temporary, resulting from a slower response of wages than prices to changes in nominal demand. According to this view, as elaborated by Milton Friedman 1/ in his presidential address to the American Economic Association, monetary expansion results in an initial fall in real wages, which will indeed stimulate employment, expanding it over the natural rate. However, as expectations begin to adjust to higher rates of inflation, the increase in nominal wages will start accelerating. Due to the presence of excess demand for labor in the economy, the rise

1/ Friedman, 1968.

in nominal wages will exceed the rate of inflation, pushing real wages up. Real wages will continue to rise until equilibrium is restored at the natural rate of employment with a higher rate of price and wage inflation.

In a similar fashion, monetarists argue that the impact of changes in monetary policy upon the nominal interest rate is related to the rate at which expectations adjust to the new economic policy. In view of the expansionary monetary policy, the public comes to expect a higher rate of inflation, nominal interest rates will increase rather than fall because, as first proposed by Fisher, lenders, in anticipation of inflation, will demand and borrowers will be willing to pay a higher rate of interest. 1/ It is clear, however, that Fisher, in formulating the relationship between the money rate of interest and the value of money, was mainly thinking in theoretical terms. He believed that in actual practice, the changes in the value of money would not be fully reflected in the nominal interest rates, mainly due to a lack of foresight. Furthermore,

---

1/ Fisher's ideas on the relationship bewtween the purchasing power of money and the nominal interest rates were stated clearly in his book The Theory of Interest, especially in Chapters II and XIX. In writing about the effects of changes in the purchasing power of money, (p.37) he stated that "the influence of such changes in the purchasing power of money on the money rates of interest will be different according to whether or not that change is foreseen. If it is not clearly foreseen, a change in the purchasing power of money will not, at first, greatly affect the rate of interest expressed in terms of money." However, to the extent that changes in the purchasing ower of money are foreseen, it is possible, at least theoretically, to make allowances for the expected change in the unit of value. "To offset a foreseen appreciation, therefore, it is necessary only that the rate of interest be correspondingly lower, and to offset a foreseen depreciation, that it be correspondingly higher."

Fisher, (p. 38) however, concedes that because of ignorance and indifference, appreciations and depreciations are never fully foreknown and therefore they are only partially provided against in the rate of interest itself.

It is interesting that Keynes' criticism of Fisher's theory of the distinction between the money rate of interest and the real rate of interest is based on a false assertion. Keynes asserts that "It is difficult to make sense of this theory as stated, because it is not clear whether the change in the value of money is or is not assumed to be foreseen." (Keynes, 1964, p. 142.) Fisher, however, was a very careful writer who made it clear whether the changes in the value of money were, or were not, assumed to be foreseen.

he suggested that "...so far as there exists any adjustment of money rate of interest to the changes in the purchasing power of money, it is for the most part (1) lagged and (2) indirect." 1/

## 2. Measuring the Fisher effect

The renewed prominence of Fisher's proposition following Friedman's presidential address stimulated a substantial amount of research on the relationship of nominal interest rates to expectations of inflation. Many of the earlier studies were based on the theoretical expression whereby the nominal interest rate is expressed as the sum of the real interest rate, the expected rate of inflation and a residual term distributed independently of the real rate and expectations of inflation. A fundamental difficulty with empirical implementation of this theoretical expression is the lack of independent data on either the real rate of interest or expectations of inflation. The approach adopted by Fisher and others was to maintain specific hypotheses about the real rates and the formation of expectations by the public. Generally, it was assumed that the real rate of interest, being a function of deeper economic variables, like the rate of time preference, and various marginal rates of substitutions and transformations, moved slowly and therefore could be approximated as constant. Expectations of inflation, on the other hand, were modeled within an error-learning framework, in which the public was assumed to form its expectations about future rates of inflation as a distributed lag of past rates of inflation. Much of the earlier econometric work has proceeded along these lines differing mainly in terms of the exact specification of the distributed lag which was presumed to generate the expectations of inflation, and the presence of other economic variables which may play a role in the determination of interest rates.

In these earlier studies the objective was to estimate the magnitude of the "Fisher effect," where the Fisher effect was defined as the amount of change in nominal interest rates as a result of a change in the expected rate of inflation. Given this objective it was natural to proceed by regressing nominal interest rates on a measure of the expected rate of inflation. If the estimated coefficient was significantly different from unity, it was argued, then the monetary authority could cause systematic variations in real interest rates, and therefore stabilize fluctuations in output. A number of studies conducted along these lines estimated that the magnitude of the Fisher effect was less than unity, thus rejecting Fisher's hypothesis. 2/ These findings were interpreted as suggesting

---

1/ Fisher, op. cit., pp. 493-4.
2/ For examples, see Feltstein and Eckstein (1973), Gordon (1970), and Roll (1972).

that an expansionary monetary policy leads to a fall in real interest rates because the nominal interest rates fail to adjust sufficiently to the higher rate of anticipated inflation. As real interest rates fall, aggregate demand is stimulated by the ensuing increase in investment demand.

Recent studies suggest, however, that the method used to estimate the magnitude of the Fisher effect is not entirely appropriate and that the results are likely to be biased. Often, the mechanism of expectation formation was proxied by a distributed lag of the actual rate of inflation, where the sum of distributed lag coefficients was constrained to be 1. Under this restriction, one can easily show that in an unrestricted regression of nominal interest rates on current and lagged rates of inflation, the sum of estimated coefficents is indeed an estimate of the magnitude of the Fisher effect. This particular restriction, however, is arbitrary and does not follow from any firm economic reasoning. It is in the nature of a neutrality result whereby a permanent increase in the actual rate of inflation leads to an equivalent increase in the expected rate of inflation after a sufficiently long period of adjustment. There is little dispute about the requirement that permanent increases in the actual rate of inflation ought to lead to permanent increases in the expected rate of inflation; this is a consistency requirement. What is disputed is that consistency implies the sum-to-unity restriction. As it turns out, this is not the case and the sum of the distributed lag coefficents in the formation of expectations could be anything without violating the consistency requirement.

Apart from the questions regarding identification, a more important issue is that it is doubtful that the magnitude of the Fisher effect can shed any light on the ability of the monetary authority to implement systematic changes in real interest rates. This point may be illustrated by the following interpretation of the test procedure.

Consider an economy in which borrowers and lenders can draw contracts denominated either in terms of money or in terms of a composite commodity which shall be called the consumption basket. Let the interest on loans denominated in terms of the consumption basket be $\rho_t$ and in terms of monetary units by $r_t$. Furthermore, suppose that economic agents anticipate that the price of the consumption basket in terms of money will increase at the rate $\pi_t$ over the period, at the end of which the loan contracts mature. If these markets are dominated by risk-neutral agents and are efficient, then the anticipated real yields will differ from each other only by a random term. Let $\rho_t^b$ be the <u>ex ante</u> real rate of interest on a contract denominated in nominal terms. Risk is expressed by $\rho_t^b - \rho_t = u_t$, where $u_t$ is a random term orthogonal 1/ to the information available to economic agents at the time the contracts are drawn. The efficiency of the

1/ i.e, completely uncorrelated with the information variables determining expectations.

market can be tested by regressing $\rho_t^b - \rho_t$ on the anticipated rate of inflation. The estimated equation will be of the form:

$$\rho_t^b - \rho_t = \alpha\pi_t + u_t. \qquad (1)$$

and the null hypothesis to be tested is $\alpha = 0$. If we add $\pi_t$ to both sides of the equation and rearrange the terms we get:

$$r_t = \rho_t + (1 + \alpha)\pi_t + u_t \qquad (2)$$

which reduces to a regression equation under the assumption that the yield on contracts denominated in units of the consumption basket is constant. Under the present interpretation, an estimate of $\alpha$ which is significantly different from zero will indicate that the joint hypothesis of risk neutrality and market efficiency could be rejected. Such a result implies that, at equilibrium, the real interest rate differential between the two assets varies systematically with the anticipated rate of inflation, but will not convey much information regarding the effectiveness of monetary policy to induce changes in real rates of interest. In fact, unless we entertain some additional assumptions, it is not possible to infer, from the estimates of $\alpha$, the direction of change in either $\rho_t^b$ or $\rho_t$.

Under an alternative, and somewhat more orthodox, interpretation, a regression of nominal interest rates on a measure of the expected rate of inflation derives its content from the definition of the anticipated real yield on an instrument as being equal to the difference between the fixed nominal yield and the anticipated rate of inflation.

$$r_t = \rho_t + \pi_t \qquad (3)$$

In equation (3), $r_t$ and $\rho_t$ are the nominal and <u>ex ante</u> real yields on the <u>same</u> instrument, and $\pi_t$ is the expected rate of inflation. Note that equation (3), being the definition of <u>ex ante</u> real yield, is exact when $\pi_t$ is the true (but unobserved) expected rate of inflation. The projection of <u>ex ante</u> real yield on a constant and on expected rate of inflation is given by:

$$\rho_t = c + \alpha\pi_t + u_t \qquad (4)$$

where $u_t$ is orthogonal to the constant and $\pi_t$. Substituting equation (4) into equation (3), one obtains a representation of nominal interest rate as follows:

$$r_t = c + (1 + \alpha)\pi_t + u_t \tag{5}$$

Because $u_t$ is orthogonal to $\pi_t$, equation (5) is the projection of nominal interest rates on a constant and on the expected rate of inflation, and consequently c and $\alpha$ can be estimated consistently by ordinary least squares through equation (5). If $\alpha$ is significantly different from zero then it is concluded that Fisher's hypothesis is rejected by the data for the particular instrument in question.

Often these conclusions are extended to imply that variations in the expected rate of inflation cause variations in real rates of interest, and that monetary policy is a potent instrument of stabilization because alternative policy measures result in variations in real interest rates and thus affect real variables. Neither of these extensions are justified, however. The fact that the estimate of $\alpha$ in equation (5) is significantly different from zero implies only that anticipated inflation and real interest co-vary, without providing any evidence as to the direction of causality. Similarly, it is not possible to pass judgement on the efficacy of monetary policy from the estimates of $\alpha$. For example, if both real interest rates and expectations of inflation respond to a set of real exogenous variables, whereas only expectations respond to nominal variables, real interest rates will be correlated with the expected rate of inflation resulting in an estimate of $\alpha$ which is significantly different from zero, even though monetary policy is ineffective in controlling real interest rates. 1/

The foregoing considerations indicate that alternative formulations aimed at measuring the magnitude of the Fisher effect are best considered as variations on a single theme: measuring the correlation between the real rate of interest and the expected rate of inflation, both of which are unobserved by the econometrician. Moreover, Fisher's proposition that an x per cent increase in the expected rate of inflation results in an x per cent increase in the nominal interest rate, is vacuous unless some hypothesis is maintained regarding the behavior of real interest rates. 2/ In order to convert Fisher's proposition into a hypothesis

---

1/ In fact, in a class of rational expectations models one can easily demonstrate that the ex ante real interest rate will be correlated with the expected rate of inflation, even though the monetary policy cannot induce systematic movements in output or the real interest rate. Conversely, one can construct models where monetary policy is a potent instrument of stabilization but the estimates of $\alpha$ in equation (5) will be zero.

2/ See Dwyer (1981) for an elaboration of this point.

with testable implications it is sufficient to assume that real interest rates are orthogonal to the expected rate of inflation. This is an interesting hypothesis with some implications for economic behavior, although, it must be emphasized, lacking any information regarding the potency of monetary policy. A test of this hypothesis is formulated in Section IV and implemented for France, Germany, Japan, the United Kingdom, and the United States.

### 3. The role of expectations

A further set of issues regarding the expectation proxies emerged from analysis of the rational expectations hypothesis, which shifted the focus of investigations from estimation of the magnitude of the Fisher effect to the determination of criteria that will help decide what patterns of distributed lag coefficients are plausible. A major econometric lesson arising from this analysis is that any identifying restrictions should be derived from the same principles that are assumed to guide economic behavior. If such behavior derives from a process of optimization, then one ought to assume that agents optimize not only with respect to their decision variables but also with respect to their techniques of forecasting. Such an assumption implies that forecast errors should have no systematic behavior relative to the information set which formed the basis of the expectations in the first place. In the more specific case of linear decision rules, the foregoing implication translates into the orthogonality principle which states that forecast errors must be orthogonal to those variables, a linear combination of which gives the forecast itself.

This orthogonality principle was a key ingredient in Fama's 1975 article, which generated substantial controversy. His formulation is based on expressing the *ex post* rate of inflation as the sum of the expected rate of inflation and a forecast error. Hence the expected rate of inflation is equal to the actual rate of inflation minus the forecast error.

In this formulation equation (3) can be written as:

$$r_t = \rho_t + \psi(x_{t+1} - e_{t+1}) \qquad (6)$$

where $x_{t+1}$ is the *actual* rate of inflation between times t and t+1, and $e_{t+1}$ is the forecast error. The coefficient $\psi$, whose theoretical value is 1, is added to the equation in order to have an estimable equation. Under the maintained hypothesis that the real rate of interest, $\rho_t$, is constant, equation (6) reduces to:

$$r_t = \rho + \psi(x_{t+1} - e_{t+1}) \qquad (7)$$

Equation (7), in turn, can be solved for $x_{t+1}$ yielding the form which was actually estimated:

$$x_{t+1} = -(1/\psi)\rho + (1/\psi)r_t + e_{t+1}$$

$$x_{t+1} = c + \beta r_t + e_{t+1} \qquad (8)$$

The orthogonality principle, jointly with the maintained hypothesis of constant real rate of interest, ensures that equation (8) can be consistently estimated by ordinary least squares. Fama's results, based on monthly data for the United States and covering the period 1953 through 1971 support the hypothesis that ex ante real rate of interest is constant. Furthermore, his estimate of $\psi$ is not significantly different from unity and therefore fails to reject Fisher's hypothesis.

Fama's paper was criticized for a number of points, particularly for an apparent lack of robustness in his results with respect to the sample period. 1/ Subsequent work, using identical data and methodology, indicates that as the period of estimation is extended, the estimates of $\psi$ decline, accompanied by an increase in R-square and the estimated first order serial correlation of residuals. 2/ This observed pattern of estimates is symptomatic of a bias which is varying slowly but systematically over time. Such a bias could result if ex ante real interest rate is not exactly equal to a constant, but has a stochastic component, $u_t$. It can be easily shown that in the presence of a stochastic component, $u_t$, the least squares estimate of $\beta$ in equation (8) will be biased by an amount which is equal to $-\beta\sigma_u^2/\sigma_r^2$. These criticisms notwithstanding, Fama's paper was influential because of its novel application of the orthogonality principle.

In addition to the orthogonality principle, the rational expectations hypothesis has a number of other concrete implications, that can be used to derive testable propositions from maintained hypotheses in general, and from the Fisher hypothesis, in particular. Under the rational expectations hypothesis, subjective expectations of the public are equal to the conditional expected values of the same variables, i.e., conditional on the information available to the public at the time expectations are formed. Consequently, under rational expectations, the expected rate of inflation, $\pi_t$, is equal to $E(x_{t+1}|\Omega_t)$, where $x_{t+1}$ is the rate of inflation between times t and t+1, $\Omega_t$ is the information set available to, and utilized by, the public in forming its expectations, and E is the conditional expectations operator. In general, it must be

---

1/ Examples are Carlson (1977), Joines (1977), Nelson and Schwert (1977), and Begg (1977). Also See Fama (1977) for his response to some of the criticisms.

2/ Saracoglu (1980), page 565.

recognized that $\Omega_t$ includes historical values, up to and including period t, of readily available time series such as interest rates, rates of inflation, money supply, etc. It must also be recognized that $\Omega_t$ probably includes other variables some of which may not be quantifiable. However, for the purpose of hypothesis testing, it is not necessary to know the contents of $\Omega_t$; it is sufficient to have knowledge on a subset of $\Omega_t$, say $H_t$.

With these preliminaries in hand, let us rewrite equation (3), using the new notation.

$$r_t = \rho_t + E(X_{t+1}|\Omega_t) \qquad (9)$$

If we take conditional expectations on both sides of equation (9), conditional on $H_t$, we get:

$$E(r_t|H_t) = E(\rho_t|H_t) + E(E(x_{t+1}|\Omega_t)|H_t) \qquad (10)$$

It is a well known result in statistics that $E(E(x_{t+1}|\Omega_t)|H_t) = E(x_{t+1}|H_t)$ whenever $H_t$ is a subset of $\Omega_t$. 1/ Under the assumption of a constant real rate of interest we obtain:

$$E(r_t|H_t) = c + E(x_{t+1}|H_t) \qquad (11)$$

The message conveyed by equation (11) is clear: projection of nominal interest rate, $r_t$, on a set of variables should be equal, up to an additive constant, to the projection of the rate of inflation between times t and t+1, $x_{t+1}$, on the same set. This is clearly a testable proposition. 2/ All that is necessary is to regress $r_t$ and $x_{t+1}$ on a given set of variables and test whether the two regressions are identical. Of course, the power of such a test depends upon the extent to which information contained in $\Omega_t$ but excluded from $H_t$ helps to predict the rate of inflation. Let $I_t$ be the complement of $H_t$ in $\Omega_t$. Then, under the null hypothesis, residuals from the regression of nominal interest rates on $H_t$ are equal to $E(x_{t+1}|I_t)$, and those from the regression of inflation rate on $H_t$ are equal to $E(x_{t+1}|I_t) + e_{t+1}$, where $e_{t+1}$ is the forecast error in inflation when

---

1/ This result is known as the law of iterated projections or the law of nested conditional expectations. Its proof is simple and can be found in any statistics textbook.

2/ The test will be valid as long as the contemporaneous interest is excluded from $H_t$. Otherwise the left hand side of equation (11) will reduce to the singular projection of $r_t$ on itself.

these forecasts are based on $\Omega_t$. If the variance of $E(x_{t+1}|I_t)$ is high (indicating that some important variables are excluded from $H_t$) the power of test is reduced accordingly.

*This line of reasoning was initially developed and empirically implemented* by Shiller (1972). Although he did not conduct a formal statistical test, Shiller's results suggest that Fisher's hypothesis is a reasonable working hypothesis and seems to work fairly well for post war U.S. data. 1/

4. *The nature of the problem*

Although Fama and Shiller both used certain principles derived from the rational expectations hypothesis, they failed to take into account the general equilibrium nature of Fisher's hypothesis. As Sargent (1973), and subsequently Begg (1977), have forcefully argued, the proper framework to study Fisher's hypothesis belongs to the domain of macroeconomic analysis and the econometric methodology must take into account the general equilibrium nature of the problem. A principal reason for this follows from the maintained hypothesis that expectations are formed rationally. Under rational expectations, it is presumed that the agents understand the economic environment in which they operate and the implications of alternative policies upon this environment. If, for example, alternative monetary policies induce alternative time paths for the price level, then the agents will try to predict the likely courses of action by the policymakers and will incorporate such predictions into their expectations. Consequently, the mechanism through which the forecasts are generated is intertwined with the linkages between the various sectors of the economy.

Once the macroeconomic nature of the relationship between the interest rate and expectations of inflation is realized, the appropriate econometric methodology to investigate this relationship becomes more complex. Generally, tests based on single equation methods are inappropriate. This follows from the fact that macroeconomic models are characterized by simultaneities and, unless we have strong a priori beliefs to the contrary, it must be assumed that all contemporaneous variables are correlated with all structural disturbances. Under these circumstances, single equation methods, where some of the regressors are dated the same as the dependent variable, will yield biased estimates.

There are, of course, ways to alleviate the problem and obtain consistent paramter estimates. One can use any one of the family of instrumental variable estimators. The basic idea here is to substitute linear

1/ See Shiller (1972), especially page 113. The distributed lag coefficients he reports in Tables 5-22 through 5-24, tend to support Fisher's hypothesis.

combinations of some exogenous variables for the endogenous variables that appear as regressors. If the exogenous variables are uncorrelated with the residuals, then so will any linear combination of them. Thus, replacement of the endogenous variables with linear combinations of exogenous variables results in consistent parameter estimates. For the application of these methods, the choice of instruments is crucial, particularly when the exogenous variables of the system are stochastic.

An alternative, and somewhat more attractive strategy to overcome the problems inherent in single equation methods would be to investigate Fisher's hypothesis within a general equilbrium framework. The strategy in conducting such a test would be to construct a complete macroeconomic model that incorporates Fisher's hypothesis as a restriction and test the model against data. To the extent that the model is not rejected, the hypothesis will be verified.

This approach was used by Sargent (1973) to test Fisher's hypothesis jointly with the natural rate hypothesis and the rational expectations hypothesis. Within the context of a macroeconomic model, Sargent conducts four alternative tests, 1/ two of which call for rejection of the composite hypothesis. Although there is some degree of ambiguity in interpreting his results, the evidence against the composite hypothesis seems to be stronger owing to the fact that the tests which call for rejection are more powerful. 2/

Although the strategy of testing the Fisher hypothesis within the context of a macroeconomic model is appealing, it has a number of drawbacks. First of all, when a complete macroeconomic model is specified, it is comprised of a number of maintained hypotheses and what is being tested is the resultant composite hypothesis. If the data does not call for its rejection, then there are no problems. However, if any one of the individual hypotheses is at variance with the data, the composite hypothesis will be rejected without providing the researcher with a clue as to what aspect, or aspects, are causing the rejection.

Secondly, for the purposes of hypothesis testing, it is necessary to identify structural parameters. Identification of the structure, however, is not easily accomplished. The inappropriate manner in which econometricians approach structural identification was elucidated recently by Sims (1980). He argues that in order to be able to achieve identification of the structure, it is common practice to make shrewd aggregations and exclusion restrictions. Such restrictions are of an ad hoc nature without having firm foundations in economic theory. As a result of these

---

1/ These tests, however, are indirect tests of Fisher's hypothesis because the statistics are formulated in terms of the natural rate hypothesis.

2/ Nevertheless, Sargent states that "...the call for rejection...is not strong enough to persuade someone to abandon a strongly held prior belief in the natural rate hypothesis." (Sargent, 1973, p. 462.) Also see comments and discussion following Sargent's paper.

identification restrictions, macroeconomic models tend to be parsimoniously parameterized. Although such restrictions and parsimonious parameterizations may be reasonable for partial equilibrium analysis, the general equilibrium properties of models thus identified may be extremely distorted. The fact that identification of large scale macro models lacks credibility and therefore should not be taken seriously, renders such models inappropriate for testing alternative economic theories.

An equally damaging assessment emerges regarding the classification of variables as exogenous and endogenous. Upon closer examination it is seen that variables that have traditionally been considered as exogenous turn out not to be so. For example, policy variables are determined not in isolation from the economic environment but rather in response to it. Furthermore, the rational expectations theorists have forcefully argued that policy formulation should take the form of setting rules for systematically changing the policy variables as the economic conditions change. Doing this will amount to explicitly endogenizing the policy variables and further weaken the credibility of identification.

Nevertheless, macroeconomic models are useful tools for policy analysis and forecasting, as Sims carefully stresses, in spite of the fact that reduced forms of such models will be distorted by false identification restrictions. The reason is that the restricted estimators can actually produce forecasts or projections with a smaller error than unrestricted estimators, provided that the restrictions are not grossly inappropriate.

The foregoing considerations have important implications for research methodology. The macroeconomic nature of Fisher's hypothesis renders partial equilibrium analysis inappropriate. Simultaneity is an essential feature of macroeconomic analysis and it necessitates a general equilibrium treatment of macroeconomic questions. By extension, the econometric methodology used in empirical work on macroeconomic questions must also reflect their general equilibrium nature. The methodology of traditional macroeconometric analysis, however, is also inappropriate for the purposes of hypothesis testing. 1/

In this paper, we shall adopt an alternative methodology suggested by Sims. This methodology, discussed in some detail in the next section, involves estimating systems of equations as unrestricted reduced form equations, treating all variables as endogenous. The main advantage of this approach is that it minimizes the amount of a priori economic theorizing that is necessary to perform the estimation. Treating all variables as endogenous relieves us of the burden of classifying them as exogenous or endogenous, whereas treating all equations as unrestricted reduced form equations eliminates the need to impose identifying restrictions. Following this preliminary stage of estimation, specific hypothesis can

1/ This basically reflects Sim's point of view. For an excellent paper in defense of some of the traditional methodology see Malinvaud (1981).

be formulated and tested in terms of the restrictions they imply over the parameter space. 1/

## III. Methodological Issues 2/

### 1. Vector autoregressions

The statistical work in this paper is based on vector autoregressions. Every component of a vector of economic variables is regressed (projected) on lagged values of every variable in the system. In principle, the regressions are unconstrained. The resulting model of the economy is that of a linear stochastic difference equation in the vector of economic variables. The model is dynamic because of the dependence of the current vector of variables to their lagged values. Dynamics of the model arise solely from the autoregressive components and not from the serial correlation properties of the residuals.

The main advantage of vector autoregressive specification lies in the fact that it is very general, capable of exhibiting a wide variety of dynamic behavior. Any covariance stationary vector stochastic process can be represented arbitrarily well by a vector autoregression. 3/ Moreover, many of the traditional macroeconomic models may be viewed as vector autoregressions, subject to sets of restrictions which are implied either by exclusionary identifying restrictions on the structural equations or by the separation of variables into exogenous and endogenous groups. These restrictions effectively limit the admissible parameter space and result in parsimonious parameterizations. Conversely, when vector autoregressions are unrestricted, they tend to be profligately parameterized. This, in fact, is the main weakness of vector autoregressions. The number of free parameters increases with the square of the number of variables in the system, and even for moderately sized systems, degrees of freedom are exhausted rapidly. Nevertheless, it is still feasible to estimate small vector autoregressive systems, particularly if the serial correlation of the variables declines rapidly.

Formally, we propose to estimate the folowing system of vector autoregressions:

$$y_t = \sum_{i=1}^{m(T)} A_i y_{t-i} + u_t \qquad (12)$$

---

1/ As examples of this see Sargent (1979), Saracoglu (1980) and Dwyer (1981).

2/ For alternative expositions of methodology see Litterman (1979) and Sims (1980).

3/ Let $y_t$ be a vector stochastic process with $E(y_t) = 0$. The autocovariance of $y_t$ is given by $K(s,t) = E(y_t y_s')$. If $K(s,t) = K(s-t)$ for all s and t then $y_t$ is a *covariance stationary stochastic process*.

where $y_t$ is the vector of variables in which we are interested, $A_i$s are square matrices, conformable in size with the vector $y_t$, and $u_t$ is the vector of disturbances whose properties are to be specified shortly. The order of autoregression, m(T) is in principle a function of the number of observations which increases with the sample size in such a way that the ratio of the number of estimated parameters to the sample size approaches zero. The estimation of vector autoregresions constitutes the first stage and essentially amounts to a preliminary data transformation, whereby the information contained in the time series could be extracted and interpreted more readily. After this is done, at the second stage, specific hypotheses could be formulated and tested. At a third stage, outcomes of hypothesis tests could be used to restrict the parameter space, and the system of equations could be re-estimated under these restrictions.

Systems of equations like (12) should best be viewed as approximations to infinite projections given by:

$$y_t = \sum_{i=1}^{\infty} A_i y_{t-i} + u_t \qquad (13)$$

The sequence $u_t$ will have the following properties: 1/

$$Eu_t = 0$$

$$E\,[u_t y'_{t-s}] = 0 \qquad \text{if } s > 0$$

$$E\,[u_t u'_{t-s}] = 0 \qquad \text{if } s \neq 0$$

$$E\,[u_t u'_t] = \Omega$$

where ' denotes transposition. In statistical terminology $y_t$ is a linear regression on its past, $y_{t-1}$, $y_{t-2}$, plus an innovation $u_t$ at time t. Thus equation (13) describes an autoregressive scheme. It is a well known result that innovations from a finite m'th order autoregression, $u_t^m$, converge to $u_t$ in the mean square sense as m increases. 2/ This congence forms the basis of the approximation. Now, let us define the following matrix valued function:

$$A(z) = I - \sum_{i=1}^{\infty} A_i z^i \qquad (14)$$

1/ For expositional purposes, we are assuming that the vector $y_t$ has an expected value of 0.

2/ See Sargent (1979a), pp. 256-262.

The determinant of the matrix A(z) will be a scalar valued function of z expressed in the form of a power series $b(z) = \sum_{i=0}^{\infty} b_i z^i$. If the zeros of the function b(z) are all outside of the unit circle then there exists a matrix valued function $D(z) = \sum_{i=0}^{\infty} D_i z^i$ converging on the unit disk such that D(z)A(z) = I. D(z) is called the inverse of A(z) under convolution and satisfies the following system of equations:

$$D_0 = I$$

$$D_j = \sum_{k=0}^{j-1} D_k A_{j-k} \qquad j > 0 \tag{15}$$

If we covolute both sides of equation (13) by D(z) we get:

$$\sum_{j=0}^{\infty} D_j y_{t-j} = \sum_{j=0}^{\infty} \sum_{i=1}^{\infty} D_j A_i y_{t-i-j} + \sum_{j=0}^{\infty} D_j u_{t-j} \tag{16}$$

which can equivalently be expressed as:

$$\sum_{j=0}^{\infty} D_j y_{t-j} = \sum_{j=1}^{\infty} \left(\sum_{i=0}^{j-1} D_i A_{j-i}\right) y_{t-j} + \sum_{j=0}^{\infty} D_j u_{t-j} \tag{17}$$

However by virtue of equation (15) all the terms involving $y_{t-j}$, j > 0 will cancel out and equation (17) will reduce to:

$$y_t = \sum_{j=0}^{\infty} D_j u_{t-j} = D(z)u_t \tag{18}$$

This is the moving average representation of $y_t$. The element on the j'th row and k'th column of the matrix $D_i$ measures the impact on variable j of an innovation in variable k after i periods.

The moving average representation shows the source of the dynamic behavior of the system. The vector of variables $y_t$, are represented as linear combinations of serially uncorrelated random variables which lack any dynamics of their own. Consequently, any dynamic behavior of the vector $y_t$ comes from the impulse response function D(z). This function, through delays and scaling, distributes the impact of innovations over time, giving rise to the observed amplitude variations and phase shifts in the original variables.

The moving average coefficent matrices, $D_j$s, in equation (18) trace out the system's response to innovations of unit magnitude, whereas for comparison purposes we need to find the system's response to "typical" innovations. Because each element of the vector $u_t$ can have a different variance, innovations of unit magnitude are not equiprobability events and therefore they are not typical. In addition, the innovations are contemporaneously correlated. For these reasons, it is necessary to transform the innovations so that they are orthogonal across equations and have unit variances. There is no unique way of doing this. One way to proceed, which has a natural interpretation, is to obtain normalization by representing $u_t$ as a linear combination of standard variates with a triangular coefficent matrix.

$$u_t = Le_t \tag{19}$$

where $E(e_t e_t') = I$ and L is lower triangular. Using equation (19), the covariance matrix of innovations can be represented as:

$$E(u_t u_t') = E(Le_t e_t' L') = LL' = \Omega \tag{20}$$

Thus the coefficient matrix L corresponds to the factor in the Choleski decomposition of the covariance matrix of innovations. The moving average respresentation of $y_t$ in terms of the normalized innovations becomes:

$$y_t = \sum_{j=0}^{\infty} D_j Le_t = \sum_{j=0}^{\infty} D_j^* e_t \tag{21}$$

2. Expectations mechanism

In order to investigate the relationship between the nominal interest rates and expectations of inflation, it is necessary to generate reasonable proxies for public's subjective expectations. This we propose to do by using estimated vector autoregressions. As we remarked earlier in Section II, part 3, reasonable proxies for expected inflation series could be generated on the basis of a subset of the actual information set used by the public. We assume that observations on the vector $y_t$ are included in the information set $\Omega_t$ and take these observations as the subset of $H_t$ to proxy the expectations. 1/

---

1/ If information is valued by market participants, it is reasonable to assume that they utilize any relevant information which is readily and costlessly available.

Let $r_t$ be the nominal interest rate on bonds with N periods to maturity. The real rate of return on this bond is defined as the difference between the nominal interest rate and the expected rate of inflation over its lifetime. Thus the real interest rate can be expressed as:

$$\rho_t = r_t - N^{-1} \sum_{j=1}^{N} E_t X_{t+j} \tag{22}$$

The expression $E_t x_{t+j}$ is the expected rate of inflation between periods t+j-1 and t+j, based on information available at time t. If we assume that expectations are formed on the basis of $H_t = \{y_t, y_{t-1}, y_{t-2}, ..\}$ then the real interest rate series, $\rho_t$, could be generated using the estimated vector autoregressions. One period ahead forecasts could be formed using equation (12). Let the first element of the vector $y_t$ be the rate of inflation. Then, one period ahead forecast of inflation is given by:

$$E_t x_{t+1} = f_1 E_t y_{t+1} = f_1 \sum_{j=1}^{m} A_i y_{t+1-j} \tag{23}$$

where $f_1$ is a row vector of the same size as $y_t$, and has 1 as its first element and 0 otherwise. 1/ Forecasts of increasing horizons could be formed by using the chain rule of forecasting in conjunction with equation (12).

In our statistical work, equation (22) is approximated as follows:

$$\rho_t = r_t - \left( (1-\lambda)/(\lambda) \right) \sum_{j=1}^{\infty} \lambda^j E_t x_{t+j} \tag{24}$$

with $0 < \lambda < 1$. This approximation is done primarily for computational reasons. By replacing the term $N^{-1} \sum_{j=1}^{N} E_t x_{t+j}$ with $((1-\lambda)/(\lambda)) \sum_{j=1}^{\infty} \lambda^j E_t x_{t+j}$ it is possible to obtain simple closed form expressions which facilitate computations. Moreover, it is well known that forecasts lose their precision as they are extended into the future. Therefore equation (24) has the appealing implication that in determining current real interest rate, expectations for which there is more confidence, have a higher weight than those for which there is less confidence.

1/ Vector $f_1$ "picks" the rate of inflation from the vector $y_t$.

The choice of $\lambda$ is dictated by the need to have the mean horizon covered by equation (24) to be the same as that covered by equation (22). The mean horizon of equation (22) is equal to $(N+1)/2$ whereas that of equation (24) is equal to $1/(1-\lambda)$. In order to have the same mean horizon, $\lambda$ should be set equal to $(N-1)/(N+1)$.

With these preliminaries, we now derive closed form expressions to represent ex ante real interest rates in terms of the observed variables. Consider the vector autoregression of $y_t$.

$$y_t - \sum_{i=1}^{m} A_i y_{t-i} = A(z)y_t = u_t \tag{25}$$

with the moving average representation given by:

$$y_t = A(z)^{-1}u_t = D(z)u_t \tag{26}$$

Optimal forecasts of vector y are given by:

$$E_t y_{t+k} = [z^{-k}D(z)]_+ u_t, \; k > 0 \tag{27}$$

where $[\ ]_+$ means ignore negative powers of z. 1/ The term in brackets in equation (27) can be expressed as:

$$[z^{-k}D(z)]_+ = z^{-k}D(z) - \sum_{j=0}^{k-1} D_j z^{j-k} \tag{28}$$

If we multiply both sides of equation (27) by $\lambda^k$ and sum over k we get

$$\sum_{k=1}^{\infty}\lambda^k E_t y_{t+k} = \sum_{k=1}^{\infty}\lambda^k [z^{-k}D(z)]_+ u_t$$

$$= \left(\sum_{k=1}^{\infty}\lambda^k z^{-k}D(z) - \sum_{k=1}^{\infty}\lambda^k \sum_{j=0}^{k-1} D_j z^{j-k}\right)u_t \tag{29}$$

1/ For a detailed exposition of representation theory and optimal forecasting see Whittle (1963).

One can simplify the right hand side of equation (29) so as to obtain:

$$\sum_{k=1}^{\infty}\lambda^k E_t y_{t+k} = \frac{\lambda z^{-1}}{1 - \lambda z^{-1}} (D(z) - D(\lambda))u_t \tag{30}$$

Using the relationship between the autoregressive and moving average representations, equation (30) can be simplified to:

$$\sum_{k=1}^{\infty}\lambda^k E_t y_{t+k} = \frac{\lambda z^{-1}}{1-\lambda z^{-1}} (A(z)^{-1} - A(\lambda)^{-1})A(z)y_t \tag{31}$$

$$\sum_{k}\lambda^k E_t y_{t+k} = -A(\lambda)^{-1}B(z)y_t \tag{32}$$

where $B(z) = \sum_{j=0}^{m-1} B_j z^j$ and the coefficent matrices $B_j$ satisfy the following recursion equation:

$$B_{m-1} = -\lambda A_m$$

$$B_j = \lambda(B_{j+1} - A_{j+1}) \qquad j = 0, 1, \ldots, m-2$$

The real rate of interest is then given by:

$$\rho_t = r_t + ((1-\lambda)/(\lambda))\; f_1 A(\lambda)^{-1}B(z)y_t \tag{33}$$

without loss of generality we can assume that the nominal interest rate is the second variable in the vector $y_t$ so that equation (33) reduces to:

$$\rho_t = [f_2 + ((1-\lambda)/(\lambda))f_1 A(\lambda)^{-1}B(z)]y_t \tag{34}$$

or equivalently:

$$\rho_t = h(z)y_t = \sum_{j=0}^{m-1} h_j y_{t-j} \tag{35}$$

Equation (35) gives the representation of ex ante real interest rate as a linear combination of vectors $y_t$ through $y_{t-m+1}$. Note, however, that equation (35) is not a regression equation; it is an exact relationship. This reflects the fact that the relationship between the nominal interest rate, the real interest rate, and expectations of inflation is exact. Whenever any two of these variables are given, the third one is determined by equation (3).

## IV. Some Empirical Results

In this section, we utilize the methodology which was described in the previous section to analyze the behavior and derive the properties of real interest rates in five industrial countries--the United States, France, Germany, Japan, and the United Kingdom. (The data used in the empirical analysis is discussed in Appendix I.) *Ex ante* real interest rates are computed as the difference between the nominal interest rates and the optimal forecasts of inflation. For each country, the optimal forecasts of inflation are generated on the basis of estimated systems of autoregressions using short-term nominal interest rates and annual rates of change in monetary base, narrow money, index of industrial production, and the consumer price index. The estimated systems of autoregressions are not presented here, due to reasons discussed in Appendix I, except for summary statistics which are reported in Table 1.

The empirical questions addressed in this paper can be categorized into three groups.

a. There are questions pertaining to movements in real interest rates over time, such as what stylized facts can be established and how they relate to significant economic developments. In this connection, subsection 1 examines historical movements in real interest rates with a view toward understanding their macroeconomic significance more clearly.

b. There are questions regarding the statistical relationship between real interest rates and expectations of inflation; more specifically, the extent to which real interest rates are correlated with rates of inflation and whether such correlations can be used to affect systematic changes in real interest rates through policy changes. To this end, in subsection 2, the co-movements betwen real interest rates and expectations of inflation are studied and it is suggested that although these two variables are correlated, such correlation cannot be used to formulate monetary policy and to influence the time path of real interest rates.

c. Finally, there are questions regarding the variability of real interest rates: what are the sources of real interest rate fluctuations and what is the relative contribution of each source? To address these questions the sources of interest rate variability are identified by means of a decomposition of variances of real interest rates in subsection 3.

Table 1. Somme Summary Statistics of Vector Autoregression

| | Dependent variable | | | | | F-Statistics 1/ |
|---|---|---|---|---|---|---|
| | M1 | Inflation rate | Interest rate | Industrial production | Base money | System F marginal significance |
| **United States** | | | | | | |
| Standard error | 1.35 | 0.33 | 0.69 | 1.11 | 2.60 | 48.3 |
| Serial correlation | 0.01 | -0.03 | 0.01 | 0.01 | -0.01 | 1.0 |
| **France** | | | | | | |
| Standard error | 2.28 | 0.26 | 0.67 | 2.30 | 5.44 | 49.3 |
| Serial correlation | 0.03 | 0.04 | 0.03 | 0.01 | -0.01 | 1.0 |
| **Germany** | | | | | | |
| Standard error | 1.31 | 0.27 | 0.53 | 2.22 | 4.90 | 45.8 |
| Serial correlation | -0.05 | 0.08 | 0.00 | 0.04 | -0.03 | 1.0 |
| **Japan** | | | | | | |
| Standard error | 2.33 | 1.02 | 0.41 | 1.61 | 1.97 | 77.4 |
| Serial correlation | 0.00 | 0.01 | 0.00 | 0.00 | 0.01 | 1.0 |
| **United Kingdom** | | | | | | |
| Standard error | 1.73 | 0.76 | 0.59 | 2.26 | 1.61 | 84.7 |
| Serial correlation | 0.00 | 0.01 | 0.01 | -0.02 | 0.04 | 1.0 |

1/ All system F-statistics have 150 and 715 degress of freedom. These F-statistics can be converted to system R-square, using the formula R-square = (150F)/(150F + 715). They range from 0.91 for Germany to 0.95 for the U.K.

## 1. Movements in real interest rates

In macroeconomic analysis, it is common practice to postulate real interest rates as being constant, particularly in medium and long term analyses. Results obtained in this paper refute that premise stongly. The estimated real interest rate series, which are plotted in Charts 1 through five, show substantial short and long run fluctuations in all five countries under investigation, with standard deviations ranging from a low of 2.1 for France to a high of 4.6 for the United Kingdom. Moreover, real interest rate series are very highly autocorrelated in all of the countries. (See Chart 6.) Autocorrelation coefficients decay slowly, indicating that unexpected changes in real interest rates persist over time. Such persistent behavior is inconsistent with the characterization of real interest rates as random movements around a constant.

A number of other salient features of real interest rates are also worth mentioning. Starting approximately in the middle of 1973 there was a sharp fall in real interest rates in all five countries, resulting in negative real rates. Although among these countries the extent to which interest rates became negative in real terms and the duration over which negative real interest rates persisted differed, their combined experience during 1973-75 was clearly out of line with their experience during 1968-72. For example in the United States real interest rates averaged 0.7 per cent during 1968-72 but fell to -1.8 per cent during 1973-75. Although real interest rates recovered partially by the end of 1975, they remained at negative levels until the second half of 1980. France and Germany had a broadly similar experience, expect in the case of Germany real interest rates became positive again relatively rapidly. In Japan and the United Kingdom the fall in the real interest rates was much sharper with rates as low as -10 per cent not being uncommon during much of the 1973-75 period.

A second episode of sharp changes in real interest rates occurred starting approximately in the middle of 1980, when real interest rates rose rapidly. The speed with which real interest rates rose was particularly pronounced in France and the United States. By 1981 real interest rates were positive in all five countries. Moreover, the increase in real rates of interest took place against a background of high rates of inflation and therefore nominal interest rates climbed to unprecedented levels. Accompanying these high interest rates was also an apparent increase in the "volatility" of interest rates as manifested by large month-to-month, even week-to-week fluctuations in the levels of interest rates.

It should be recalled that both episodes of large movements in real interest rates occurred at times of high rates of inflation. Moreover, both episodes also coincided with substantial increases in oil prices and large reductions in industrial production. In contrast to these similarities, the two episodes are distinguished by the speed at which the rate of inflation declined following large movements in real interest rates. After the 1973-75 episode, underlying inflationary pressures persisted, notwithstanding a gradual deceleration in prices, whereas following the 1979-81 episode inflation rate declined rapidly. It is reasonable to conjecture that movements in real interest rates would provide an understanding of the differences between the two episodes.

Both in 1973-74 and 1979-80 there were large increases in oil prices which took place against a background of weakening real economic activity and strengthening inflationary pressures in industrial countries. The magnitude of oil price increases and the scale of relative price adjustment that was called for led to a sudden fall in aggregate output. This was very much in the nature of an unexpected aggregate supply shock. In the first episode, the supply shock was followed, after a short lag, with a sharp fall in real interest rates, inducing an increase in current expenditures and a reduction in saving. The resulting increase in aggregate demand at a time when aggregate supply was falling, led to the intensification and entrenchment of the inflationary process. Only when the momentum of the initial aggregate supply shock declined sufficiently, did the expansionary effect of negative real interest rates started to contribute to economic recovery.

In contrast, following the second round of oil price increases, industrial countries adopted non-accomodative economic policies with the objective of bringing inflation under control. Interest rates rose and attained significantly positive levels in real terms. The increase in real interest rates discouraged current expenditures and stimulated savings, leading to a fall in aggregate demand in tandem with aggregate supply and thereby eliminating inflationary pressures. Thus, the increase in real interest rates after the second round of oil price increases was largely responsible for the speed at which inflation slowed down. 1/

The stylized facts suggest that variations in real interest rates lead primarily to changes in aggregate demand by inducing shifts in time paths of expenditures. Moreover, aggregate supply shocks result in relatively persistent movements in real output. Consequently, following

---

1/ There were, of course, a number of other related factors. The accomodative policies of 1973-75 period were probably unexpectedly lax and therefore contributed to the subsequent economic upturn. Similarly the policies adopted during the latter episode were probably unexpectedly tight, particularly in view of the policies of the earlier episode, and are likely to have contributed to the length and severity of the economic slowdown. I am grateful to D.F.I. Folkerts Landau who brought this point to my attention.

Chart 1.

UNITED STATES

REAL INTEREST RATE

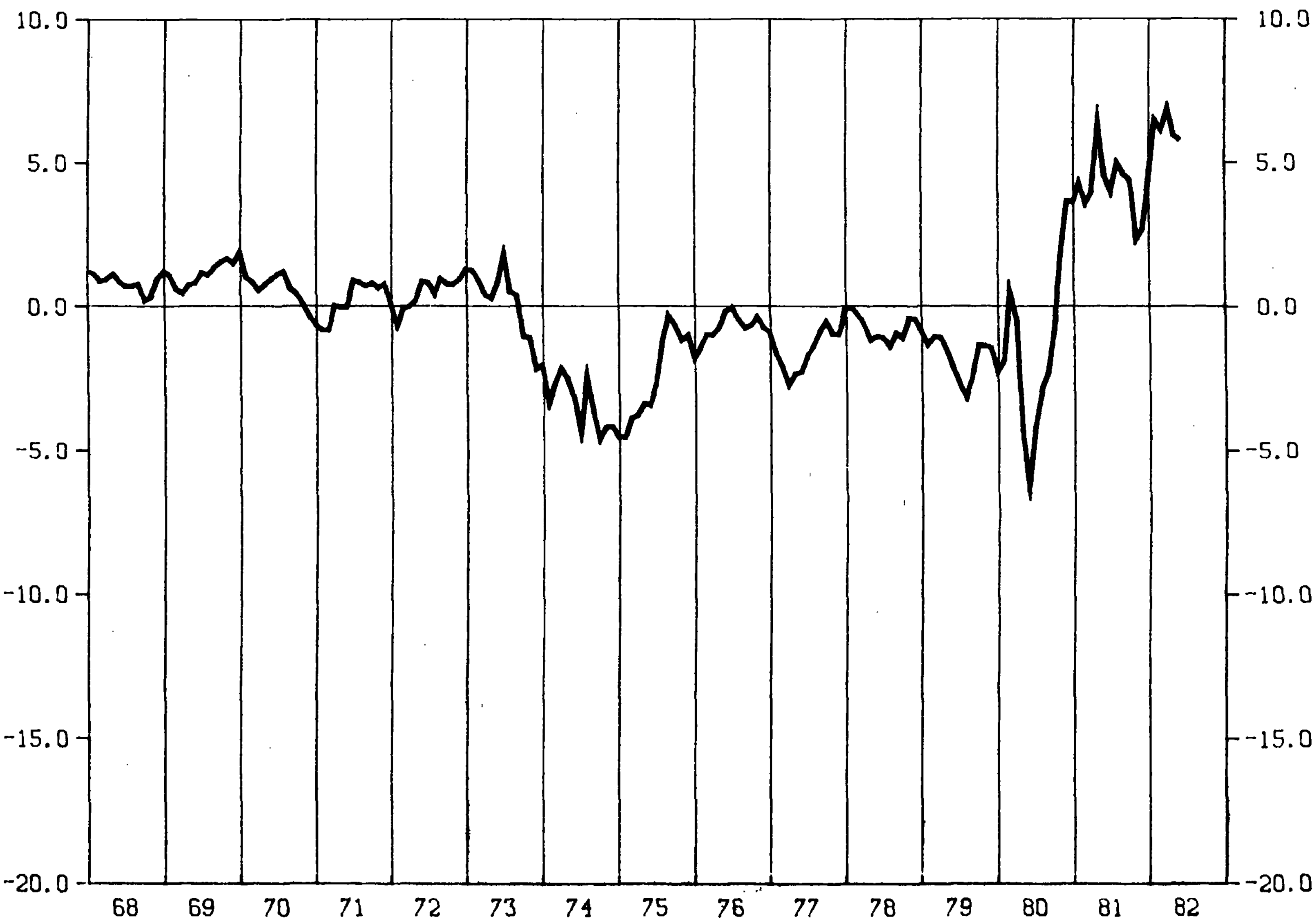


Chart 2.

FRANCE
REAL INTEREST RATE

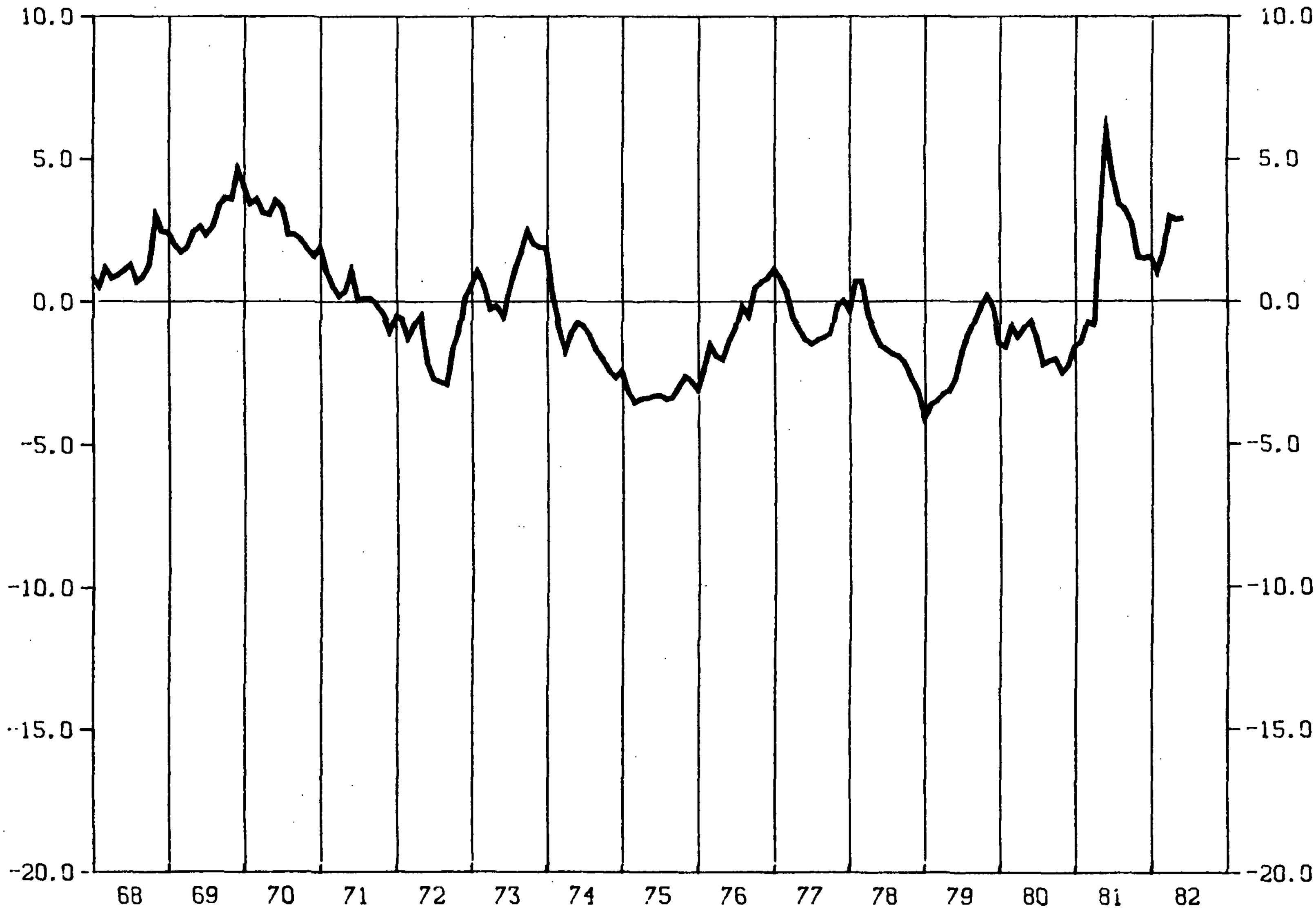


Chart 3.

GERMANY
REAL INTEREST RATE

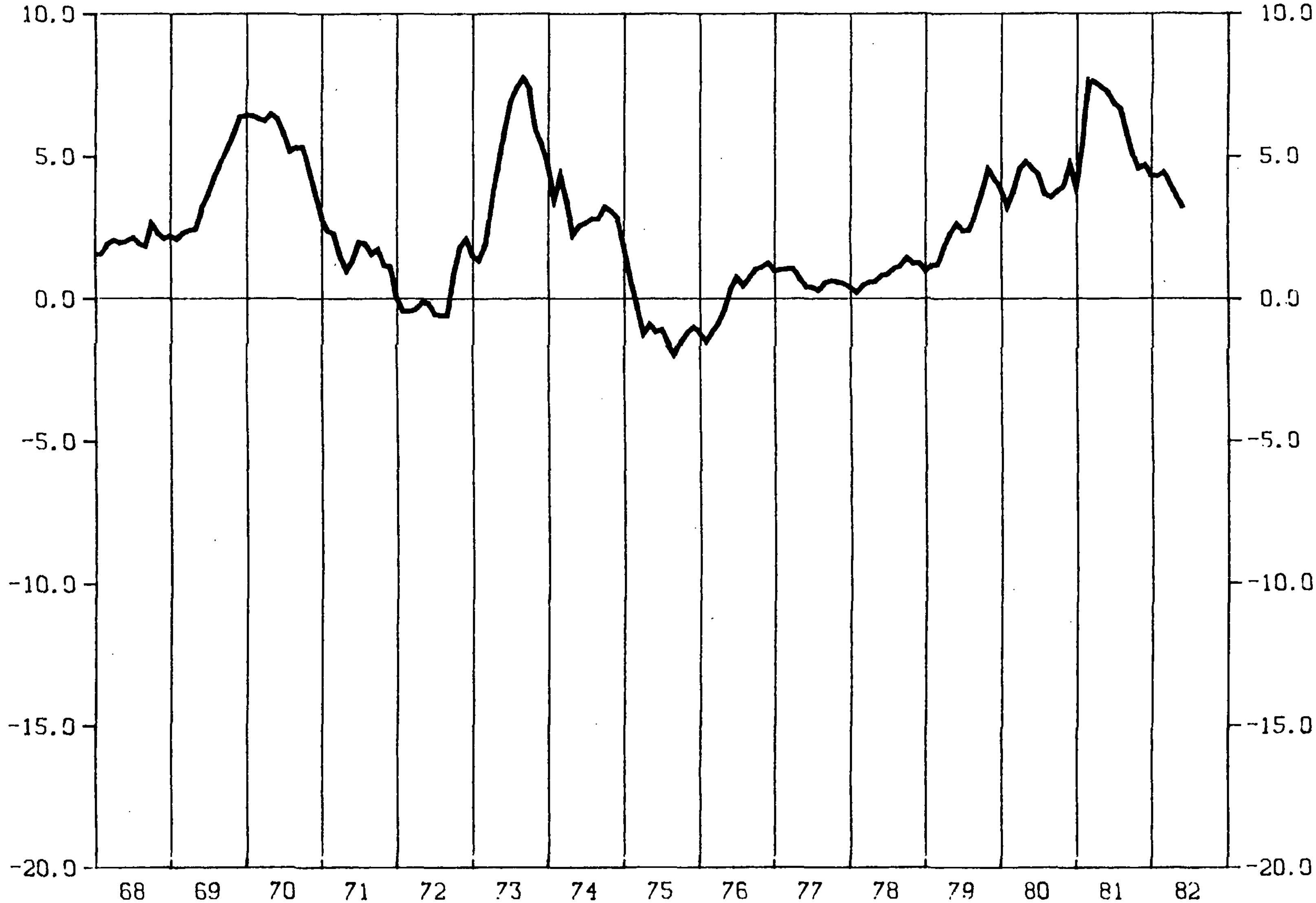


Chart 4.

JAPAN
REAL INTEREST RATE

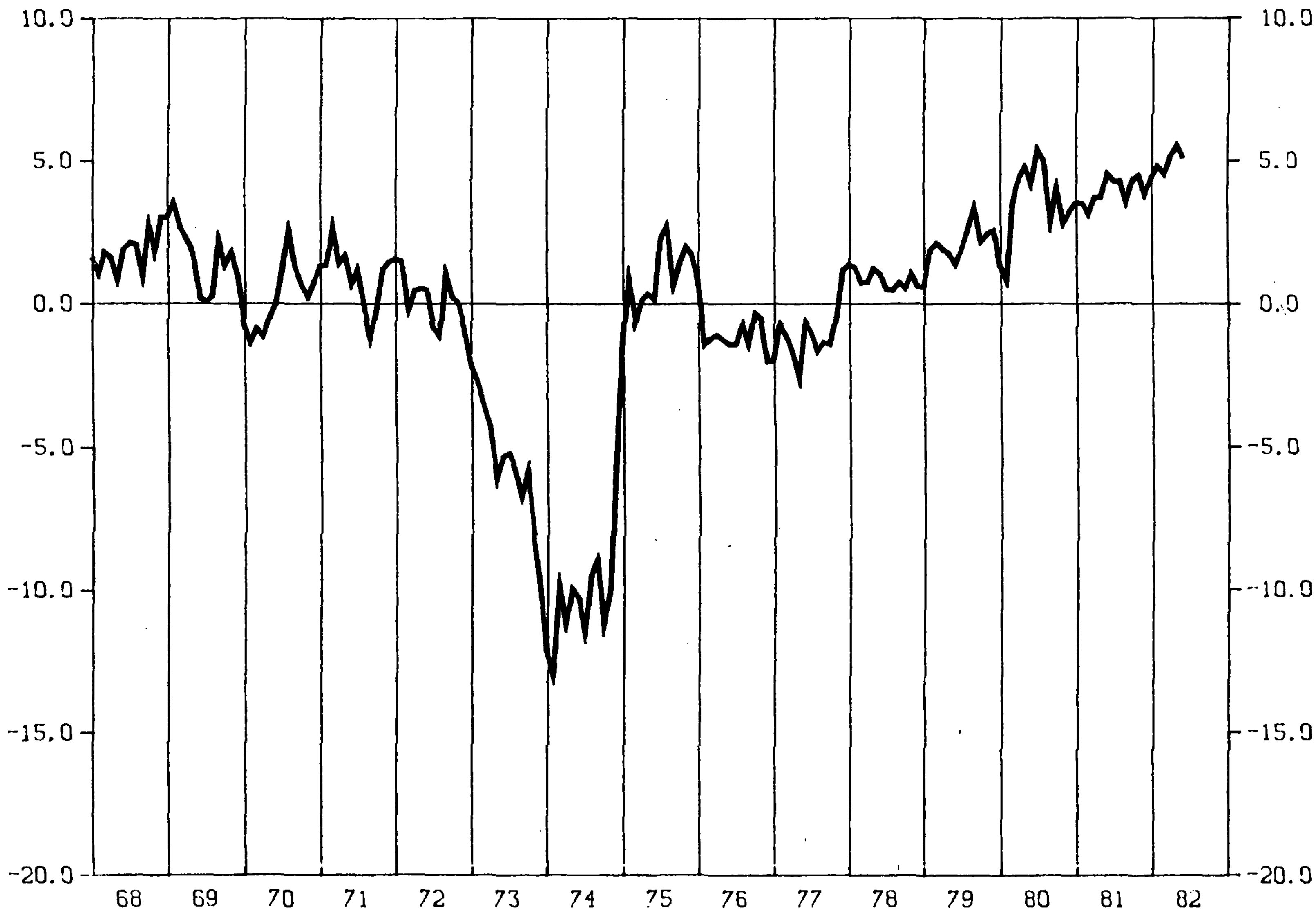


Chart 5.

UNITED KINGDOM
REAL INTEREST RATE

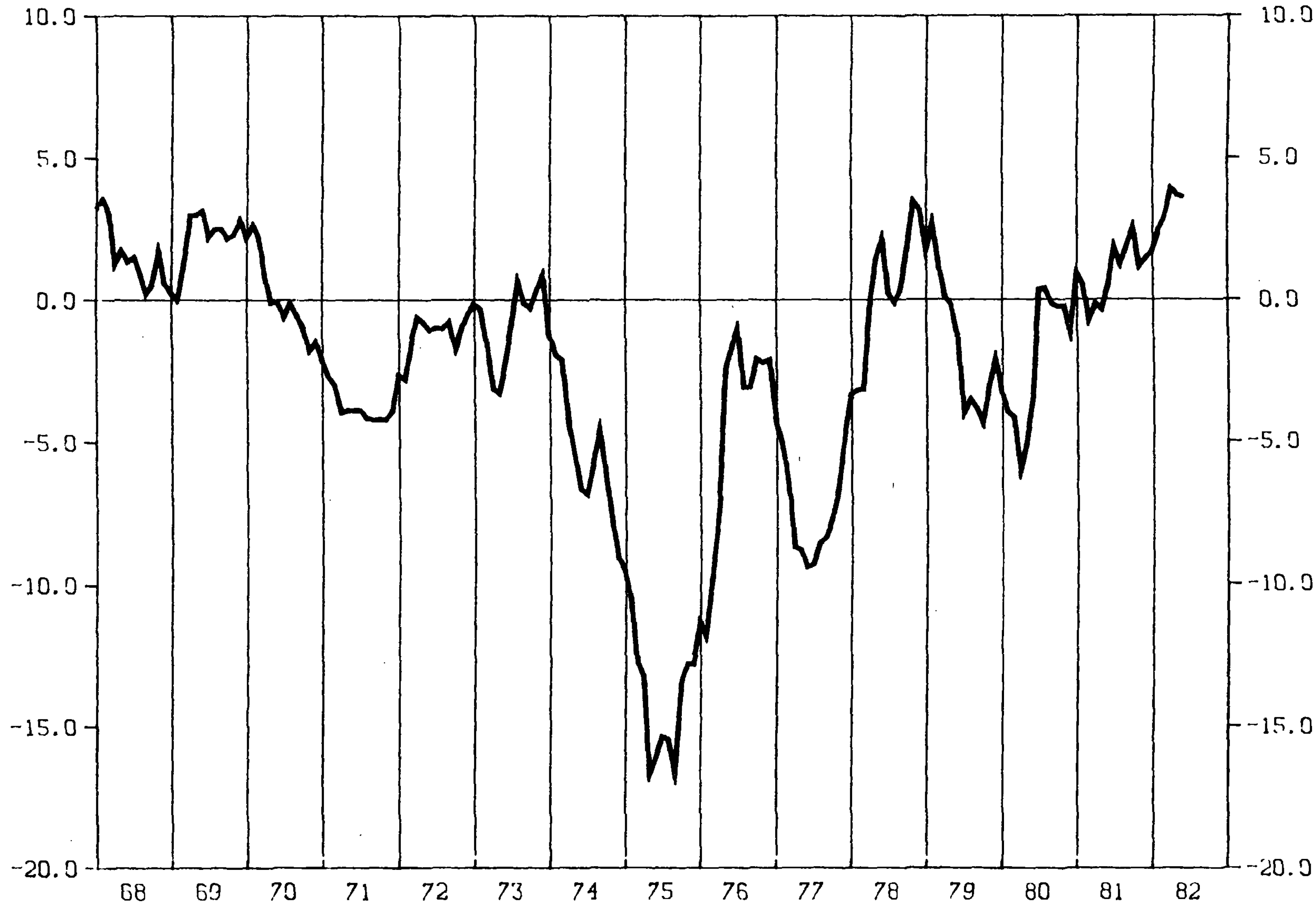
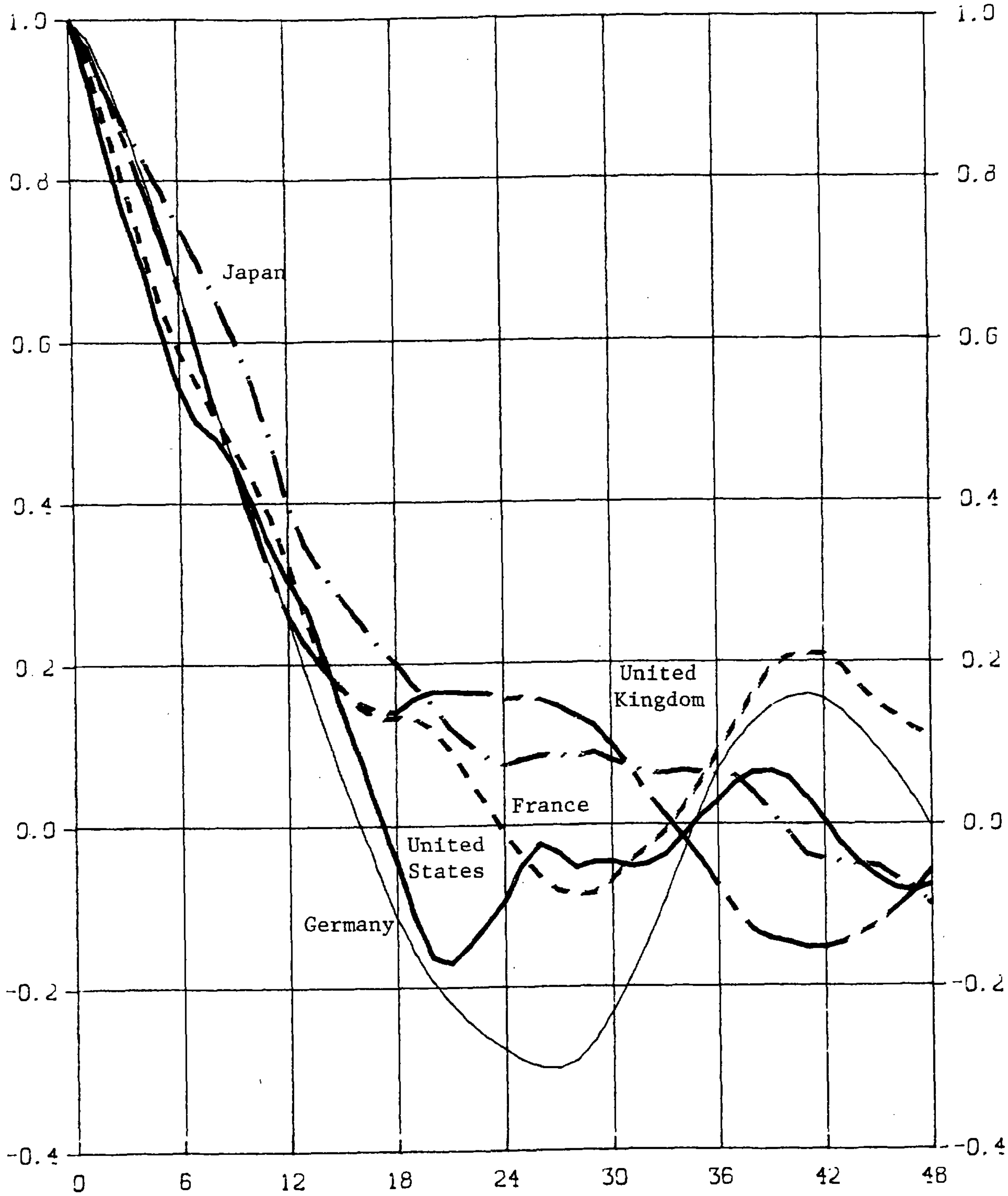


Chart 6. Serial Correlation Coefficients of Real Interest Rates



negative aggregate supply shocks, policies that temporarily reduce real interest rates are likely to be inflationary without having a significant positive impact on real output.

## 2. Correlation between the real interest rate and the expected rate of inflation

We now return to a topic that was discussed earlier in Section II, part 2, namely the test of whether real interest rates are correlated with expected rates of inflation. If market participants' subjective expectations of inflation were observed, the computation of correlation coefficients would have been a straightforward exercise. However, given the fact that "true" expectations are not observed a more roundabout procedure is necessary. Lack of observed time series on expectations implies that optimal forecasts of inflation which are based on estimated vector autoregressions would be representative of true expectations subject to an error of measurement. This error of measurement, in turn, will render the correlation coefficient that is computed on the basis of optimal forecasts of inflation, a biased estimate of the true correlation coefficient. Nevertheless, it is still possible to formulate a test in the form of upper and lower bounds under the assumption that errors of measurement are independent from the true expectations. Technical details of this test are presented in Appendix II. Because it is computationally more convenient to do so, we formulated the test in the form of bounds on the regresion coefficient of real interest rate on expected rate of inflation. rather than on the coefficient of correlation between these two variables. 1/

In Table 2, we present upper and lower bounds on the regression coefficient of real interest rate on expected rates of inflation, together with the estimated standard errors of these bounds. These results show very strongly that real interest rates are correlated with expected rates of inflation in all the countries, and the hypothesis that real interest rates are independent from expected rates of inflation is strongly rejected by the data. 2/

However, caution must be exercised before reaching conclusions about the structure of the economies or deriving policy prescriptions. It is well known that a property of Keynesian economic models is a contemporaneous correlation between the real rate of interest and expectations of inflation. Nevertheless, this property cannot be used to distinguish between alternative economic models. For example, it is possible to show that a class of rational expectations models in which monetary policy is

---

1/ The regression coefficient, as is well known, is proportional to the correlation coefficient.

2/ From the estimated bounds it is possible to form a range for the magnitude of the Fisher effect. In the United States, this range is 0.673 to 0.746. These values are in complete accord with those reported by Makin and Tanzi (1982), Table 2.

Table 2. Upper and Lower Bounds on Regression Coefficients of Real Interest Rate on Expected Rates of Inflation

(Standard errors in paranthesis)

| Country | Lower Bound | Upper Bound |
|---|---|---|
| United States | -0.327<br>(0.024) | -0.254<br>(0.021) |
| France | -0.255<br>(0.020) | -0.196<br>(0.016) |
| Germany | 0.158<br>(0.036) | 0.269<br>(0.033) |
| Japan | -0.721<br>(0.045) | -0.601<br>(0.034) |
| United Kingdom | -0.763<br>(0.039) | -0.629<br>(0.032) |

not a potent instrument of stabilization also permits a contemporaneous correlation between the real rate of interest and the expected rate of inflation.

More importantly, at least from a policy point of view, is whether nominal interest rates adjust fully to changes in expected rates of inflation, and, if they do, what is the duration of the adjustment process. 1/ In order to shed some light on these important questions we regressed nominal interest rates on lagged values of real interest rates and current and lagged values of expected rates of inflation with the objective of using the sum of distributed lag coefficients as an indicator of the degree of adjustment process. In Table 3, we present some summary statistics of these regressions, the details of which are omitted here. 2/ First of all it is obvious that the fit of regressions is very satisfactory with high correlation coefficients, the absence of serial correlation in residuals, and the expected sign and magnitude of the sum of distributed lag coefficients. In all the countries, the sum of distributed lag coefficients of both expected rates of inflation and real interest rates is very close to unity. Given the fact that the highest order lag that was included in the regressions is six, these results very strongly support the hypothesis that nominal interest rates adjust almost fully to changes in both the expected rates of inflation and real rates of interest, and that the adjustment process is completed in six months.

The hypothesis that the sum of the distributed lag coefficients on expected rates of inflation is equal to unity cannot be rejected at any conventional levels of significance for the United States, France, and Germany. In the case of Japan and the United Kingdom, the sum is significantly different from one at 5 percent level of significance, but this difference is unimportant from a policy perspective in view of the closeness of these sums to one. 3/ The sum of the distributed lag coefficients on lagged real rates of interest, on the other hand, are all significantly different from one at the 5 per cent level, though again the differences are rather small.

---

1/ The length of time it takes for nominal interest rates to adjust for changes in expected rates of inflation is important because if it takes a long time for the adjustment to be completed, then alternative policies will have semi-permanent effects even though nominal interest rates fully adjust eventually.

2/ They are, however, available from the author.

3/ Note that Japan and the United Kingdom are also the two countries where the contemporaneous impact of expected rates of inflation on real interest rates is largest as evidenced in Table 2.

Table 3. Summary Statistics of Regressions of Nominal Interest Rates on Real Interest Rates and Expected Rates of Inflation

| | Sum of Coefficients | | R- Square | D.W. Statistic |
|---|---|---|---|---|
| | $\pi$ | $\rho$ | | |
| United States | 1.00 | 0.95 | 0.95 | 1.97 |
| France | 0.99 | 0.94 | 0.96 | 2.00 |
| Germany | 0.99 | 0.93 | 0.98 | 1.98 |
| Japan | 0.96 | 0.94 | 0.98 | 1.99 |
| United Kingdom | 0.97 | 0.96 | 0.97 | 2.02 |

## 3. Sources of real interest rate variability

As discussed earlier, from a policy perspective, it is also important to know the extent to which variations in real interest rates are attributable to different sources, particularly to monetary factors, M1 and base money in our case. In order to be able to discuss this question, we must first define the sense in which variations in real interest rates can be attributed to various sources, and then proceed to compute these proportions.

Our starting point is equation (35) which defines the real interest rate in terms of the observable variables that were included in vector autoregressions, i.e., nominal interest rate and rates of change in money stock, base money consumer price index, and the index of industrial production. We repeat equation (35) here for convenience.

$$\rho_t = h(z)y_t = \sum_{j=0}^{m-1} h_j y_{t-j} \tag{35}$$

Substituting for $y_t$ from equation (26) we get:

$$\rho_t = h(z)A(z)^{-1}u_t = h(z)A(z)^{-1}Le_t \tag{36}$$

Using equation (36) and by virtue of the fact that the covariance matrix of $e_t$ is identity, the variance of $\rho_t$ can be decomposed into parts each of which is attributable to one of the elements of $e_t$, which are standardized innovations in the information variables. 1/ These innovations are the unexpected changes in the information variables, standardized so that they are uncorrelated with each other, and that they have unit variances. Because standardized innovations are uncorrelated with each other, the variance of real interest rate can be written as the sum of components each of which is attributable to one of the innovations. This decomposition takes the following form.

$$\sigma_\rho^2 = \sum_{j=1}^{5} v_j^2$$

*where* $v_j^2$ *is the contribution of normalized innovation in variable j to* the variance of real interest rate. Results of this decomposition are presented in Table 4.

---

1/ Actual computations are done by first writing the inverse matrix in terms of determinant and the adjoint. This results in an expression where the real interest rate is the sum of distributed lags of the elements of $e_t$, where the distributed lags are generated by rational functions. Each component of the variance is then computed by contour integration, using the method of residues.

Table 4. Sources of Real Interest Fluctuations

(In per cent)

| | Money Stock | Inflation Rate | Interest Rate | Industrial Production | Base Money |
|---|---|---|---|---|---|
| United States | 6.7 | 36.8 | 38.3 | 10.5 | 7.7 |
| France | 1.6 | 15.5 | 72.6 | 3.2 | 7.1 |
| Germany | 6.8 | 9.5 | 69.3 | 11.8 | 2.6 |
| Japan | 31.1 | 26.5 | 22.2 | 9.2 | 10.9 |
| United Kingdom | 9.8 | 66.1 | 6.0 | 6.6 | 11.6 |

What is the interpretation of these results? Clearly, in the United States, France, Germany and the United Kingdom innovations in money supply and base money account for only a small proportion of the variation in real interest rates. The percentage of variation in real interest rates, due to unexpected movements in monetary variables ranges from 8.7 per cent for France to 21.4 per cent in the United Kingdom. It must be emphasized that these proportions, though they are small, could, neverthless, constitute statistically significant contributions to the variation of real interest rates. However, the fact that the contribution of monetary innovations is small indicates that volatility of real interest rates can not be attributed to monetary policies that result in high variability of monetary variables. In Japan, on the other hand, monetary factors account for 42 per cent of the variation in real interest rates. Moreover, unexpected change in the money stock is the single most important variable, accounting for 31.1 per cent of the variation in real interest rates. In view of our earlier discussion, the results suggest very strongly that variations in expected rates of inflation due to monetary factors have only small and transitory effects on real interest rates except in Japan.

## V. Concluding Remarks

One major conclusion that emerges from the findings of this paper is that real interest rates are not constant over time. In all five countries examined, real interest rates exhibit substantial variation and serial correlation. Presence of both variability and serial correlation are inadmissible under the hypothesis of constant real interest rates. The fact that real interest rates are not constant over time is further verified by the presence of correlation between real interest rates and expectations of inflation. If ex ante real interest rates were constant, the correlations would have been zero.

A second conclusion is that policies that change expected rates of inflation in a systematic manner leave real interest rates unaltered, with nominal interest rates adjusting fully to the new time path of expected rates of inflation. By extension, it is highly unlikely that expansionary monetary policies will succeed in lowering nominal interest rates, provided that the implications of such policies are well understood by economic agents and therefore result in an upward adjustment of their expectations of inflation.

A third conclusion is that, for all the countries studied except Japan, monetary factors account for only a small proportion of the variation in real interest rates. This finding casts doubt on some monetarist claims that, at least for the United States, erratic behavior of monetary policy is the major source of real interest rate variability. By extension, it seems unlikely that increased monetary discipline could be effective in reducing the volatility of real interest rates. In the United States, for example, halving the variance of innovations in M1 will reduce the variance of real interest rates from 7.1 to 6.9, whereas halving the variance of innovations in both M1 and base money will lower it to 6.6.

Finally, consideration should be given to investigate the determination of real interest rates within a richer specification, to account for the impact of other important variables, such as wages, the exchange rate, fiscal deficit, etc. In estimating these larger systems of vector autoregresions, parameter space must be restricted, perhaps using Bayesian methods, in order to preserve the degrees of freedom. Such an approach seems to be a promising area of future research.

# Appendix I. Some Notes on Data and Estimated Autoregressions

## 1. The data

In order to implement the methodology which is outlined in Section III, we estimated vector autoregressions for five countries: the United States, France, Germany, Japan, and the United Kingdom. The variables included in the vector autoregressions are the annual rate of change in the money stock, the annual rate of change in the monetary base, the annual rate of change in the index of industrial production, the annual rate of change in the consumer price index, and a short term interest rate. The interest rate variable for the United States and the United Kingdom is the three-month Treasury Bill rate, for France and Germany it is the three-month interbank lending rate and for Japan it is the two-month lending rate. The source of data is the International Financial Statistics, except for the interest rates which is the Board of Governors of the United States Federal Reserve System. The period of estimation is 1968 through mid-1982.

The data is not seasonally adjusted despite the fact that some of the variables do display some seasonal variation. It is assumed here that seasonal variation is part of the information set used by economic agents. Rational behavior on the part of such agents dictates that they utilize this information in forming their forecasts and such behavior should manifest itself in the observed behavior of the variables. As demonstrated by Saracoglu and Sargent (1978) seasonal behavior on the part of the information variables results in a complicated pattern for the response variables, whereby the former is mapped into the latter with their phase shifted and amplitude scaled. It is true that the resulting response variables also display some seasonal variation. However this variation is substantially different from that of the information variables.

## 2. Estimated vector autoregressions

In estimating any autoregressive process, a decision has to be made about the order of autoregression. There are no straightforward ways of accomplishing this. Because the statistical foundations of vector autoregressions are based on asymptotic distribution theory, a crucial considerations is the degrees of freedom. When the order of autoregression is such that the number of estimated parameters is the same order of magnitude as the remaining degrees of freedom, the applicability of asymptotic theory becomes doubtful. 1/ On the other hand, the order of autoregression should be high enough to capture the dynamic behavior of variables. Our experiments with different orders suggested that six lags accounted sufficiently

1/ See Sims (1980) for a detailed discussion of these issues.

for the serial correlations while still leaving enough degrees of freedom to render attributions to asymptotic theory meaningful. 1/

There is a problem in reporting the results of estimated vector autoregressions. For each country the model to be estimated has 170 free parameters including 150 autoregressive coefficients, 5 constants, and 15 parameters for the residual covariance matrix. It seems unreasonable to report all the estimated parameters for all the countries. The autoregressive coefficients, themselves, are relatively uninformative. They tend to oscillate and many of them are not significantly different from zero which makes them difficult to interpret. Moreover, the estimated coefficients have little relevance to Fisher's hypothesis. In view of these considerations reported in Table 1, standard error of estimate and the first order serial correlation coefficient of residuals for each equation together with the F-statistics to test the goodness of fit for each country. These statistics indicate that the estimated vector autoregressions fit the data closely in every country. First order serial correlation statistics of residuals show that they are spectrally white, as required by the theory. Moreover, the estimated residual covariance matrices are approximately diagonal indicating that innovations in different variables are contemporaneously uncorrelated. In all of the countries residuals of inflation and interest rates have substantially smaller variances than the residuals of other variables suggesting that large unexpected fluctuations in the rate of inflation and the rate of interest are much less likely than in money stock, industrial production, or base money.

The estimated systems of vector autoregressions were also tested for stability. Stability is a necessary condition for the transformation of the autoregressive representation into the moving average representation. To test for stability, we computed the determinental polynominals of the autoregressive systems and calculated their roots. These roots were uniformly outside the unit circle indicating that the estimated systems were stable. As for stationarity, the presence of complex roots which are close in magnitude and with small imaginary parts, suggests the possibility of mild nonstationarity in all of the countries. No attempt was made, however, to either investigate this nonstationary behavior or to account for it in the estimation. 2/ Following these preliminary tests, the estimated vector autoregressions were used to compute vectors $h_j$ of equation (35) in order to generate time series on ex ante real interest rates for all five countries.

1/ No attempt was made, however, to find the "optimal" order of autoregressions for each variable of every country. This would require a systematic and costly search without effecting the results substantially.

2/ It is possible to estimate vector autoregressions with time varying coefficients. See, for example, Sims (1982).

## Appendix II. Derivation of Bounds of the Regression Coefficient of Real Interest Rate on the Expected Rate of Inflation

In this appendix tests are formulated in the form of upper and lower bounds around the regression coefficient of real interest rates on expected rates of inflation.

Let $x_t^*$ be the public's true (and unobserved) expectations of inflation based on the complete information set $\Omega_t$. Let $\pi_t$ be a proxy for $x_t^*$ based on the information set $H_t$, which is a subset of $\Omega_t$. Then $x_t^*$ can be written as:

$$x_t^* = \pi_t + n_t \qquad \text{(A1)}$$

where $n_t$ is orthogonal to $H_t$ and therefore to $\pi_t$. Furthermore, we have the relationship:

$$r_t = \rho_t + x_t^* \qquad \text{(A2)}$$

In equations (A1) and (A2) only $r_t$ and $\pi_t$ are observed. From equation (A2) the covariance between $r_t$ and $x_t^*$ is given by:

$$\sigma_{rx^*} = \sigma_{\rho x^*} + \sigma_{x^*}^2 \qquad \text{(A3)}$$

whereas from equation (A1) we have:

$$\sigma_{rx^*} = \sigma_{r\pi} + \sigma_{rn} \qquad \text{(A4a)}$$

$$\sigma_{x^*}^2 = \sigma_{\pi}^2 + \sigma_{n}^2 \qquad \text{(A4b)}$$

If nominal interest rate is included in $H_t$ then $\sigma_{rn} = 0$, with the result that:

$$\sigma_{\rho x^*} = \sigma_{r\pi} - \sigma_{x^*}^2 \qquad \text{(A5)}$$

From equation (A4b) and the fact that actual rate of inflation, $x_{t+1}$, is equal to the sum of expected rate of inflation, $x_t^*$, and the forecast error $u_{t+1}$, we obtain the following inequalities:

$$\sigma_{\pi}^2 \leq \sigma_{x^*}^2 \leq \sigma_{x}^2 \qquad \text{(A6)}$$

Equations (A5) and (A6) jointly imply the following inequalities:

$$(\sigma_{r\pi} - \sigma_x^2)/\sigma_x^2 \leq \sigma_{\rho x}^*/\ \sigma_x^{2*} \leq (\sigma_{r\pi} - \sigma_\pi^2)/\sigma_x^2 \qquad \text{(A7a)}$$

$$(\sigma_{r\pi} - \sigma_x^2)/\sigma_\pi^2 \leq \sigma_{\rho x*}/\sigma_x^{2*} \leq (\sigma_{r\pi} - \sigma_\pi{}^2)/\sigma_\pi{}^2 \qquad \text{(A7b)}$$

Moreover, if $\sigma_{\rho x}^* > 0$ then:

$$\sigma_{\rho x*}/\sigma_\pi^2 > \sigma_{\rho x*}\sigma_{x*}^2 > \sigma_{\rho x*}\sigma_x^2 \qquad \text{(A8a)}$$

otherwise:

$$\sigma_{\rho x*}/\sigma_\pi^2 \leq \sigma_{\rho x*}/\sigma_{x*}^2 \leq \sigma_{\rho x*}/\sigma_x^2 \qquad \text{(A8b)}$$

Note that the regression coefficient of real interest rate on expected rate of inflation is given by $\sigma_{\rho x}^*/\sigma_{x*}^2$. By virtue of equations (A7a), (A7b), (A8a), and (A8b), this coefficient is bounded both from above and from below by observable magnitudes.

## Bibliography

Barro, R.J., "Rational Expectations and the Role of Monetary Policy," Journal of Monetary Economics, (April 1976).

Begg, D.K.H., "Rational Expectations of Inflation and the Behaviour of Asset Returns in a Stochastic Macro Model," Ph.D. dissertation, (Massachusetts Institute of Technology, 1977).

Carlson, J.A., "Short-term Interest Rates as Predictors of Inflation: Comment," American Economic Review, (June 1977).

Dwyer, G.P., "Are Expectations of Inflation Rational? Or Is Variation of the Expected Real Interest Rate Unpredictable," Journal of Monetary Economics, (1981).

Eckstein O., and M.S. Feldstein "The Fundamental Determinants of Interest Rates," Review of Economics and Statistics, (1973).

Fama, E.F., "Short Term Interest Rates as Predictors of Inflation," American Economic Review, (June 1975).

____, "Inflation Uncertaintly and Expected Returns on Treasury Bills," Journal of Political Economy, (June 1976).

____, "Interest Rates and Inflation: The Message it Entails," American Economic Review, (June 1977).

Fisher, I., The Theory of Interest (New York, Macmillan, 1930).

Friedman, M., "The Role of Monetary Policy," American Economic Review, (March 1968).

Gordon, R.J., "The Recent Acceleration of Inflation and Its Lessons for the Future," Brookings Papers on Economic Activity, (1970).

Joines, D., "Short-Term Interest Rates as Predictors of Inflation: Comment," American Economic Review, (June 1977).

Keynes, J.M., The General Theory of Employmennt, Interest, and Money Harbinger ed. (New York, Harcourt, Brace and World Inc., 1964).

Litterman, R.B., "Techniques of Forecasting Using Vector Autoregressions," mimeo (1979).

Makin, J.H., and V. Tanzi, "The Level and Volatility of Interest Rates in the United States: The Roles of Expected Inflation, Real Rates, and Taxes," IMF DM/82/81.

Malinvaud, E., "Econometrics Faced with the Needs of Macroeconomic Policy," Econometrica (November 1981).

Mishkin, F.S. "The Real Interest Rate: An Empirical Investigation," Carnegie-Rochester Conference Series on Public Policy, (Autumn, 1981).

Modigliani, F., and R.J. Shiller, "Inflation, Rational Expectations of the Term Structure of Interest Rates," Economica (February 1973).

Mundell, R.A., "Inflation and Real Interest," Journal of Political Economy, (June 1963).

Nelson, C.R., and G.W. Schwert, "Short-term Interest Rates as Predictors of Inflation: On Testing the Hypothesis that the Real Rate of Interest is Constant," American Economic Review, (June 1977).

Roll, R., "Interest Rates on Monetary Assets and Commodity Price Index Changes," Journal of Finance, (May 1972).

Saracoglu, R., "Two Essays on Interest Rates and Inflation," Ph.D. dissertation, (University of Minnesota, 1980).

Saracoglu, R., and T.J. Sargent, "Seasonality and Portfolio Balance Under Rational Expectations," Journal of Monetary Economics, (1978).

Sargent, T.J., "Interest Rates and Prices in the Long Run," Journal of Money, Credit and Banking, (February 1973).

_______, "The Fundamental Determinants of the Interest Rate: A Comment," Review of Economics and Statistics, (1973).

_______, "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment," Brookings Papers on Economic Activity, (1973).

_______, "Interest Rates and Expectations of Inflation," Occasional Paper No. 3, NBER Vol. 3, (1976).

_______, "A Note on the Maximum Likelihood Estimation of the Rational Expectations Model of the Term Structure," Journal of Monetary Economics, (1979).

_______, Macroeconomic Theory, Academic Press, (New York, 1979).

Shiller, R.J., "Rational Expectations and the Structure of Interest Rates," Ph.D. dissertation, (Massachusetts Institute of Technology, 1972).

_______, "Can the Fed Control Real Interest Rates?" Rational Expectations and Economic Policy, S. Fischer, editor (Chicago, The University of Chicago Press, 1980).

Sims, C.A., "Macroeconomics and Reality," Econometrica, (January 1980).

____, "Policy Analysis with Econometric Models," Brookings Papers on Economic Activity, Vol. 1 (1982).

Tanzi, V., "Inflationary Expectation, Economic Activity, Taxes, and Interest Rates," American Economic Review, (March 1980).

Whittle, P., Prediction and Regulation by Linear Least Squares Methods (Princeton, D. Van Nostrum Publications, 1963).