

IMF Working Paper

© 1998 International Monetary Fund

This is a *Working Paper* and the author(s) would welcome any comments on the present text. Citations should refer to a *Working Paper of the International Monetary Fund*. The views expressed are those of the author(s) and do not necessarily represent those of the Fund.

WP/98/93

INTERNATIONAL MONETARY FUND

Research Department

Bank Lending and Interest Rate Changes in a Dynamic Matching Model

Prepared by Giovanni Dell’Ariccia and Pietro Garibaldi¹

Authorized for distribution by Donald Mathieson and Eduardo Borensztein

June 1998

Abstract

This paper presents theory and evidence on the dynamic relationship between aggregate bank lending and interest rate changes. Theoretically, it proposes and solves a stochastic matching model where credit expansion and contraction are time consuming. It shows that the response of bank lending to changes in money market rates is likely to be asymmetric and depends crucially on two structural parameters: the speed at which new loans become available, and the speed at which banks recall existing loans. Empirically, it provides evidence that bank lending in Mexico and the United States responds asymmetrically to positive and negative shocks in money market rates.

JEL Classification Numbers: E44, G21

Keywords: Bank Lending, Monetary Transmission Mechanism, Matching Models

Authors’ E-Mail Address: gdellariccia@imf.org, pgaribaldi@imf.org

¹We benefited from comments and suggestions of Fabio Bagliano, Eduardo Borensztein, Luis Cubeddu, Ilan Goldfajn, Vincent Hogan, Sunil Sharma, and seminar participants at the IMF. We are particularly indebted to Robert Marquez, Michael Mussa, and Miguel Savastano. Address for correspondence: Giovanni Dell’Ariccia (or Pietro Garibaldi), International Monetary Fund, 700 19th Street, N.W., Washington D.C.

	Page
Summary	4
I. Introduction	5
II. Description of the Model	7
III. The Model: Steady State	10
IV. Stochastic Shock to the Money Market	15
A. The Model	15
B. The Aggregate Dynamics: Simulations and Discussions	17
V. Empirical Evidence	20
A. Methodology	20
B. Results and Robustness Checks	22
VI. Policy Relevance and Alternative Interpretations	23
VII. Concluding Remarks	25
Tables:	
1. Baseline Parameter Values	26
2. U.S. Federal Fund Rate	27
3. Mexico Money-Market Rate	28
4. Mexico: Aggregate Lending	29
5. US: Aggregate Lending	30
6. Tests of Asymmetry in Aggregate Lending	31
7. Variable Deletion Tests	31
8. Long Run Coefficients	32
9. US: Restricted Regression	32
10. US: Robustness Checks	33
11. Mexico: Robustness Checks	34
Figures:	
1. Banks' Capital Allocation over a easy-tight-easy cycle; $\sigma > \alpha$	35
2. Average Interest Rate over an easy-tight-easy cycle; $\sigma > \alpha$	35
3. Banks' Capital Allocation over a easy-tight-easy cycle; $\sigma = \alpha$	36
4. Average Interest Rate over an easy-tight-easy cycle; $\sigma = \alpha$	36

Appendices:

I.	Sufficient Conditions for the Existence of an Equilibrium	37
II.	Monotonicity of Surplus Functions	40
III.	Comparative Static Results	42
IV.	On the Timing of the Bankruptcy Procedure	44
	References	45

SUMMARY

This paper proposes theory and gives evidence on the dynamic relationship between aggregate bank lending and changes in money market rates. Theoretically, it proposes a matching model of the market for loans and argues that lending expansion and contraction are time-consuming activities. Investment opportunities might be difficult to find, and screening potential applicants might impose a time constraint on the banking system's ability to issue new loans. Similarly, recalling nonperforming loans often requires an uncertain and time-consuming legal procedure that limits the banks' ability to rapidly recover their capital. As a result, the speed at which lending opportunities become available and banks can recall existing loans are important determinants of the dynamic response of bank lending to changes in money market rates. In particular, when banks can rapidly recall nonperforming loans but experience technological delays in expanding credit, the response of bank lending to interest rate changes is likely to be asymmetric: positive changes result in the immediate contraction of bank loans, whereas negative changes produce only a gradual expansion of bank lending. More generally, the speed of credit contraction and expansion are determined by different structural and institutional factors, and econometric procedures that impose aggregate lending to respond symmetrically to interest rate changes are likely to be overrestrictive.

Empirically, the paper investigates whether the response of bank lending to interest rate changes is indeed asymmetric, as most parameterizations of the model would suggest. Empirical evidence for Mexico and the United States confirms the theoretical intuition and suggests that bank lending reacts asymmetrically to exogenous money market perturbations. In particular, banks react more rapidly to market interest rate increases in both countries.

I. INTRODUCTION

In a world with asymmetric information and other market imperfections, financial intermediaries provide credit to otherwise liquidity constrained agents. If lending without screening and monitoring entails large deadweight losses, and if market financing is prevented by free-rider problems, banks emerge as the only source of external financing for potentially productive agents (Diamond, 1984).¹ As a result, the relationship between monetary perturbations and aggregate economic activity is necessarily linked to bank lending behavior.² However, the response of bank lending to positive and negative interest rate changes may be inherently different, and potentially asymmetric. Even though several papers have studied the asymmetric effects of monetary policy on real economic activity, little attention has been paid to the asymmetric response of bank lending to interest rate changes.³

This paper has two aims. First, it proposes and solves a dynamic matching model, where credit expansion and contraction are time consuming, and shows that bank lending is likely to respond asymmetrically to interest rate changes.⁴ Second, it provides empirical evidence that bank lending in Mexico and the US responds asymmetrically to positive and negative innovations in money market rates.

The paper argues that lending expansion and contraction are time consuming activities, whereas buying and selling money market funds takes place without time delays. In reality, there are several reasons why lending expansion may be a time consuming process: investment opportunities might be difficult to find, or screening potential applicants might impose a time constraint on the banking system's ability to issue new loans. In the case of existing bank-client relationships, these problems are potentially less severe. However, financial institutions still need to evaluate the profitability of expanding an existing loan. Similarly, recovering non-performing loans may require a time consuming legal procedure.⁵ As a result, the speed at which lending opportunities become available, and the speed at which banks can recall existing loans are important determinants of the *dynamic response of bank lending to shocks in the money market*. In particular, when banks can rapidly recall non-performing loans, but experience technological delays in expanding credit, the response of bank lending to interest rate changes is likely to be asymmetric: positive changes result in the immediate contraction of bank loans, whereas negative changes produce only a gradual expansion

¹In the rest of the paper we use the term financial intermediaries and banks interchangeably.

²See Bernanke and Gertler (1995), Bernanke and Blinder (1988, 1992), Hubbard (1994), and Kashyap and Stein (1993).

³See section VI for a brief review of the existing literature.

⁴The model follows the most recent developments of the matching literature (Burdett and Wright (1998), and Mortensen and Pissarides, 1998).

⁵In reality, a significant component of aggregate bank lending is represented by line of credits, which banks can close without time delays. However, recalling the portion of funds actually withdrawn by clients requires a time consuming procedures.

of bank lending.

A large literature in theoretical macroeconomics has emphasized the role of search frictions in the labor market, and the existence of a matching problem between vacant jobs and unemployed workers. Similar frictions are also relevant in intermediated capital markets, where banks are the main source of productive capital. As vacancies and workers search for each other in a world with imperfect information, so do bank's capital and idle projects. Theoretically, we model the market for lending as a matching environment, where banks and entrepreneurs search for each other with a view toward establishing profitable relationships. A large microeconomic literature has shown that asymmetric information may lead to equilibrium credit rationing in the banking system.⁶ In this paper, even though we do not directly deal with informational asymmetries in bank-client relationships, we do model an aggregate form of credit-rationing. Throughout the analysis we assume that there is a positive probability that bank funds and idle projects *do not succeed* in finding each other in a given period. In other words, we assume that new loan contracts can profitably take place only after a bank and an entrepreneur have been randomly matched. This over-simplification, while extreme from the perspective of microeconomic theory, is meant to capture in an aggregate model the time consuming process of credit formation.

Banks are endowed with a given quantity of capital to be invested in two assets: money market funds and entrepreneurial projects. Technologically, the two assets differ in various respects. First, investments in the money market are risk free, whereas loans have an idiosyncratic probability of default, and are heterogeneous in terms of risk and return. Second, the technology to issue and recall loans is time consuming, while investing in the money market is not. From the banks' perspective, the probability of forming a credit relationship, and the probability of recovering non-performing loans are two exogenous and independent stochastic processes⁷. Conversely, the timing for investing and disinvesting in the money market is deterministic and immediate. Hence, banks' evaluation of entrepreneurial projects will reflect not only the immediate return generated by the associated loan, but also their value as an asset that might be difficult to replace.⁸

We show that banks monotonically rank individual projects, and optimally choose a reservation project below which buying money market funds is strictly preferred to project financing. Since the value of the marginal project depends on the return on the banks' alternative asset, in equilibrium, any shock to the money market (securities) rates affects the optimal allocation of bank capital. The paper shows that

⁶See in particular Stiglitz and Weiss (1981). Bhattacharya and Thakor (1993) provide an extensive survey.

⁷Within the related labor market literature, Garibaldi (1998) proposes a model in which firing is stochastic and time consuming.

⁸In this sense this paper relates to Greenbaum, Kanatas and Venezia (1989).

the dynamic response of bank lending to interest rate shocks depends crucially on two structural parameters: the speed at which lending opportunities become available, and the speed at which banks can recall existing loans. From this perspective, econometric estimates that force lending to respond symmetrically to interest rate changes are likely to impose undue restrictions.

Empirically, we investigate whether the response of bank lending to interest rate changes is indeed asymmetric, as most of the parameterization of the model would suggest. Empirical evidence for the US and Mexico confirms our intuition: bank lending reacts asymmetrically to exogenous money market perturbations. In particular, we find that banks react more rapidly to market interest rate increases in both countries.⁹

The paper proceeds as follows. Section II introduces concepts and notations, while section III presents and solves the steady-state model, where returns to the money market are fixed and time invariant. Section IV solves a stochastic dynamic version of the model with shocks to market returns, provides numerical simulations, and briefly highlights the model's empirical implications. Section V develops the empirical analysis and provides evidence of asymmetry in the response of aggregate lending to money market shocks in Mexico and in the United States. Section VI discusses policy implications and alternative theoretical interpretations. Section VII briefly summarizes and concludes the paper.

II. DESCRIPTION OF THE MODEL

We consider an economy populated by a fixed number of risk neutral banks and risk neutral entrepreneurs. Entrepreneurs are endowed with projects of different qualities and seek project financing. Banks are endowed with liquid funds and seek investment opportunities. Entrepreneurs have no private source of funds, and have no access to market financing, so that bank loans are their only source of external capital. Banks, however, may invest their capital in two alternative assets: money market bonds and project-loans. For simplicity, we assume that each entrepreneur is endowed with an indivisible project requiring an initial investment of \$1 that is productive only when it is matched to a unit of bank capital. Since our focus is on loans, rather than on banks, we assume that each bank has a single unit of funding capital. As a result, we abstract from issues related to market structure in the banking system.

Throughout the analysis, the aggregate capital available to the banking sector, and the aggregate number of entrepreneurs/projects are constant and time invariant. However, we assume that there are n different types of projects, and k projects of each

⁹These results are partly related to the literature that analyzes empirically the relationship between bank lending rates and money market rates. See Hannan and Berger (1991), Neumark and Sharpe (1992) and Scholnick (1996).

type, so that the aggregate number of projects is nk . For analytical convenience, we let nk be also the fixed number of banks or, alternatively, the stock of capital in the banking system.

Each bank can invest its indivisible unit of capital in two different assets: money market funds or project loans. We let the return on the money-market investment be risk-free, and we let r_d indicate its instantaneous return. Conversely, a project of type i , with $i \in (1, 2, \dots, n)$ is characterized by a pair (y_i, λ_i) , where y_i is a time-invariant return, and λ_i is an instantaneous probability of destruction, a Poisson process that measures the project's idiosyncratic risk. Since most models predict that riskier projects must have higher returns, we assume that the elements of the pair (y_i, λ_i) are positively correlated, and strictly increasing in i . Furthermore, in order to obtain a monotonic ranking of the projects, and reasonable aggregate results, we need to assume that $\frac{y_i}{\lambda_i} > \frac{y_j}{\lambda_j}$, $\forall i > j$. Thus, dividends grow faster than destruction rates, and the index i is a proxy of a project's quality. Finally, each project can be in two different states, depending on whether or not it is matched to a bank. A financed project is active and it produces an idiosyncratic dividend y_i , while an unfinanced project is idle and does not yield any dividend.

We model credit expansion and contraction as time-consuming processes. When time elapses in a continuous way, as we assume in the rest of the analysis, money market investment and disinvestment can be undertaken immediately, whereas loan expansions and contractions are time consuming. Formally, an analytically convenient way to model credit formation as a time-consuming process can be borrowed from the traditional matching literature (Diamond, 1982, and Mortensen and Pissarides, 1994, 1998). In what follows, we assume that the number of credit applications that are fully screened and evaluated in a given interval of time is described by a unique function of few aggregate variables: the stock of capital in the money market, and the number of idle projects.¹⁰ Thus, a function $x(v, m)$, where v is the number of idle projects, and m is the stock of capital invested in the money market, *records the number of loan applications completely screened and evaluated in a given period*. In terms of the function $x(\cdot)$, the assumption that screening and evaluating projects are time consuming activities is equivalent to assuming that

$$x < \min(v, m).$$

Furthermore, since every credit relationship involves one unit of funds and a single project, in our simple set-up, $v = m$. If we also assume that credit formation can be described by a constant return technology, as we do in the rest of the paper, x can be written simply as

¹⁰This assumption implies that the amount of capital invested in existing loans does not affect the number of applications screened. Relaxing this assumption would make the analytic of the model much more cumbersome, but it would not alter its conclusions.

$$x = \alpha v, \tag{1}$$

where $\alpha \equiv x(1, 1) < 1$, is the instantaneous probability that a bank screens a project in a infinitesimal time interval. Alternatively, given our symmetric structure, α is the probability that a credit application is completely screened.¹¹ In other words, we are assuming that in the economy, during a short period of time δt , there is a positive probability $1 - \alpha\delta t$ that a unit of capital and an idle project do not succeed in finding each-other. As a result of equation (1), the economy is characterized by *aggregate (and stochastic) credit rationing*. Formally, we do not need to specify whether banks meet entrepreneurs randomly over time, or whether banks find new projects at an infinite speed but their screening technology is intrinsically time-consuming. Nevertheless, aggregate credit formation is time consuming, and the parameter α captures this property in a simple way.

When a unit of bank capital and an idle project match, all uncertainty is resolved, and the bank immediately knows the type of the project. An active loan is subject to idiosyncratic risk of destruction at rate λ_i . For the entrepreneur, the realization of the shock represents immediate bankruptcy, and its life-time utility immediately drops to zero. For the bank, the bankruptcy of a project brings to an end the income generated by the associated loan. Immediately thereafter, the bank initiates a costly and time consuming bankruptcy procedure. For analytical simplicity, we assume that recovering capital out of a bankrupt project involves a flow cost f , and an instantaneous probability of success equal to σ , where σ is the arrival rate of a simple Poisson process.¹² This assumption captures the idea that bankrupt firms have assets that can potentially be liquidated, but only via a time consuming and (stochastic) device. Furthermore, to keep symmetry between banks' aggregate capital and the entrepreneurial population, we assume that a new type i project appears idle only when the bank has successfully completed the bankruptcy procedure.

The existence of credit rationing and a finite α generate a *pure economic rent* to be split between entrepreneurs and banks that successfully match. As a result, to formally close the model, we need a sharing rule that determines the interest rate charged to different projects. We follow the standard matching literature and assume that the total surplus generated by an active project is continuously shared in fixed proportions, and we let β represents the bank's share.

Banks choose a search strategy that maximizes the expected value of their capital; they select a decision rule that describes whether to finance a specific project, whenever it becomes available. Since the present value of financing each loan is monotonic in i , the bank's decision rule satisfies a "reservation" property. We show that

¹¹W can also say that the banking system issues new loans with an average waiting time $\frac{1}{\alpha}$.

¹²This assumption rule out banks' effort as a determinant of the probability of recovery.

in equilibrium each bank selects a cut-off quality i^* , such that for projects of quality lower than i^* banks prefer to invest in the money market. In the model, an equilibrium is a reservation rule i^* , or alternatively, a stationary allocation of capital (a distribution of bank capital among project financing, recovery loans, and money market investments) that is consistent with the optimal reservation rule.

III. THE MODEL: STEADY STATE

This section presents and solves the steady state model, with a fixed and time invariant money market rate (r_d). In what follows, we shall indicate with V_i and J_i the present discounted values for a type i entrepreneur of, respectively, an idle project and an active project, while $\rho < 1$ shall be the discount rate, which is assumed to be the same for the banks and the entrepreneurs. Even though idle projects do not yield any dividend, their present discounted value may still be positive, by virtue of an expected capital gain associated with successful matching.¹³ More formally, if α is the entrepreneur's probability of having his or her project screened, the valuation of an idle type i project is

$$\rho V_i = \alpha (\max [V_i; J_i] - V_i), \quad (2)$$

where the *max* operator in equation (2) indicates that an entrepreneurs has always the option to leave his or her project inactive. An active, or financed, type i project yields an instantaneous dividend y_i , and is characterized by an instantaneous destruction probability λ_i . If a destroyed project yields a zero value to the entrepreneur, the present discounted value of an active project reads

$$\rho J_i = y_i - r_i - \lambda_i J_i, \quad (3)$$

where r_i is the interest that a type i project pays to the bank.

Similarly, we shall indicate with C_i the bank's value of a unit of capital invested in a type i loan, and with D the value of investing in the money market. Finally, B shall indicate the present discounted value of a bad loan (a unit of capital in the recovery state). If the interest rate on the money market is r_d , and α is the probability of completely screening a project, the value of investing a unit of capital in the money market reads

$$\rho D = r_d + \alpha \left[\frac{1}{n} \sum_{j=1}^n \max (C_j; D) - D \right]. \quad (4)$$

Equation (4) is one of the key equations of the model, and the *max* operator in the capital gain term encodes the bank's choice between investing in project loans or in money market bonds. Once a bank has successfully (or luckily) screened a project j , it will convert its funds into a type- j loan as long as C_j is greater than D . Ex-ante,

¹³We will show that in equilibrium some project are never financed, and have zero value.

however, the bank does not know the quality of the project, but only its overall distribution, which we have assumed uniform for analytical convenience. Hence, the summation term in equation (4) represents the conditional value of a loan.

The asset valuation of a type i loan depends on the interest rate charged, r_i , and the capital loss suffered by the bank in case of bankruptcy. Since the bank will start the bankruptcy procedure when the project is hit by an idiosyncratic shock¹⁴, the value of a type i loan reads

$$\rho C_i = r_i + \lambda_i (B - C_i), \quad (5)$$

where B is the present discounted value of a bad loan. In the recovery state the bank pays a flow cost f , and expects to successfully recover its capital with instantaneous probability σ . Since the bank will immediately invest the capital that it has successfully recovered in the money market, B solves

$$\rho B = -f + \sigma (D - B). \quad (6)$$

For the bank, establishing a credit relationship is optimal as long as $C_i > D$. Similarly, for the entrepreneur, having an active project is optimal as long as $J_i > V_i$. As a result, the surplus from a credit-match can be formally represented as the net value of establishing a credit relationship, and reads

$$W_i = J_i - V_i + C_i - D, \quad (7)$$

where W_i , the surplus of a type i loan, is the measure of the *quasi-rent* generated by the match. To formally close the model, we need a rule that determines how the bank and the entrepreneur will divide the surplus generated by a match. Since we assume that the bank gets a fixed share β of the total surplus, it must be true that

$$C_i - D = \beta W_i, \quad (J_i - V_i) = \frac{1 - \beta}{\beta} (C_i - D). \quad (8)$$

Equation (8) reflects two characteristics of the model's equilibrium: first, it is profitable for the bank and the entrepreneur to establish a credit relationship as long as the total surplus is positive; second, there is agreement between the bank and the entrepreneurs on which projects should be financed. Making use of equations (5), (3), (2) and (4), the surplus from the credit relationship (7) can be conveniently written as

$$\begin{aligned} (\rho + \lambda_i) W_i &= y_i - a_i r_d - \alpha \beta a_i \left[\frac{1}{n} \sum_{j=1}^n \max(W_j; 0) \right] \\ &\quad - \alpha (1 - \beta) b_i \max(W_i; 0) - \frac{\lambda_i f}{\sigma + \rho}, \end{aligned} \quad (9)$$

¹⁴The appendix shows that the alternative banks' policy of entering the recovery state when the loan is still viable is never optimal when time is continuous, and the idiosyncratic shock λ_i is independent of the probability of recovering the capital.

where

$$\begin{aligned} a_i &= \frac{\rho + \sigma + \lambda_i}{\rho + \sigma}, \\ b_i &= \frac{\rho + \lambda_i}{\rho}. \end{aligned}$$

After dividing both sides by $(\rho + \lambda_i)$, the right hand side of equation (9) is a mapping. Unfortunately, equation (9) does not satisfy the Blackwell sufficient conditions for a contraction. Nevertheless, Appendix I shows that a simple restriction on the parameters provides us with a sufficient condition for the mapping of equation (9) to be a contraction, and thus, to guarantee the existence of a fixed point.¹⁵

The surplus function W_i is a monotonic increasing function of i . Intuitively, as long as the dividends of higher quality projects grow faster than the associated bankruptcy probability, the total rent generated by a project should be monotonic in i . This result, albeit analytically intuitive, is formally derived in Appendix II. By virtue of (8), monotonicity of projects' surplus implies monotonicity of the bank's surplus, which allows us to characterize banks' behavior through the reservation property. In other words, banks select a marginal type i^* , or a cut-off value W_{i^*} , below which investment in the money market is the optimal policy. More formally, i^* solves

$$i^* : W_{i^*} \geq 0, \quad W_{i^*-1} < 0. \quad (10)$$

In the case in which $W_{i^*} \equiv 0$, from equation (9), the marginal returns of the project can be conveniently written as

$$y_{i^*} = a_{i^*} r_d + \alpha \beta a_{i^*} \left[\frac{1}{n} \sum_{j=1}^n \max(W_j; 0) \right] + \frac{\lambda_{i^*} f}{\sigma + \rho}. \quad (11)$$

Equation (11) describes in a simple way the return on the marginal project. In order to be worthwhile to grant a loan, the marginal project i^* must at least compensate the bank for three different elements: the per period return on the money market ($a_{i^*} r_d$), the expected value of an alternative project (the expression in square brackets), and the expected cost of recalling the loan if conditions turn bad ($\frac{\lambda_{i^*} f}{\sigma + \rho}$).

Before analyzing several comparative static results, we solve for the equilibrium interest rate r_i . Rearranging equation (3), and making use of equation (8), the interest rate on a type i loan reads

$$r_i = y_i - (\rho + \lambda_i) \beta W_i. \quad (12)$$

¹⁵The proof in the Appendix is in the spirit of Sharma (1987), who shows the existence and uniqueness of a fixed point in traditional dynamic matching models.

Finally, making use of equations (5) and (4), the interest rate can be simply written as

$$r_i = \rho D + \frac{\lambda_i \rho D}{\sigma + \rho} + (\rho + \lambda_i) \beta W_i \quad (13)$$

Equation (13) shows that the interest rate on a type i project is equal to the return on the alternative banks' asset (ρD), augmented by a strictly positive spread. Furthermore, it is possible to show that the spread component of equation (13) is monotonic in the quality i , so that banks earn higher returns on riskier projects. For this purpose, it is sufficient to show that the interest rate is increasing in i . Making use of equation (13), the spread between two different quality projects can be conveniently written as

$$r_i - r_{i-j} = \rho \beta (W_i - W_{i-j}) + \beta (\lambda_i W_i - \lambda_{i-j} W_{i-k}) + (\lambda_i - \lambda_{i-j}) \left(\frac{\rho D}{\sigma + \rho} \right). \quad (14)$$

Since $W_i > W_{i-j}$ and $\lambda_i > \lambda_{i-j}$, it immediately follows that $r_i - r_{i-j} > 0$ for any positive integer j . As a result, the interest rate spread between a type i loan and an alternative investment grows monotonically with the quality i .

We are now in the position to analyze the most important comparative static results of this section, namely the relationships between the money market rate, r_d , the interest rates on loans, r_i , and the banks' optimal portfolio allocation, as described by the reservation quality i^* . In general, changes in r_d will affect both the amount of lending and the interest rate charged on loans. First, following an increase (decrease) in r_d , it is quite likely that some project becomes immediately unprofitable (profitable) relative to the money market investment. Since the surplus of a type i match is decreasing in the money market rate,¹⁶

$$\frac{\partial W_i}{\partial r_d} < 0 \quad \forall i, \quad (15)$$

an increase in r_d may affect the bank's equilibrium allocation between project loans and money market investments. If $W_{i^*} \approx 0$, equation (15), by virtue of (10) produces an immediate change in i^* . In general, for any marginal increase (decrease) in r_d , if n is sufficiently large, the bank will change its optimal asset allocation and it will decrease (increase) its share of loans. Similarly, making use of equation (13), the effect of a change of the money market rate on the interest rate charged to a type i project is

$$\frac{\partial r_i}{\partial r_d} = \frac{(\rho + \alpha)(1 - \beta)(\rho + \sigma + \lambda_i)}{[\rho + \alpha(1 - \beta)](\rho + \sigma)} \left(1 + \alpha \beta \frac{1}{n} \sum_{j|W_j \geq 0} \frac{\partial W_j}{\partial r_d} \right) > 0 \quad (16)$$

Appendix III. shows that both terms in the brackets of equation (16) are positive. Thus, there is a positive *pass-through* effect between money market rates and interest rate

¹⁶See Appendix III.

changes. Equation (16) shows that the magnitude of the pass-through, albeit strictly positive, crucially depends on several structural parameters. First, $\frac{\partial r_i}{\partial r_d}$ is a decreasing function of β . If we take β as a proxy for the bank's market power, this result implies that the pass through is larger in less concentrated markets, a result familiar in the industrial organization literature (Tirole, 1988). Second, when β is zero and σ tends to infinity, the pass-through is exactly one. In this case, bank lending to private entrepreneurs does not entail any recovery risk, and banks have no market power. Hence, all projects pay to the bank the money market return.

Now we can specify the equilibrium allocation between alternative investments. In equilibrium, banks will finance only projects of type $i \geq i^*$, leaving unfunded the remaining types. Nevertheless, the economy experiences continuous turnover of projects, and to complete the model we need to specify the distribution of banks' asset among active loans, bad loans and money market investment. If we indicate with v_i , b_i and c_i the steady state quantity of type i projects which are, respectively, idle, bankrupt, and fully active, the flow balance conditions are

$$\begin{aligned} \alpha v_i &= \lambda_i c_i & i \geq i^* \\ \sigma b_i &= \lambda_i c_i & i \geq i^* \\ c_i + b_i + v_i &= k & \forall i. \end{aligned} \tag{17}$$

Solving the system of equations (17) yields, for $i \geq i^*$, the following equilibrium quantities

$$\begin{aligned} c_i &= \frac{k\alpha\sigma}{\sigma(\alpha + \lambda_i) + \alpha\lambda_i} \\ b_i &= \frac{k\alpha\lambda_i}{\sigma(\alpha + \lambda_i) + \alpha\lambda_i} \\ v_i &= \frac{k\sigma\lambda_i}{\sigma(\alpha + \lambda_i) + \alpha\lambda_i}, \end{aligned} \tag{18}$$

while for $i < i^*$, all projects are idle and $v_i = k$. The comparative static properties of the quantities in equations (18) show that v_i is a decreasing function of α . The easier the screening process, the larger the proportion of funds kept in the money market. Similarly, higher σ leads to a lower quantity of projects in the bankruptcy state. Finally we aggregate equation (18) over different quality indices to obtain

$$\begin{aligned} c &= \sum_{i \geq i^*} \frac{k\alpha\sigma}{\sigma(\alpha + \lambda_i) + \alpha\lambda_i} \\ b &= \sum_{i \geq i^*} \frac{k\alpha\lambda_i}{\sigma(\alpha + \lambda_i) + \alpha\lambda_i} \\ v &= nk - c - b \end{aligned} \tag{19}$$

From equations (19), it is clear that an increase in market rates, by raising i^* , reduces aggregate lending c and the quantity of capital in the bankruptcy state. As a result, the capital invested in the money market rises. The next section looks at interest rate changes in a stochastic setting.

IV. STOCHASTIC SHOCKS TO THE MONEY MARKET

We now extend the analysis of the previous section and consider a world in which money market returns are stochastic and time variant. For analytical simplicity, we assume that the money market rate jumps stochastically between two different values, r_d^T and r_d^E , with $r_d^T > r_d^E$. Since the money market can be in two different states, in what follows we shall call r_d^T the *tight* state, and r_d^E the *easy* state. We also assume that the state of the money market moves according to a symmetric Markov process, and we shall indicate with μ the instantaneous probability of a state switch. To solve the model, we need to specify a full set of state contingent value functions, which makes the analytic of the model particularly cumbersome. Hence, we do not provide a close form solution of the model, or comparative static results. However, as a way to maintain the discussion at an intuitive level, we describe the dynamics of the model through numerical simulations.

A. The Model

Since there are now two states of the money market, each bank's behavior is described by two indices of reservation quality, that we shall indicate by i^{T*} and i^{E*} , depending on whether the state of the market is easy or tight. Furthermore, since $r_d^T > r_d^E$, it will be generally true that $i^{T*} > i^{E*}$. More formally, if the probability of a state switch is μ , the value of an inactive project of quality i when conditions are "easy" is

$$\rho V_i^E = \alpha \max [J_i^E; 0] + \mu (V_i^T - V_i^E), \quad (20)$$

where the capital gain term $\mu (V_i^T - V_i^E)$ reflects the possibility that the state of the money market switches to tight. With respect to equation (3), the value of a type i project to an entrepreneur when the market is easy is

$$\rho J_i^E = y_i - r_i^E - \lambda_i J_i^E + \mu [\max (V_i^T; J_i^T) - J_i^E] \quad (21)$$

where the *max* operator in the last term reflects the fact that, following a state switch, an entrepreneur might be better off inactive. For the bank, the value of a type i loan when the market is easy reads

$$\rho C_i^E = r_i^E + \lambda_i (B^E - C_i^E) + \mu [\max (B^T; C_i^T) - C_i^E]. \quad (22)$$

The last term in equation (22) suggests that, conditional on the rate r_D^E switching to tight, the bank might find it profitable to recall the capital, in such a way that the corresponding loan enters the bankruptcy state. This term, absent in the steady-state version of the model, is the novel feature of this section. From equation (22) it is clear that a loan enters the recovery state B^E when there is a shock λ_i , and it might enter the state B^T when there is a state switch. The bank's value function of a type i loan in the bankruptcy state reads

$$\rho B^T = -f + \sigma(D^T - B^T) + \mu(B^E - B^T),$$

when the state of the money market is tight, and

$$\rho B^E = -f + \sigma(D^E - B^E) + \mu(B^T - B^E),$$

when the state of the money market is easy. Finally, the asset value of money market investment is

$$\rho D^E = r_d^E + \alpha \left[\frac{1}{n} \sum_{j=1}^n \max(C_j^E; D^E) - D^E \right] + \mu [D^T - D^E]. \quad (23)$$

When the state of the market is tight, the value functions for the banks and the entrepreneurs are very similar to those presented for the steady state. In particular, they do not embed any new element of choice in correspondence to the possible state switch, but only an extra capital gain term. The value function of the entrepreneur when the money market is tight is

$$\rho V^T = \alpha \max [V^T; 0] + \mu (V^E - V^T), \quad (24)$$

for an idle project, and

$$\rho J^T = y_i - r_i^T - \lambda_i J^T + \mu [J^E - J^T] \quad (25)$$

for an active project. Similarly, for the bank, the value of an active loan is

$$\rho C^T = r_i^T + \lambda_i (R_i^T - C^T) + \mu [C^E - C^T], \quad (26)$$

and the value of investing in the money market is

$$\rho D^T = r_d^T + \alpha \left[\frac{1}{n} \sum_{j=1}^n \max(C^T; D^T) - D^T \right] + \mu [D^E - D^T]. \quad (27)$$

Proceeding as in the previous section, it is possible to derive an expression that describes the surplus from the match in the tight and in the easy state¹⁷. As μ grows,

¹⁷We do not report the expressions for the match's surplus in tight and easy states, but the detailed expressions are available from the authors upon request.

the money market shock becomes less persistent and, at the limit, the value function ceases to be state contingent. The intuition for this result is as follows. As the expected duration of each state tends to zero, so does the capital gain (loss) associated with a discrete change in money market conditions, and in the limit case, the bank's allocation decision becomes time invariant.¹⁸

To complete the dynamic model, and to describe bank lending behavior in different market states, we need a set of differential equations. Since it is always true that $c_i + b_i + v_i = K$, we have only to specify the dynamics of active loans and bad loans (loans in recovery state), obtaining money market investment as a residual. The differential equations are

$$\begin{aligned} \frac{db_i^S}{dt} &= -\sigma b_i^S + \lambda_i b_i^S \\ \frac{dc_i^S}{dt} &= -\lambda_i c_i^S + \alpha v_i^S, \quad S = T, E. \end{aligned} \tag{28}$$

The next section presents numerical simulations of the model's dynamics.

B. The Aggregate Dynamics: Simulations and Discussions

This section simulates the aggregate dynamic response of bank lending to changes in market conditions. In what follows we assume that time is discrete and that the state of the money market (*tight*, r_d^T or *easy*, r_d^E) is realized at the beginning of the period, and constant throughout. The timing of the decision is as follows. Banks observe the money market realization, and immediately select a reservation i^{*T} or i^{*E} . Thereafter, each entrepreneur learns whether his or her project has been screened, the credit allocation (c, r, v) is determined, interest rates are paid, and the period is completed.

In a well behaved equilibrium, given the n project qualities, there is always a subset of project types whose quality at time t is above the highest reservation quality ($i \geq i^{*T}$), and another subset of projects whose quality is below the lowest one ($i < i^{*E}$). For these projects, the allocation of capital is not affected by regime switches, even though the interest rate charged on the associated loans will be conditional on the state of the market. More formally, for $i \geq i^{*T}$, the shares of projects that are financed, not-financed, or are in the recovery state are those described by equations (18).

¹⁸Consider, for example, equation 20. We can rewrite it as:

$$V_i^E = \frac{\alpha}{\rho + \mu} \max[V^E; 0] + \frac{\mu}{\rho + \mu} (V_i^T)$$

so that we obtain:

$$\lim_{\mu \rightarrow \infty} V_i^E = V_i^T$$

and similarly for the other value functions.

Conversely, for types $i < i^{*E}$ no project is ever financed, and we just have $v_i = k$. However, for those projects whose quality lies between the two reservation thresholds ($i^{*E} \leq i < i^{*T}$), the composition of projects that are financed, not-financed, or are in the recovery state is state dependent. If we let $x(t)$ be an indicator function

$$x(t) = \begin{cases} i^{*T} & \text{if } r_d = r_d^T \\ i^{*E} & \text{if } r_d = r_d^E \end{cases}$$

that records the state of the money market at time t , the dynamics of a type i project is

$$c_{i,t+1} = (1 - \Phi_1)(1 - \lambda_i dt)c_{i,t} + \Phi_2 \alpha v_{i,t} dt \quad (29)$$

$$b_{i,t+1} = (1 - \sigma dt)b_{i,t} + (1 - \Phi_1)\lambda_i c_i dt + \Phi_1 c_{i,t} \quad (30)$$

$$c_{i,t} + b_{i,t} + v_{i,t} = k \quad \forall i, t,$$

where

$$\Phi_1 = \begin{cases} 1 & \text{if } x(t+1) = i^{*T}, x(t) = i^{*E}, W_i^T < -\frac{\rho D^t}{\rho + \sigma} \\ 0 & \text{otherwise} \end{cases} \quad (31)$$

and

$$\Phi_2 = \begin{cases} 1 & \text{if } x(t+1) = i^{*E} \\ 0 & \text{otherwise} \end{cases} \quad (32)$$

Consider first the dynamics of equations (29) and (30). If the state switches from easy to tight ($\Phi_1 = 1$), all active projects enter the bankruptcy state b_i , while when there is no state switch the flow from active to recovery is governed by the natural turnover λ_i . Conversely, successfully screened projects are converted into active loans only when the market is easy, and the indicator function Φ_2 keeps track of this condition in equation (29).

In what follows, we simulate the dynamic response of the aggregate economy using the baseline parameter values specified in Table 1. We focus on two aggregate statistics, the aggregate credit (c_t) and the average interest rate on active loans, which we formally indicate with

$$\bar{r}_t = \frac{\sum_{i>x(t)}^n r_{it} c_{it}}{\sum_{i>x(t)}^n c_{it}},$$

where c_{it} is described by equation (29) for $i^{*E} \leq i < i^{*T}$, and by equation (18) for $i \geq i^{*T}$.

Figures 1 to 4 plot the dynamics response of c_t and \bar{r}_t for different values of α and σ . Figure 1 plots the dynamics response of c_t for $\sigma > \alpha$ along a full easy-tight-easy cycle. σ is the institutional parameter that describes the average speed of recovering a loan, while α is the technological parameter that describes the average screening period of a credit application. In Figure 1, when the market state turns tight (approximately at period 9 in Figure 1) i^{*E} jumps to i^{*T} , and all projects whose idiosyncratic quality lies

between the two reservation qualities $i^{*E} \leq i < i^{*T}$ are rapidly recalled. Since σ is relatively high, the average duration of the recovery state is very short, and the bank rapidly recovers the invested capital. When the market state switches to tight (approximately at period 30 in Figure 1), i^{*T} jumps immediately to i^{*E} , but the dynamic response of aggregate credit is time consuming. As the interest rate falls, banks immediately change the reservation quality, but they still have to process the additional loan applications, and the associated credit expansion takes place only at rate α . Thus, *when $\sigma > \alpha$, there is a short-run asymmetric response of aggregate lending to interest rate changes*. However, as the projects are successfully screened, credit returns to its original level, *without any long-run asymmetry*. These results represent a first set of empirical implications of our model. Figure 2 plots the dynamic response of the average interest rate when $\sigma > \alpha$. In general, the dynamic response of \bar{r}_t to changes in the money market rate is affected by two different effects. First, there is an instantaneous *pass-through effect*, linked to the translation of the new r_d into the existing r_i , as was formally described by equation (16). Second, there is a *portfolio effect*, induced by the modification of the bank's optimal portfolio towards more profitable projects. When r_d switches to tight in figure 2, the average interest rate immediately jumps to a new level, as a result of the portfolio effect and the pass-through effect. However, when policy switches to easy, on impact, the change in r_d is only related to the pass-through effect, since the portfolio effect is time-consuming. From figure 2, is clear that the asymmetric response of the average interest rate depends entirely on the *portfolio effect*.

Figures 3 and 4 plot the dynamic response of c_t and \bar{r}_t along a full easy-tight-easy cycle when $\sigma = \alpha$. Qualitatively, the dynamic profile of Figure 3 is clearly different from the dynamic profile of Figure 1. In Figure 3, when the market turns tight (approximately at period 9) i^{*E} jumps to i^{*T} , but nothing happens on impact, neither to aggregate credit nor to the recovery state. To properly understand figure 3 it is necessary to go back to equation (22). When r_d switches to tight, and σ is very low, banks expect a very long and costly recovery process, and may well prefer to keep active some of the projects with $i < i^{*T}$. When the latter is true, banks prefer to wait for the natural bankruptcy of the project, rather than go through a costly separation. Nevertheless, during the tight phase banks choose not to finance projects that are successfully screened but have an idiosyncratic quality below i^{*T} . Basically, projects of quality i , with $i^{*E} \leq i < i^{*T}$, are destroyed at rate λ_i , but are not replaced at rate α . As a result, aggregate credit gradually falls. When the policy switches to easy (approximately at period 30 of figure 3), banks begin to finance all projects with reservation quality greater than i^{*E} , and aggregate credit rises smoothly. Overall, there is no asymmetry over the full cycle. Comparing Figures 3 and Figure 1, it is clear that *the dynamic response of aggregate credit to interest rate changes depends crucially on the relative values of σ and α* . Finally, figure 4 plots the dynamic behavior of the average interest rate. When $\sigma = \alpha$, there is no portfolio reallocation and, as a result, there is almost no asymmetry in the dynamic response of the average interest rate.

V. EMPIRICAL EVIDENCE

In this section we provide some empirical evidence on the effects of interest rate changes on bank lending, focusing on the difference between positive and negative shocks. We consider two countries, in representation of mature and emerging markets: the United States and Mexico. The data for the US are quarterly, and run from 1960 to 1997. For Mexico we have monthly data, from 1982 to 1997. The source of all the data is the *International Financial Statistics*. Market rates are taken from line 60b, which records the most representative short term rates.¹⁹ We use “*claims on the private sector*” (line 22d) as a proxy for loans. This choice, as was the use of *IFS* data, was mainly motivated by the need of international comparability. For the US we use the Consumer Price Index (CPI) and GDP growth as a way to construct exogenous interest rate shocks. For Mexico we use Industrial Production in the absence of monthly GDP data.

A. Methodology

We employ a two-step procedure similar to that applied by Cover (1992) and Garibaldi (1997), but adapted to consider the stationarity problems of aggregate credit. First, we estimate the money market rate processes, and we use the associated residuals for constructing positive and negative interest rate shocks. Second, we estimate the effect of those shocks on a properly defined specification of aggregate credits.²⁰

In the first step we estimate an *autoregressive-distributed lag* model (ARDL) of the money market rate, CPI inflation, and GDP (or industrial production). If MM is the money market rate, the ARDL regression reads

$$MM_t = a + \sum_{i=1}^M \alpha_i MM_{t-i} + \sum_{i=0}^N \beta_i G_{t-i} + \sum_{i=0}^Q \gamma_i P_{t-i} + \varepsilon_t, \quad (33)$$

where G is the real output growth, and P is the CPI inflation. When the lags are properly chosen, ε_t mimics a simple white noise process, and we take its estimated values as exogenous shocks to the money market rate. As a way to test the robustness of our generated shocks, we use the Akaike and Schwartz criteria to optimally select M , N , and Q (the order of the ARDL)²¹. Results of regression (33) are reported in Table (2) for the US and Table (3) for Mexico. Next, we divide the residuals generated in regression (33) into positive and negative values, and we introduce two different variables. More specifically, we define a positive shock to the money market interest rate as

$$tight_t = \max(\varepsilon_t, 0),$$

¹⁹For the U.S. we use the Federal Fund Rate.

²⁰To obtain meaningful results we have to take into account all those factors that affect both the money market interest rate and bank lending, and that, if excluded, would bias our estimates. An alternative procedure could have been to include the variables that influence the money-market rate directly in the final regression.

²¹For Mexico both criteria gave the same results, so that we have only one set of shocks.

and a negative shock as

$$easy_t = \min(\varepsilon_t, 0).$$

In the second regression we use “*claims on the private sector over CPI*” as a proxy for aggregate real loans, and we regress this variable on its lagged values, the lagged values of real deposits, and the lagged values of positive and negative market rate shocks. Real deposits are included as a control variables to take into account changes in the amount of funds available to the banking system.²² The ARDL model of the second step is

$$L_t = b + \sum_{i=1}^R \delta_i L_{t-i} + \sum_{i=0}^S \vartheta_i D_{t-i} + \sum_{i=1}^U \varphi_i tight_{t-i} + \sum_{i=1}^V \psi_i easy_{t-i} + \xi t + \eta_t \quad (34)$$

where L_t is the log of real claims on the private sector, D_{t-i} is the log of real deposits, t is a time trend, and η is a white noise error. Since we need to allow credit to react to interest rate changes we do not include the contemporaneous values of *easy* and *tight*, and the variables *tight* and *easy* in equations (34) are only lagged.

To check the stability of the model, we employ the two-stage procedure proposed in Pesaran *et al.* (1996). We use the error correction specification of regression (34), and perform a variable deletion test on the coefficients of the lagged levels of L , D , *tight*, and *easy*. The null hypothesis is the non-existence of a long-run relationship, or in other words, the instability of the model. This procedure allows us to avoid the pre-testing problems associated with standard cointegration analysis, which requires the classification of variables into $I(1)$ and $I(0)$. Fortunately, we are able to reject the null hypothesis of instability (no-cointegration) for both countries.²³

²²Our theoretical framework assumes that the banking system is endowed with a fixed amount of capital.

²³After writing regression (34) in the error correction form

$$\begin{aligned} \Delta L_t = & c + \zeta_0 L_{t-1} + \gamma_0 D_{t-1} + \nu_1 tight_{t-2} + \mu_1 easy_{t-2} + \\ & \sum_{i=1}^{R-1} \zeta_i L_{t-i} + \sum_{i=1}^{S-1} \gamma_i D_{t-i} \\ & + \sum_{i=2}^{U-1} \nu_i tight_{t-i} + \sum_{i=2}^{V-1} \mu_i easy_{t-i} \\ & + \chi t + \omega_t, \end{aligned}$$

we test the following null hypothesis of instability: $\zeta_0 = \gamma_0 = \nu_1 = \mu_1 = 0$. The F-statistic for this test is non-standard, and its critical values are reported in Pesaran *et al.* (1996). In this case the critical bound values at the 99 percent level are 5.315 and 6.414. The computed F -statistic for Mexico and for the US are, respectively, 7.7 and 6.5 when the shocks are obtained with the Akaike criterion, and 6.6 when the US shocks are obtained through the Schwartz criterion. Thus, we comfortably reject the null of instability at the 99 percent level.

B. Results and Robustness Checks

The results of our estimation indicate that the response of bank lending to interest rate shocks is asymmetric. More specifically, a structural interpretation of the results would suggest that frictions that slow down credit formation are particularly important. Indeed, bank lending seems to react more rapidly to positive interest rate shocks than to negative interest rate shocks. For both countries we are able to identify a negative effect of interest rate increases on aggregate loans, while we find only weak evidence of the effect of interest rate cuts. Tables 4 and 5 report the results of regression (34) for the US and Mexico. These results suggest that any regression that does not take explicitly into account the asymmetric structure of the different shocks is likely to be over-restrictive.

To test the aggregate significance of the coefficients *easy* and *tight* in equation (34), we run “variable deletion tests” in the main ARDL regression. For both Mexico and the US we can easily reject the null for *tight*, but not for *easy*. This result suggests that bank lending is more sensitive to interest rate increases than decreases. We also directly test for asymmetry, by performing a Wald restriction test in two different forms. First we focus on the marginal impact of *easy* and *tight* on aggregate lending, by testing the null hypothesis that $\varphi_1 = \psi_1$ in equation (34). Second we test the asymmetry of the overall dynamic structure, and we test the hypothesis that $(\varphi_i = \psi_i \forall i)$ in equation (34). The results of these tests are reported in Table 6. Clearly, we are able to reject the null of symmetry of all the coefficients for both countries. Moreover, for Mexico we also reject the null of a symmetric marginal impact. Table 7 reports tests of the long-run effects of *tight* and *easy* on aggregate lending. Our theory has a clear long-run prediction. Eventually, the cumulative effects on lending of a positive and negative change in interest rate should be symmetric. Even though most of the coefficients in Table 8 are correctly signed, their overall significance is low. In particular, there is no evidence of a long-run negative relationship between *tight* and lending, while there is some evidence of a long-run negative relationship between *easy* and lending. One reason for the lack of evidence of our long-run prediction may well be linked to the fact that in the long-run banks are able to substitute deposits with other liabilities. To that extent, deposits are no longer a good proxy for the banks’ lending capacity.

In order to test the robustness of our results, we explicitly take into account the possibility of structural breaks in the interest rate processes. For the US we perform two robustness checks. First, we run the regressions (33) and (34) for a limited sample -1961 to 1979- so as to exclude the Volcker period; second, we include a Dummy for the period 1979:4-1982:4, when the Federal Reserve deemphasized the funds rate. For Mexico, we limit the sample to the period 1985-1997, so as to exclude the financial turmoils of the early 80s, and we include a dummy for the 1994 crisis.²⁴ The results of these regressions checks are reported in Table 10 and 11. Overall, the asymmetry between lending

²⁴See Aspe (1993) for discussion of developments in Mexico economy during the 80s.

expansion and contraction is confirmed by these different specifications.

Finally, to check the robustness of our approach, we test a restricted version of the model, by exogenously imposing symmetry on the coefficients of the interest rate shocks. Results in Table 9 confirm that not taking into account the possibility of an asymmetric response may lead to misleading results. The shock coefficients in the regression of Table 9 perform worse than the coefficients of the unrestricted regression, reported in Table 5. Furthermore, if the shocks are generated with the Akaike criterion, we can not reject a variable deletion test, while in the Schwartz case we reject it at the 10 percent level. Overall, the results in this section confirm an empirical prediction of our theory: the response of bank lending to interest rate shocks is gradual and asymmetric. However, as we discuss in the next section, these results are also consistent with other models, and further empirical research is needed for identifying competing theories.

VI. POLICY RELEVANCE AND ALTERNATIVE INTERPRETATIONS

If the shocks to the money market rate are interpreted as policy innovations, our framework provides some insights on the dynamic effects of monetary policy. However, before entering into a policy discussion, it is useful to position our theory in the context of the standard monetary transmission channels (Mishkin, 1995). Even though lending plays a key role in the transmission of policy shocks, our approach has important differences from the “lending” channel of monetary policy transmission (also known as the “credit view”). As in our approach, the credit view considers bank loans as important determinants of the real economic activity. Our entrepreneurs do not have access to market financing, and bank lending is the only element through which monetary policy may affect economic activity. However, the “credit view” focuses on the existence of a *direct* link between policy impulses and lending response, beyond the indirect effects induced by the classical interest rate channel. If monetary policy affects the supply of deposits to the banking system, the banks’ ability to issue new loans may be directly affected, independently of what happens to the interest rate.²⁵ This effect is certainly absent in our model, which limits the analysis of bank lending to the asset side of the banks’ balance sheet, and abstracts from the relationship between policy impulses and the capital available to the banking system. In our approach, changes in bank lending are induced by changes in interest rate, in a way that is consistent with the traditional interest rate channel.

In particular, we have shown that the dynamic response of aggregate lending to interest rate shocks depends crucially on two independent processes: the banking system’s ability to find and screen new projects, and the banking system’s ability to

²⁵This idea relies on the particular role of deposits as a source of funds for the banks. See Kashyap and Stein (1993).

recall existing loans. Since these processes are inherently different, it is quite likely that bank lending will respond asymmetrically to positive and negative innovations. This theoretical intuition was also confirmed by our empirical evidence on lending behavior in Mexico and the US. Hence, the design and implementation of monetary policy should take into account the asymmetric lag structure between policy contractions and policy expansions, and react consequently.

An extensive literature has addressed the issue of the asymmetric effects of monetary policy on real economic activity. The traditional explanations focus on price rigidity, on informational asymmetries, and on the structure of the labor market. If prices are more flexible upward than downward, monetary contractions will produce mainly quantity adjustments, while monetary expansions will cause prices to rise with a smaller effect on output²⁶. If informational asymmetries produce binding liquidity constraints, monetary contractions induce reduction in consumption for liquidity constrained agents, in a way that does not have a counterpart during monetary expansions.²⁷ Finally, if hiring is costly and time consuming, while firing is immediate, job creation and destruction respond asymmetrically to changes in money market rates²⁸. In this paper, we have argued that the technological and institutional structure of the credit markets are likely to produce asymmetry in monetary policy, in a way that has been previously neglected.

The relationship between deposit (and lending) rates and changes in money market rates has been the focus of an extensive banking literature. In that literature, the evidence of asymmetries in upward and downward adjustments of banks' interest rate is linked to the existence of oligopolistic behavior in the banking industry (Hannan and Berger, 1991). In our approach, asymmetries in banks' interest rate are completely independent of the market structure, and are linked to "technological" characteristics of bank lending. Since the processes that describe credit formation and contraction are completely independent, albeit time-consuming, prices and volumes of bank lending may adjust asymmetrically to interest rate changes, regardless of the underlying competitive structure. Even though in reality the "technological" and "oligopolistic" approaches may both be relevant, they provide us with very different implications. In our model, the dynamic and static equilibria are entirely determined by technological and institutional factors, and by the structural parameters that describe the speed of loan formation and of loan contraction. In order to affect those equilibria, it is necessary to deal with technological and institutional innovations. Conversely, in the oligopolistic view, the credit allocation may be affected by changes in the market structure, and by policies aimed at increasing competition. Hence, determining the empirical relevance of these two frameworks would certainly be important, at least from a regulatory

²⁶See Ball and Mankiew (1994), or Caplin and Leahy (1991).

²⁷See Jackman and Sutton (1982).

²⁸Garibaldi (1997) analyses the asymmetric effects of monetary policy on job creation and destruction in a dynamic matching model.

standpoint. Unfortunately, aggregate data do not allow us to discriminate between these two models.²⁹

Finally, we need to deal with efficiency considerations. The previous discussion has suggested that institutional and technological changes may affect the equilibria of the model. Certainly, the existence of stochastic lags in the banks' ability to recall its invested capital is a measure of inefficiency, and policies aimed at speeding up this process (e.g. improvements in bankruptcy laws, increased enforcement of property rights) are likely to be welfare improving. Other things equal, the higher is σ , the more efficient is the banking system. Efficiency considerations on the bank's ability to screen projects are more subtle. In a first-best world with perfect information, α is infinitely large, lending is instantaneous and bank loans lose the special role played in this paper. However, in a second-best world, α is a physiological parameter, related only to information technology. Changing the latter is certainly an economic activity, but its discussion is beyond the topic of the present paper.

VII. CONCLUDING REMARKS

This paper has proposed and solved a dynamic model of bank lending, where loan expansion and contraction are time consuming activities. We have shown that the speed at which lending opportunities become available, and the speed at which banks can recall existing loans are key determinants of the dynamic response of bank lending to exogenous innovations in the money markets. Since these two processes are inherently different, bank lending is likely to respond asymmetrically to interest rate changes. This empirical implication is broadly consistent with the evidence on aggregate lending in Mexico and the US. In both countries, positive innovations to money market rates take time to produce credit expansion, while negative innovations appear to produce immediate contractions. As a result, the design and implementation of monetary policy should take these asymmetries into consideration, and react accordingly.

Further research is needed. Theoretically, we have focused on the probability that a credit application is accepted, while we have not modeled the way in which banks select the optimal screening time. A natural extension of the model would consider a world in which banks endogenously invest resources in the screening process. Such resources are likely to be dependent on the state of the market, and their proper consideration would further enrich the dynamics of the model. Empirically, the availability of disaggregated data on specific loan contracts would help to distinguish between our technological asymmetries, and asymmetries linked to banks' oligopoly power it would be necessary to use disaggregated data.

²⁹Our theory is consistent with asymmetric adjustment of lending volumes at the individual and aggregate level, while it predicts a symmetric adjustment of lending rates at the individual loan level. Microdata on banks' lending would allow us to test this empirical implication.

Table 1. Baseline Parameter Values

Variables	Notation	Value
Screening probability	α	0.050
Recall probability (low)	σ	0.050
Recall probability (high)	σ	1.000
Tight Money market rate	r_d^T	0.050
Easy Money market rate	r_d^E	0.020
discount rate	ρ	0.100
quality index	n	12.000
bankruptcy rate (min)	λ_1	0.070
bankruptcy rate (max)	λ_n	0.020
idiosyncratic return (min)	y_1	0.100
idiosyncratic return (min)	y_n	0.000
banks' surplus share	β	1.000
<i>Source: Authors' calculation</i>		

Table 2. U.S Federal Fund Rate

Dependent Variable: Federal Fund Rate			
Quarterly Data: 1960(1) to 1997(4)			
(Ordinary Least Square, First stage regression)			
Regressor ^a	Notation ^a	Akaike ^b	Schwartz ^c
MM_{t-1}	α_1	0.99*** (0.087)	0.91*** (0.034)
MM_{t-2}	α_2	-0.32*** (0.124)	-
MM_{t-3}	α_3	0.27*** (0.090)	-
G_t	β_0	0.23** (0.102)	0.31*** (0.100)
G_{t-1}	β_1	0.25 (0.102)	0.31 (0.100)
G_{t-2}	β_2	0.14 (0.106)	-
G_{t-3}	β_3	0.15 (0.095)	-
P_t	γ_0	0.91*** (0.19)	0.81*** (0.174)
P_{t-1}	γ_1	-0.63*** (0.226)	-0.67*** (0.217)
P_{t-2}	γ_2	0.73*** (0.222)	0.51*** (0.182)
P_{t-3}	γ_3	-0.39* (0.211)	-
$Cost$	a	-0.83*** (0.304)	-0.58** (0.259)

Notes: Standard errors are in parentheses
 One, two and three asterisks indicate significance at the 10, 5, and 1 percent respectively.
^a See equation (33)
^b ARDL order selected with Akaike criterion
^c ARDL order selected with Schwartz criterion
 Source: Authors' calculation

Table 3. Mexico Money-Market Rate

Dependent Variable: Money Market Rate Monthly Data: 1982(1) to 1997(12) (Ordinary Least Square, First stage regression)		
Regressor ^a	Notation ^a	Akaike-Schwartz ^b
MM_{t-1}	α_1	1.24 *** (0.076)
MM_{t-2}	α_2	-0.59 *** (0.114)
MM_{t-3}	α_3	0.29 *** (0.070)
G_t	β_0	0.1 (0.118)
P_t	γ_0	2.21 *** (0.363)
P_{t-1}	γ_1	-3.26 *** (0.705)
P_{t-2}	γ_2	1.5 * (0.769)
P_{t-3}	γ_3	-2.00 *** (0.747)
P_{t-4}	γ_3	1.54 *** (0.402)
$Cost$	a	3.76 ** (1.766)

Notes: Standard errors are in parentheses
One, two and three asterisks indicate significance at the 10, 5, and 1 percent respectively.
^a See equation (33)
^b ARDL order selected with Akaike criterion
^b ARDL order selected with Schwartz criterion
Source: Authors' calculation

Table 4. Mexico: Aggregate Lending

Dependent Variable: Real Lending Monthly Data: 1982(1) to 1997(12) (Ordinary Least Square, Second stage Regression ^a)			
Regressor	Notation ^b	Coefficient	T-Ratio
L_{t-1}	δ_1	1.002***	13.21
L_{t-2}	δ_2	0.017	0.17
L_{t-3}	δ_3	0.081	0.78
L_{t-4}	δ_4	-0.201*	-1.89
L_{t-5}	δ_5	0.169	1.57
L_{t-6}	δ_6	0.066	0.63
L_{t-7}	δ_7	-0.037	-0.35
L_{t-8}	δ_8	-0.005	-0.05
L_{t-9}	δ_9	-0.145**	-2.00
D_t	θ_0	0.175***	5.99
D_{t-1}	θ_1	-0.178***	-3.94
D_{t-2}	θ_2	-0.051	-1.12
D_{t-3}	θ_3	0.017	0.39
D_{t-4}	θ_4	0.070	1.57
D_{t-5}	θ_5	-0.040	-0.89
D_{t-6}	θ_6	-0.007	-0.17
D_{t-7}	θ_7	0.034	0.72
D_{t-8}	θ_8	-0.013	-0.28
D_{t-9}	θ_9	0.018	0.57
$Tight_{t-1}$	ψ_1	-0.003***	-4.20
$Tight_{t-2}$	ψ_2	-0.002**	-3.22
$Tight_{t-3}$	ψ_3	-0.000	-0.13
$Tight_{t-4}$	ψ_4	0.000	0.04
$Tight_{t-5}$	ψ_5	-0.000	-0.19
$Tight_{t-6}$	ψ_6	-0.002*	-1.65
$Tight_{t-7}$	ψ_7	-0.000	-0.16
$Easy_{t-1}$	φ_1	-0.001	-1.37
$Easy_{t-2}$	φ_2	0.000	0.65
$Easy_{t-3}$	φ_3	-0.000	-0.32
$Easy_{t-4}$	φ_4	-0.001**	-2.19
$Easy_{t-5}$	φ_5	-0.001*	-1.62
$Easy_{t-6}$	φ_6	-0.000	-0.42
$Easy_{t-7}$	φ_7	-0.002***	-3.32
<i>trend</i>		0.000***	3.96
<i>const</i>	<i>b</i>	0.572***	4.39

Notes: One, two and three asterisks indicate significance at the 10, 5, and 1 percent respectively.

^a Shocks in the first stage selected with Akaike and Schwartz criteria

^b See equation (34)

Source: Authors' calculation

Table 5. US: Aggregate Lending

Dependent Variable: Real Lending Quarterly Data: 1960(1) to 1997(4) (Ordinary Least Square, Second stage Regression)					
Regressor	Notation ^c	Schwartz Shocks ^a		Akaike Shocks ^b	
		Coefficient	T-Ratio	Coefficient	T-Ratio
L_{t-1}	δ_1	1.117***	13.2	1.099***	13.3
L_{t-2}	δ_2	0.279**	2.3	0.292**	2.4
L_{t-3}	δ_3	-0.466***	-5.8	-0.474***	-6.0
D_t	θ_0	0.369***	7.8	0.381***	8.2
D_{t-1}	θ_1	-0.587***	-8.3	-0.580***	-8.3
D_{t-2}	θ_2	0.134	1.5	0.116	1.3
D_{t-3}	θ_3	0.159**	2.4	0.178***	2.7
$tight_{t_1}$	ψ_1	-0.005**	-2.1	-0.004**	-2.1
$tight_{t-2}$	ψ_2	-0.004**	-2.1	-0.002	-0.8
$tight_{t-3}$	ψ_3	0.004*	1.9	0.005***	2.6
$easy_{t-1}$	φ_1	0.001	0.4	0.002	0.7
$easy_{t-2}$	φ_2	-0.002	-0.9	-0.006**	-2.1
$easy_{t-3}$	φ_3	-0.003	-1.3	-0.005**	-2.0
$trend$		0.000	-0.4	0.001***	2.8
$cost$		-0.041**	2.5	-0.091	-0.8

Notes: One, two and three asterisks indicate significance at the 10, 5, and 1 percent respectively.

^a Shocks in the first stage selected with the Akaike criterion

^b Shocks in the first stage selected with the Schwartz criterion

^c See equation (34)

Source: Authors' calculation

Table 6. Tests of Asymmetry Aggregate Lending

Wald Restriction Tests				
country	$\varphi_1 = \psi_1^a$	P value	$\varphi_i = \psi_i \forall i^b$	prob.
US - Akaike ^c	2.52	0.11	9.50 ***	0.02
US - Schwartz ^d	2.31	0.13	6.23 *	0.10
Mexico ^e	3.29 ***	0.07	15.45 ***	0.03

Notes: One, two and three asterisks indicate significance at the 10, 5, and 1 percent respectively.

^a Wald Statistic under the null that the impact coefficient on *tight* (ψ_1) is equal to the impact coefficient on *easy* (φ_1) Distributed as $\chi(1)$.

^b Wald Statistic under the null that the each coefficient on *tight* (ψ_i) is equal to the corresponding coefficient on *easy* (φ_i) Distributed as $\chi(7)$ for Mexico and $\chi(3)$ for the US.

^c Shocks generated with the Akaike criterion, see Table (5)

^d Shocks generated with the Schwartz criterion, see Table (5)

^e Shocks generated with the Schwartz and Akaike criterion, see Table (4)

Source: Authors' calculation

Table 7. Variable Deletion Tests

Variable Deletion Tests				
country	<i>tight</i> ^a	<i>pvalue</i>	<i>easy</i> ^b	prob.
US - Akaike ^c	3.585**	0.016	2.94 ***	0.036
US - Schwartz ^d	4.08***	0.008	1.07	0.366
Mexico ^d	4.93 ***	0.000	4.04 ***	0.000

Notes: One, two and three asterisks indicate significance at the 10, 5, and 1 percent respectively.

^a F-statistics under the null that the joint coefficients on *tight* (ψ_i) are equal to zero

^b F-statistics under the null that the joint coefficients on *easy* (φ_i) are equal to zero

^c Shocks generated with the Akaike criterion, see Table (5)

^d Shocks generated with the Schwartz criterion, see Table (5)

^e Shocks generated with the Schwartz and Akaike criterion, see Table (4)

Source: Authors' calculation

Table 8. Long Run Coefficients

Long Run Coefficients				
country	<i>tight</i> ^a	<i>pvalue</i>	<i>easy</i>	<i>pvalue</i>
US - Akaike	-0.01	0.807	-0.10 **	0.036
US - Schwartz	-0.68	0.294	-0.62	0.245
Mexico	1.35	0.480	-0.11 ***	0.002

Notes: One, two and three asterisks indicate significance at the 10, 5, and 1 percent respectively.

^a t statistics of the long-run effect of *tight* in Table (5)

^b t statistics of the long-run effect of *easy* in Table (5)

^c Shocks generated with the Akaike criterion, see Table (5)

^d Shocks generated with the Schwartz criterion, see Table (5)

^e Shocks generated with the Schwartz and Akaike criterion, see Table (4)

Source: Authors' calculation

Table 9. US: Restricted Regression

Dependent Variable: Real Lending Quarterly Data: 1960(1) to 1997(4) (Ordinary Least Square, Second stage Regression)					
		Schwartz Shocks ^a		Akaike Shocks ^b	
Regressor	Notation	Coefficient	T-Ratio	Coefficient	T-Ratio
L_{t-1}	δ_1	1.099***	13.1	1.107***	13.1
L_{t-2}	δ_2	0.245**	2.0	0.259**	2.1
L_{t-3}	δ_3	-0.418***	-5.4	-0.439***	-5.5
D_t	θ_0	0.394***	8.3	0.386***	8.2
D_{t-1}	θ_1	-0.593***	-8.3	-0.587***	-8.3
D_{t-2}	θ_2	0.132	1.5	0.130	1.5
D_{t-3}	θ_3	0.145**	2.2	0.148**	2.3
$shock_{t-1}$	^c	-0.001	-1.1	-0.002	-1.4
$shock_{t-2}$	^c	-0.003**	-2.1	-0.002**	-2.2
$shock_{t-3}$	^c	0.000	-0.4	0.000	0.6
<i>trend</i>		0.0003**	2.6	0.0003***	2.6
<i>cost</i>		-0.050	2.5	-0.053	-0.5

Notes: One, two and three asterisks indicate significance at the 10, 5, and 1 percent respectively.

^a Shocks in the first stage selected with the Akaike criterion

^b Shocks in the first stage selected with the Schwartz criterion

^c $shock_{t-j} = easy_{t-j} + tight_{t-j}$ $j = 1, 2, 3$.

Source: Authors' calculation

Table 10. US: Robustness Checks

Tight and easy shocks generated with different specifications of the Federal Fund Process.					
Lags in the first stage selected with the Schwartz criterion					
Dependent Variable: Real Lending					
(Ordinary Least Square, Second stage Regression)					
		Limited Sample:		Full Sample:	
		1961:1	1979:3	Dummy for 1979:4 1982:4	
Regressor	Notation ^a	Coefficient	T-Ratio	Coefficient	T-Ratio
L_{t-1}	δ_1	0.887***	7.44	1.100***	13.30
L_{t-2}	δ_2	0.428**	2.69	0.276**	2.28
L_{t-3}	δ_3	-0.500***	-5.32	-0.457***	-5.92
D_t	θ_0	0.331***	5.28	0.397***	8.58
D_{t-1}	θ_1	-0.544***	-5.58	-0.600***	-8.59
D_{t-2}	θ_2	0.151	1.23	0.111	1.24
D_{t-3}	θ_3	0.310**	3.07	0.170***	2.82
$tight_{t-1}$	ψ_1	-0.009**	-2.09	-0.004**	-1.98
$tight_{t-2}$	ψ_2	-0.007**	-1.82	-0.002	-1.16
$tight_{t-3}$	ψ_3	0.003*	0.78	0.004**	2.02
$easy_{t-1}$	φ_1	0.002	0.58	0.001	0.40
$easy_{t-2}$	φ_2	-0.002	-0.53	-0.005**	-1.82
$easy_{t-3}$	φ_3	-0.007*	-1.72	-0.005**	-1.99
<i>trend</i>		-0.530***	-2.97	0.001**	2.56
<i>cost</i>		-0.041**	2.95	-0.130	-1.18
Dummy				0.681*	1.88
Tests of Asymmetry:					
	$\varphi_1 = \psi_1$ ^b	$\varphi_i = \psi_i$ ^c			
Limited Sample	2.68*	5.5			
Dummy for 1979:1 1982:4	1.83	6.83*			
Variable Deletion Tests:					
	<i>tight</i> ^d	<i>easy</i> ^e			
Limited Sample	2.67**	1.31			
Dummy for 1979:1 1982:4	2.66**	2.45**			
Notes: One, two and three asterisks indicate significance at the 10, 5, and 1 percent respectively.					
^a See equation (34)					
^b Wald Statistic under the null that the impact coefficient on <i>tight</i> (ψ_1) is equal to the impact coefficient on <i>easy</i> (φ_1) Distributed as $\chi(1)$.					
^c Wald Statistic under the null that the each coefficient on <i>tight</i> (ψ_i) is equal to the corresponding coefficient on <i>easy</i> (φ_i) Distributed as $\chi(3)$					
^d F-statistics under the null that the joint coefficients on <i>easy</i> (φ_i) are equal to zero					
^e F-statistics under the null that the joint coefficients on <i>tight</i> (ψ_i) are equal to zero					
Source: Authors' calculation					

Table 11. Mexico: Robustness Checks

Limited Sample			
Lags in the first stage selected with the Schwartz criterion			
Dependent Variable: Real Lending			
(Ordinary Least Square, Second stage Regression)			
		Limited Sample:	
		1985:1 1997:5	
Regressor	Notation ^a	Coefficient	T-Ratio
<i>tight</i> _{t-1}	ψ_1	-0.003***	-3.01
<i>tight</i> _{t-2}	ψ_2	-0.003**	-2.70
<i>tight</i> _{t-3}	ψ_3	0.000	-0.01
<i>tight</i> _{t-4}	ψ_4	0.001	0.44
<i>tight</i> _{t-5}	ψ_5	0.001	1.15
<i>tight</i> _{t-6}	ψ_6	0.000	0.08
<i>tight</i> _{t-7}	ψ_7	0.001	0.82
<i>easy</i> _{t-1}	φ_1	-0.001	-0.89
<i>easy</i> _{t-2}	φ_2	0.001	1.45
<i>easy</i> _{t-3}	φ_3	0.000	0.21
<i>easy</i> _{t-4}	φ_4	0.000	-0.49
<i>easy</i> _{t-5}	φ_5	0.000	-0.37
<i>easy</i> _{t-6}	φ_6	0.000	-0.38
<i>easy</i> _{t-7}	φ_7	-0.002**	-2.49
Tests of Asymmetry:			
	$\varphi_1 = \psi_1$ ^b	$\varphi_i = \psi_i$ ^c	
Limited Sample	1.63	16.6 **	
Variable Deletion Tests:			
	<i>tight</i> ^d	<i>easy</i> ^e	
Limited Sample	2.98***	1.48	
<p>Notes: One, two and three asterisks indicate significance at the 10, 5, and 1 percent respectively.</p> <p>Regression includes 3 lags of real lending, real deposits, a constant and a trend.</p> <p>The first stage regression included a dummy for the 1994 crisis.</p> <p>^a See equation (34)</p> <p>^b Wald Statistic under the null that the impact coefficient on <i>tight</i> (ψ_1) is equal to the impact coefficient on <i>easy</i> (ψ_1) Distributed as $\chi(1)$.</p> <p>^c Wald Statistic under the null that the each coefficient on <i>tight</i> (ψ_i) is equal to the corresponding coefficient on <i>easy</i> (ψ_i) Distributed as $\chi(3)$</p> <p>^d F-statistics under the null that the joint coefficients on <i>easy</i> (φ_i) are equal to zero</p> <p>^e F-statistics under the null that the joint coefficients on <i>tight</i> (ψ_i) are equal to zero</p> <p>Source: Authors' calculation</p>			

Figure 1. Banks' Capital Allocation over a easy-tight-easy cycle; $\sigma > \alpha$

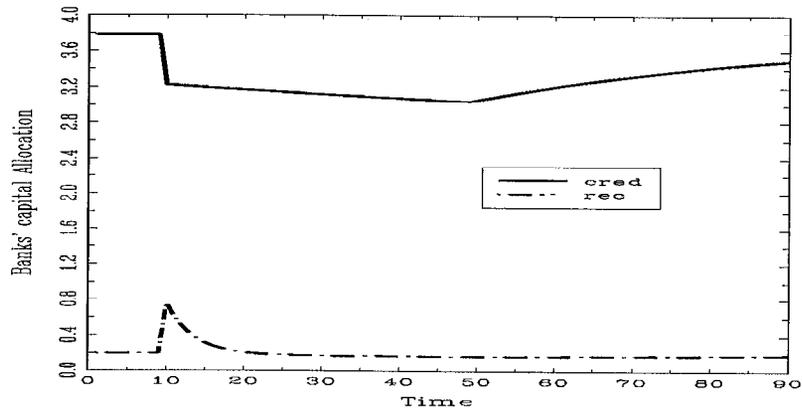


Figure 2. Average Interest Rate over an easy-tight-easy cycle; $\sigma > \alpha$

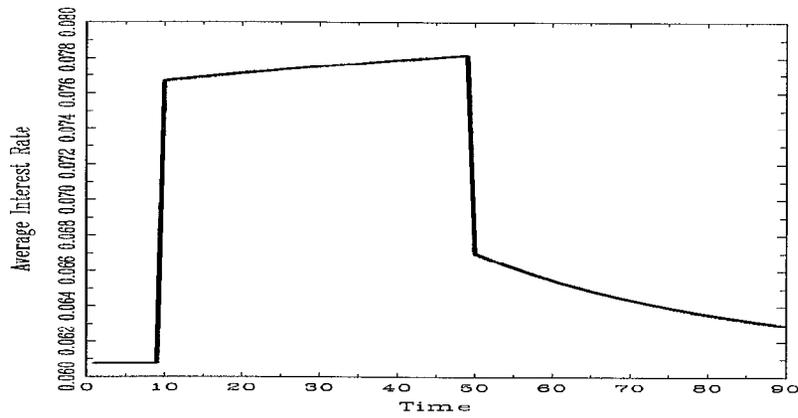


Figure 3. Banks' Capital Allocation over an easy-tight-easy cycle; $\sigma = \alpha$

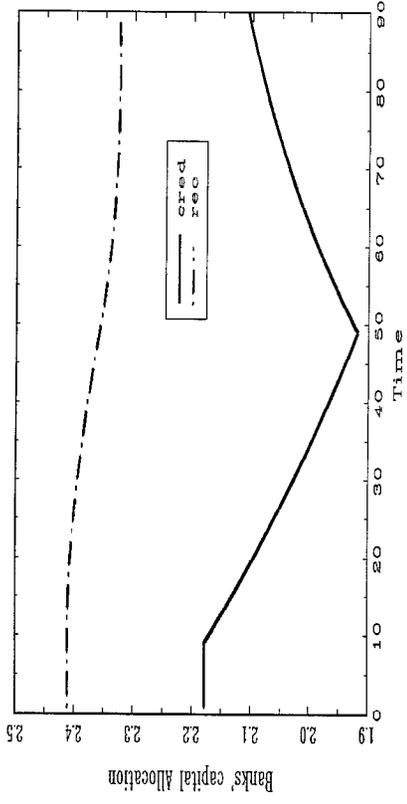
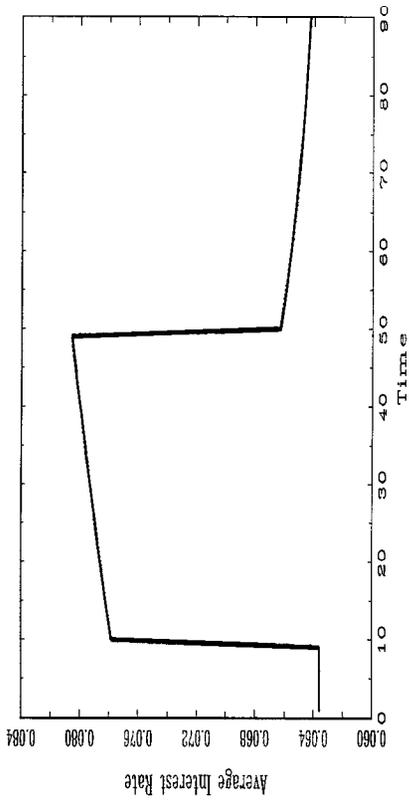


Figure 4. Average Interest Rate over an easy-tight-easy cycle; $\sigma = \alpha$



Sufficient Conditions for the Existence of an Equilibrium

The function implied by Eq. 9:

$$W_i = \frac{y_i - a_i r_d - \alpha \beta a_i \left[\frac{1}{n} \sum_{j=1}^n \max(W_j; 0) \right] - \alpha (1 - \beta) b_i \max(W_i; 0)}{(\rho + \lambda_i)} \quad (1)$$

does not satisfy the Blackwell's sufficient conditions for a contraction. However, it is easy to show that, under certain restrictions on the parameters, that mapping is a contraction mapping, that guarantees the existence of a unique fixed point.

We can define Eq. 1 as a function $T : R^n \rightarrow R^n$. We can also define a sort of aggregate average discount factor, ψ , dependent on both the actual discount factor, ρ , and the parameters describing death processes, λ_i , as:

$$\psi = \frac{\alpha (1 - \beta)}{\rho} + \frac{1}{n} \sum_{i=1}^n \frac{\alpha \beta (\rho + \sigma + \lambda_i)}{(\rho + \sigma) (\rho + \lambda_i)}$$

Now consider two vectors X and Y elements of R^n and a metric d , where $d(X, Y) = \|X - Y\| = \sum_{i=1}^n |x_i - y_i|$, then we can write the following proposition:

Equilibrium Existence

If $\psi < 1$, then for any $X, Y \in R^n$, $\exists \varphi \in (0, 1)$ such that: $\|T(X) - T(Y)\| \leq \varphi \|X - Y\|$

The i^{th} element of $\Delta T = T(X) - T(Y)$ reads as:

$$\Delta T_i = -A_i \left[\frac{1}{n} \sum_{j=1}^n \max(x_j; 0) - \frac{1}{n} \sum_{j=1}^n \max(y_j; 0) \right] - B_i [\max(x_i; 0) - \max(y_i; 0)]$$

where:

$$A_i = \frac{a_i \alpha \beta}{(\rho + \lambda_i)} = \frac{\alpha \beta (\rho + \sigma + \lambda_i)}{(\rho + \sigma) (\rho + \lambda_i)} > 0$$

$$B_i = \frac{b_i}{(\rho + \lambda_i)} = B = \frac{\alpha (1 - \beta)}{\rho} > 0$$

then:

$$\|T(X) - T(Y)\| = \sum_{i=1}^n \left| -B [\max(x_i; 0) - \max(y_i; 0)] - A_i \left[\frac{1}{n} \sum_{j=1}^n \max(x_j; 0) - \frac{1}{n} \sum_{j=1}^n \max(y_j; 0) \right] \right|$$

changing all signs within the absolute value, and applying the triangular inequality we get:

$$\|T(X) - T(Y)\| \leq \sum_{i=1}^n |B [\max(x_i; 0) - \max(y_i; 0)]| \quad (2)$$

$$+ \sum_{i=1}^n \left| A_i \left[\frac{1}{n} \sum_{j=1}^n \max(x_i; 0) - \frac{1}{n} \sum_{j=1}^n \max(y_i; 0) \right] \right| \quad (3)$$

now, we know that:

$$|\max(x_i; 0) - \max(y_i; 0)| \leq |x_i - y_i|$$

thus:

$$\sum_{i=1}^n |B [\max(x_i; 0) - \max(y_i; 0)]| =$$

$$\sum_{i=1}^n B |[\max(x_i; 0) - \max(y_i; 0)]| \leq \sum_{i=1}^n B |x_i - y_i|$$

and finally:

$$\sum_{i=1}^n B |x_i - y_i| = B \cdot \sum_{i=1}^n |x_i - y_i| \quad (4)$$

Now consider the second term in Eq. 2, given $A_i > 0$ for any i , we have:

$$\begin{aligned} \sum_{i=1}^n A_i \left| \frac{1}{n} \sum_{j=1}^n \max(x_i; 0) - \frac{1}{n} \sum_{j=1}^n \max(y_i; 0) \right| \\ = \sum_{i=1}^n A_i \left| \frac{1}{n} \sum_{j=1}^n [\max(x_i; 0) - \max(y_i; 0)] \right| \end{aligned}$$

for the triangular inequality we have:

$$\begin{aligned} \sum_{i=1}^n A_i \left| \frac{1}{n} \sum_{j=1}^n [\max(x_i; 0) - \max(y_i; 0)] \right| \\ \leq \sum_{i=1}^n A_i \cdot \frac{1}{n} \sum_{j=1}^n |\max(x_i; 0) - \max(y_i; 0)| \end{aligned}$$

and, as before:

$$\sum_{i=1}^n A_i \cdot \frac{1}{n} \sum_{j=1}^n |\max(x_i; 0) - \max(y_i; 0)| \leq \left(\sum_{i=1}^n A_i \right) \cdot \left(\frac{1}{n} \sum_{j=1}^n |x_i - y_i| \right) \quad (5)$$

Now, putting together Eq. 4 and Eq. 5 we obtain:

$$\|T(X) - T(Y)\| \leq B \cdot \sum_{i=1}^n |x_i - y_i| + \left(\sum_{i=1}^n A_i \right) \cdot \left(\frac{1}{n} \sum_{j=1}^n |x_i - y_i| \right)$$

and a sufficient condition for T to be a contraction mapping can be written as:

$$B \cdot \sum_{i=1}^n |x_i - y_i| + \left(\sum_{i=1}^n A_i \right) \cdot \left(\frac{1}{n} \sum_{j=1}^n |x_j - y_j| \right) \leq \varphi \sum_{j=1}^n |x_j - y_j|$$

that is equivalent to:

$$B + \frac{1}{n} \sum_{i=1}^n A_i \leq \varphi$$

or:

$$\frac{\alpha(1-\beta)}{\rho} + \frac{1}{n} \sum_{i=1}^n \frac{\alpha\beta(\rho + \sigma + \lambda_i)}{(\rho + \sigma)(\rho + \lambda_i)} \leq \varphi < 1$$

q.d.e.

Monotonicity of the Surplus Function

In the following proposition we state the conditions for the monotonicity of the surplus function.

Monotonicity

$\forall i \in [1, 2, \dots, n]$, and $\forall k < i$, if:

$$\begin{aligned} \lambda_i &> \lambda_{i-k} \\ y_i &> y_{i-k} \\ \frac{y_i}{y_{i-k}} &> \frac{\lambda_i}{\lambda_{i-k}} \end{aligned}$$

then $W_i > W_{i-k}$.

Assume that $W_i < W_{i-k}$ then:

$$\begin{aligned} &= \frac{y_{i-k} - a_{i-k}r_d - \alpha\beta a_{i-k}\frac{1}{n}\sum_{j=1}^n \max(W_j; 0) - \alpha(1-\beta)b_{i-k}\max(W_{i-k}; 0)}{\rho + \lambda_{i-k}} && W_{i-k} - W_i && (6) \\ &= \frac{y_i - a_i r_d - \alpha\beta a_i \frac{1}{n} \sum_{j=1}^n \max(W_j; 0) - \alpha(1-\beta)b_i \max(W_i; 0)}{\rho + \lambda_i} > 0 && && (7) \end{aligned}$$

substituting we can see that for any j :

$$\frac{\alpha(1-\beta)b_j \max(W_j; 0)}{\rho + \lambda_j} = \frac{\alpha(1-\beta)\max(W_j; 0)}{\rho}$$

then, by assumption:

$$\frac{\alpha(1-\beta)\max(W_{i-k}; 0)}{\rho} > \frac{\alpha(1-\beta)\max(W_i; 0)}{\rho}$$

that means that:

$$W_{i-k} - W_i < \frac{y_{i-k} - a_{i-k}r_d - \alpha\beta a_{i-k}\frac{1}{n}\sum_{j=1}^n \max(W_j; 0)}{\rho + \lambda_{i-k}} - \frac{y_i - a_i r_d - \alpha\beta a_i \frac{1}{n} \sum_{j=1}^n \max(W_j; 0)}{\rho + \lambda_i}$$

Also by assumption, we know that:

$$\frac{y_i}{\rho + \lambda_i} > \frac{y_{i-k}}{\rho + \lambda_{i-k}}$$

thus:

$$W_{i-k} - W_i < \frac{-a_{i-k}r_d - \alpha\beta a_{i-k}\frac{1}{n}\sum_{j=1}^n \max(W_j; 0)}{\rho + \lambda_{i-k}} - \frac{-a_i r_d - \alpha\beta a_i \frac{1}{n} \sum_{j=1}^n \max(W_j; 0)}{\rho + \lambda_i} \quad (8)$$

we can rewrite the right hand side of Eq.8 as:

$$\left[\frac{1}{n} \sum_{j=1}^n \max(W_j; 0) + r_d \right] \cdot \left(\frac{a_i}{\rho + \lambda_i} - \frac{a_{i-k}}{\rho + \lambda_{i-k}} \right)$$

now, we can show that the second term in this expression is negative, and that contradicts the initial assumption $W_i < W_{i-k}$.

$$\frac{a_i}{\rho + \lambda_i} < \frac{a_{i-k}}{\rho + \lambda_{i-k}} \Leftrightarrow \frac{\rho + \sigma + \lambda_i}{(\rho + \sigma)(\rho + \lambda_i)} < \frac{\rho + \sigma + \lambda_{i-k}}{(\rho + \sigma)(\rho + \lambda_{i-k})}$$

that is equivalent to:

$$\frac{\rho + \sigma + \lambda_i}{\rho + \lambda_i} < \frac{\rho + \sigma + \lambda_{i-k}}{\rho + \lambda_{i-k}}$$

that, given $\lambda_i > \lambda_{i-k}$, is verified for any ρ , and σ positive.

Comparative Static Results

Effects of r_d on Surplus

$\frac{\partial W_i}{\partial r_d} < 0$ for any i .

Start from the following equation:

$$(\rho + \lambda_i) W_i = y_i - a_i r_d - \alpha \beta a_i \left[\frac{1}{n} \sum_{j=1}^n \max(W_j; 0) \right] - \alpha (1 - \beta) b_i \max(W_i; 0) \quad (9)$$

First consider that for any $W_i < 0$, we have $\frac{\partial \max(W_i; 0)}{\partial r_d} = 0$, so that we can write:

$$\frac{1}{n} \sum_{j=1}^n \frac{\partial \max(W_j; 0)}{\partial r_d} = \frac{1}{n} \sum_{j|W_j \geq 0} \frac{\partial \max(W_j; 0)}{\partial r_d} = \frac{1}{n} \sum_{j|W_j \geq 0} \frac{\partial W_j}{\partial r_d} \quad (10)$$

for the $W_i \geq 0$ Eq. 9 can be rewritten as:

$$W_i = y_i - \frac{a_i}{c_i} r_d - \alpha \beta \frac{a_i}{c_i} \left[\frac{1}{n} \sum_{j=1}^n \max(W_j; 0) \right]$$

where:

$$c_i = (\rho + \lambda_i) + \alpha (1 - \beta) b_i$$

and, using Eq. 10 we have:

$$\frac{\partial W_i}{\partial r_d} = -\frac{a_i}{c_i} - \alpha \beta \frac{a_i}{c_i} \frac{1}{n} \sum_{j|W_j \geq 0} \frac{\partial W_j}{\partial r_d} \quad (11)$$

summing for all the non negative W_i , and solving, we get:

$$\sum_{j|W_j \geq 0} \frac{\partial W_j}{\partial r_d} = -\frac{\sum_{j|W_j \geq 0} \frac{a_j}{c_j}}{1 + \frac{\alpha \beta}{n} \sum_{j|W_j \geq 0} \frac{a_j}{c_j}} < 0 \quad (12)$$

from Eq. 11 we can write:

$$\frac{\partial W_i}{\partial r_d} \cdot \frac{c_i}{a_i} = -1 - \alpha \beta \frac{1}{n} \sum_{j|W_j \geq 0} \frac{\partial W_j}{\partial r_d} \quad (13)$$

that is $\frac{\partial W_i}{\partial r_d} \cdot \frac{c_i}{a_i}$ is a constant respect to i . Thus, given $\frac{c_i}{a_i} > 0$, from Eq. 12 and 13, it has to be:

$$\frac{\partial W_i}{\partial r_d} < 0$$

q.d.e.

Effect of r_d on r_i $\frac{\partial r_i}{\partial r_d} > 0$ for any i .

We can write the lending rate for projects of type i as:

$$r_i = (\rho + \lambda_i) \beta W_i + \left(1 + \frac{\lambda_i}{\sigma + \rho}\right) \rho D \quad (14)$$

then we have:

$$\frac{\partial r_i}{\partial r_d} = (\rho + \lambda_i) \beta \frac{\partial W_i}{\partial r_d} + \left(1 + \frac{\lambda_i}{\sigma + \rho}\right) \rho \frac{\partial D}{\partial r_d} \quad (15)$$

now, from Eq. 4 and the previous proposition we know that:

$$\frac{\partial \rho D}{\partial r_d} = 1 + \alpha \beta \frac{1}{n} \sum_{j|W_j \geq 0} \frac{\partial W_j}{\partial r_d} > 0 \quad (16)$$

then, using Eq. 11, Eq. 15 becomes:

$$\frac{\partial r_i}{\partial r_d} = \left(1 + \frac{\lambda_i}{\sigma + \rho} - \frac{a_i}{c_i} (\rho + \lambda_i) \beta\right) \left(1 + \alpha \beta \frac{1}{n} \sum_{j|W_j \geq 0} \frac{\partial W_j}{\partial r_d}\right) \quad (17)$$

where we can rewrite the first term in Eq. 17 as:

$$\frac{(\rho + \alpha)(1 - \beta)(\rho + \sigma + \lambda_i)}{[\rho + \alpha(1 - \beta)](\rho + \sigma)} > 0$$

which, together with Eq. 16, means that:

$$\frac{\partial r_i}{\partial r_d} > 0$$

q.d.e.

On The Timing of The Bankruptcy Procedure

In this section we show that in the presence of a positive flow cost f , banks never choose to start the recovery process before a project is bankrupt. Let L_i be the value to the bank of an active project for which the bank started a bankruptcy procedure involving a flow cost f , the equation reads:

$$\rho L_i = r_i - f + \lambda_i (B - L_i) + \sigma (\max [D, L_i] - L_i) \quad (18)$$

where D is the value of one unit of deposits:

$$\rho D = r_d + \alpha \left[\frac{1}{n} \sum_{j=1}^n \max(L_j; D) - D \right]$$

Then we have the following proposition:

For dt arbitrary small, any $f > 0$, implies $C_i > L_i$.

Consider the problem in discrete time, Eq. 18 reads as

$$\begin{aligned} \rho dt L_i &= r_i dt - f dt + \lambda_i dt (B - L_i) \\ &+ \sigma dt (\max [D, L_i] - L_i) + \lambda_i dt \sigma dt (D - L_i) \end{aligned}$$

or, dividing by dt

$$\rho L_i = r_i - f + \lambda_i (B - L_i) + \sigma (\max [D, L_i] - L_i) + \lambda_i \sigma dt (D - L_i)$$

that means:

$$\lim_{dt \rightarrow 0} \rho L_i = r_i - l + \lambda_i (R_i - L_i) + \sigma (\max [D, L_i] - L_i) \quad (19)$$

However, in steady state, the last term in Eq. 19 is always zero, so that we have:

$$\rho L_i = r_i - l + \lambda_i (R_i - L_i)$$

that is smaller than ρC_i for any $l > 0$. q.d.e.

REFERENCES

- Aspe, P., (1993), *Economic Transformation the Mexican Way*, MIT Press: Cambridge.
- Ball, L., and N. G. Mankiew, (1994), "Asymmetric Price Adjustment and Economic Fluctuations", *Economic Journal* Vol. 104.
- Bernanke, B and M. Gertler, (1995), "Inside the Black Box: The Credit Channel of Monetary Policy Transmission" *The Journal of Economic Perspectives*, Vol. 9, Num. 4, pp. 27-48.
- Bernanke, B. and A. Blinder, (1988), "Credit, Money, and Aggregate Demand", *American Economic Review Papers and Proceedings*, pp.435-439.
- Bernanke, B. and A. Blinder, (1992), "The Federal Funds Rate and the Channels of Monetary transmission", *American Economic Review*, Vol. 82 pp.901-921.
- Burdett, K, and R. Wright, (1998), "Two-Sided Search", *Review of Economic Dynamics*, forthcoming.
- Bhattacharya, S. and A, Thakor, (1993), "Contemporary Banking Theory" *Journal of Financial Intermediation*, 3, pp. 2-50.
- Caplin, A. and J. Leahy (1991), "State-Dependent Pricing and the Dynamics of Money and Output", *Quarterly Journal of Economics*, August, pp. 683-708.
- Cover, J.P. (1992), "Asymmetric Effects of Positive and Negative Money-Supply Shocks", *Quarterly Journal of Economics*, Vol. CVII, pp. 1261-82
- Diamond, D. (1984), "Financial Intermediation and Delegated Monitoring", *Review of Economic Studies*, Vol. 51, pp. 393-414.
- Diamond, P. (1982), "Wage Determination and Efficiency in Search Equilibrium." *Review of Economic Studies*. Vol. 49: 217-227.
- Garibaldi,P.(1997), "The Asymmetric Effect of Monetary Policy on Job creation and Destruction", *IMF Staff Papers*, Vol.44, pp. 557-84.
- Garibaldi,P.(1998), "Job Flow Dynamics and Firing restrictions" *European Economic Review*, Vol. 42, No. 2 245-275.
- Greenbaum, S., Kanatas, G. and I. Venezia (1989), "Equilibrium Loan Pricing under the Bank-Client Relationship." *Journal of Banking and Finance*. Vol. 13, pp. 221-235.
- Hannan, T. and A. Berger(1991), "The Rigidity of Prices: Evidence form the Banking Industry." *American Economic Review*. Vol. 81, pp. 938-945.

- Hubbard, R. (1994), "Is There a 'Credit Channel' for Monetary Policy", *NBER Working Paper*, No. 4977.
- Jackman, R. and J. Sutton, (1982), "Imperfect Capital Markets and the Monetarist Black Box: Liquidity Constraints, Inflation and the Asymmetric Effects of Interest Rate Policy," *The Economic Journal*, Vol. 92, pp. 108-28.
- Kashyap A. and J. Stein,(1993), "Monetary Policy and Bank Lending", *NBER Working Paper*, No. 4317.
- Mishkin, F. (1995) "Symposium on the monetary transmission mechanism" *Journal of Economic Perspectives* Vol. 9 pp. 3-10.
- Mortensen, D. and C. Pissarides,(1994), "Job Creation and Job Destruction in the Theory of Unemployment", *Review of Economic Studies*, Vol. 61 (July) pp. 397-415.
- Mortensen, D. and C. Pissarides,(1998), "New Developments in Models of Search in the Labor Market", forthcoming *Handbook of Macroeconomics*, North Holland, Amsterdam.
- Neumark, D. and S. Sharpe, (1992), "Market Structure and the Nature of Price Rigidity: Evidence from the Market for Consumer Deposits", *Quarterly Journal of Economics* pp. 657-681.
- Pesaran, M.H., Y. Shin, and R. Smith (1996), "Testing for the Existence of a Long-Run relationship", DAE Working Paper No. 9622, Department of Applied Economics, University of Cambridge.
- Scholnick, B. (1996), "Asymmetric adjustment of commercial bank interest rates: evidence from Malaysia and Singapore", *Journal of International Money and Finance*, Vol. 15, No. 3, pp. 485-96.
- Sharma, S. (1987), "On the Existence and Uniqueness of Value Functions in Models of Labor Market Dynamics", *Economic Letters* Vol. 24, pp. 349-352.
- Stiglitz, J. and A. Weiss, (1981), "Credit Rationing in Markets with Imperfect Information", *American Economic Review*, Vol.71, pp.393-410.
- Tirole, J. (1988), *The Theory of Industrial Organization* , Cambridge: MIT Press.