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**Unemployment, Capital-Labor Substitution, and Economic Growth**

Prepared by Robert Rowthorn<sup>1</sup>

Authorized for distribution by Graham Hacche

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**Abstract**

This paper discusses the influence of economic growth on the equilibrium unemployment rate (NAIRU). It examines how income distribution and the NAIRU are influenced by capital formation, technical progress, and labor force expansion, and how these factors' impact depends on the elasticity of substitution between capital and labor. The paper distinguishes between the short-run NAIRU when capital stock is exogenous, and the long-run NAIRU when it is endogenous. It also considers how the analysis must be modified to take into account Keynesian ideas concerning the role of aggregate demand. It concludes that unless the capital stock grows in line with labor supply in efficiency units, the short-run NAIRU will increase, reducing the scope for demand stimulation.

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Author's E-Mail Address: Bob.Rowthorn@econ.cam.ac.uk

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## I. INTRODUCTION: THE NAIRU APPROACH

The current analysis of unemployment in Europe is based on what used to be called the “conflict” theory of inflation, renamed by Blanchard (1986) the “battle of the mark-ups”. The origins of this theory go back at least to Rowthorn (1977) and probably earlier. Indeed, a version of this theory is implicit in the work of Friedman (1968) and Lucas and Rapping (1969).

The theory can be summarized as follows:

- Unanticipated inflation is the outcome of inconsistent claims on total output.
- Unanticipated inflation cannot be permanently sustained because it leads to accelerating and ultimately explosive price increases.
- To prevent unanticipated inflation, ex-ante claims on total output must be mutually consistent and add-up to ex-post total output. Consistency is brought about through variations in the level of economic activity, in particular through unemployment and its influence on wage (and price) formation.
- The “Non-accelerating Inflation Rate of Unemployment” (NAIRU) is that level of unemployment which eliminates unanticipated inflation.

There are many different claims on total output, such as taxes or payments for imports, but we shall focus exclusively on factor income: wages and profits. The inconsistency of wage and profit claims arises when wage setting and price setting are not fully coordinated. Given their expectations about future prices in general, bargainers in the labor market negotiate money wages, and when this has been done individual firms then set their own prices to maximize profits.

The collective outcome of these individual decisions is the price level which workers face when they come to spend their wages. If the new price level is higher than was anticipated in the wage bargain, then workers will experience a lower than expected real wage and, by implication, real profits will be higher. Thus, unanticipated inflation arises when the pricing behavior of firms is collectively incompatible with what has been agreed in the labor market.

Unanticipated inflation can be eliminated by increasing unemployment and thereby inducing (unionized) labor to accept a lower real wage. When unemployment is equal to the NAIRU, the real wages negotiated in the labor market are exactly consistent with the real profits which firms obtain in the product market.

Most discussion of the NAIRU is short-term in character and does not depend in any obvious way on longer-term factors, such as technical progress and the growth of capital

stock and labour supply. The purpose of this paper is to analyse the circumstances under which such factors are important. The paper focuses particularly on the work of Layard, Nickell and Jackman (1991)), where longer term issues are finessed by means of very particular assumption, and that of Blanchard (1997, 1998), who confronts longer term issues directly, but relies on a similar unrealistic assumption. In both cases, the elasticity of substitution between capital and labour ( $\sigma$ ) turns out to be a crucial parameter. The analysis of Layard Nickell and Jackman (LNJ) is predicated on the assumption that  $\sigma = 1$ , whilst that of Blanchard presumes that  $\sigma \geq 1$ . Yet the evidence strongly suggests that  $\sigma$  is considerably less than unity. This finding has important implications which are explored at length in this paper.

The structure of the paper is as follows. Since the magnitude of  $\sigma$  plays such an important role in the subsequent analysis, the first section contains a survey of the relevant econometric evidence. Next, a simple model is presented which captures the main features of the NAIRU approach and allows us to explore the influence of longer-term factors on unemployment and profits. This model reveals the crucial importance of the parameter  $\sigma$ . This is followed by a discussion of the long-run relationship between growth, unemployment and profitability. There is then a careful examination of Blanchard's recent work on the "medium-term". The paper concludes with a discussion of how the analysis ties in with Keynesian ideas concerning the role of demand.

## II. ELASTICITY OF SUBSTITUTION: THE EVIDENCE<sup>2</sup>

Rowthorn (1999) reports the results of 33 econometric studies which have estimated the value of  $\sigma$ , or from which estimates of this elasticity can be derived. Most of these studies contain a large number of estimates referring to different industries, regions or countries, or to alternative equation specifications. Their findings are summarize using employment-weighted averages or medians. Out of a total of 33 studies, in only 7 cases does the summary value exceed 0.8, and the overall median of the summary values (median of the medians) is equal to 0.58.<sup>3</sup>

Additional evidence can be gleaned from published econometric studies which estimate the relationship between employment and wages. With given capital stock, suppose that a one

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<sup>2</sup> This section draws heavily on Rowthorn (1996, 1999)

<sup>3</sup> Note that substitution between capital and labor may occur indirectly because consumers switch between goods whose techniques of production have different capital-intensities. Such a switch may be induced because relative output prices alter when relative factor prices change. The possibility of indirect substitution should mean that the aggregate elasticity of substitution is greater than engineering-type studies at the firm-level suggest. However, even highly aggregated studies normally reveal an elasticity of substitution which is well below unity.

percent increase in the real wage rate leads to a long-run reduction of  $\epsilon$  percent in employment. Then, if the production function is CES, the following relationships holds

$$\epsilon = \frac{\sigma(1 - 1/\eta)}{(s - 1/\eta)} \geq \frac{\sigma}{s} \quad (2)$$

where  $s$  is the share of profits in output. This relationship has implications which are not widely recognized. With the values of  $\sigma$  normally assumed by economists, it implies a small reduction in wages will lead to a large increase in employment. For example, many economists, including Blanchard, assume that  $\sigma = 1$  and  $s = 0.3$ . With perfect competition in the product market ( $\eta$  is infinite), these assumptions imply that  $\epsilon = 3.3$ . With a modest imperfection in the product market, so that  $\eta$  is equal to 10, the implied value of  $\epsilon$  is 7.4. These values of  $\epsilon$  are totally implausible and are much larger than the figures derived from econometric estimates of labor demand. They imply that a reduction in the real wage rate of between 1½ and 3 percent would be enough to eliminate the whole of European unemployment using the existing quantity of capital and existing technology.

Table 1 reports the estimates of  $\epsilon$  derived from three major econometric studies of employment in industrial countries.<sup>4</sup> Using assumed values for  $s$  and  $\eta$ , we can convert these into estimates of  $\sigma$ , using the following formula,

$$\sigma = \frac{\epsilon(s - 1/\eta)}{(1 - 1/\eta)} \geq \epsilon s \quad (3)$$

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<sup>4</sup>These studies estimate labor demand equations of the form

$$\log(N/K) = \text{constant} - \sum_{i=0}^m \hat{a}_i \log(W/P)_{-i} + \sum_{j=1}^n \hat{b}_j \log(N/K)_{-j} + \sum_{k=1}^s \hat{c}_k z_k$$

where  $N$ ,  $K$ , and  $W/P$  are employment, capital and the real wage rate, respectively, and the  $z_k$  are other variables; the symbol  $\hat{\phantom{x}}$  indicates that coefficients are estimated. The 'other variables' are time, the deviation of output from trend, and the acceleration rate of nominal wages. This equation implies that, holding  $K$  constant, the (absolute) elasticity of employment ( $N$ ) with respect to the real wage ( $W/P$ ) is given by

$$\hat{\epsilon} = \frac{\sum \hat{a}_i}{1 - \sum \hat{b}_j}$$

Table 1. Estimates of the Elasticity of Labor Demand ( $\epsilon$ )

	L N J	N S	B L N
Australia	0.62	0.59	0.77
Austria	0.27	0.75	0.73
Belgium	0.59	2.38	0.88
Canada	5.00	2.11	0.42
Denmark	0.69		0.61
Finland	0.06	0.56	-0.71
France	0.28	0.50	0.61
Germany	1.71	2.17	0.83
Ireland	0.53	0.35	1.03
Italy	0.30	0.35	0.37
Japan	0.73	0.88	1.03
Netherlands	0.60	0.78	1.10
New Zealand	0.87		
Norway	0.43	0.07	0.19
Spain	1.38		
Sweden	0.17	1.36	0.65
Switzerland	1.68	3.41	0.63
United Kingdom	0.97	1.50	0.63
United States	0.32	0.70	0.48
Median	0.60	0.76	0.63

Note: Elasticities are calculated from estimated marginal revenue product equations as reported in the Appendix to Chapter 9 of Layard, Nickell and Jackman (1991), using the formula given in footnote 14.

Key: L N J = Layard, Nickell and Jackman (1991); N S = Newell and Symons (1985); B L N = Bean, Layard and Nickell (1986).

Table 2 shows the result of this calculation. Two sets of estimates for  $\sigma$  are shown. One assumes that  $s = 0.4$  and  $\eta = \infty$ , and the other that  $s = 0.3$  and  $\eta = 10$ . The picture revealed by this table is remarkable. The values of  $\sigma$  are extremely low and in almost every case this parameter is well below unity. In the first panel, only three out of fifty-two estimates of  $\sigma$  exceed 0.5 and in the second panel only nine exceed this figure. It is possible that these estimates are biased systematically downwards, but the error would have to be truly gigantic to justify the conventional assumption that  $\sigma$  is greater than or equal to unity.

Table 2. Implied Values of the Elasticity of Substitution between Labor and Capital ( $\sigma$ )

	(a)			(b)		
	LNJ	NS	BLN	LNJ	NS	BLN
Australia	0.14	0.13	0.17	0.25	0.24	0.31
Austria	0.06	0.17	0.16	0.11	0.30	0.29
Belgium	0.13	0.53	0.20	0.24	0.95	0.35
Canada	1.11	0.47	0.09	2.00	0.84	0.17
Denmark	0.15		0.14	0.28		0.24
Finland	0.01	0.12	-0.16	0.02	0.22	-0.28
France	0.06	0.11	0.14	0.11	0.20	0.24
Germany	0.38	0.48	0.18	0.68	0.87	0.33
Ireland	0.12	0.08	0.23	0.21	0.14	0.41
Italy	0.07	0.08	0.08	0.12	0.14	0.15
Japan	0.16	0.20	0.23	0.29	0.35	0.41
Netherlands	0.13	0.17	0.24	0.24	0.31	0.44
New Zealand	0.19			0.35		
Norway	0.12	0.02	0.04	0.21	0.03	0.08
Spain	0.31			0.55		
Sweden	0.04	0.30	0.14	0.07	0.54	0.26
Switzerland	0.37	0.76	0.14	0.67	1.36	0.25
United Kingdom	0.22	0.33	0.14	0.39	0.60	0.25
United States	0.07	0.16	0.11	0.13	0.28	0.19
Median	0.13	0.17	0.14	0.24	0.30	0.25

The estimates in this table are derived from Table 1 using the following parameter values

	panel (a)	panel (b)
profit share ( $s$ )	0.3	0.4
price elasticity ( $\eta$ )	10	infinite

### III. THE BASIC MODEL

To facilitate the exposition, we shall assume an economy in which all firms are identical<sup>5</sup>. They have the same amount of capital and the same price elasticity of demand for their output. Each firm has the following constant returns to scale CES production function,

$$Y = \left[ \alpha (\Lambda_N N)^{(\sigma-1)/\sigma} + (1 - \alpha) (\Lambda_K K)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \quad (4)$$

where  $\sigma > 0$  is the elasticity of substitution between labor and capital;  $N$  and  $K$  are labor and capital, and  $\Lambda_N$  and  $\Lambda_K$  are indices of the technical efficiency with which these factors are utilized. Labor-augmenting and capital-augmenting technical progress are indicated by an increase in  $\Lambda_N$  and  $\Lambda_K$  respectively. With no loss of generality, we shall assume that  $\alpha$  remains constant, and that biased technical progress is indicated by a change in the ratio  $\Lambda_N/\Lambda_K$ .<sup>6</sup> Apart from notational differences, this is equivalent to the production function used by Blanchard (1997, 1998) and also Rowthorn (1996, 1999).

#### A. The Demand for Labor

Suppose that for each firm the (absolute price) elasticity of demand for its output is equal to  $\eta$ . Thus, a one percent fall in the this firm's relative price will lead to an  $\eta$  percent increase in the amount sold. Equilibrium requires that  $\eta > 1$ . To maximize profits the firm will produce to the point where the ratio of price to marginal cost is equal to  $(1+m)$ , where  $m = 1/(\eta - 1)$ . Assuming

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<sup>5</sup> A more detailed description of the model and derivation of the main equations is contained in the appendix.

<sup>6</sup> Provided  $\sigma \neq 1$  and is constant, the precise value of  $\alpha$  is arbitrary. Any modification in  $\alpha$  can be expressed by a suitable change in the indices  $\Lambda_N$  and  $\Lambda_K$ . Thus, when considering biased technical progress we can regard  $\alpha$  as constant. Note this is not true when  $\sigma$  is exactly equal to unity. In this case the production function takes the form  $Y = \Lambda N^\alpha K^{(1-\alpha)}$ , and capital augmenting technical progress cannot be distinguished from labor augmenting technical progress.



$$\frac{W}{\Lambda_N} = \frac{\alpha}{(1+m)} \left( \frac{Y}{\Lambda_N N} \right)^{\frac{1}{\sigma}} \quad (5)$$

that all firms follow this rule, real wages and employment will satisfy the following equation, where  $W$  is the real wage rate. This can be interpreted as an implicit labor demand function. It implies that the share of wages is given by,

$$\frac{WN}{Y} = \frac{\alpha}{(1+m)} \left( \frac{Y}{\Lambda_N N} \right)^{\frac{(1-\sigma)}{\sigma}} \quad (6)$$

The above equations can be also be expressed as follows

$$\frac{W}{\Lambda_N} = \frac{\alpha}{(1+m)} \left[ \alpha + (1 - \alpha) \left( \frac{k}{1-u} \right)^{(\sigma-1)/\sigma} \right]^{1/(\sigma-1)} \quad (7)$$

and

$$\frac{WN}{Y} = \frac{\alpha}{(1+m)} \left[ \alpha + (1 - \alpha) \left( \frac{k}{1-u} \right)^{(\sigma-1)/\sigma} \right]^{-1} \quad (8)$$

where  $L$  is the total labor force,  $u = 1 - N/L$  is the unemployment rate, and

$$k = \frac{\Lambda_K K}{\Lambda_N L} \quad (9)$$

Note that  $k$  is the ratio of capital stock to labor force (i.e. employment plus unemployment) with measured in efficiency units. As  $\sigma \rightarrow 1$ , the right-hand side of equation (5) converges to  $\alpha/(1+m)$ .

## B. Real Wages and Unemployment

Suppose the objective is to maintain a constant rate of unemployment. The labor demand equations indicate how wages must respond to achieve this objective when there is a change in  $k$ . Suppose that  $k$  falls for some reason, such as faster growth in the labor supply, slower growth in capital stock or a labor-saving bias in technical progress. Equation (6) indicates that to prevent unemployment rising, there must be a reduction in the real wage rate *per efficiency unit of labor*  $W/\Lambda_N$ . This is true no matter what is the

value of  $\sigma$ .<sup>7</sup> The evolution of  $\Lambda_N$  through time can be seen as the main trend factor underlying the growth of real wages per worker, whilst variations in the ratio  $W/\Lambda_N$  are modifications in response to events such as capital scarcity or expansion of the labor force. The term “wage cuts” in the following discussion must be interpreted with this observation in mind. This term will be used to signify a reduction in wages in relation to their underlying trend. This may or may not imply an absolute reduction in the real wage per worker.

Equation (7) indicates what happens to the *share* of wages in total output. The answer depends on the elasticity of substitution. If  $\sigma < 1$ , the lower wage rate required to preserve employment implies a lower share of wages in total output. Thus, there is no ambiguity in a call for “wage moderation”. The situation is more complex if  $\sigma > 1$ , for in this case the wage rate and the wage share move in opposite directions. To maintain employment requires a reduction in the real wage rate, but this will be accompanied by an increased share of wages in total output. Trade unions will (correctly) see themselves as exercising restraint by holding down wages, whilst national income statistics will suggest a continuing squeeze on the profit share. Indeed, the greater the wage cuts which unions accept, the higher will be the share of wages and the lower the share of profits. Since empirical estimates suggest that  $\sigma$  is normally less than unity, this paradoxical situation is unlikely to arise in practice (see below).

### C. Two Versions of Wage Determination

In his recent papers on unemployment, Blanchard (1997, 1998) postulates a wage-setting function of the following kind<sup>8</sup>,

$$\frac{W}{\Lambda_N} = -\beta u + z \quad (10)$$

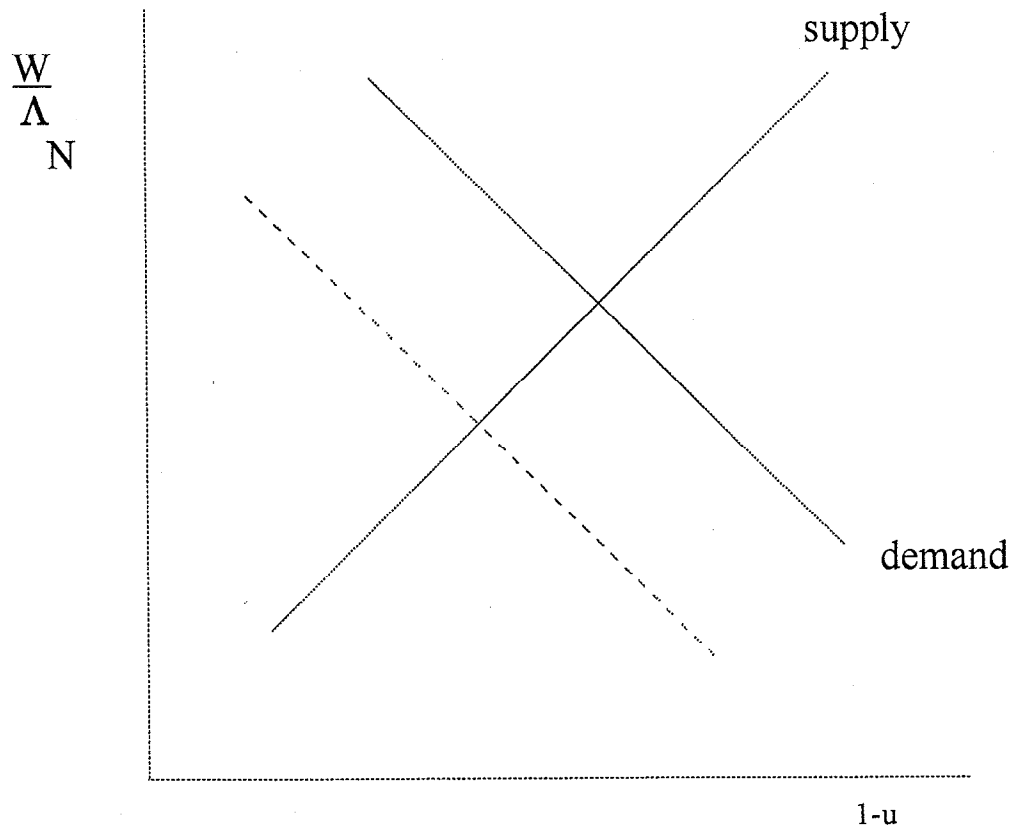
where  $z$  is an index of trade union pushiness and other pertinent variables. This function is intended to summarize the outcome of bargaining between firms and workers under given labor market conditions. Combining equations (6) and (9) we can determine  $W/\Lambda_N$  and  $1 - u$  as functions of  $k$ . If  $k$  is reduced, the result is a lower “efficiency” wage and more unemployment. This is illustrated in Fig. 1. Note that in this and other diagrams higher unemployment is indicated by a move to the left.

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<sup>7</sup> Note that the total wage  $W$  which a worker receives may increase even though remuneration per efficiency unit is falling. This will be the case if the reduction in  $W/\Lambda_N$  is accompanied by an even greater increase in the parameter  $\Lambda_N$ .

<sup>8</sup> Blanchard himself uses the term “labor supply function” to denote equation (7). We prefer the term “wage setting function” because it ties in better with other models of wage bargaining, such as that of LNJ which is discussed below.

Fig. 1: Effect of a lower  $k$  (Blanchard - style bargaining)



The above conclusion holds for all values of  $\sigma$ . This is because equation (9) assumes that trade union bargainers take no account of what impact wage settlements will have on the employment decisions of individual firms. However, there are more sophisticated bargaining models in which the unions are forward-looking and do consider how employers will respond to the wage settlement. In such models, a lower  $k$  may have a positive or negative impact on unemployment depending on the value of  $\sigma$ . To illustrate this point we shall follow the approach of Layard, Nickell and Jackman (LNJ), who assume a Nash bargaining framework for wage determination<sup>9</sup>. Each firm bargains separately with its workers, and the outcome is the wage which maximizes a Nash product of the following type,

$$\Omega = (V - \bar{V})^\beta (\Pi - \bar{\Pi}) \quad (11)$$

where  $V$  and  $\Pi$  are the utility functions of the unions and employers respectively. The bar above a variable indicates the outside option if the firm shuts down, and  $\beta$  is an index of relative bargaining power which reflects the ability of the two sides to halt production. The utility of each union depends on the real wage rate, the vulnerability of insiders to job loss, and the costs resulting from loss of employment. The cost of job loss depends, in turn, on the probability of getting another job outside the firm, the rates of pay elsewhere in the economy and the level of unemployment benefit. The utility of a firm is equal to its profits. Further details are given in the appendix.

When the above utility function is maximized with respect to  $W$ , we obtain the following first-order condition,

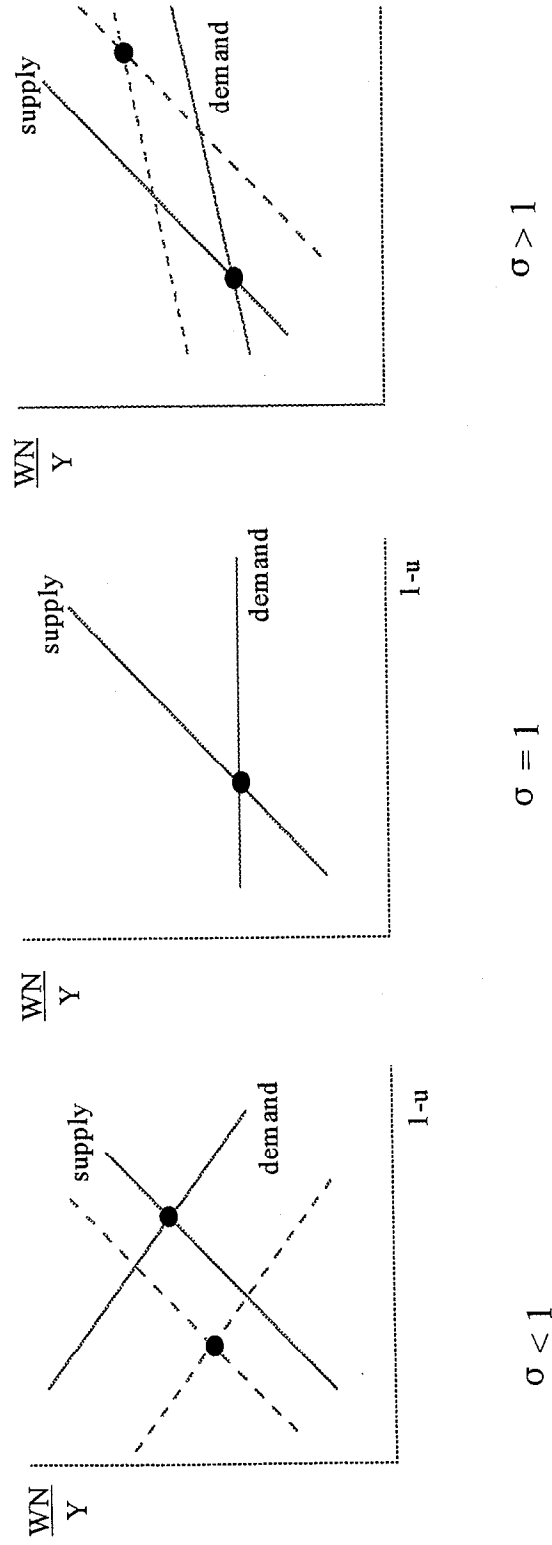
$$\frac{WN}{Y} = 1 - \left[ 1 - \beta \epsilon_{SN} \left( \sigma + \frac{\alpha \left( 1 - \frac{\sigma}{\eta} \right)}{\left[ \frac{\alpha}{\eta} + \frac{(1-\alpha)}{\sigma} \left( \frac{k}{1-u} \right)^{(\sigma-1)/\sigma} \right]} \right) + \frac{\beta}{\phi u(1-b)} \right]^{-1} \quad (12)$$

where  $\epsilon_{SN}$  and  $\phi$  are constants reflecting the vulnerability of insiders to job loss and the rate of outflow from unemployment; while  $b$  is the ratio of unemployment benefits to wages (replacement ratio).

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<sup>9</sup> Layard, Nickell and Jackman (1991)

Fig. 2: Effect of a lower  $k$  (LNU - style bargaining)



Suppose that benefits are fully indexed to wages so that  $b$  is constant. This assumption ensures that the relative cost of unemployment is independent of the wage level. The model then behaves as though workers are bargaining over their share in output rather than over absolute wages. For given  $k$ , we can then solve equations (7) and (11) to obtain  $WN/Y$  and  $1-u$ . The solution is illustrated diagrammatically in Figs. 2 a-2c, which also show what happens when there is a reduction in  $k$ . We assume that  $\eta > \sigma$ . For stability the wage setting curve derived from equation (11) must be steeper than the demand curve based on equation (7). This is always the case for  $\sigma \leq 1$ .

When  $\sigma < 1$ , the demand curve is downward sloping and the wage setting curve is upward sloping. A reduction in  $k$  shifts the demand curve downwards and the supply curve upwards, thereby increasing the unemployment rate. It is shown in Rowthorn (1996, 1999) that the wage share also falls. These are plausible results. They imply that the unions accept some reduction in the real "efficiency" wage rate when investment slows down or there is an exogenous increase in the labor supply; this leads to a decline in the wage share but is not sufficient to prevent some rise in the unemployment rate.

When  $\sigma = 1$  we have the Cobb-Douglas case considered by LNJ. In this case the labor demand curve is horizontal when the dependent variable is the wage share  $WN/Y$ . Since we are assuming that benefits are indexed to wages, neither curve shifts when  $k$  varies. Thus, the share of wages and the unemployment rate are unaffected by changes in this variable. To grasp what this result would mean in practice, suppose technology is given and that some of the existing capital stock is scrapped, so that workers who operate this equipment lose their jobs. The unions respond by accepting a cut in real wages, thereby stimulating job creation elsewhere in the economy. When  $\sigma = 1$ , LNJ-style bargaining ensures that this cut in wages is *always* such that the number of new jobs created is *exactly* equal to the number of old jobs lost through scrapping. Thus, total employment is unaffected by the loss of capital. Alternatively, suppose that there is an influx of new workers from abroad, that technology is given, and that no additional capital is installed. The unions will again respond by accepting a wage cut. When  $\sigma = 1$ , LNJ-style bargaining ensures that this cut in wages is exactly enough to keep the unemployment rate constant. Thus, utility-maximizing trade unions automatically respond in such a way that neither capital scrapping nor immigration affect the unemployment rate. This is an implausible result. The fault may lie in the assumptions about wage bargaining, but a more likely explanation is that  $\sigma$  is normally less than unity.

Even more implausible is the outcome when  $\sigma > 1$ . The demand curve is then upward sloping. A reduction in  $k$ , for whatever reason, will shift this curve upwards and the wage-setting curve downwards. The result will be a lower real wage rate and less unemployment. Thus, if there is an influx of new workers from abroad or a slowdown in investment, unions will respond by accepting such a large wage cut that the unemployment rate actually falls. Moreover, the increase in employment will be so large that the *share* of wages in total output will rise despite the fact that individual workers receive a lower wage than before.

#### IV. EMPLOYMENT AND GROWTH

Assume that the parameters  $\alpha$ ,  $\sigma$  and  $m$  are invariant through time. Then, in order to maintain simultaneously both a constant wage share and a constant unemployment rate,  $k$  must also remain constant. This follows directly from the labor demand equation (7) and is true no matter what assumptions are made about wage bargaining. Constancy of  $k$  implies that the following relationship must be satisfied,

$$g_K + g_{\Lambda_K} = g_L + g_{\Lambda_N} \quad (13)$$

where  $g$  denotes the growth rate of the variable concerned. Thus, capital measured in efficiency units must grow just fast enough to keep up with the growth in labor supply also measured in efficiency units. The latter is conventionally known as the "natural" growth rate.

Suppose that capital measured in efficiency units is growing at the natural rate. With a Blanchard-style wage-setting function, this will ensure that wages rise exactly in line with labor-augmenting technical progress, and that both the unemployment rate and the share of wages in total output remain constant. If benefits are indexed to wages, the same is also true with LNJ-style bargaining. In each case, the following equations will be satisfied

$$\begin{aligned} g_Y &= g_L + g_{\Lambda_N} \\ g_{K/Y} &= -g_{\Lambda} \end{aligned} \quad (14)$$

where  $Y$  is output and  $K/Y$  is the capital-output ratio. Thus, if capital measured in efficiency units grows at the natural rate, physical output will also grow at this rate, and the conventional capital-output ratio will fall in line with capital-augmenting technical progress.

Alternatively, suppose that for some reason capital is growing slowly and that  $k$  is falling. To maintain a constant unemployment rate under these conditions, the real wage per efficiency unit of labor must also fall at an appropriate rate. This does not occur spontaneously with a Blanchard-style wage-setting function, nor does it occur with LNJ-style bargaining when  $\sigma < 1$ . In each case, there is some decline in the "efficiency" wage, but this is not sufficient to prevent unemployment from rising.

##### A. Endogenous Capital Formation

The discussion so far has assumed that capital stock is exogenous. We shall now consider what happens if capital is endogenous. Blanchard assumes that each firm has a given amount of capital and that growth takes place through the entry of new firms. Thus, as capital accumulates the number of firms grows indefinitely. This seems

inconsistent with the assumption that the degree of monopoly in the product market remains unchanged through time. We shall assume that the number of firms remains fixed and that growth occurs through each firm increasing its own capital stock. However, the following analysis can be readily modified to embody Blanchard's assumption without affecting the basic results.

For the individual firm the rate of return on marginal investment is given by is given by <sup>10</sup>

$$\pi = \frac{1 - \alpha}{1 + m} \left( \frac{Y}{\Lambda_K K} \right)^{1/\sigma} \Lambda_K \quad (15)$$

This can also be written as follows

$$\pi = f(k, u), \quad f_k < 0, \quad f_u < 0 \quad (16)$$

where

$$f(k, u) = \frac{1 - \alpha}{1 + m} \left[ \alpha \left( \frac{k}{1 - u} \right)^{(1-\sigma)/\sigma} + (1 - \alpha) \right]^{1/(\sigma-1)} \quad (17)$$

The capital of the firm is at the desired level when the rate of return on investment is equal to the exogenously determined cost of borrowing given by

$$\pi^* = r + \delta + \mu \quad (18)$$

where  $r$  is the real interest rate,  $\delta$  is depreciation and  $\mu$  is a risk premium<sup>11</sup>.

The economy will be in long-run equilibrium when the capital stock is fully adjusted and unemployment is equal to the NAIRU corresponding to the pertinent capital-labor ratio. Using '\*' to denote long-run equilibrium values, the following equation must therefore hold,

$$\pi^* = f(k^*, u^*) \quad (19)$$

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<sup>10</sup> Note that intra-marginal units of capital will earn a higher return than  $\pi$ . These intra-marginal profits will be capitalized in the market value of the firm.

<sup>11</sup> A more sophisticated model would take into account the possibility of raising capital through the issue of shares. This would not affect the basic argument.



where  $\pi^*$  is exogenously given by equation (17). This determines the long-run equilibrium value of  $k/(1-u)$  as a function of  $\pi^*$ .

The value of  $u^*$  depends on what assumptions are made about wage determination. With a Blanchard-style wage-setting function, we find that<sup>12</sup>

$$-\beta u^* + z = \frac{\alpha}{(1+m)} \left[ \alpha + (1 - \alpha) \left( \frac{k^*}{1-u^*} \right)^{(\sigma-1)/\sigma} \right]^{1/(\sigma-1)} \quad (20)$$

Solving (18) and (19) we obtain  $k^*$  and  $u^*$  as functions of  $\pi^*$ . It can be shown that

$$\frac{\partial k^*}{\partial \pi^*} < 0, \quad \frac{\partial u^*}{\partial \pi^*} > 0 \quad (21)$$

Thus, unless labor is willing to adjust its reservation wage downwards, an increase in the external cost of capital leads to less capital-intensive production and more unemployment.

The above discussion is based on a Blanchard-style labor supply function, but the conclusions may be rather different with LNJ-style bargaining. With such bargaining it is always the case that a higher cost of capital leads to less capital intensive production, but the implications of for unemployment depend on the value of  $\sigma$ . When  $\sigma < 1$  long-run unemployment increases as capital becomes more expensive, but the opposite occurs when  $\sigma > 1$ .

## B. The Investment Function

To endogenise capital we must specify an investment function. One plausible candidate is the following,

$$g_K = g_L + g_{\Lambda_N} - g_{\Lambda_K} + \gamma(\pi - \pi^*) \quad (22)$$

This investment function has a forward-looking dimension because entrepreneurs take into account technical progress and growth of the labor force. At the same time their investment decision is influenced by current profitability in the economy. In long-term equilibrium,  $\pi = \pi^*$  and the above equation implies that capital in efficiency units is growing at the natural rate.

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<sup>12</sup>This equation is derived from equations (5) and (9).

An alternative investment function, based on the capital-stock adjustment principle, is as follows,

$$g_K = g_L + g_{\Lambda_N} - g_{\Lambda_K} + \gamma(\log k^* - \log k) \quad (23)$$

This equation is based on the idea that the rate of investment is influenced by the desire of firms to achieve some target capital-labor ratio. Since  $\pi$  and  $k$  are closely related, this investment function is similar to the one given in equation (21), and has identical implications for long-run growth.

Both of the above equations imply that a scarcity of capital will be self-correcting. This raises a number of policy-related questions. How long does the “automatic” economic recovery take? If the coefficient  $\gamma$  is very small, several decades may be required to get back to anything like long-run equilibrium following a major shock. Could economic policy accelerate the recovery, and what kind of measures might be used? With either of the above functions, a reduction in the cost of capital or in wage pressure (from the trade unions) would encourage faster economic growth. Moreover, if such changes were permanent the result would be a permanent reduction in unemployment. Since the cost of capital includes a risk premium, this suggests that unemployment would be lower in a less risky economic environment.

## V. COMMENTARY ON BLANCHARD (1997, 1998)

This section examines in detail the arguments which Blanchard has presented in two original papers on European unemployment. In most continental countries there has been a strong upward trend in unemployment since 1973; and after an initial decline, the profit share in many of these countries is now higher than before the onset of the crisis.<sup>13</sup> Blanchard regards the combination of higher unemployment and a higher profit share as surprising, and he constructs rather a complex argument to explain it. He concludes that this combination was due to either a “capital-using shift” in technology or a shake-out of labor caused by the termination of feather-bedding. Neither of these explanations is adequate. The former is implausible, whilst the latter is at best incomplete.

Before examining the details of Blanchard’s argument, we should mention another potential explanation which he does not consider. Since 1973 the growth rate of the capital stock in continental Europe has slowed down dramatically. If the elasticity of substitution  $\sigma$  is less than unity, such a slow down could explain both the rise in the profit share and the rise in unemployment. As we shall see below, most empirical studies

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<sup>13</sup> Blanchard’s figures on profits have been criticized as misleading (Nordhaus, 1998), but more reliable statistics for the manufacturing sector alone reveal a similar pattern (see Glyn, 1997).

suggest that  $\sigma$  is less than unity (as do some of Blanchard's own estimates). Yet his discussion is based on the assumption that  $\sigma$  is equal to or greater than unity. This assumption excludes the possibility that the strong recovery in the continental profit share could have been due to a slow down in the growth rate of capital.

Blanchard begins by estimating the rate of labor-augmenting technical progress using the following formula,

$$x = [g_Y - (1-s)g_N - sg_K] (1-s)^{-1} \quad (24)$$

where  $s$  is the share of profits in total output and, as above,  $g$  indicates the growth rate of the relevant variable. The expression in square brackets is the Solow residual. He then integrates the above equation to construct an index for  $\Lambda_N$ . This procedure assumes capital-augmenting technical progress to be zero and, as Blanchard recognizes, it also ignores imperfections in the product market. Numerical simulations suggest that the combined bias from these sources could be up to 5 or 10 percent over a twenty five year period (i.e. around 0.3 percent per annum)<sup>14</sup>. Although not massive, such a bias could be important.

Blanchard's next step is to calculate what he, rather confusingly, calls the "mark-up". The labor demand equation (4) can be written as follows,

$$\frac{(1+m)}{\alpha} = \left( \frac{W}{\Lambda_N} \right)^{-1} \left( \frac{Y}{\Lambda_N N} \right)^{\frac{1}{\sigma}} \quad (25)$$

Apart from notational differences and lags, the right-hand side of this equations is identical to the formula which Blanchard uses to estimate the "mark-up". Having previously estimated  $\Lambda_N$ , he evaluates the right-hand side on the assumption that  $\sigma$  is equal to either 1 or 2. This provides two distinct estimates of the ratio  $(1+m)/\alpha$ . Using either of these estimates, Blanchard's detailed calculations for France imply that this ratio has risen strongly over the past twenty five years. More cursory calculations with  $\sigma = 1$  give the same picture for most other continental countries, although not for the Anglo-Saxon countries where the estimated ratio did not change much over the period.

Blanchard ascribes the increase in  $(1+m)/\alpha$  in continental countries to a shift in the demand curve for labor, resulting from either a "capital-using" shift in technology or a "shake-out of labor". To assess the plausibility of this explanation it is useful to write the this ratio in terms of the original parameters of the model,

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<sup>14</sup> See the Appendix .

$$\frac{(1 + m)}{\alpha} = \frac{\eta}{\alpha(\eta - 1)} \quad (26)$$

where  $\eta$  is the (absolute) price elasticity of demand for the output of the typical firm. It is clear from the right-hand expression that Blanchard's "mark-up" is not affected by the efficiency with which labor is utilized. If this ratio has genuinely risen in continental Europe, it *must* be because there has been either (1) a fall in  $\eta$  resulting from less competition in continental product markets, or (2) a reduction in  $\alpha$  (which Blanchard refers to as a "capital-using shift" in technology). No other explanation is possible.

Neither of these explanations is convincing. The growth of international trade and foreign direct investment have increased the competitive pressure on continental firms over the period in question, suggesting that the parameter  $\eta$  may have risen, rather than fallen as the first explanation requires. As Nordhaus (1997) has pointed out, the technology hypothesis is equally implausible. It cannot explain why the "mark-up" should have increased in continental Europe but not in the Anglo-Saxon countries. To account for such a contrast, the capital-using shift in technology would have to be confined to continental Europe and not affect the Anglo-Saxon area<sup>15</sup>.

If neither  $\eta$  nor  $\alpha$  has changed in the required direction, the conclusion must be that the continental "mark-up" has not risen dramatically as Blanchard's estimates imply. These estimates must therefore be wrong. The explanation could be that Blanchard has estimated  $\Lambda_N$  incorrectly. It may also be that the true value of  $\sigma$  is much lower than he assumes. Continental Europe has experienced a large increase in the unemployment rate and a rising profit share. The obvious explanation for higher unemployment is that the variable  $k$  has fallen, indicating that capital accumulation has been too slow in relation to labor force growth and technical progress. Provided the basic parameters of the model are constant, inadequate capital growth is only consistent with the observed increase in the continental profit share if  $\sigma$  is considerably less than unity. Such a conclusion is independently confirmed by the empirical estimates of this parameter reported above.

We have suggested that continental Europe's combination of simultaneously rising unemployment and profits can be explained by inadequate growth in capital stock. This explanation is consistent with Blanchard's hypotheses regarding changing technology and work-practices. In his analytical framework, a labor shake-out will manifest itself in the data as labor-augmenting technical progress, and unless it is accompanied by additional investment, will cause the ratio  $k$  to fall. A capital-using shift in technology will have the same effect. Provided  $\sigma < 1$ , the result will be rising

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<sup>15</sup> Note that capital-using technical progress is not the same as the substitution of capital for labor induced by higher wages. The latter has certainly occurred in continental Europe, possibly on a greater scale than in the Anglo-Saxon countries..

unemployment and a rising profit share. Thus, if  $\sigma$  is less than unity, Blanchard's capital-using and labor shake-out hypotheses are both consistent with the view that investment has been inadequate in continental Europe.

## VI. KEYNESIAN COMPLICATIONS

In this section, we consider Keynesian ideas concerning the role of demand and their bearing on the preceding analysis. The first point to note is that, despite later misinterpretations, there is no doubt that Keynes himself in the General Theory accepts the existence of a trade-off between wages and employment. The General Theory assumes that product market are perfectly competitive and that individual firms are therefore price-takers who can sell as much as they want at the going price. Moreover, each firm produces at the point where the marginal product of labor measured at current prices is equal to the money wage rate; or to put it differently, where the marginal product of labor is equal to the real product wage. Thus, firms are always on their labor demand curve. In this analytical framework, the *only* way in which a change in aggregate demand can be transmitted to the individual firm is through a change in product prices. There is no other signal connecting the firm to the product market. If money wages are given and the general price level falls, firms throughout the economy will produce less because marginal output has become unprofitable. They will cut back production until the marginal product has risen sufficiently to offset lower prices. Since money wages are given and all prices are lower than before, the real consumption wage is higher. Thus, in the General Theory, a reduction in aggregate demand is accompanied by a fall in prices, an increase in both the real wage rate and the marginal product of labor, and a reduction in employment. The opposite occurs if aggregate demand increases. The connections between these various phenomena are not mere accidents as many of his followers have claimed, but are the logical outcome of his assumptions regarding product market competition and the behavior of firms.

These assumptions were not unique to Keynes, nor was his view that increased demand could stimulate output by raising prices and thereby reducing real wages. One of his main contributions was to argue that this would not necessarily lead to explosive inflation. Following the experience of hyperinflation immediately after the First World War, many economists believed that any attempt to stimulate demand would set off an explosive wage-price spiral because the trade unions would resist a reduction in their real wages. Keynes himself accepted that this might be the case under conditions of high employment, but he rejected such an argument for an economy operating under *depression conditions*. He believed that during a depression the unions are likely to accept an increase in prices without demanding extra money wages to compensate. Thus, in a depression, a demand stimulus will lead to a once and for all price increase, but will not set off an unsustainable inflationary spiral.

Keynes also argued that the willingness of trade unions to accept uncompensated price increases is consistent with their refusal to accept money wage cuts. This

proposition is often dismissed on the grounds that it assumes “money illusion” on the part of workers. Such a claim is incorrect. Keynes argued that the unions oppose money wage cuts because their impact is inevitably very uneven and they disturb existing wage relativities. In theory it is possible to cut money wages by the same proportion throughout the economy, but in reality this is difficult to implement. In any actual programme of money wage cuts, some workers will experience a large reduction in pay whilst others will be almost unaffected. By contrast, a general price increase is universal in its impact and leaves existing wage relativities unchanged. Thus, it is quite rational for the unions to oppose money wage cuts whilst accepting uncompensated price increases. Such a response is fully justified by the modern theory of transactions costs. Keynes also opposed money wage cuts because they would penalize debtors and depress the sales expectations of firms, leading to a deflationary spiral of falling wages and prices, higher real interest rates and less investment, thereby depressing demand even further. This was the experience of the deflations around 1920 and 1930.

Although Keynes himself assumed that product markets are perfectly competitive, his argument can easily be recast within the framework of imperfect competition used in this paper. The logic of Keynes’ argument is illustrated in Fig.3. Suppose that wage bargainers normally follow the behavior encapsulated in equation (9), which establishes an upward-sloping wage-setting curve relating real wages to unemployment. There is also a downward-sloping relationship given by the labor demand equation (6). The equilibrium unemployment rate is at the point A where these curves intersect. Suppose there is a negative demand shock. According to the General Theory, this will cause product prices to fall and real wages to rise. Since firms are always on their labor demand curve, they will move up this curve to some point C. This point is off the labor supply curve, indicating that unions are in principle prepared to accept a real wage cut. However, they are not willing to accept a cut in money wages, for the reasons mentioned above. The solution proposed by Keynes is to stimulate expenditure, so that prices rise and the economy moves down the labor demand curve back to the original point A.

The above argument can be modified in a variety of ways. Following the disciples of Keynes, let us drop the assumption that firms are always on their labor demand curve<sup>16</sup>. Fig. 4 illustrates what this might imply. In this diagram prices and money wages (per efficiency unit) are both assumed to be rigid downwards, so that a negative shock in the product market will move the economy horizontally from A to some new point B. Suppose the government now responds by stimulating the demand for output. The first panel assumes that money wages and prices remain constant, so the economy simply retraces its original path back to A. The second panel shows what happens if the government or unions follow the “underconsumptionist” strategy of stimulating the output demand by raising real wages. This may work for a time but eventually the economy will reach the labor demand curve at some point D where real wages are

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<sup>16</sup> The following discussion draws on the work of Malinvaud (1977).

Fig. 3: The General Theory

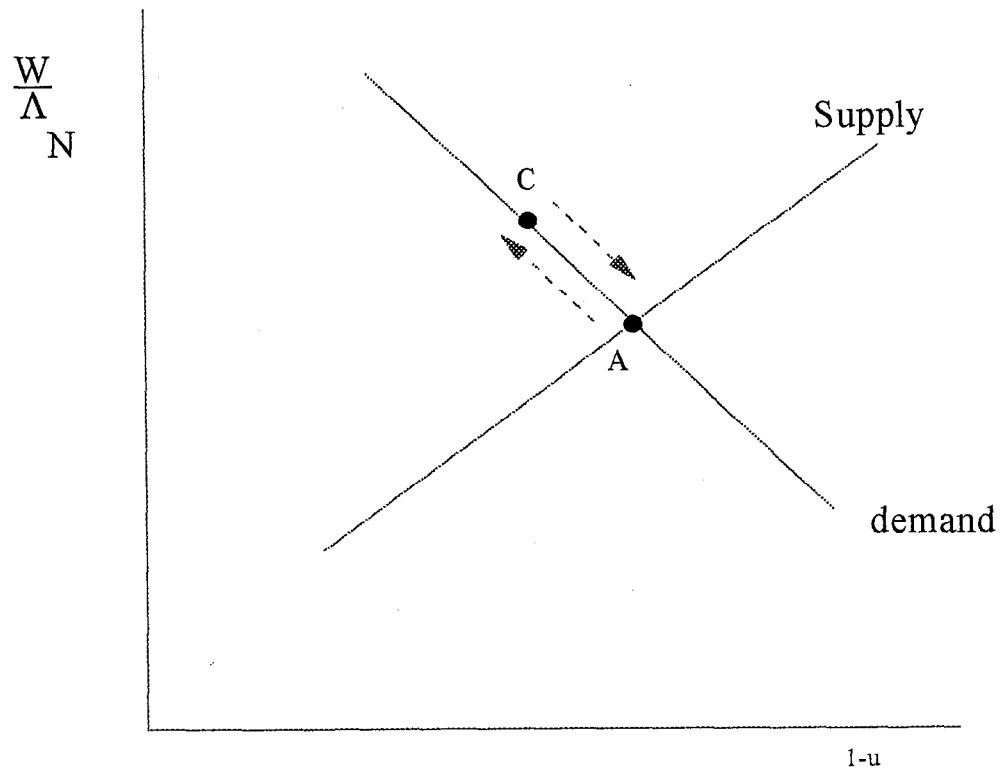
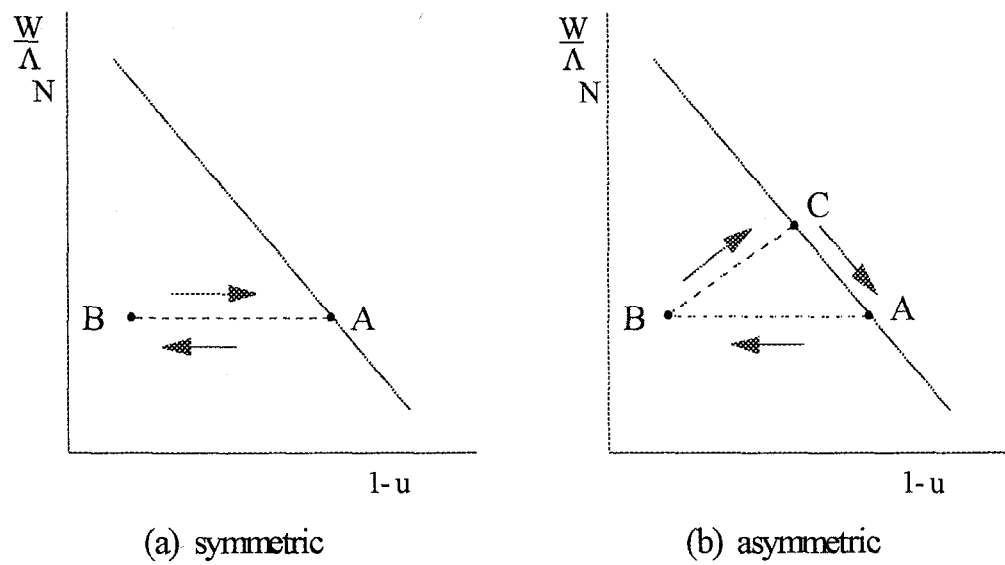


Fig. 4: Slump and recovery : alternative paths





higher than before and employment is lower. To reduce unemployment still further now requires a reduction in real wages, which in the spirit of Keynes fashion might be achieved through a once and for all increase in the price level. Note that the diagram shows real wages per efficiency unit of labor. If there is no labor-augmenting technical progress "efficiency" wages rise or fall in line with actual wages. However, if such progress is occurring, then the reduction in efficiency wages along the path DA may be consistent with a stationary or even increasing real income per worker.

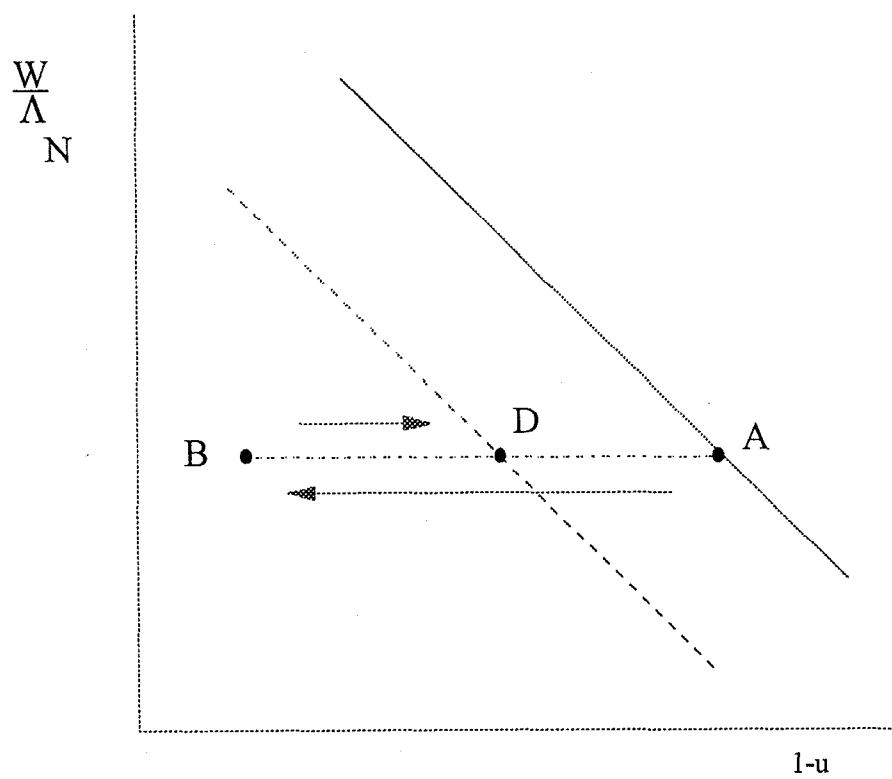
A central concern of this paper has been the relationship between capital stock and employment. Our discussion of the topic assumed that firms are always on their labor demand curve, but this is not always the case in reality. Fig. 5 illustrates the role which capital stock may play in the case where prices and money wages (per efficiency unit) are constant. As in the previous diagram, a negative demand shock in the product market will move the economy from the point A on the labor demand curve to a new point B off the curve. In the new situation firms are operating with excess capacity and depressed profits, so they are likely to respond by reducing investment. If this happens, the variable  $k$  will fall and the labor demand curve will shift inwards. Suppose that the demand for output now starts to recover. This will cause firms to take on more workers and employment will shift back towards the right.

As capacity utilization and profits rise, firms will respond by increasing investment. This will cause the labor demand curve to shift outwards again. The combined effect of these two shifts depends on the relative speed with which they occur. Given the initial excess capacity and depressed profits, there may be a considerable lag before new capacity comes on stream, so the outward shift of the labor demand curve may be quite slow. Thus, if the economic recovery is rapid, employment will catch up with the labor demand curve whilst unemployment is still relatively high. This is shown by the point E. At this point firms no longer have excess capacity, the economy is facing a "capital scarcity" and both output and employment are cost-constrained. To reduce unemployment still further at the existing "efficiency" wage would require enough additional investment to shift the labor demand curve further outwards. Workers can encourage such an outcome by accepting a temporary increase in profits and reduction in the efficiency wage. This might also induce firms to employ more workers on existing capital stock.

## VII. CONCLUDING REMARKS

A central theme of this paper is that capital formation affects the equilibrium unemployment rate. Two notions of equilibrium unemployment can be distinguished: the short-run NAIRU, when capital stock is taken as given, and the long-run NAIRU when capital stock is endogenous. To maintain a constant short-run NAIRU, capital must grow fast enough to offset growth in the labor supply and the labor-augmenting bias in technical progress. If capital formation is inadequate, the short-run NAIRU will increase and the scope for demand stimulation will be limited by scarcity of capital.

Fig. 5: Capital scarcity in a Keynesian framework



Capital formation can be encouraged by measures which reduce the risk-adjusted cost of capital or increase the expected profitability of investment. Such measures might include lower real interest rates, wage restraint, fiscal incentives for investment, or a more predictable economic environment. These would increase the rate of investment and cause the short-run NAIRU to fall. They would also reduce the long-run NAIRU and allow the economy to enjoy a permanently lower rate of unemployment than would otherwise be the case.

Capital formation is to some degree endogenous. If there is a prolonged period of low investment, the short-run NAIRU will rise, and given appropriate assumptions about substitution elasticities and the nature of wage bargaining, this will eventually be reflected in higher profits. As profits increase there will be a gradual recovery in investment. However, these spontaneous forces for recovery may operate very slowly, so that policy measures may be helpful to accelerate the process.

The policy implications of this discussion are as follows. Wage restraint can help to reduce unemployment in two ways - by encouraging more employment on existing capital stock, and by encouraging more investment so as to increase the amount of capital stock. Given the low elasticity of substitution between capital and labor, the latter may have a greater role to play than many economists currently recognize. However, wage restraint is only one of the means through which investment can be encouraged. Lower real interest rates, fiscal incentives for investment, and risk-reducing policies may also have a role to play. An exclusive emphasis on labor market reforms is therefore misplaced.

This appendix describes the model underlying the paper and derives the main equations.

### Production

There is a large, but fixed number of identical firms indexed by the subscript  $I$ . Their common CES production function is

$$Y_i = \left( \alpha(\Lambda_N N_i)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\Lambda_K K_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (\text{A1})$$

where  $Y_i$ ,  $N_i$ , and  $K_i$  are output, employment, and capital;  $\Lambda_N$  and  $\Lambda_K$  are indices of technological efficiency;  $\alpha$  and  $\sigma$  are constants.

Differentiating we obtain

$$\frac{\partial Y_i}{\partial N_i} = \alpha \left( \frac{Y_i}{N_i} \right)^{\frac{1}{\sigma}} \Lambda_N^{\frac{\sigma-1}{\sigma}} \quad (\text{A2})$$

$$\frac{\partial Y_i}{\partial K_i} = (1-\alpha) \left( \frac{Y_i}{K_i} \right)^{\frac{1}{\sigma}} \Lambda_K^{\frac{\sigma-1}{\sigma}} \quad (\text{A3})$$

### The Demand for Output

The inverse demand form facing each firm is given by

$$\frac{P_i}{P} = \left( \frac{Y_i}{Y} \right)^{-\frac{1}{\eta}} \quad (\text{A4})$$

where  $\eta$  is the absolute price elasticity of demand, and  $P$  and  $Y$  are the price and output of the average firm.

The mark-up of price over marginal cost is given by

$$m = \frac{1}{\eta-1} \quad (\text{A5})$$

Profits are given by

$$\Pi_i = P_i Y_i - W_i N_i \quad (\text{A6})$$

where  $W_i$  is the wage rate.

### Labor Demand Equations

Wages are settled first and then the firm chooses its output. Maximization of profits implies that

$$\begin{aligned} W_i &= \frac{\partial(P_i Y_i)}{\partial N_i} \\ &= \frac{\alpha P}{1+m} \left( \frac{Y_i}{Y} \right)^{-\frac{1}{\eta}} \left( \frac{Y_i}{\Lambda_N N_i} \right)^{\frac{1}{\sigma}} \Lambda_N \end{aligned} \quad (\text{A7})$$

In equilibrium all firms have the same output, prices, wages, and capital. For simplicity we assume that  $P = 1$ . Thus,  $Y_i = Y$ ,  $W_i = W$ ,  $K_i = K$ , and  $P_i = 1$ . From (A7) it follows that

$$\frac{W}{\Lambda_N} = \frac{\alpha}{1+m} \left( \frac{Y}{\Lambda_N N} \right)^{\frac{1}{\sigma}} \quad (\text{A8})$$

and

$$\frac{WN}{Y} = \frac{\alpha}{1+m} \left( \frac{Y}{\Lambda_N N} \right)^{\frac{1}{\sigma}-1} \quad (\text{A9})$$

These are the labor demand equations.

Define,

$$\begin{aligned} k &= \frac{\Lambda_K K}{\Lambda_N L} \\ u &= 1 - \frac{N}{L} \end{aligned} \quad (\text{A10})$$

where  $L$  is the total labor force. Then

$$\frac{\Lambda_K K}{\Lambda_N N} = \frac{k}{1-u} \quad (\text{A11})$$

Hence,

$$\frac{Y}{\Lambda_N N} = \left[ \alpha + (1-\alpha) \left( \frac{k}{1-u} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (\text{A12})$$

Substituting in (A8) and (A9) we get

$$\frac{W}{\Lambda_N} = \frac{\alpha}{1+m} \left[ \alpha + (1-\alpha) \left( \frac{k}{1-u} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} \quad (\text{A13})$$

$$\frac{WN}{Y} = \frac{\alpha}{1+m} \left[ \alpha + (1-\alpha) \left( \frac{k}{1-u} \right)^{\frac{\sigma-1}{\sigma}} \right]^1 \quad (\text{A14})$$

### Profit Rate

For the individual firm the marginal rate of return is given by

$$\begin{aligned} \pi_i &= \frac{\partial(P_i Y_i)}{\partial K_i} \\ &= \frac{(1-\alpha)P}{1+m} \left( \frac{Y_i}{Y} \right)^{-\frac{1}{\eta}} \left( \frac{Y_i}{\Lambda_K K_i} \right)^{-\frac{1}{\sigma}} \Lambda_K \end{aligned} \quad (\text{A15})$$

In equilibrium  $P = 1, Y_i = Y, K_i = K$  and  $\pi_i = \pi$ .

Hence,

$$\begin{aligned} \pi &= \frac{1-\alpha}{1+m} \left( \frac{Y_i}{\Lambda_K K} \right)^{\frac{1}{\sigma}} \Lambda_K \\ &= \frac{1-\alpha}{1+m} \left[ \alpha \left( \frac{k}{1-u} \right)^{\frac{1-\sigma}{\sigma}} + (1-\alpha) \right]^{\frac{1}{\sigma-1}} \end{aligned} \quad (\text{A16})$$

### LNJ-Style Bargaining

The objective of the union in firm  $I$  is to maximize the expected income of 'insiders'. This is given by

$$V_i = S_i W_i + (1 - S_i) A \quad (\text{A17})$$

where  $S_i$  is the proportion of insiders who keep their jobs and  $A$  is the expected income of those lose their jobs unemployed. The latter is given by

$$A = (1 - \phi u) W + \phi u B \quad (\text{A18})$$

where  $B$  is unemployment benefit and  $\phi$  is a constant. There are no taxes.

Wages are determined by maximizing the Nash product,

$$\Omega_i = (V_i - \bar{V})^\beta (\Pi_i - \bar{\Pi}) \quad (\text{A19})$$

where  $\bar{V}$  and  $\bar{\Pi}$  are the outside options. We assume  $\bar{V} = A$ ,  $\bar{\Pi} = 0$ . Thus

$$\begin{aligned} \Omega_i &= (V_i - A)^\beta \Pi_i \\ &= S_i^\beta (W_i - A)^\beta \Pi_i \end{aligned} \quad (\text{A20})$$

Differentiating, we obtain

$$\begin{aligned} \frac{W_i}{\Omega_i} \frac{\partial \Omega_i}{\partial W_i} &= \beta \frac{W_i}{S_i} \frac{\partial S_i}{\partial W_i} + \frac{\beta W_i}{W_i - A} + \frac{W_i}{\Pi_i} \frac{\partial \Pi_i}{\partial W_i} \\ &= \beta \epsilon_{SN} \frac{W_i}{N_i} \frac{\partial N_i}{\partial W_i} + \frac{\beta W_i}{W_i - A} - \frac{W_i N_i}{\Pi_i} \end{aligned} \quad (\text{A21})$$

where  $\epsilon_{SN} = \frac{N_i}{S_i} \frac{\partial S_i}{\partial N_i}$ . Assume that  $\epsilon_{SN}$  is a constant which is the same for all firms. The

first order condition for maximization requires that the right-hand side is zero, and hence

$$\frac{W_i N_i}{\Pi_i} = \beta \epsilon_{SN} \frac{W_i}{N_i} \frac{\partial N_i}{\partial W_i} + \frac{\beta W_i}{W_i - A} \quad (\text{A22})$$

From (A17) and (A18) it follows that

$$\frac{W_i}{W_i - A} = \frac{W_i}{(W_i - W) + \phi u \left( 1 - \frac{B}{W} \right) W} \quad (\text{A23})$$

In equilibrium,  $W_i = W$  and hence

$$\frac{W_i}{W_i - A} = \frac{1}{\phi u (1 - b)} \quad (\text{A24})$$

where

$$b = \frac{B}{W} \quad (\text{A26})$$

From (A6),

$$\begin{aligned} \frac{\partial \Pi_i}{\partial W_i} &= \frac{\partial (P_i Y_i)}{\partial N_i} \frac{\partial N_i}{\partial W_i} + W_i \frac{\partial N_i}{\partial W_i} - N_i \\ &= -N_i \end{aligned} \quad (\text{A26})$$

From (A7)

$$\begin{aligned} \frac{N_i}{W_i} \frac{\partial W_i}{\partial N_i} &= \left( \frac{1}{\sigma} - \frac{1}{\eta} \right) \frac{N_i}{Y_i} \frac{\partial Y_i}{\partial N_i} - \frac{1}{\sigma} \\ &= \alpha \left( \frac{1}{\sigma} - \frac{1}{\eta} \right) \left( \frac{Y_i}{\Lambda_N N_i} \right)^{\frac{1-\sigma}{\sigma}} - \frac{1}{\sigma} \\ &= - \frac{\left[ \frac{\alpha}{\eta} + \frac{1-\alpha}{\sigma} \left( \frac{\Lambda_K K_i}{\Lambda_N N_i} \right)^{\frac{\sigma-1}{\sigma}} \right]}{\alpha + (1-\alpha) \left( \frac{\Lambda_K K_i}{\Lambda_N N_i} \right)^{\frac{\sigma-1}{\sigma}}} \end{aligned} \quad (\text{A27})$$

Inverting, we get



$$\begin{aligned} \frac{W_i}{N_i} \frac{\partial N_i}{\partial W_i} &= - \frac{\left[ \alpha + (1-\alpha) \left( \frac{\Lambda_K K_i}{\Lambda_N N_i} \right)^{\frac{\sigma-1}{\sigma}} \right]}{\frac{\alpha}{\eta} + \frac{1-\alpha}{\sigma} \left( \frac{\Lambda_K K_i}{\Lambda_N N_i} \right)^{\frac{\sigma-1}{\sigma}}} \\ &= - \left[ \sigma + \frac{\alpha \left[ 1 - \frac{\sigma}{\eta} \right]}{\frac{\alpha}{\eta} + \frac{1-\alpha}{\sigma} \left[ \frac{\Lambda_K K_i}{\Lambda_N N_i} \right]^{\frac{\sigma-1}{\sigma}}} \right] \end{aligned} \quad (\text{A28})$$

In equilibrium  $K_i = K$ ,  $N_i = N$  and hence

$$\frac{W_i}{N_i} \frac{\partial N_i}{\partial W_i} = - \left[ \sigma + \frac{\alpha \left[ 1 - \frac{\sigma}{\eta} \right]}{\frac{\alpha}{\eta} + \frac{1-\alpha}{\sigma} \left[ \frac{k}{1-u} \right]^{\frac{\sigma-1}{\sigma}}} \right] \quad (\text{A29})$$

From (A22), (A24), and (A29) we obtain the following equilibrium condition

$$\frac{WN}{\Pi} = -\beta \epsilon_{SN} \left[ \sigma + \frac{\alpha \left( 1 - \frac{\sigma}{\eta} \right)}{\frac{\alpha}{\eta} + \frac{1-\alpha}{\sigma} \left( \frac{k}{1-u} \right)^{\frac{\sigma-1}{\sigma}}} \right] + \frac{1}{\phi u (1-b)} \quad (\text{A30})$$

To obtain the wage share in total output note that

$$\frac{WN}{Y} = 1 - \frac{1}{1 + \frac{WN}{\Pi}} \quad (\text{A31})$$

### Estimation of $\Lambda_N$

The conventional formula for estimating labor-augmentary technical progress is

$$x = (g_y - (1-s)g_N - sg_k)(1-s)^{-1} \quad (\text{A32})$$

where  $s$  is the share of profits and  $g$  denotes a growth rate.

In equilibrium

$$Y^{\frac{\sigma-1}{\sigma}} = \alpha(\Lambda_N N)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\Lambda_K K)^{\frac{\sigma-1}{\sigma}} \quad (\text{A33})$$

Differentiating with respect to time,

$$Y^{\frac{\sigma-1}{\sigma}} g_y = \alpha(\Lambda_N N)^{\frac{\sigma-1}{\sigma}} [g_{\Lambda_N} + g_N] + (1-\alpha)(\Lambda_K K)^{\frac{\sigma-1}{\sigma}} [g_{\Lambda_K} + g_K] \quad (\text{A34})$$

Hence,

$$\left( \frac{Y}{\Lambda_N N} \right)^{\frac{\sigma-1}{\sigma}} g_y = \alpha(g_{\Lambda_N} + g_N) + (1-\alpha) \left( \frac{\Lambda_K K}{\Lambda_N N} \right)^{\frac{\sigma-1}{\sigma}} [g_{\Lambda_K} + g_K] \quad (\text{A35})$$

and

$$g_y = \alpha \left( \frac{Y}{\Lambda_N N} \right)^{\frac{1}{\sigma-1}} [g_{\Lambda_N} + g_N - g_{\Lambda_K} - g_K] + [g_{\Lambda_K} + g_K] \quad (\text{A36})$$

Since  $s = 1 - \frac{WN}{Y}$  we can write this as follows,

$$g_y = (1+m)(1-s)[g_{\Lambda_N} + g_N - g_{\Lambda_K} - g_K] + [g_{\Lambda_K} + g_K] \quad (\text{A37})$$

From (A15) and (A32) we obtain,

$$(1-s)x = (1+m)(1-s)[g_{\Lambda_N} + g_N - g_{\Lambda_K} - g_K] + g_{\Lambda_K} + g_K - (1-s)g_N - sg_K \quad (\text{A38})$$

From which it follows that

$$g_{\Lambda_N} = \left[ x + m(g_K - g_N) + \left( m - \frac{s}{1-s} \right) g_{\Lambda_K} \right] (1+m)^{-1} \quad (\text{A39})$$

and, thus,

$$x - g_{\Lambda_N} = \left[ m(x + g_N - g_K) + \left( \frac{s}{1-s} \right) g_{\Lambda_K} \right] (1+m)^{-1} \quad (\text{A40})$$

This equation indicates the bias from using  $x$  as an estimator of labor-augmenting technical progress. Table A1 illustrates the possible scale of this bias.

Table A1. Bias From Using  $x$  To Estimate  $g_{\Lambda_N}$

$x$	$g_K - g_N$	$g_{\Lambda_K}$	$m$	$s$	$x - g_{\Lambda_N}$ (annual)	$x - g_{\Lambda_N}$ (25 years)
2.0	1.0	0.0	0.0	0.3	0.0	0.0
2.0	1.0	0.0	0.2	0.3	0.2	4.3
2.0	1.0	0.5	0.0	0.3	0.2	5.5
2.0	1.0	0.5	0.2	0.3	0.3	9.0
2.0	2.0	0.0	0.0	0.3	0.0	0.0
2.0	2.0	0.0	0.2	0.3	0.0	0.0
2.0	2.0	0.5	0.0	0.3	0.2	5.5
2.0	2.0	0.5	0.2	0.3	0.2	4.6

Note:  $x$ ,  $g_K$ ,  $g_N$ ,  $g_{\Lambda_N}$  and  $g_{\Lambda_K}$  are annual percentage growth rates.

### Elasticity of Labor Demand With Given Capital

Equation (A8) can be written as follows,

$$\log W = \frac{1}{\sigma} \log Y - \frac{1}{\sigma} \log N + \text{other terms} \quad (\text{A41})$$

Differentiating with respect to  $N$  we get, using (A2) and (A9),

$$\begin{aligned} \frac{N}{W} \frac{\partial W}{\partial N} &= \frac{1}{\sigma} \left[ \frac{N}{Y} \frac{\partial Y}{\partial N} - 1 \right] \\ &= \frac{1}{\sigma} \left[ \alpha \left( \frac{Y}{\Lambda_N N} \right)^{\frac{1}{\sigma}-1} - 1 \right] \\ &= \frac{1}{\sigma} \left[ (1+m) \frac{WN}{Y} - 1 \right] \end{aligned} \quad (\text{A42})$$

Let

$$\epsilon = - \frac{W}{N} \frac{\partial N}{\partial W} \quad (\text{A43})$$

Since  $\frac{WN}{Y} = 1-s$  and  $m = \frac{1}{\eta-1}$  we can write equation (A42) as follows,

$$\begin{aligned} \sigma &= \epsilon \left[ 1 - \left( 1 + \frac{1}{\eta-1} \right) (1-s) \right] \\ &= \frac{\epsilon [s\eta - 1]}{\eta - 1} \\ &= \frac{\epsilon \left[ s - \frac{1}{\eta} \right]}{\left[ 1 - \frac{1}{\eta} \right]} \end{aligned} \quad (\text{A44})$$

This formula indicates how  $\sigma$  is related to  $\epsilon$ .

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