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Growth Accounting and Growth Processes

Prepared by Jahangir Aziz¹

Authorized by Graham Hacche

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Abstract

The standard growth accounting framework, which weights various inputs by their factor shares to measure their contributions to output growth, is known to underestimate the contribution of inputs in the presence of externalities and increasing returns. This paper develops a model in which, in the absence of such departures from the standard neoclassical framework, growth can occur through either embodied technological progress or firms replication of existing technology. The standard growth accounting framework fails to distinguish between these contrasting development processes. This failure thus reveals another limitation to the use of growth accounting in identifying the processes of economic developments.

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Contents	Page
Summary	3
I. Introduction	4
II. Growth Accounting	5
III. Growth by Replication	7
A. Technology, Preferences, and Demographics	7
B. The Balanced Growth Path	9
Properties of the Balanced Growth Path	10
Optimality of the Balanced Growth Path	11
IV. Growth Through Technological Progress	12
A. The Balanced Growth Path	14
B. The Observational Equivalence of Growth Paths	14
V. Concluding Remarks	16
References	17

Summary

In this paper, it is argued that the growth accounting framework may not be suitable as a “development accounting” framework. It is known that, in the presence of externalities and increasing returns, the growth accounting framework underestimates the contribution of inputs to the growth process. In such a situation, either competitive markets break down or prices under competitive conditions no longer reflect the marginal product of factors. With social and private valuations diverging, growth accounting using factor shares as weights fails to capture the true contribution of resource mobilization.

This paper identifies another limitation of growth accounting. Growth occurring through embodied productivity gains may be generated by multiple processes that have significant qualitative differences but cannot be differentiated using the growth accounting methodology. The paper provides descriptions of growth processes in which, although increasing returns and externalities are not present and the competitive market structure is preserved, growth accounting exercises using aggregate production functions are unable to distinguish between contrasting processes of development. In one such process of development, growth occurs through firms replicating the existing technology; in another, existing firms continually increase productivity by adopting more advanced technology.

I. Introduction

Since Solow (1957), aggregate production functions have often been used to measure the relative importance of various factors that contribute to the growth of an economy. This aggregate growth-accounting framework has formed the basis of numerous studies that have attempted to quantify the relative importance of technological progress vis-a-vis resource mobilization in generating growth. While such measurements are clearly important as they are central both to the evaluation as well as the appropriate direction of government policies¹, their use in identifying the process by which growth is achieved, i.e., the process of development, is problematic.

The appropriateness of the Solow growth accounting technique hinges on how well the assumption of perfectly competitive markets approximates the real economy at the aggregate level. If such an approximation is not close then one cannot use factor prices to approximate marginal products of inputs and consequently, weighting growth rates of various contributing factors by their factor shares in national income to account for total growth becomes problematic. This is particularly true for descriptions of growth processes that rely on increasing returns or externalities. In such descriptions, which are part of what has been collectively called "endogenous growth models", either competitive markets break down or prices under competitive conditions no longer reflect the marginal product of factors. With social and private valuations diverging, growth accounting using factor shares as weights fails to account adequately for technological progress (Romer 1989).

In the case of growth occurring through embodied productivity gains, apart from possible divergence of social and private valuations of inputs, there may be multiple processes of generating growth that have significant qualitative differences but cannot be differentiated using the Solow growth accounting methodology. The paper provides descriptions of contrasting growth processes that do not rely on increasing returns or externalities such that the competitive market structure is preserved, but where growth accounting exercises, using aggregate production functions, may fail to distinguish them from one another. This paper does not discuss any new method of growth accounting as it would involve, in way or another, generating measures that adjusted inputs for qualitative changes - an area of research where a large body work already exists (Darby 1984, Norsworthy 1984). Instead, the paper provides another reason why the aggregate production function approach may not be a good accounting framework for development - the process by which growth is achieved.

In this paper, an environment is constructed where, in equilibrium, growth takes place through the replication of the existing technology by an ever increasing number of production units or firms. All factors of production are assumed to be reproducible and are represented by a single input. The production technology is such that the average cost curve is "U" shaped at the level of the individual firm or plant. Each firm also incurs an overhead cost which allows the measure of participating firms to be finite in a competitive equilibrium. Given this structure, the natural market-determined aggregation process leads to an aggregate technology that displays constant returns to scale in the reproducible input such that the economy displays positive long-run growth. By construction there is no technological

¹ Sarel (1995) discusses in more detail such issues in the context of the newly industrialized East Asian economies.

progress in this environment and all growth occurs through the continuous replication of the existing technology.

By grafting the model onto a version of the Romer (1986) economy, it is possible to allow for technological progress to take place. Each firm has the ability to decide on the degree of vertical depth i.e., the number of intermediate inputs or the degree of specialization at which it conducts its production. Unlike the Romer model, the final good producers themselves produce the intermediate goods from the primary input. Consequently, the commodity as well as the input markets remain competitive. In this environment, it is possible for growth to occur both through technological progress and through replication of the existing technology. It is shown that there may exist multiple balanced growth paths, each reflecting a different pattern of development. In particular, there may be one in which the number of firms remains constant and each firm continuously increases the degree of specialization and another where the level of technology remains constant with growth occurring through the expansion of the number of firms engaged in production – each firm replicating the existing technology. Moreover, and this is the main result of the paper, these growth paths, which are in sharp contrast to one another, generate aggregate output, consumption, investment and factor share paths that are identical. As a result of this observational equivalence, the standard growth accounting framework fails to distinguish between these two processes of growth i.e., paths of development

The remainder of the paper is organized as follows. In section II, the standard growth accounting framework and its limitations in accounting for growth processes are discussed. The next section describes a model where there is no technological progress by construction, but where growth occurs through replication. In section IV, the environment is expanded to allow for technological progress. It is shown that development can take place both through replication and through a deepening of the production process. The section also discusses the main result of the paper, namely, that the growth accounting framework may fail to distinguish contrasting development processes.

II. Growth Accounting

Following Solow (1957) assume that the aggregate technology is given by $Y = AK^\alpha L^{1-\alpha}$ where K and L are capital and labor respectively. The aggregate growth rate is then given by

$$g_Y = g_A + \alpha g_K + (1 - \alpha)g_L$$

Under perfectly competitive conditions the shares of capital and labor in national income are given by α and $1 - \alpha$ respectively. Therefore, one can use the growth rate of capital and labor and weigh them by their respective income shares and compare this with the actual growth rate of the economy. By implication the residual is technological progress. One can therefore use information from aggregate measures of capital and labor to determine the extent of the contribution of resource mobilization to growth i.e., the amount accounted for by increasing inputs and the fraction of growth that occurs due to increases in productivity. There are two reasons why this accounting is important. First, if there is no technical progress i.e., growth

results from increasing resources only, then such processes will, in all likelihood, come to a halt (in the absence of any exogenous increases such as population growth) as returns to the accumulation of each of the factor diminishes with increased mobilization. Second, growth promoting policies are significantly different depending on whether growth is more likely to occur through greater resource mobilization or increased efficiency.

There is however, an important reason why such a framework may not be suitable for accounting for development - the growth generating process. Suppose that technological progress comes embodied in the inputs i.e., through changes in the quality of labor and capital. This occurs in descriptions of growth processes that rely on increasing returns or externalities. In this case, the social valuation of the inputs - the marginal contribution of various inputs to overall growth - may differ from their private valuation, the market price of the inputs, so that relative income shares may not reflect the relative contributions of these factors. As a result, when productivity gains are embodied in inputs, what growth accounting counts as pure technological progress may actually be due to increased usage of inputs. The contribution of increasing returns or externalities is to drive a wedge between the production technology at the individual firm level and that at the aggregate level i.e., the adding up of individual technologies creates an aggregate technology that is functionally different (Romer 1989).

The Romer (1986) model is a useful illustrative example of such a problem. Assume that in the final good sector each firm uses the following constant returns to scale technology,

$$y = l^{1-\alpha} \int_0^M x(i)^\alpha di$$

where l is labor input and $x(i)$ is the amount of the i th intermediate good used in production and M is the number of such intermediate inputs in operation. The intermediate good sector produces x using a reproducible primary good called capital Z . Let the production technology of each of these goods be such that one unit of the primary input produces a unit of an intermediate good. Now, note that for each firm in the final good sector $y = l^{1-\alpha} \int_0^M \left(\frac{Z}{M}\right)^\alpha di$. Consequently, $y = l^{1-\alpha} M^{1-\alpha} Z^\alpha$ such that

$$g_Y = \alpha g_K + (1 - \alpha)g_L + (1 - \alpha)g_M$$

In this case, g_M measures the rate of technical progress. However, if M is itself a function of Z and l then, in equilibrium, technological change is embodied in the inputs and the aggregate technology will only be a function of Z and l . It can be shown that if this technology is such that to produce x amount of the i th good it requires $c_0 + c_1 x$ amount of Z , then, in equilibrium, $M = \frac{1-\alpha}{c_0} Z$ and $x(i) = \frac{\alpha c_0}{(1-\alpha)c_1}$. Consequently, the aggregate technology will be of the form $Y = BL^{1-\alpha}Z$, such that $g_Y = g_K + (1 - \alpha)g_L$. Note that technological progress, i.e., the increase in M , is embodied in capital, K . However, the shares of capital and labor will

still be α and $1 - \alpha$ respectively. Consequently, using the standard technique for measuring technological progress by weighting the growth rates of capital and labor by their respective shares in national income will underestimate the contribution of capital and overestimate that of technology. Moreover, for policy purposes, in the Romer economy, greater resource mobilization leads to greater efficiency as the size of the market, which limits innovation, is increased.

The failure of the Solow growth accounting framework in environments where technological progress is embodied also applies to other endogenous growth models. In the following sections, it is shown that the problem may be further compounded. Even if income shares reflect the contribution of factors, the level of aggregation at which the standard growth accounting framework is applied may not be able to discern one process of development from another. In other words, paths of development which differ dramatically from one another may be observationally the same at the aggregate level.

III. Growth by Replication

In this section an economic environment is described where the economy is capable of generating unlimited positive growth although the average cost function is “U” shaped. The development process, however, is characterized by new firms being continually formed each replicating the existing technology.

A. Technology, Preferences and Demographics

Consider an environment where a single final good y_t is produced by the following technology:

$$y_t = (z_t - \phi)^\alpha$$

where z_t is the amount of capital used in production and ϕ is the amount of fixed capital required to start production and $0 < \alpha < 1$. z_t is used to summarize all reproducible factors like physical capital, human capital etc., and there are no non-reproducible factors of production like land required for production. To complete the model, assume there is a unit measure of households residing in the interval $[0, 1]$. Each household has an infinite life span and the representative household has preferences that can be represented by the function $U = \sum_{t=0}^{\infty} \beta^t \log C_t$, where C_t is the amount of t -period consumption. All factors of production are owned by households and at $t = 0$ each household is endowed with z_0 amount of the reproducible factor. This factor may be a composite of both physical and human capital. In what follows, I shall adopt the convention of using lower case letters to denote firm level variables and upper case letters for aggregate variables.

Both commodity and factor markets are assumed to be competitive. A representative firm (owner of a plant) maximizes profits such that its choice problem is given by

$$\max \pi_t = A(z_t - \phi)^\alpha - R_t z_t$$

where A is a positive constant and R_t is the price of capital. Competitive conditions imply that,

$$\alpha A(z_t - \phi)^{\alpha-1} = R_t \quad (1)$$

Consequently, the firm's profit is given by

$$\pi_t = A(z_t - \phi)^{\alpha-1}[(1 - \alpha)z_t - \alpha\phi]$$

Now suppose that in equilibrium N_t is the number of firms. Also let Z_t be the aggregate capital stock in the economy. Market clearing in the input market implies that $N_t z_t = Z_t$. Therefore in equilibrium,

$$\pi_t = A\left(\frac{Z_t}{N_t} - \phi\right)^{\alpha-1}\left[(1 - \alpha)\frac{Z_t}{N_t} - \phi\right]$$

Entry and exit of firms under competition imply that, in equilibrium, profits must be zero,

$$\pi_t = 0$$

which results in,

$$N_t = (1 - \alpha)\frac{Z_t}{\phi} \quad (2)$$

Using the above expression, the scale of an individual plant is given by,

$$z_t = \frac{\phi}{(1 - \alpha)}$$

which is constant. This result is similar to the Romer model where the quantity of each intermediate good used in production is also constant. Aggregating over plants, output at the

economy level is given by,

$$Y_t = B \frac{1}{\phi^{1-\alpha}} Z_t \quad (3)$$

where, $B = A\alpha^\alpha(1-\alpha)^{1-\alpha}$.

The aggregate technology is now of the AK variety (Rebelo 1991) with a constant marginal product for capital. Consequently even if $Z_t \rightarrow 0$, the marginal product remains finite. This property plays a crucial role in determining whether unlimited growth is possible (see Jones and Manuelli, 1992). To complete the model consider the household's problem of

$$\max \sum_{t=0}^{\infty} \beta^t \log C_t$$

subject to

$$C_t + Z_{t+1} \leq R_t Z_t + (1-\delta)Z_t + \int_0^{N_t} \pi_t$$

$$C_t, Z_t \geq 0$$

The first-order conditions are:

$$\frac{\beta^t}{C_t} = \lambda_t \quad (4)$$

$$\lambda_t = \lambda_{t+1}[R_{t+1} + (1-\delta)] \quad (5)$$

B. The Balanced Growth Path

Letting $\gamma = \beta[A\alpha^\alpha(\frac{1-\alpha}{\phi})^{1-\alpha} + (1-\delta)]$, the balanced growth path is characterized by²

² Combining the first-order conditions one gets $\frac{C_{t+1}}{C_t} = \beta[R_{t+1} + (1-\delta)]$. Using the budget constraint at equality $\frac{C_{t+1}}{C_t} = \beta \frac{[R_{t+1} + (1-\delta)]Z_{t+1}}{Z_{t+1}} = \beta \frac{C_{t+1} + Z_{t+2}}{Z_{t+1}}$. Rearranging the expressions gives $\frac{Z_{t+1}}{C_t} = \beta \frac{C_{t+1} + Z_{t+2}}{C_{t+1}} = \beta(1 + \frac{Z_{t+2}}{C_{t+1}})$. Letting $\frac{Z_{t+1}}{C_t} = X_t$, one gets $X_t - \beta X_{t+1} = \beta$. Since $\beta < 1$, it implies that $X_t = \frac{\beta}{1-\beta}$. Substituting this back in the budget constraint one gets $Z_{t+1} = \beta[R_t + (1-\delta)]Z_t$. Replacing R_t by its equilibrium expression implies that $Z_{t+1} = \beta[\alpha^\alpha(\frac{1-\alpha}{\phi})^{1-\alpha} + (1-\delta)]Z_t$. Therefore, along the equilibrium path, the growth rate is given by $\frac{Z_{t+1}}{Z_t} = \beta[\alpha^\alpha(\frac{1-\alpha}{\phi})^{1-\alpha} + (1-\delta)]$.

$$\left. \begin{aligned} Z_t &= Z_0 \gamma^t \\ Y_t &= \hat{Y} Z_0 \gamma^t \\ C_t &= \hat{C} Z_0 \gamma^t \end{aligned} \right\} \quad (6)$$

where $\hat{Y} = A \frac{1}{\phi^{1-\alpha}}$, and $\hat{C} = \frac{1-\beta}{\beta}$.

Properties of the Balanced Growth Path

Depending on the values of A , ϕ , α and β , unlimited growth is just as likely as perpetual regression. Whether the economy grows or regresses depend on whether $\gamma > 1$. However, the irreversibility of investment implies that the lowest possible growth is bounded by $-\delta$, which corresponds to the path along which net investment is zero. Despite the presence of decreasing returns to scale at the micro-level, long-run growth is possible with growth occurring through the continuous increase in the number of production units in the economy. In this economy, increases in the stock of capital are used up in creating, at the margin, more units of production, all operating at the same scale. As a result, for each production unit the marginal product of capital is kept constant. Although there is no change in quality of capital at the level of the firm, at the aggregate level the efficiency of the economy's capital stock is preserved.

In models of growth that rely on externalities or increasing returns, growth accounting fails primarily because the output elasticity of capital at the micro-level is different from that at the aggregate level. For example in the Romer model, the output elasticity of capital is α while at the aggregate level it is 1. In this model, the elasticity at the micro level is $\frac{\alpha z_t}{z_t - \phi}$, which varies with z_t . In equilibrium, however, the elasticity is 1 (using $z_t = \frac{\phi}{(1-\alpha)}$) which is identical to that at the macro level. Consequently, unlike other endogenous growth models, in this environment it would be appropriate to use the share of z in national income to approximate its contribution to growth.

Like other AK models, the economy has no transitional dynamics; capital always grows at the rate γ , starting from any arbitrary initial level Z_0 . As a result, countries with different initial endowments of capital do not show any tendency to converge. The standard neoclassical growth model predicts that regardless of their initial conditions if economies have the same structural characteristics then their output levels tend to converge over time — the *conditional β -convergence hypothesis* (Sala-i-Martin, 1996). Put differently, once structural characteristics are controlled for countries with lower initial per capita output will grow faster than those with higher per capita output. Behind this notion is the assumption of diminishing marginal productivity of factors of production. Suppose the aggregate production technology is given by AK_t^α , where $\alpha < 1$. Then over time capital cannot increase indefinitely. In the absence of any other effects, increased capital accumulation will cause the marginal product of capital to fall continually, such that the economy must converge to a unique globally stable steady state equilibrium regardless of the initial stock of capital. In the environment discussed

in this section economies will not display any tendency towards conditional β –convergence.

Based on the data set in Summers and Heston (1991) over the period 1960–85 the following three principal findings or development facts emerge (for a summary see Parente and Prescott, 1993):

1. There is a huge disparity in the wealth of nations. The per capita output of the five richest countries in 1985 was 29 times greater than of the five poorest countries.
2. The range of relative wealth distribution has remained fairly constant though there have been large movements within the distribution.
3. While the distribution of relative wealth has remained fairly stationary, the level of wealth has shifted upwards over time.

Romer (1986) and Lucas (1988) used these observations to reject the standard neo-classical growth model and introduce endogenous growth models. These models led to another version of the convergence hypothesis — the *club convergence hypothesis* (Galor, 1996) — which argues that countries that have similar structure converge to one another if they have similar initial conditions. This convergence argument seems consistent with the second and third fact i.e., that the relative distribution of wealth among nations have remained fairly stable. The model described here does in fact display these characteristics i.e., that countries with similar structure and initial conditions will converge and a stochastic version of the model (one with an exogenous technological shock that follows, for example, a stationary Markov process) may even display movements within the distribution while keeping the distribution stable. However, such an argument is quite vacuous - if economies are in fact similar to one another both in terms of structure as well as history, then it is not surprising that they should have similar performance. What makes this model different is that such a non-convergence result follows without appealing to increasing returns or externalities.

In this model, both C and I grow at the same rate and their relative share in Y are constant. The capital-output ratio remains constant at \hat{Y} . Thus all the Kaldorian stylized facts about growth that pertain to the model are satisfied. Differences in ϕ across countries will give rise to perpetual differences in productivity and growth. Such differences may stem from variations in institutional arrangements, government policy etc. among countries.

Optimality of the Balanced Growth Path

Whether the equilibrium rate of growth is optimal depends on the ability of the zero-profit condition in determining the optimal number of participating firms. Like the Romer model, in this environment individual decisions i.e., whether or not to set up an extra plant at the margin, produce a positive externality at the aggregate level. Note that aggregate output is given by $Y = AN^{1-\alpha}(Z - N\phi)^\alpha$. If N is chosen so as to maximize Y , then $\frac{\partial Y}{\partial N} = AN^{-\alpha}(Z - N\phi)^{\alpha-1}[(1 - \alpha)Z - N\phi]$. Consequently, the socially optimal number of firms is the same as the one dictated by the zero-profit condition, namely $(1 - \alpha)\frac{Z}{\phi}$. This is in direct contrast to the Romer economy where the positive externality remains unexploited and

therefore government intervention improves efficiency.

IV. Growth Through Technological Progress

In the previous section, by construction individual participants did not have the option of bringing about changes in the level of the technology and consequently firms simply replicated the existing technology. To allow firms to choose the level of technology, following Romer (1986), technological change is introduced as a vertical deepening of the production process. In the Romerian world the final good sector's technology is given by

$$y_t = A \int_0^{M_t} x_t^\alpha(i) di$$

where M is the number of intermediate inputs and $x_t(i)$ is the quantity of the i th input used in production. Each of the intermediate input is produced from a primary input capital Z_t . Since each intermediate input enters the production technology in an identical manner, if intermediate goods could be produced by a constant returns to scale technology then output of the final good is given by

$$y_t = AM_t^{1-\alpha} Z_t^\alpha$$

such that even unlimited growth is possible by continually increasing M_t i.e., increasing the vertical depth of the production process. In order to achieve a determinate solution for M_t , Romer imposes a fixed cost of producing the intermediate good. But since the final goods technology is subject to constant returns to scale, under competitive conditions, the sector would exhaust its total output by paying labor and capital their marginal products. Without any profits the fixed cost of producing intermediate inputs cannot be covered. In order to avoid this problem, Romer assumes that the intermediate goods are produced by a separate sector i.e., firms producing the final good are not vertically integrated.

For reasons discussed earlier, in this paper the final goods sector is such that even after paying capital its marginal product, positive profit is possible. As a result, one can allow the final goods sector to be vertically integrated i.e., firms can produce their own intermediate inputs from the primary input Z_t even with fixed costs of production. With this in mind, the technology in the Romer model is modified such that firms in the final good sector can choose their own level of vertical depth. The modified technology is given by

$$y_t = A \int_0^{M_t} x_t^\alpha$$

subject to

$$\int_0^{M_t} (c_1 x_t + c_0) \leq z_t$$

Again, since the intermediate inputs enter the production process in an identical manner one can rewrite the technology as

$$y_t = A \left(\frac{1}{c_1} \right)^\alpha m_t^{1-\alpha} (z_t - c_0 m_t)^\alpha$$

An interpretation of this technology is that the overhead cost of producing output is no longer fixed as in the earlier section but depends on the level of vertical deepening of the production process – the more complex the process, higher is the overhead cost.

As before, under competitive conditions, a representative firm maximizes profits such that its choice problem is given by

$$\max \pi_t = A \left(\frac{1}{c_1} \right)^\alpha m_t^{1-\alpha} (z_t - c_0 m_t)^\alpha - R_t z_t$$

One can derive optimal profit as

$$\pi_t = A \left(\frac{1}{c_1} \right)^\alpha m_t^{1-\alpha} (z_t - c_0 m_t)^{\alpha-1} [(1-\alpha)z_t - c_0 m_t]$$

Market clearing in the capital market demands that $z_t = \frac{Z_t}{N_t}$, while that in the commodity market implies that existing firms do not make positive profit. Together, these conditions imply that in equilibrium

$$m_t = \frac{(1-\alpha)Z_t}{c_0 N_t} \tag{7}$$

Consequently, output at the individual firm level is given by $y_t = A \left(\frac{\alpha}{c_1} \right)^\alpha \left(\frac{1-\alpha}{c_0} \right)^{1-\alpha} \left(\frac{Z_t}{N_t} \right)$,

rate of interest $R_t = A \left(\frac{\alpha}{c_1} \right)^\alpha \left(\frac{1-\alpha}{c_0} \right)^{1-\alpha}$, and aggregate output $Y_t = A \left(\frac{\alpha}{c_1} \right)^\alpha \left(\frac{1-\alpha}{c_0} \right)^{1-\alpha} Z_t$.

A. The Balanced Growth Path

Letting $\gamma = \beta[A \left(\frac{\alpha}{c_1} \right)^\alpha \left(\frac{1-\alpha}{c_0} \right)^{1-\alpha} + (1-\delta)]$ and following the same method as in the previous section, the balanced growth competitive equilibrium is characterized by

$$\left. \begin{aligned} Z_t &= Z_0 \gamma^t \\ Y_t &= \hat{Y} Z_0 \gamma^t \\ C_t &= \hat{C} Z_0 \gamma^t \end{aligned} \right\} \quad (8)$$

where $\hat{Y} = B \frac{1}{\phi^{1-\alpha}}$, and $\hat{C} = \frac{1-\beta}{\beta}$.

For this economy aggregate output is given by

$$Y_t = A \left(\frac{1}{c_1} \right)^\alpha N_t^{1-\alpha} m_t^{1-\alpha} (Z_t - c_0 N_t m_t)^\alpha$$

If N is chosen so as to maximize Y , then

$$\frac{\partial Y}{\partial N} = A N^{-\alpha} (Z - N\phi)^{\alpha-1} [(1-\alpha)Z - N\phi]$$

Consequently, the social optimal level of firms is the same as the one dictated by the zero-profit condition namely, $(1-\alpha) \frac{Z}{\phi}$. Again, the zero-profit condition does in fact deliver the socially optimal level of participation in the economy.

B. Observational Equivalence of Growth Paths

Note that this balanced growth path is compatible with both *lateral expansion*, where the number of firms increases continually with each firm replicating the existing technology, and a path of *vertical deepening*, where the number of firms remains constant with each existing firm continually improving the efficiency of the production process by bringing into operation more complex production designs.

To see the first process, set $m_t = m_0$, the level of technology at the initial stage. Then, in equilibrium, $N_t = \frac{(1-\alpha)Z_t}{c_0 m_0}$. Each firm's output now gets fixed at a constant level $\bar{y} = A \left(\frac{\alpha c_0}{(1-\alpha)c_1} \right)^\alpha m_0$. Apart from these changes other variables in the economy do not change. Growth rate of aggregate remains the same. Aggregate output is still given by $Y_t = A \left(\frac{\alpha}{c_1} \right)^\alpha \left(\frac{1-\alpha}{c_0} \right)^{1-\alpha} Z_t$ and the share of capital income by construction will remain at 1. On the other hand, if $N_t = N_0$, the initial number of firms, then $m_t = \frac{(1-\alpha)Z_t}{c_0 N_0}$. Each firm continually increases output as productivity at the firm level improves. At the firm level, $y_t = A \left(\frac{\alpha}{c_1} \right)^\alpha \left(\frac{1-\alpha}{c_0} \right)^{1-\alpha} \left(\frac{Z_t}{N_0} \right)$. However, aggregate output, interest rate and the share of capital remain the same. Apart from these two extreme cases, along the balanced growth path described in (8) any combination of these two situations is also possible. The following proposition summarizes this result.

Proposition *Given initial conditions $\{K_0, N_0, m_0\}$, the economy has a continuum of equilibria indexed by $\{Z^j, N^j, m^j\}$, where $m^j = \frac{(1-\alpha)Z^j}{c_0 N^j}$, and $\{Y^j, Z^j, C^j\}$ is given by (8). Along all these paths the growth rate is given by $\gamma = \beta \left[A \left(\frac{\alpha}{c_1} \right)^\alpha \left(\frac{1-\alpha}{c_0} \right)^{1-\alpha} + (1-\delta) \right]$.*

The model presented here is essentially the same as in Romer (1986) with one modification. In the original model, the final good sector technology was subject to constant returns to scale such that after paying inputs their marginal product firms exhausted all their output. This assumption resulted in two consequences. One, the number of firms in the final good sector did not matter and two, final good producers could not pay for technological progress i.e., in the language of that model, increase the variety of inputs. The only difference between the model analyzed in this section and the Romer economy is that the final good technology is subject to a "U" shaped average cost function. Consequently, even after paying inputs their marginal products, firms do not use up all their output. This possibility of positive profit results in two things - one, the number of firms matters in the determination of aggregate output and two, firms can use part of their output in increasing the productivity of the technology i.e., by increasing the vertical depth of the production process. The assumption of decreasing returns to scale does not seem to be unreasonable. Moreover, since in equilibrium aggregate output is exhausted as payment to inputs, there is no positive profit. This corresponds quite closely to the observed low level of profit in the data, at least, for the developed economies.

However, as discussed, the assumption of decreasing returns to scale has quite significant effects. As far as the Romer model is concerned, it is no longer true that along the balanced growth path there will be continual improvement in productivity. Growth can take place with the number of firms increasing at a constant rate. Moreover, given any set of initial conditions, the multiple balanced growth paths of the economy all have identical paths for aggregate output with significant differences in the process that generates growth. This observational equivalence of the aggregate output paths prevents the standard growth accounting framework from isolating pure productivity changes.

V. Concluding Remarks

Growth accounting plays an important role in the evaluation of public policies. Whether countries develop through greater mobilization of resources or more efficient utilization of inputs not only allows one to choose among competing theories of development but also sheds light on the historical role of particular interventionist policies. From a forward looking perspective, such exercises may make it possible to compare the relative effectiveness of plausible policy alternatives. However, growth accounting exercises that are confined to fairly high degree of aggregation may generate results that can be at odds with the underlying development processes. In descriptions of development processes that rely on externalities and increasing returns, the standard Solow methodology may underestimate the role of resource mobilization especially since technological progress is embodied in factor inputs in such models. In this paper, it was shown that there may be another dimension to this problem. Growth accounting using aggregate data may in fact fail to distinguish different processes of development. In the example described in this paper, two contrasting processes of growth - one involving technological progress and the other with technological replication - both generated identical paths for output and the capital stock.

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