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Is the Phillips Curve Really a Curve? Some Evidence for Canada, the United Kingdom, and the United States

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Abstract

Previous tests for convexity in the Phillips curve have been biased because researchers have employed filtering techniques for the NAIRU that have been fundamentally inconsistent with the existence of convexity. This paper places linear and nonlinear models of the Phillips curve on an equal statistical footing by estimating model-consistent measures of the NAIRU. After imposing plausible restrictions on the variability in the NAIRU we find that the nonlinear model fits the data best. The implications for the macroeconomic policy debate is that policymakers that are unsuccessful in stabilizing the business cycle will induce a higher natural rate of unemployment.

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Summary

Previous tests for convexity in the Phillips curve have been biased because researchers have employed filtering techniques for the nonaccelerating inflation rate of unemployment (NAIRU) that have been fundamentally inconsistent with the existence of convexity. A preferred statistical methodology would place both the linear and nonlinear models on an equal statistical footing by estimating model-consistent measures of the NAIRU.

This paper proposes a Kalman filter maximum likelihood procedure to simultaneously estimate the parameters of the model along with model-consistent estimates of the NAIRU. A novel feature of the study is that information is used from bond markets to develop measures of inflation expectations. With plausible restrictions imposed on the variance of the NAIRU, it is found that the nonlinear model fits the data better for Canada, the United Kingdom, and the United States.

The paper demonstrates that asymmetry in the unemployment-inflation process has important policy implications. Stabilization policies that are not very successful at reducing the variability in the business cycle can have very undesirable consequences, not only for the variance of unemployment but also for the natural rate of unemployment. Uncertainty about the true level of the NAIRU reinforces the case for adopting a cautionary strategy of raising interest rates before the economy reaches potential. It may thus be optimal in the context of uncertainty to develop a strategy that attempts to avoid large boom-bust cycles, rather than one that attempts to fine-tune policy to ensure that all resources are fully employed at all points in time.

I. INTRODUCTION

Assessments of the rate of unemployment that is consistent with holding inflation stable (at a low rate) represent an integral part of the monetary policy framework in most industrial countries. Whether the current rate of unemployment lies above or below the rate that is consistent with inflation stability is a key input into the monetary policy decision-making process. Unfortunately the rate of unemployment that is consistent with inflation stability is not directly observable; policymakers can only estimate it by using other, more observable pieces of information about the state of the economy.

For the United States, estimates of Phillips curves and the NAIRU were regularly published in the Brookings Papers during the 1970s and early 1980s by Robert Gordon (1970, 1975, 1977). After a period of inactivity in the 1980s when Phillips curves were generally assumed to have broken down, a number of recent papers (Gordon (1994), Fuhrer (1995)) have sought to re-estimate the Phillips curve and derive new estimates of the natural rate of unemployment. The general approach has been to regress inflation on a measure of expected or lagged inflation, unemployment gaps and dummy variables to control for various supply shocks such as the oil shocks and the Nixon price controls.

Despite the variety of techniques used, the common feature of these recent estimates is their use of a linear Phillips curve. A separate strand of the literature has presented evidence supporting the concept of a nonlinear Phillips curve (Macklem (1996), Clark, Laxton and Rose (1996), Turner (1995), and Laxton, Meredith and Rose (1994)). A convex nonlinear Phillips curve implies that a given fall in the unemployment rate below the NAIRU causes a larger rise in inflation than does a rise of the same magnitude produce a fall in inflation.

The literature on the relationship between inflation and unemployment frequently refers to both the nonaccelerating inflation rate of unemployment (NAIRU) and the natural rate of unemployment. With linear specifications of the Phillips curve, the two terms are often used interchangeably. More generally, when allowing for the possibility of a nonlinear Phillips curve and stochastic variability in demand and supply it is useful to distinguish the NAIRU at a point in time from the expected value over time of the unemployment rate that would be consistent with nonaccelerating inflation, given the stochastic distributions of shocks. Following Friedman (1968), this paper uses the term "natural rate of unemployment" to refer to the latter concept. The nonlinearity in the model implies that policymakers who are less successful in stabilizing the business cycle will induce a larger natural rate of unemployment in their economies (De Long and Summers (1988) and Laxton, Rose and Tetlow (1993b)).

Laxton, Rose and Tetlow (1993a) demonstrate that previous tests for nonlinearity have been severely compromised because researchers have tended to rely upon ad hoc prefiltering techniques that are grossly inconsistent with the key implications of a nonlinear Phillips curve. In this paper, we derive estimates of the NAIRU and natural rate of unemployment for three countries—Canada, the United Kingdom and the United States—by extracting information present in a nonlinear model of the Phillips curve. As in Kuttner

(1992,1994), we use a Kalman filter and a maximum likelihood procedure to simultaneously estimate the parameters of the model along with model-consistent estimates of the NAIRU. We also estimate a linear model of the Phillips curve using the Kalman filter. We show that the nonlinear model fits the data better than the linear model for all three of these countries when plausible restrictions are imposed on the variance of the natural rate.

The next section describes the linear and the nonlinear models that we estimate and describes the Kalman filtering technique that we use in the estimation. The derivation of the inflation expectations series, which is a key component of our model is described in Section III. Section IV presents the results while Sections V and VI discuss the uncertainty and sensitivity of the results. Section VII concludes.

II. MODELS AND ESTIMATION TECHNIQUE

This section describes the models and estimation procedure that we use to estimate the NAIRU. The standard linear expectations-augmented Phillips curve has the following functional form:

$$\pi_t=\pi_t^e+\gamma(u^*-u_t)+\epsilon_t \qquad (1)$$

where π is inflation, u is the unemployment rate and u^* is the NAIRU. Inflation expectations π^e are a linear combination of a backward and forward-looking component (see the discussion in Section III). The backward-looking component could also reflect inertia in the inflation process. For example, an overlapping contracts model such as Fischer (1977), could motivate such a result.

$$\pi_t^e = \lambda\ A^{-1}(L)\pi_{t-i} + (1-\lambda)\ B(L)\pi_{t-i} \qquad (2)$$

where A(L) and B(L) are polynomial lag operators.

Equations such as (1) were estimated by Gordon in the 1970s and early 1980s—see Gordon (1970, 1975, 1977). To proxy inflation expectations, Gordon used lags of the inflation rate of up to two years and also controlled for a number of supply side influences such as price controls, relative price changes and real exchange rate changes. Furthermore, Gordon imposed the constraint that the sum of the coefficients on lagged inflation sum to one. Evidence in favor of this restriction is still sometimes interpreted as support for the long-run natural rate hypothesis despite the fact that Sargent (1971) explained years ago that this restriction has nothing to do with the long-run natural rate hypothesis. The restriction is inappropriate because it would imply that, given a stationary unemployment gap (u^*- u), the natural rate hypothesis could only be true in those countries where inflation has a unit root. In the case of the United States, the natural rate of unemployment has either been assumed to be constant over the sample period or some small shifts have been estimated to control for changes in the composition of the labor force.

More recently, Gordon (1994), Tootell (1994) and Fuhrer (1995), have estimated similar models, which also assume that the natural rate is constant over the sample period. This assumption is supported by both within-sample and out-of sample tests that fail to reject the hypothesis of a constant natural rate. While this assumption may be difficult to reject in the United States, it is likely to be rejected in countries where the natural rate has clearly moved over time (such as the United Kingdom and Canada).

Staiger, Stock and Watson (1996) employ a variety of techniques to derive estimates of the natural rate based on Gordon's approach and on univariate analysis and highlight the uncertainty associated with the estimates. They obtain estimates of the natural rate by assuming, alternatively, that it is a constant over the sample period, a constant with occasional shifts, an unobserved random walk, and a specific function of labor market variables. A typical estimate of the degree of uncertainty associated with the estimates is a 95 percent confidence interval of 5.1–7.7 percent (around a point estimate of 6.2 percent) in 1990. In addition, the point estimates vary quite substantially across the different techniques.

Kuttner (1992, 1994) adopts a strategy that is the closest to the one adopted here. He estimates a model of the Phillips curve allowing for time variation in the level of potential output. He employs a Kalman filter to extract an estimate of the level of potential output where potential output is assumed to be a random walk with positive drift. However, once again, Kuttner assumes that the Phillips curve has a linear specification.

The key difference between this paper and the previous literature is that we estimate the natural rate of unemployment in the context of a non-linear Phillips curve. The nonlinear Phillips curve we use here is assumed to have a simple structure of the form:

$$\pi_t = \pi_t^e + \gamma \frac{(u_t^* - u_t)}{u_t} + \epsilon_t \qquad (3)$$

where u is the observed unemployment rate, and u* is the time-varying unemployment rate at which inflation is constant (the NAIRU).[2]

A nonlinear Phillips curve may be motivated by the traditional concept of an upward sloping aggregate supply curve. As the unemployment rate falls below the NAIRU, bottlenecks start to develop which result in further increases in demand having even larger inflationary consequences. In the limit, once the unemployment rate reaches some lower bound, inflation will increase at an almost infinite rate.

As mentioned above, in the nonlinear model, there is a distinction between the NAIRU and the natural rate which is not present in the linear model. If one defines the natural rate as the expected value of unemployment in the stochastic steady state, then the convexity of the nonlinear Phillips curve implies that the natural rate of unemployment will lie above the NAIRU, by a constant α that embodies the degree of convexity and the nature of the stochastic shocks. In the traditional Phillips curve model, the linearity ensures that the NAIRU and the natural rate are the same.

This distinction is most easily demonstrated if we assume that the Phillips curve takes a slightly different functional form:

$$\pi_t = \pi_t^e + \exp(\gamma(u^* - u_t)) - 1 + \epsilon_t \qquad (4)$$

The NAIRU is given by u*—when the rate of unemployment is equal to u*, inflation is equal to inflation expectations and is neither rising nor falling. If we assume that the error term is normally distributed with zero mean, then the average rate of unemployment is given by u*+ var(u_t)/2. That is, the stochastic steady state rate of unemployment, which we interpret as the natural rate (in the sense of Friedman (1968)), is greater than the NAIRU. Furthermore, policies that reduce the variance of unemployment, will reduce the natural rate of unemployment. That is, a policymaker which is more effective in stabilizing the business cycle will reduce the gap between the NAIRU and the natural rate.

The top panel of Chart 1 is a useful device for illustrating the implications that such asymmetry has for stabilization policy. The nonlinear curve in the chart, the "Phillips curve," depicts the short-run relationship between inflation (π), adjusted for inflation expectations (π^e), and the unemployment rate (UNR). The key assumption underlying the Phillips curve is that the slope of the curve, or the tradeoff between unemployment and inflation, worsens as

[2]This functional form has been used to model the unemployment-inflation process at the Australian Treasury, see Downes and Johnson (1995). It is approximately linear in the region where unemployment is equal to the NAIRU but allows for an appealing restriction that the unemployment rate cannot fall below zero.

unemployment falls significantly below the NAIRU. For illustrative purposes we assume that the NAIRU is equal to 5 percent. The chart illustrates this point by incorporating an incontrovertible assumption that there is some minimum level of unemployment that cannot be obtained through expansionary aggregate demand management policies even in the short run. For purely illustrative purposes, it is assumed that this minimum level of unemployment is equal to 1 percent of the labor force.

Because excess demand conditions are more inflationary than excess supply conditions are disinflationary, allowing the economy to enter the region of excess demand implies that the economy will have to operate longer in the region of excess supply to prevent inflation from drifting upwards over time. Thus, if disturbances to the economy cause the unemployment rate to vary over time, the natural rate of unemployment (NAT)—or the average unemployment rate that is consistent with stable inflation—will be larger than the NAIRU that enters the Phillips curve, because unemployment will have to spend more time above the NAIRU in order to offset the greater inflationary tendencies that will be associated with periods when it falls below the NAIRU. For illustrative purposes, it is assumed in the top panel of Chart 1 that the unemployment rate varies between 4 percent and 8 percent, and that the natural rate of unemployment is 6 percent, or one percentage point above the NAIRU.

This asymmetry in the unemployment-inflation process has important policy implications. Stabilization policy that is not very successful at reducing the variability in the business cycle can have very undesirable consequences, not only for the variance of unemployment, but also for the natural rate of unemployment. The bottom panel of Chart 1 illustrates this point by considering the alternative case in which unsuccessful stabilization policy allows the unemployment rate to vary over a wider range than in the top panel, which periodically subjects the economy to serious overheating. In this case the natural rate (shown as 7 percent) will be even higher than in the first case because it will take larger excess supply or recessionary states to offset the greater inflationary impetus caused from periodically subjecting the economy to serious overheating.

The concept of the NAIRU that is used in this paper is that which is generally used in discussions of the "natural rate." The NAIRU will be affected by the operation of the labor market and embodies the "actual structural characteristics of the labor and commodity markets, including market imperfections" (Friedman (1968), p. 8). Labor market policy will affect the level of the NAIRU through time. However, the distinction we are focusing on here is that macroeconomic policy will have a further impact on the natural rate through its effect on the variability of the macroeconomy.

Previous work has tried to estimate the determinants of the NAIRU directly, but with mixed success. For example, Lilien (1982) examines the effect of sectoral changes on the structural unemployment rate. Blanchard and Katz (1996) provide a recent summary of this research. While acknowledging that changes in labor market policy and institutional factors will affect the NAIRU, we do not directly identify the impact of these factors on the NAIRU in our model.

The nonlinear functional form that we employ in our empirical work allows for a hyperbolic shape which asymptotes at zero. In economic terms, this implies that as the unemployment rate approaches zero, the inflation rate increases at a higher and higher rate. Allowing the asymptote to be at zero is, perhaps, being overly conservative. It is likely that the economy will run up against insurmountable supply constraints before the unemployment rate reaches zero. Nevertheless, a zero unemployment rate provides an uncontroversial lower limit for the asymptote.

The NAIRU u* is allowed to be time-varying in both the linear and the nonlinear models. To estimate the Phillips curve, we need estimates of the NAIRU which is not directly observable. The Kalman filter allows us to estimate the model while simultaneously providing a time-series estimate of the NAIRU. The Kalman filter estimates models of the general form:[3]

$$y_t = X_t' \beta_t + \epsilon_t \qquad \epsilon_t \sim N(0,\sigma^2 H) \tag{5}$$

$$\beta_t = T * \beta_{t-1} + \mu_t \qquad \mu_t \sim N(0,\sigma^2 Q) \tag{6}$$

The parameter vector β is time-varying, in a manner determined by the transition equation (6). In our models we allow the NAIRU u* to be time-varying, and more particularly a random walk. As mentioned above, the NAIRU will be affected by structural changes in the labor market including those induced by labor market policy. However, for estimation purposes, we have not directly modeled the effects of such changes.

As Kuttner (1994) points out, estimating the natural rate using a Kalman filter has three advantages: It allows us to use the information present in the difference between inflation and inflation expectations, rather than solely relying on the univariate properties of the unemployment rate. Secondly, it allows for "smooth continuous adjustment of the estimate in real time as new data becomes available." This will be a particular advantage in the context of policy making decisions. Thirdly, we can derive estimates of the uncertainty about the natural rate as in Staiger, Stock and Watson (1996).

Operationally, we estimate the linear model with the Kalman filter, allowing the constant term γu* in equation (1) to be time-varying. We assume that the matrices H and T are identity matrices, while the matrix Q is constructed so that only the constant term is time-varying. In the nonlinear model, the coefficient on the inverse of the unemployment rate is

[3]For more information on the Kalman filter, see Harvey (1981).

time-varying. In both models, the time-series for the NAIRU is obtained by dividing the time-varying parameter obtained from the Kalman filter, by the (time-invariant) estimated coefficient γ.

Application of the Kalman filter generates two time series of the NAIRU. The first comes from the recursive estimation of the model which uses data that is available only up to the current period which Kuttner (1992) refers to as "one-sided" estimates. The second (smoothed or "two-sided") series uses data from the whole sample to estimate a time-series for the NAIRU and the parameters of the model to maximize the likelihood function. The one-sided estimates allow an assessment of how the model performs in "real time."

The information we use to identify movements in the NAIRU is the difference between inflation and inflation expectations. We proxy the backward looking component of inflation expectations by the first lag of inflation. The estimates of the forward-looking component of inflation in this paper are derived from information present in long bond rates. The technique used in this latter step is described in the next section.

III. THE DATA

All data are quarterly and are drawn from the OECD's analytical database. The unemployment and CPI series are based on official country seasonally adjusted estimates. Our approach to measuring inflation expectations is based on a simple idea that long-term interest rates may embody valuable information about policy credibility—for example, see Goodfriend (1993) and McCallum (1995). We use information from bond markets to develop a measure for long-term inflation expectations (π_t^{LTE}) and then this series is used to help identify the short-term measures of inflation expectations (π_t^E) that enter the Phillips curve.

This approach assumes that the inflation expectations of wage and price setters are related to the inflation premia that participants in the bond market demand to hold long-term fixed-income securities. Specifically, it is assumed that short-term inflation expectations are a weighted average of long-term inflation expectations and actual inflation in the previous period, $\delta\pi_t^{LTE} + (1\text{-}\delta)\pi_{t-1}$, where δ represents the weight on information from the bond market.

The measure of long-term inflation expectations for each country is constructed by taking measures of long-term interest rates in these countries and then subtracting alternative measures of the equilibrium world real interest rate. After constructing these proxies for long-term inflation expectations, we then test to see if these proxies provide significant explanatory power in the Phillips curve.

The estimates of the equilibrium world real interest rate that we employ are based on recent empirical work that suggests that the equilibrium real interest rate has been gradually rising over time in response to the large buildup in world government debt—for example, see Ford and Laxton (1995), Helbling and Wescott (1995), and Tanzi and Fanizza (1995).

The basic approach that we follow to derive estimates of the world equilibrium real interest rate is quite simple. First, in order to provide a reasonable benchmark for the equilibrium world real interest rate at the end of the sample we use data on indexed bonds. Second, after obtaining this benchmark for the end of the sample, we then rely on estimates of the effects of government debt on real interest rates to construct a time series that extends back to the early 1970s.

Our preferred set of estimates are based on the empirical work by Tanzi and Fanizza (1995). Their work suggests that a 1 percentage point increase in the gross government debt-to-GDP ratio increases the world real interest rate by about 7 basis points. Based on a benchmark estimate of 4.5 percent for the real interest rate at the end of the sample, the upward trend in gross public debt since the late 1970s would suggest that the equilibrium real interest has gradually shifted up from an average value of 2.4 percent in the 1970s to 4.5 percent by the first quarter of 1994.[4] Other estimates were also constructed using alternative values of 3.5 percent and 5.5 percent for the end-of-sample benchmark.

In addition, because the effects of government debt on real interest rates vary significantly across different empirical studies, we also calculated estimates that were based on both smaller and larger estimates of the effect of debt buildup on the equilibrium world real interest rate. Firstly, to be consistent with the estimates reported by Ford and Laxton (1995) and Helbling and Wescott (1995), we doubled the assumed effect of world government debt on the equilibrium real interest rate. Secondly, we assumed that government debt had no effect on the equilibrium real interest rate and thus that the equilibrium real interest rate has been constant since the early 1970s. This last assumption is more consistent with earlier empirical work that suggested that government debt has had no discernible effect on real interest rates—for example, see Evans (1985).

Despite the different assumptions that were used to construct the equilibrium real interest rate, we found that the resultant measures of long-term inflation expectations were highly significant in all the regressions that were considered.[5] In fact, in not one of the alternative specifications described above could we reject the hypothesis that long-term bond yields do not provide significant explanatory power for identifying historical shifts in the Phillips curve. This result is encouraging because, in theory, there should be some relationship between long-term inflation expectations of participants in the bond market and the more short-term inflation expectations that influence the decisions of wage and price

[4]The benchmark estimate at the end of the sample was based principally on examining data from indexed government bonds in Canada.

[5]We do not report all of the results in this paper but they can be obtained from the authors.

setters. In this vein, our results tend to support Goodfriend's (1993) conclusion that most of the high frequency variation in long-term bond yields are driven by inflation scares rather than by historical movements in the ex ante real rate of interest.

IV. Estimation Results

This section presents empirical estimates of Phillips curves and Phillips lines for Canada, the United Kingdom and the United States. The basic empirical strategy followed in this paper is to derive model-consistent measures of the NAIRU under the assumption that the Phillips curve is really a curve and then to compare the results of this model to an alternative model that assumes the curve is linear.

The first model that we estimate assumes that the amount that actual inflation changes relative to expected rates, $\pi - \pi^e$, is related to the proportional difference between the NAIRU and the actual unemployment rate, (NAIRU-UNR)/UNR—see the top of Table 1. We refer to this as a Phillips curve because this specification hypothesizes that the relationship between inflation and unemployment is approximately linear and symmetric in the region where unemployment is close to the NAIRU but the unemployment-inflation tradeoff worsens considerably as unemployment is further away from the NAIRU. The results where we assume a linear structure, replacing the (NAIRU-UNR)/UNR term with (NAIRU-UNR), are reported in Table 2.

The estimated parameters were obtained using the maximum-likelihood Kalman filter routine in TSP. This technique builds up model-consistent estimates of the NAIRU under the assumption that we can approximate historical shifts in the NAIRU by a random walk.[6] In both tables we include the value of the likelihood function (LLF), the estimated parameter values and their associated standard errors. We also report the maximum absolute gap between the NAIRU and the actual unemployment rate (UNR), and the maximum absolute quarterly change in the NAIRU to provide some indication of how jumpy the NAIRUs have to be to explain inflation in these countries.

A. The Phillips Curve with Model-Consistent NAIRUs

As can be seen in Table 1, the estimated parameters (δ) on the forward-looking proxy for long-term inflation expectations as well as the estimated parameters (γ) on the measure of labor market tightness are statistically significant. The estimates of δ can be interpreted as implying that wage and price setters place a weight of between 8 percent and 20 percent on

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[6] The NAIRU does not literally have to follow a random walk. This simple parsimonious process was chosen because it is flexible enough to allow for permanent shifts in the NAIRU in finite samples.

Table 1. Estimates of Phillips Curves with Model Consistent NAIRUS

(t-statistics in parentheses)

Estimated equation: $\pi_t = \pi_t^E + \gamma\,(\text{NAIRU-UNR})/\text{UNR} + \epsilon_t^{\pi}$

Where:

π Percent change in the CPI at annual rates

π_t^E Inflation expectations, $\delta\,\pi_t^{LTE} + (1-\delta)\pi_{t-1}$

π_t^{LTE} Long-term inflation expectation proxy from bond market

δ Estimated weight on inflation proxy from bond market

UNR Unemployment rate

NAIRU Estimates derived from Kalman filter

LLF Value of the Likelihood Function

σ^2 Variance of Residuals

α Average historical value of UNR-NAIRU

Ω Maximum absolute value of (NAIRU - UNR)

τ Maximum absolute Quarterly Change in NAIRU

Estimation period: 1971Q2 to 1995Q2

Country	γ	δ	LLF	σ^2	α	Ω	τ
Canada	2.37	0.15	-104.03	0.41	0.86	5.1	0.21
	(4.65)	(3.47)					
United Kingdom	2.43	0.20	-176.41	1.81	0.57	3.79	0.65
	(3.70)	(4.48)					
United States	3.55	0.08	-104.85	0.36	0.33	4.01	0.15
	(7.32)	(2.45)					

forward looking inflation expectations. A higher value of γ implies that a given amount of excess demand has a larger inflationary impact, but also that a given amount of excess supply has a larger disinflationary impact.

Table 1 also shows estimates of α[7], the difference between the historical average rate of unemployment and the NAIRU (recall Chart 1)[8]. As emphasized earlier, in economies where policymakers have been less successful in avoiding boom and bust cycles (which may depend on the nature of the shocks hitting the economy) we should expect to observe a larger difference between the average rate of unemployment and the NAIRU. The estimates of α shown in Table 1 suggest that policymakers in the United States have been more successful in stabilizing their business cycle than their counterparts in the other two countries, or that the shocks that have hit the U.S. economy have been smaller. Other things being equal, this would have contributed to a lower natural rate of unemployment (or average rate of unemployment) in the United States.

The value of α is assumed to be constant here. However, in reality it is likely to be time-varying. The value depends on how successful past policymakers were in stabilizing the business cycle. If policymakers become more successful at avoiding large boom and bust cycles over time one should expect that α would shift down somewhat over time as the economy moved to a new stochastic steady state. For example, after the recent disinflation episodes in Canada and the United Kingdom, and the subsequent relatively stable inflation process, one might expect that the value of α may decline, reflecting the attainment of a new regime with an improved stabilization performance. However, as suggested in Section V this may occur very gradually if it takes a long time to develop credibility in the new regime.

The model-consistent estimates of the NAIRU from the Phillips curve are plotted in Chart 2. This methodology for measuring the NAIRU produces estimates that are fairly smooth despite the fact that the random walk assumption in the measurement equation could, in principle, produce fairly jumpy measures of the NAIRU if this was necessary to provide a good explanation of inflation in these countries. In fact, the estimates suggest that the maximum quarterly change in the NAIRU in the United States was only 0.15 percentage points since the early 1970s. Although the jumps in the NAIRU are somewhat larger for Canada and the United Kingdom, the estimated NAIRU series are also fairly smooth in these countries. According to these estimates the increase in the NAIRU from the low levels that

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[7]Standard errors are not reported for the estimate of α. Standard errors could be obtained in an extended system which included an equation for the unemployment gap, in addition to the Phillips curve.

[8]Our estimate of α for the United States is fairly close to that obtained by Mankiw (1988).

have been measured for the 1950s and 1960s started sooner in Canada and the United States.[9] In particular, this methodology suggests that the NAIRU in the United Kingdom was fairly low at the beginning of the sample (e.g. 3.3 percent in 1973Q1) but then shows a much stronger upward trend than the other two countries over this estimation period. Indeed, according to these estimates there has been a slight tendency for the NAIRU to decline somewhat over the last decade in the United States.

The estimates of the gap between NAIRU and UNR is broadly consistent with a classical characterization of business cycles in these economies. For example, the maximum absolute unemployment gap is 5.0 percent in Canada, 3.8 percent in the United Kingdom, and 4.0 percent in the United States. All three of these estimates are excess supply gaps and are dated around the troughs of the business cycle that was related to the large disinflationary episode in the early 1980s. More precisely, the peak excess supply gaps are dated as 1982:4 in Canada and the United States and 1983:2 in the United Kingdom.

The estimates of the NAIRUs at the end of the sample (1995:2) were 8.8 percent in Canada, 8.1 percent in the United Kingdom and 6.1 percent in the United States. These can be compared to the actual unemployment rates at the time of 9.5 percent, 8.3 percent and 5.6 percent, respectively, suggesting that there was still some disinflationary pressure in the Canadian economy, the potential for some inflationary pressure in the U.S., and no pressure in the UK.[10]

Chart 3 demonstrates the fit of the model in terms of the ability of the estimated excess demand/supply term to explain historical movements in inflation. These charts were constructed by comparing the deviation of inflation from expected inflation, $\pi_t - \delta\pi_t^{LTE} - (1-\delta)\pi_{t-1}$, with our measure of the effect of labor market tightness γ (NAIRU-UNR)/UNR. The fit of the simple Phillips curve is remarkably good in all three of these countries considering the highly parsimonious functional form.[11] The chart illustrates the strong inflationary pressure around the time of the two oil shocks in the seventies, and the disinflation in 1981-82. It also illustrates the disinflation in Canada in the early 1990s.

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[9]We do not report measures for this earlier period using our methodology because data for aggregate government debt in the OECD countries were not readily available.

[10]It is important to note that although these estimates of the NAIRU may be useful for predicting short-term inflationary pressures, they do not account for the implications of future shocks for inflation. The fact that any estimates of the NAIRU are uncertain strengthens the case for cautionary policies that attempt to avoid serious overheating.

[11]For example, in Robert Gordon's latest estimate of the U.S. Phillips "curve" he includes lags of up to five years on past inflation developments and a host of other variables to control for supply shocks—see Gordon (1994).

B The Phillips Line with Model-Consistent NAIRUs

The estimation results for the linear model with model-consistent NAIRUS are reported in Table 2. The problem with the linear model is evident in the final two columns of Table 2. The maximum value of the gap takes on values that are excessively large in Canada and the United Kingdom. Furthermore, the NAIRU changes by as much as 7.7 percent and 3 percent in one quarter in Canada and the United Kingdom, respectively. Clearly such estimates are not economically sensible. This problem is illustrated by Chart 4. The series for the NAIRU are excessively volatile in both Canada and the United Kingdom, and, to a lesser extent, in the United States. The estimates for the NAIRU for Canada, which range from around -5 percent to over 20 percent are not plausible.

The nonlinear and linear models are not nested but because each model has the same number of parameters we can simply compare the values of the likelihood function to determine which model fits the data best.[12] When this comparison is done we can see that the linear model has a significantly better fit in all three of these countries. However, unlike the nonlinear model, the fit in the linear model is achieved by allowing extreme volatility in the NAIRU series for each country as shown in Chart 4. Indeed, the resulting NAIRU series from the linear model are considerably more volatile than the actual unemployment series.

One advantage of the Kalman filter approach is that it is fairly straight forward to impose priors about volatility in the NAIRU series. Because the linear model produced such implausible estimates of the NAIRUs for these countries we estimated an alternative model where we imposed some priors that restrict large jumps in the NAIRUs. In order to make the results comparable to the results for the nonlinear model we re-estimated the linear model subject to the constraint that the maximum absolute quarterly change in the NAIRU was equal to that obtained in the nonlinear model (that is, 0.21, 0.65 and 0.15 for Canada, the United Kingdom and the United States, respectively). The results are reported in Table 3. When this restriction is imposed there is a significant deterioration in the fit of the equation. Indeed, in this case a comparison of the values for the likelihood function indicates that the nonlinear model is the preferred model.

V. Uncertainty and Recursive Estimates of the NAIRU

A fundamental problem that policymakers face in attempting to stabilize the business cycle is that there is considerable uncertainty in their inferences about current excess demand pressures. Measures of the natural rate of unemployment, and hence labor market tightness, are especially uncertain for countries that have undergone major structural reforms or are

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[12]The two models have the same number of estimated parameters, so the value of the likelihood functions are directly comparable.

Table 2. Estimates of Phillips Lines with Model Consistent NAIRUS

(t-statistics in parentheses)

Estimated equation: $\pi_t = \pi_t^E + \gamma\ (\text{NAIRU-UNR}) + \epsilon_t^{\Pi}$

Where: π Percent change in the CPI at annual rates

π_t^E Inflation expectations, $\delta\pi_t^{LTE} + (1-\delta)\pi_{t-1}$
π_t^{LTE} Long-term inflation expectation proxy from bond market
δ Estimated weight on inflation proxy from bond market

UNR Unemployment rate
NAIRU Estimates derived from Kalman filter
LLF Value of the Likelihood Function
σ^2 Variance of Residuals
α Average historical value of UNR-NAIRU
Ω Maximum absolute value of (NAIRU - UNR)
τ Maximum absolute Quarterly Change in NAIRU

Estimation period: 1971Q2 to 1995Q2

Country	γ	δ	LLF	σ^2	α	Ω	τ
Canada	0.09	0.21	-103.49	0.18	0	17.47	7.66
	(0.77)	(3.23)					
United Kingdom	0.63	0.33	-173.40	0.80	0	8.99	3.06
	(2.36)	(5.20)					
United States	0.55	0.25	-99.73	0.15	0	4.06	1.33
	(0.13)	(0.06)					

Table 3. Estimates of Phillips Lines with Restricted Model Consistent NAIRUS

(t-statistics in parentheses)

Estimated equation: $\pi_t = \pi_t^E + \gamma\,(\text{NAIRU-UNR})/\text{UNR} + \epsilon_t^{\Pi}$

Where:
- π Percent change in the CPI at annual rates
- π_t^E Inflation expectations, $\delta\pi_t^{LTE} + (1-\delta)\pi_{t-1}$
- π_t^{LTE} Long-term inflation expectation proxy from bond market
- δ Estimated weight on inflation proxy from bond market
- UNR Unemployment rate
- NAIRU Estimates derived from Kalman filter
- LLF Value of the Likelihood Function
- σ^2 Variance of Residuals
- α Average historical value of UNR-NAIRU
- Ω Maximum absolute value of (NAIRU - UNR)
- τ Maximum absolute Quarterly Change in NAIRU

Estimation period: 1971Q2 to 1995Q2

Country	γ	δ	LLF	σ^2	α	Ω	τ
Canada	0.25 (4.35)	0.14 (3.33)	-104.81	0.41	0	4.76	0.21
United Kingdom	0.46 (3.56)	0.22 (4.52)	-179.75	1.83	0	5.28	0.65
United States	0.47 (7.00)	0.08 (2.27)	-106.61	0.38	0	3.68	0.15

regularly subjected to significant supply shocks that require a continuous reallocation of labor resources across sectors. An important lesson from history is that it may be counterproductive to place too high a weight on stabilizing unemployment if there is considerable uncertainty about the level around which it should be stabilized, that is, the underlying natural rate.

The difficulties in measuring the natural rate of unemployment can contribute to significant policy errors. Indeed, one popular interpretation of overheating in the 1970s is that policymakers significantly underestimated the increases in the NAIRU in their countries. This type of policy error can have rather deleterious implications for the economy if it takes considerable time for the monetary authorities to re-establish credibility in low inflation. In this sense, the asymmetric model predicts that the seeds of large contractions are sown when the monetary authority defers dealing with rising inflation and allows excess demand conditions to become entrenched in inflation expectations (see Clark and Laxton (1996)).

In order to truly assess the inflationary risks of overheating it is important to develop measures of uncertainty so that confidence bands can be established around any particular point estimate. As mentioned earlier, an advantage of the Kalman filter is that it allows the calculation of real time estimates of the model. Each period, the filter uses the new information to revise its estimates of the model's parameters and the estimate of the NAIRU. This exercise replicates to an extent, the process a policymaker would undertake in using this framework to determine inflationary pressures. One can then assess whether the advantage of hindsight indicates whether the recursive estimates give a significantly different picture of the degree of inflationary pressure than the full sample estimates which incorporate more information.

In the framework used in this paper, there are two sources of uncertainty, parameter uncertainty and uncertainty about the natural rate. The solid lines in Charts 5a-7a show the recursive or period by period estimates of the NAIRU and the model parameters for the nonlinear model. Charts 5b-7b display the same information for the linear model. The dotted lines in the upper panels of Charts 5-7 provide confidence bands of one standard error around our estimates of the natural rate of unemployment (which are shown by the bars). These standard error bands were obtained by imposing the full sample parameter estimates for γ and δ for the whole sample thus removing the effect of parameter instability, and then re-estimating the model. This estimation produces an end of sample standard error on the estimate of the NAIRU which is applied for the whole sample.[13]

The parameter estimates are generally relatively stable, and approach their full sample values relatively quickly. Each period, the filter will assign variation in the left hand side variable (in effect the difference between inflation and inflation expectations) between

[13]Technically, the standard error varies recursively over the sample. However, the variation is not great so for expositional purposes we have used the final period standard error.

variation in the NAIRU, variation in the parameters and the error term. Thus there is likely to be a negative correlation between movements in the recursive estimates of the NAIRU and movements in the recursive parameter estimates.

The recursive estimates of the NAIRU fluctuate generally within one standard error of the full sample estimate. In particular, in the United States, there is little difference between the recursive estimates of the NAIRU, and the full sample estimates. When the recursive estimate lies above the full sample estimate, the policymaker using this framework would be overestimating the extent of excess demand in the economy, and thus may be running an overly restrictive policy. Such a situation could arise when a negative shock to inflation occurs, as this will tend to pull up the recursive estimate of the NAIRU until more observations pin down whether the shock is temporary or permanent, and until the curvature of the Phillips curve is more exactly estimated.

Nevertheless, the standard error bands are wide in all three countries. Even in the United States, a confidence interval of approximately 66 percent (one standard error) for the NAIRU is 1 percentage points wide. This is comparable to the uncertainty surrounding other estimates of the NAIRU. The estimates of the NAIRU by Staiger, Stock and Watson (1996) are associated with a 66 percent confidence interval that is 1.3 percentage points wide.[14]

Turning to the linear model estimates, the recursive estimates of the NAIRU are at least as variable as the volatile full sample estimates. The 66 percent confidence interval for the Canadian NAIRU is almost 8 percentage points wide. Furthermore, in Canada and the United States, the recursive estimates often lie outside the standard error bands of the full sample estimates, implying that a policymaker using the linear model is more likely to misread the extent of excess demand/supply in the economy.

In the nonlinear model, the standard error bands also suggest that it is considerably easier to measure the NAIRU in the United States than it is in Canada or the United Kingdom. Indeed, this is perhaps one reason why policymakers in the United States have been more successful at avoiding boom and bust cycles because they have had less difficulty in obtaining reliable measures of the natural rate of unemployment. That being said, the enormous uncertainty in these estimates reinforce the view that it may be desirable for policymakers to exercise caution by setting monetary conditions in a way that guards against the serious overheating that was allowed in the 1970s.

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[14]Staiger, Stock, and Watson report substantially larger bonds that are based on a confidence level of 90 percent. We prefer to report estimates at the 66 percent level because in many situations policymakers simply could not afford to be so certain when making decisions that could result in potentially larger policy errors.

VI. The Information Content of Long-Term Inflation Expectations

As indicated earlier, our basic methodology to modeling inflation expectations (π^e) involves using information from bond markets to develop proxies for long-term inflation expectations (π^{LTE}) and then testing to see if these proxies are useful for identifying shifts in the Phillips curve. This approach to measuring inflation expectations may have some important advantages over reduced-form distributed lag models if these measures of long-term inflation expectations embody information about how wage and price setters revise their expectations in response to shocks or changes in policy regimes.

Because there is nothing really fundamental to tie down the distribution of future monetary policies—beyond the reputation of today's policymakers—it may take a considerable amount of time for agents to become convinced that governments are committed to low inflation. Furthermore, along the transition path it may be entirely rational for market participants, when confronted with a new regime to discount recent inflation performance under the new regime and to place a high weight on long moving averages of past inflation performance until it is evident that policymakers are committed to living with any adverse consequences of low inflation. Laxton, Ricketts and Rose (1993) show that along the transition path where the monetary authorities are gaining credibility one should expect to observe persistent excess supply gaps. This explanation is consistent with the results reported in Chart 3 that suggest that Canada and the United Kingdom have been operating mainly in the region of excess supply since the great disinflationary episode that started in the early 1980s.

The three panels in Chart 8 provide plots of our measures of inflation π, inflation expectations π^e, and long-term inflation expectations (π^{LTE}). These graphs are useful for identifying the role of the long-term inflation proxy in generating the statistical properties of the π-π^e measure reported earlier. As can be seen in the charts, because of the larger estimated weight on the lagged inflation component ($\delta \leq .2$ for all three countries), the quarterly variation in short-term inflation expectations that enter the Phillips curve are dominated by movements in information about recent inflation performance (π_{t-1}). However, the persistent deviation between π and π^e over longer periods of time are influenced to a significant extent by the long-term inflation proxy.

According to these estimates, market participants revise their expectations of long-term inflation very slowly in response to observed inflation performance. This interpretation of our results seem to reconcile the findings of our simple Phillips curves with other empirical work in the literature. First, as Robert Gordon and many others have found, it takes fairly long lags on past inflation and a host of other "supply shock" variables in order to save the Phillips curve. Obviously, in our highly simplistic model of the inflation process in these countries the proxy for long-term inflation expectations is fulfilling this role. Indeed, our empirical results are consistent with some recent evidence that suggests that trends in long-term interest rates have fairly long-term memory components. For example, Gagnon (1996) shows that the Fisher equation holds surprisingly well if long moving averages of past inflation are used to measure long-term inflation expectations.

Because a traditional interpretation of the demand-determined view of the business cycle requires that measures of the business cycle such as (NAIRU-UNR) must mimic measures of π-π^e, the trick of reduced-form modelers has been to search over distributed lag models in order to find specifications that fit both the story and the data. In Chart 3 presented earlier, we show that if information from the bond market is used to identify inflation expectations, one can obtain a fairly parsimonious nonlinear specification of the unemployment-inflation process in these countries without appealing to implausible estimates of the NAIRU. In other words, our measures π-π^e are consistent with both convexity in the Phillips curve and a fairly traditional interpretation of the business cycle, namely that its is driven principally by demand shocks with some moderate changes in the underlying NAIRU.

Given the problems associated with modeling inflation expectations with fixed-parameter reduced-form models we are somewhat surprised that more research has not focussed on using information from the bond market to help identify shifts in the Phillips curve. As McCallum (1996) points out, without reliable survey measures of inflation expectations in most countries, information from long-term bond yields is probably the only objective indicator of policy credibility. One possibility why more researchers have not utilized this information more in the past for identifying shifts in the Phillips curve may be related to problems with measuring the ex ante real interest rate. Although our procedure of developing proxies for the ex ante real interest rate is admittedly crude, it is interesting that many of our results are not overly sensitive to alternative assumptions.

For example, one might argue that the benchmark for the end-of-sample estimate of 4.5 percent may be too high for the United States. When we use a lower estimate of the equilibrium real interest rate of 3.5 percent there is only a slight change in our estimate of γ, and α rises from 0.33 percent to 0.48 percent in the United States.[15] Similar results are obtained for the other two countries. As we mentioned earlier several alternatives were employed where we changed the assumed effect of government debt on the equilibrium world real interest rate. In fact, in one polar case we doubled the effect to be more consistent with some recent estimates that suggest that the effects have been large in some countries. In another case, we assumed that the world ex ante real interest rate has been unaffected by world government debt. In addition, in all these cases we also used alternative assumptions about the end-of-sample benchmark that ranged from 3.5 percent to 5.5 percent. In all cases, it was impossible to reject the hypothesis that there is valuable information in the bond market that will help identify historical shifts in the Phillips curve.

[15]These results with alternative assumptions for the ex ante long-term real interest rate can be obtained from the authors.

VII. Conclusions and Policy Implications

Previous tests for convexity in the Phillips curve have been biased because researchers have employed filtering techniques for the NAIRU that have been fundamentally inconsistent with the existence of convexity. A preferred statistical methodology would place both the linear and nonlinear models on an equal statistical footing by estimating model-consistent measures of the NAIRU. This paper proposes a simple method for estimating model-consistent measures of the NAIRU that allows for either convexity or linearity in the unemployment-inflation process. After imposing plausible restrictions on the variability in the NAIRU we find that the nonlinear model fits the data best in Canada, the United Kingdom and the United States.

The implications of convexity for the macroeconomic policy debate is that policymakers that are unsuccessful in stabilizing the business cycle will induce a higher natural rate of unemployment on their economies. Uncertainty about the true level of the NAIRU reinforces the case for a cautionary strategy of acting to raise interest rates before the economy reaches potential. Thus, rather than trying to fine tune policy to the point of attempting to ensure that all resources are fully employed at all points in time, it may be optimal in the context of uncertainty to develop a strategy that attempts to avoid large boom-bust cycles.

Indeed, in order to avoid the necessity of generating large recessions to reign in inflationary forces and reestablish credibility in low inflation, it may be optimal for policymakers to provide a buffer zone to guard against the possibility of serious overheating, where the width of the buffer zone would be positively related both to the degree of uncertainty about the natural rate of unemployment and the degree of asymmetry in the unemployment-inflation process. As noted above, in the presence of an asymmetry in the unemployment-inflation tradeoff, such a buffer zone strategy can conceivably raise the average level of output, and reduce the average level of unemployment over time if it is successful in avoiding boom and bust cycles.

Chart 1a.
The Phillips Curve, the NAIRU and the Natural Rate of Unemployment

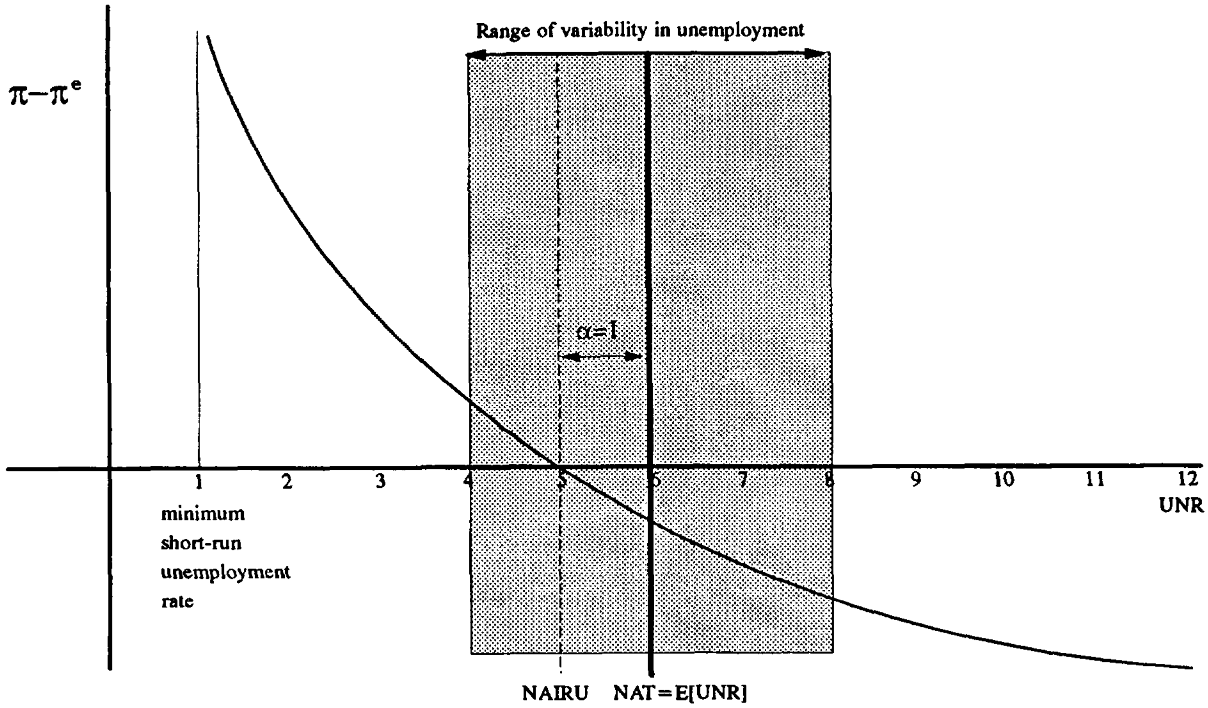


Chart 1b.
The Implications of Greater Unemployment Variability for the Natural Rate

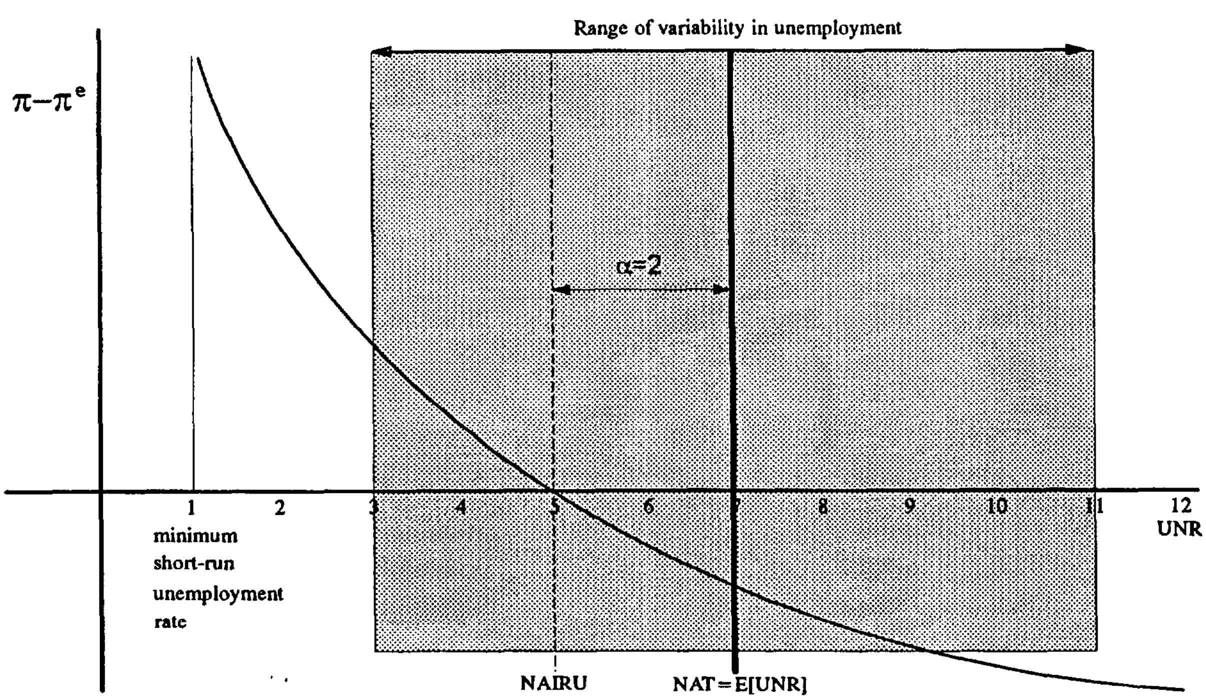
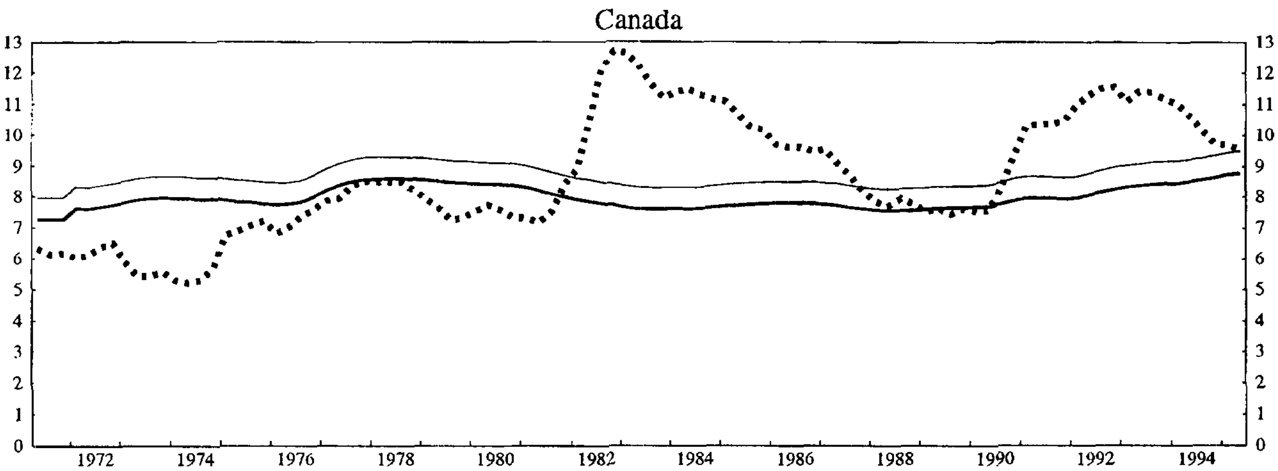
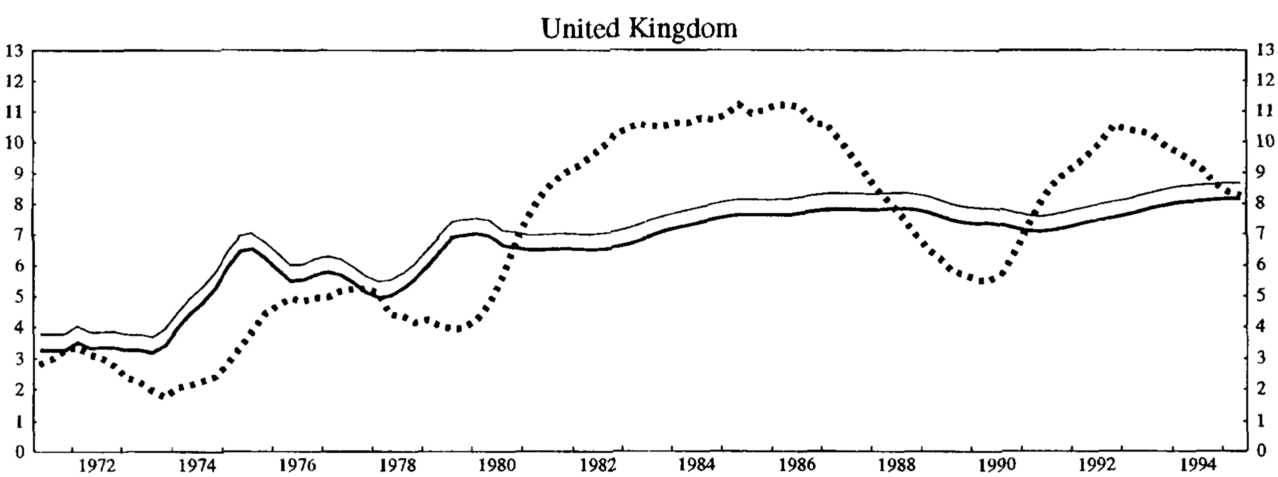


Chart 2: Unemployment, the NAIRU and the Natural Rate of Unemployment

Estimates based on the nonlinear model: 1971Q2 to 1995Q2

Unemployment Rate (dashed), NAIRU (solid thick line), Natural Rate (solid thin line)





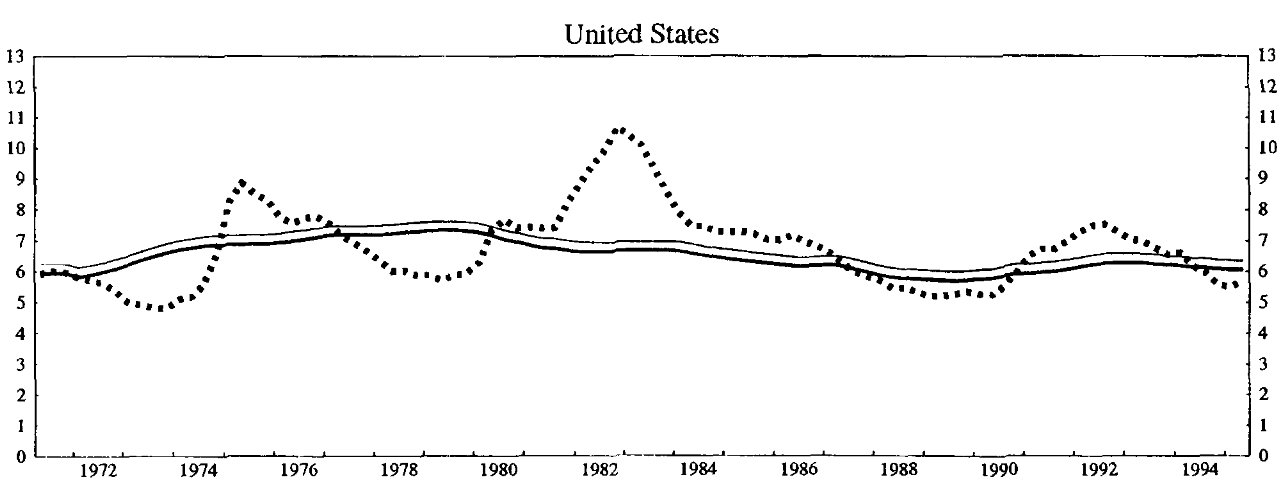
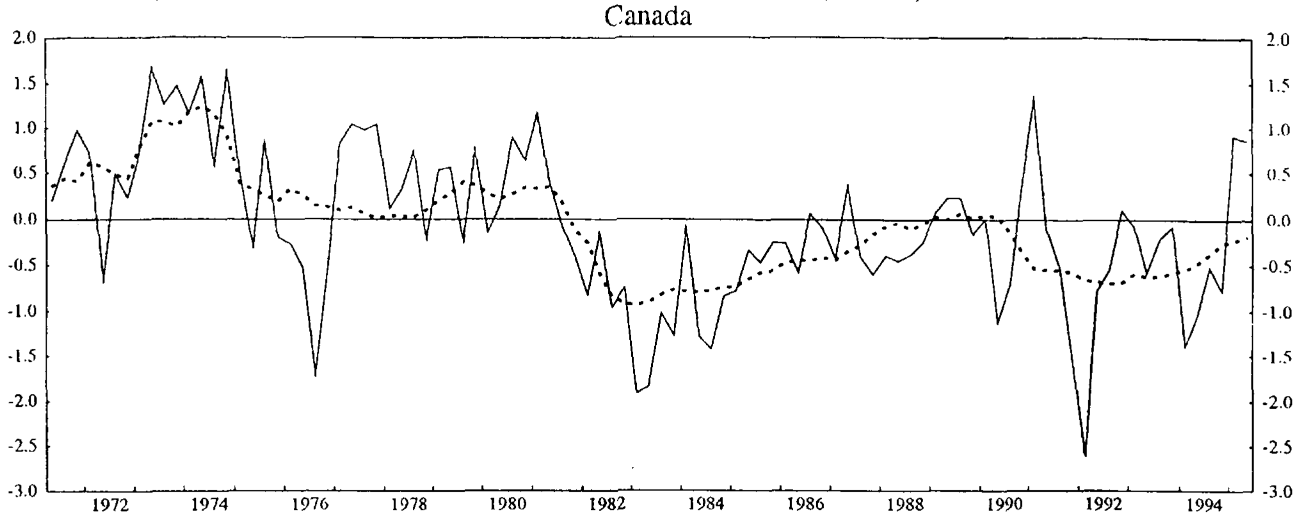
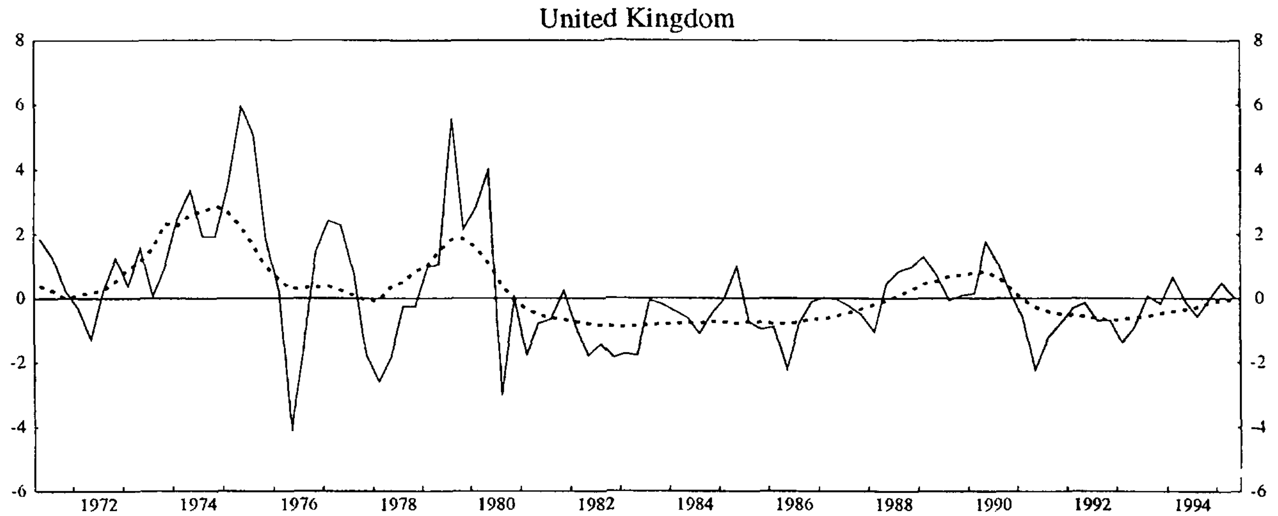


Chart 3: Historical Performance of the Nonlinear Model (1971Q2 to 1995Q2)

$\pi_t - \delta\pi_t^{LTE} - (1-\delta)\,\pi_{t-1}$ (solid line)

$\gamma^*(nairu_t - unr_t)/nairu_t$ (dashed line)





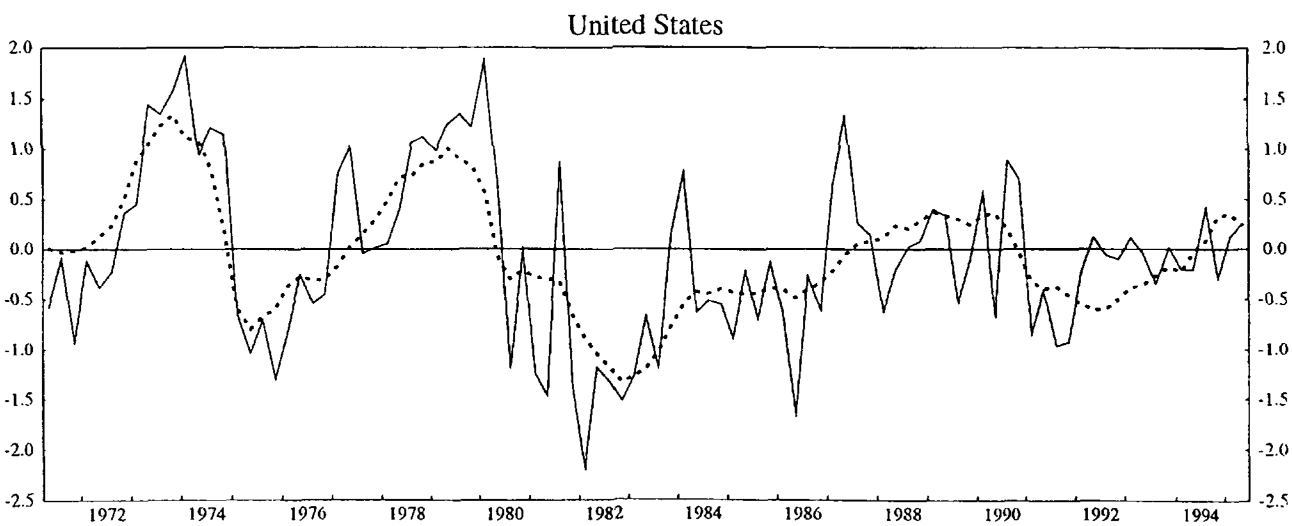
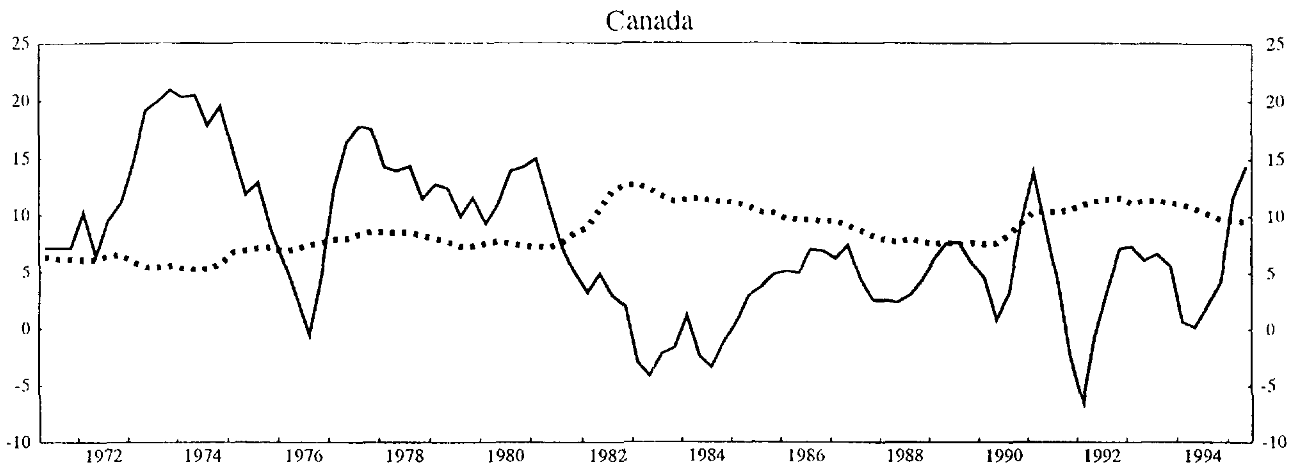
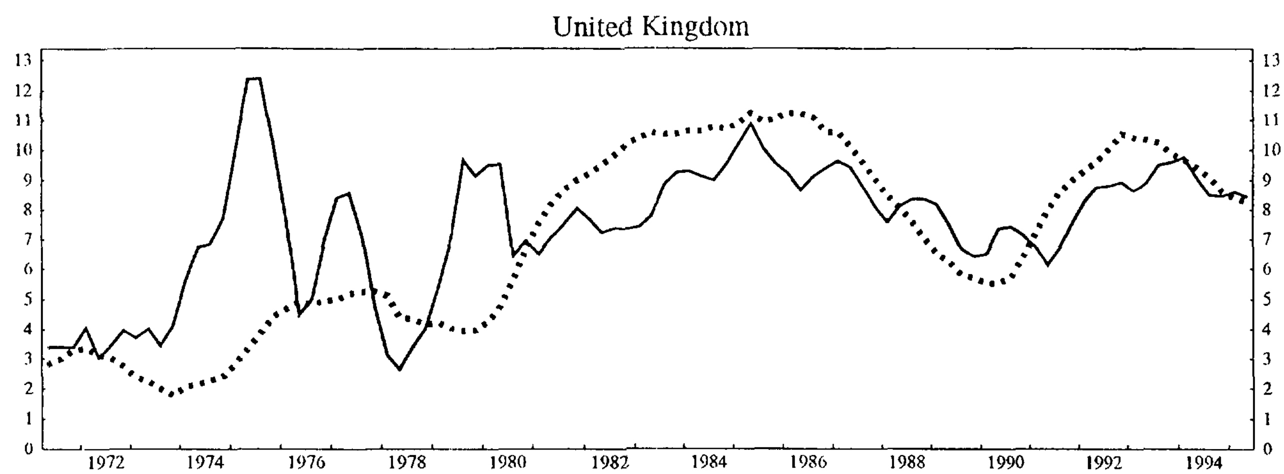


Chart 4: Unemployment and the NAIRU with a Linear Phillips Curve

1971Q2 to 1995Q2

Unemployment Rate (dashed), NAIRU (solid thin line)





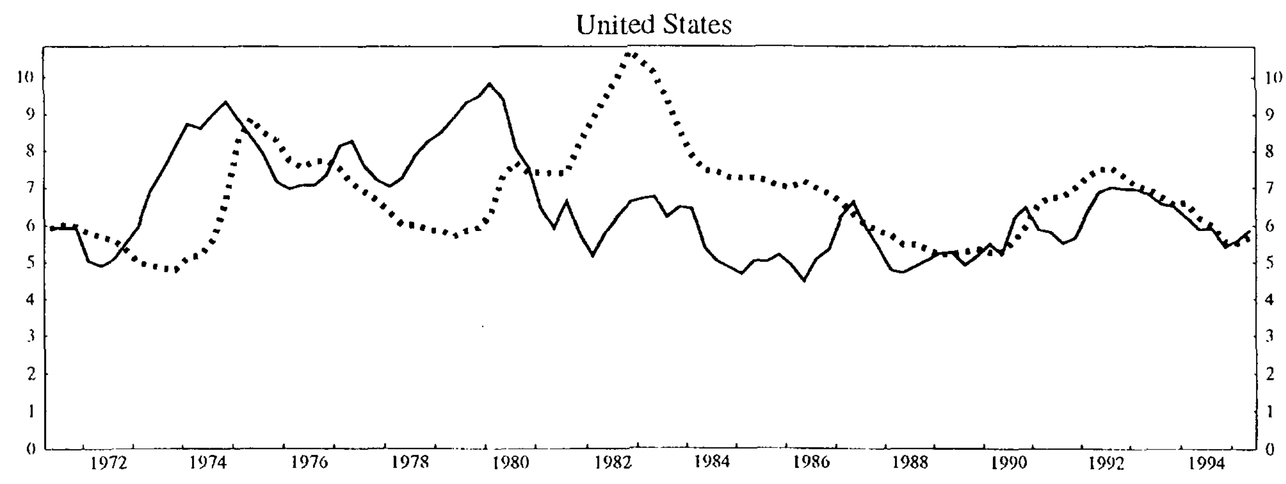
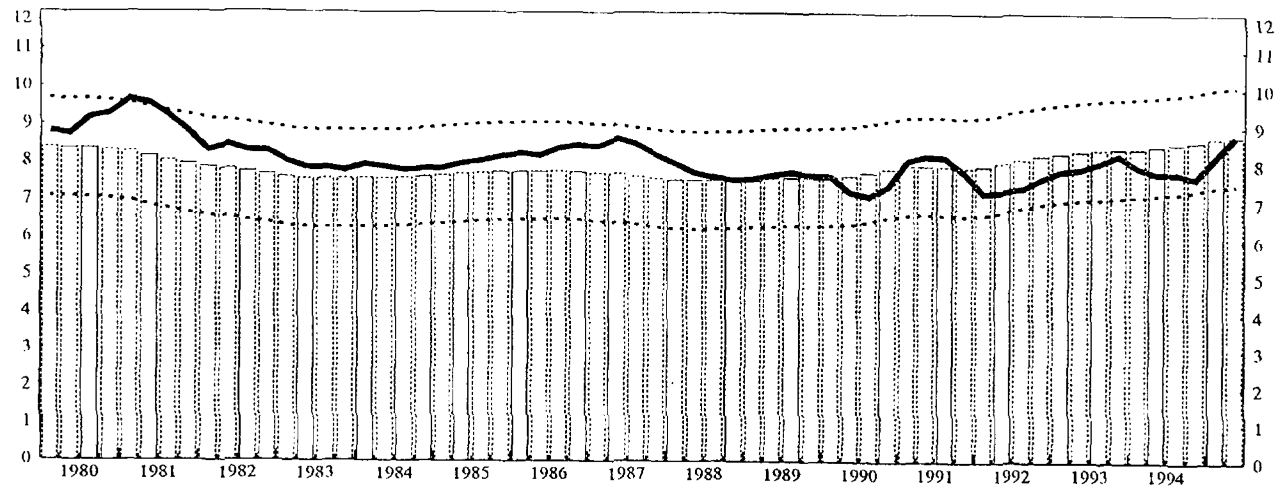


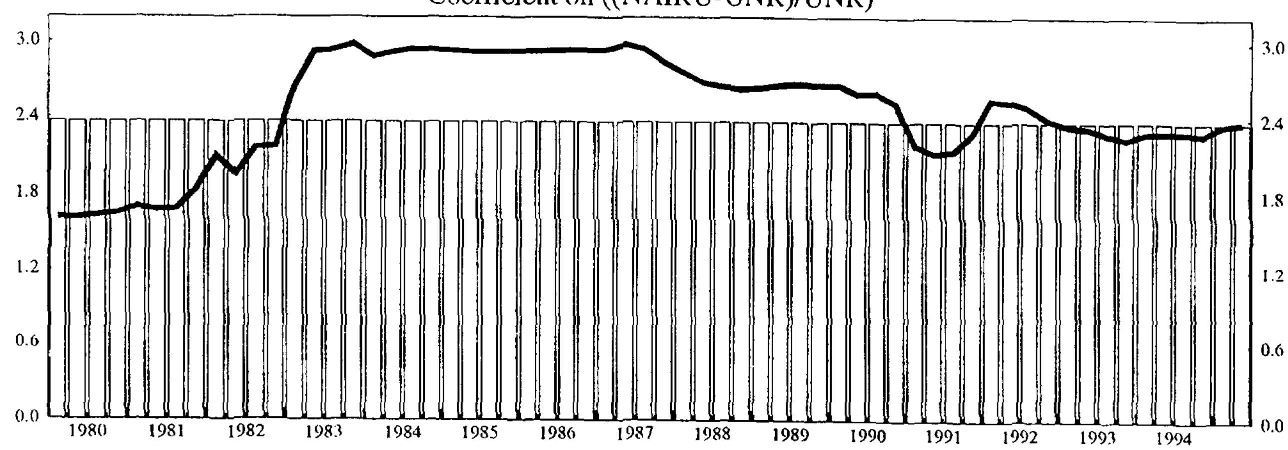
Chart 5a: Recursive Estimates of NAIRU and Parameters for Canada

Nonlinear Phillips Curve Model

Recursive Estimates (solid thick line), Full Sample Estimates (bar), Standard Error Bands (dotted line)



Coefficient on ((NAIRU-UNR)/UNR)



Coefficient on expectation proxy from bond market

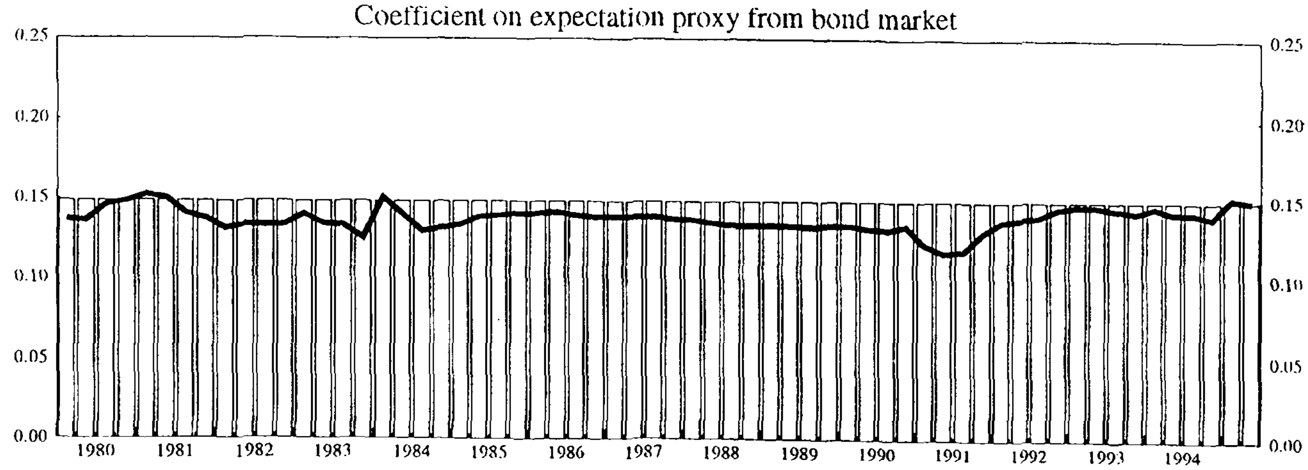
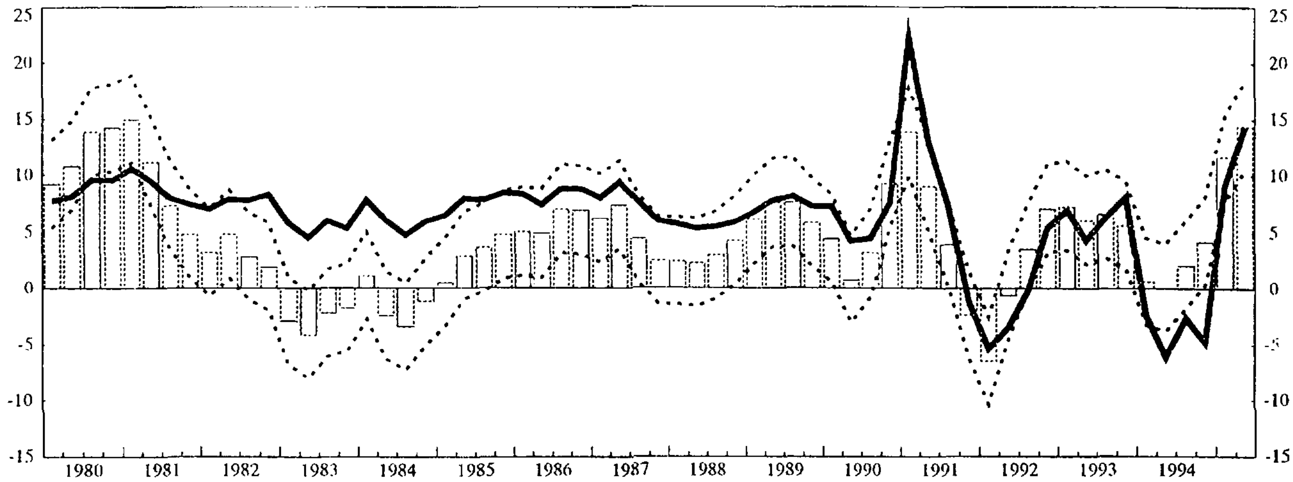


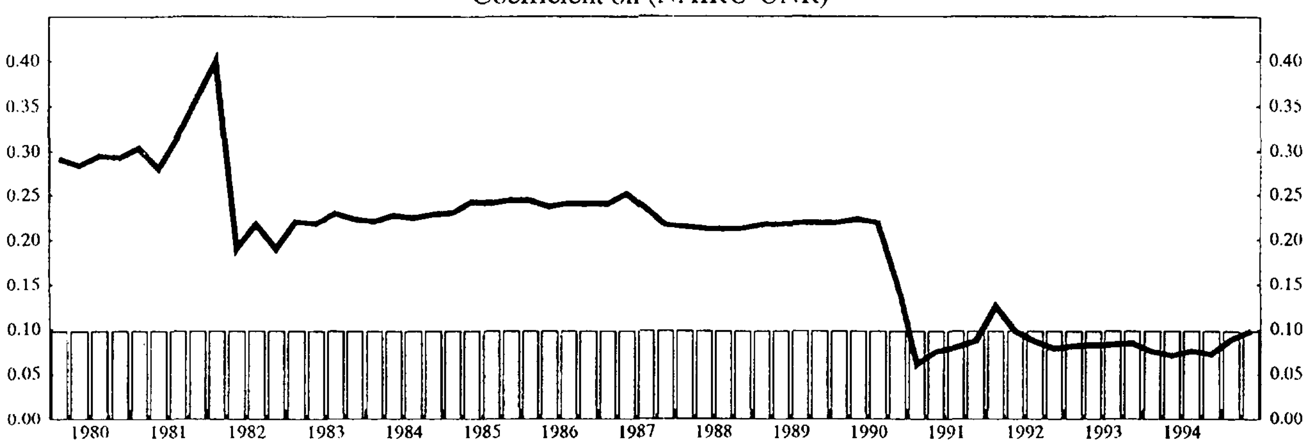
Chart 5b: Recursive Estimates of NAIRU and Parameters for Canada

Linear Phillips Line Model

Recursive Estimates (solid thick line), Full Sample Estimates (bar), Standard Error Bands (dotted line)



Coefficient on (NAIRU-UNR)



Coefficient on expectation proxy from bond market

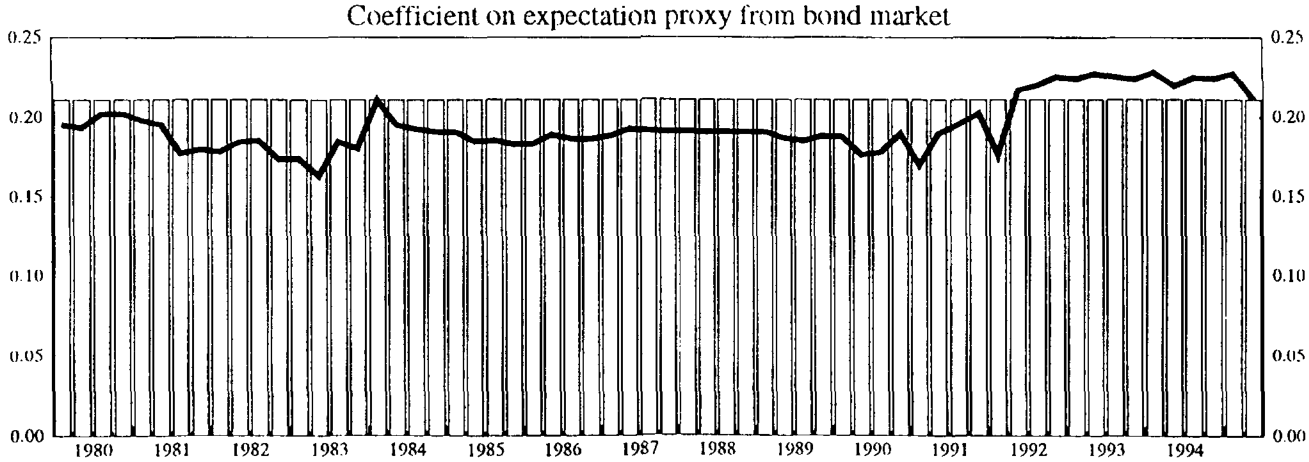


Chart 6a: Recursive Estimates of NAIRU and Parameters for United Kingdom

Nonlinear Phillips Curve Model

Recursive Estimates (solid thick line), Full Sample Estimates (bar), Standard Error Bands (dotted line)

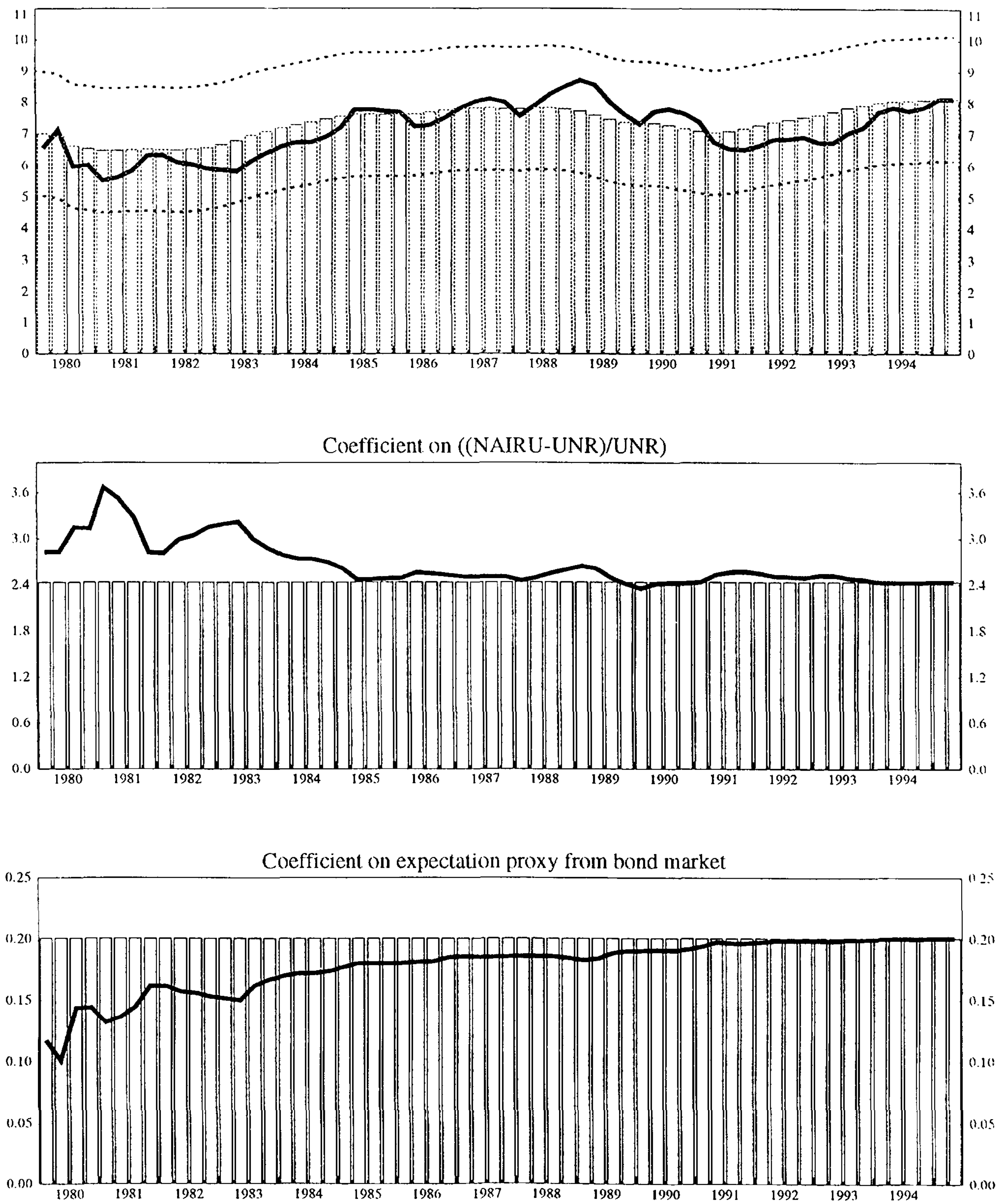
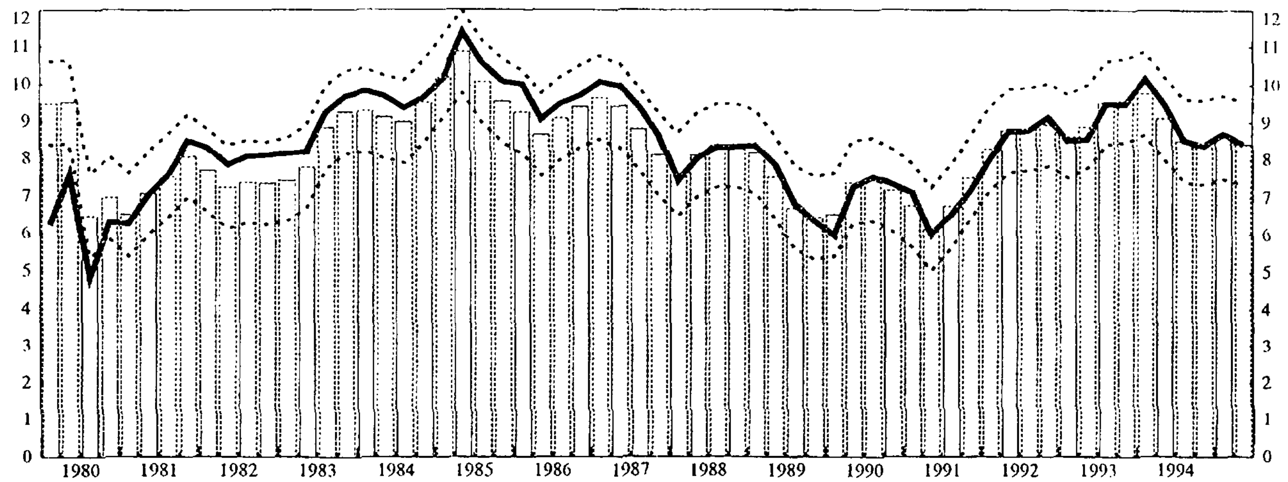


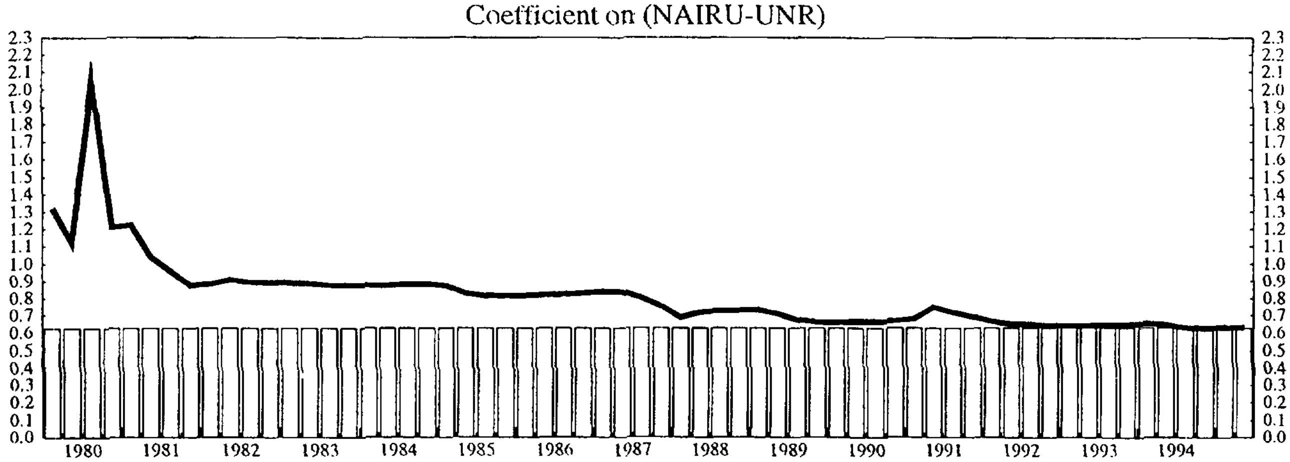
Chart 6b: Recursive Estimates of NAIRU and Parameters for United Kingdom

Linear Phillips Line Model

Recursive Estimates (solid thick line), Full Sample Estimates (bar), Standard Error Bands (dotted line)



Coefficient on (NAIRU-UNR)



Coefficient on expectation proxy from bond market

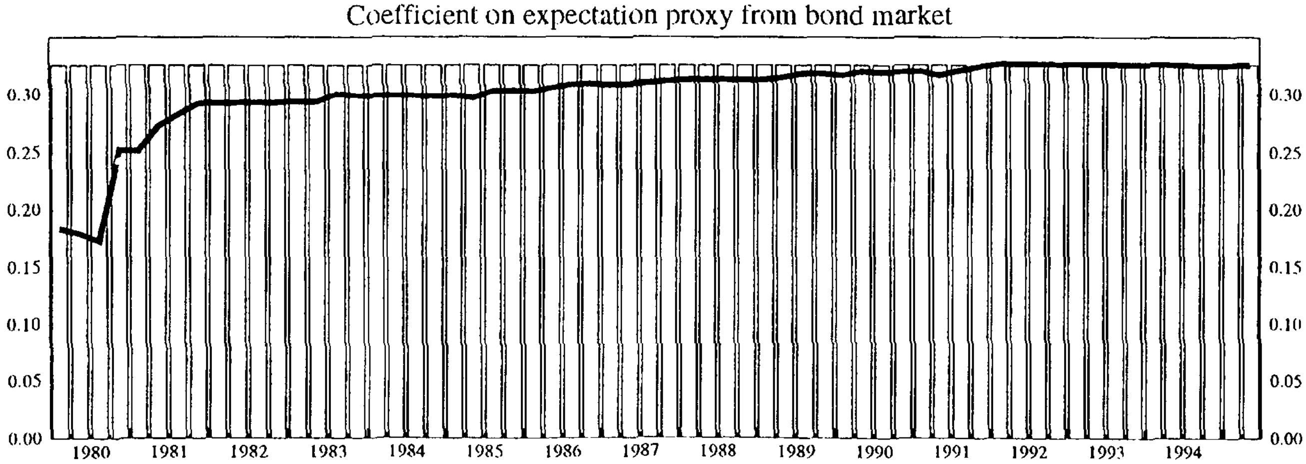
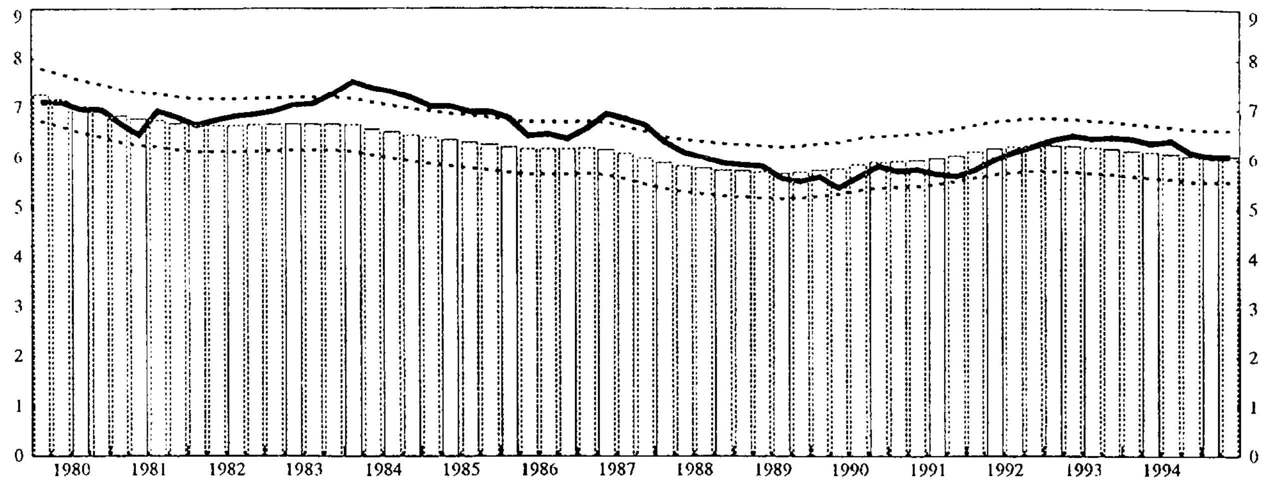
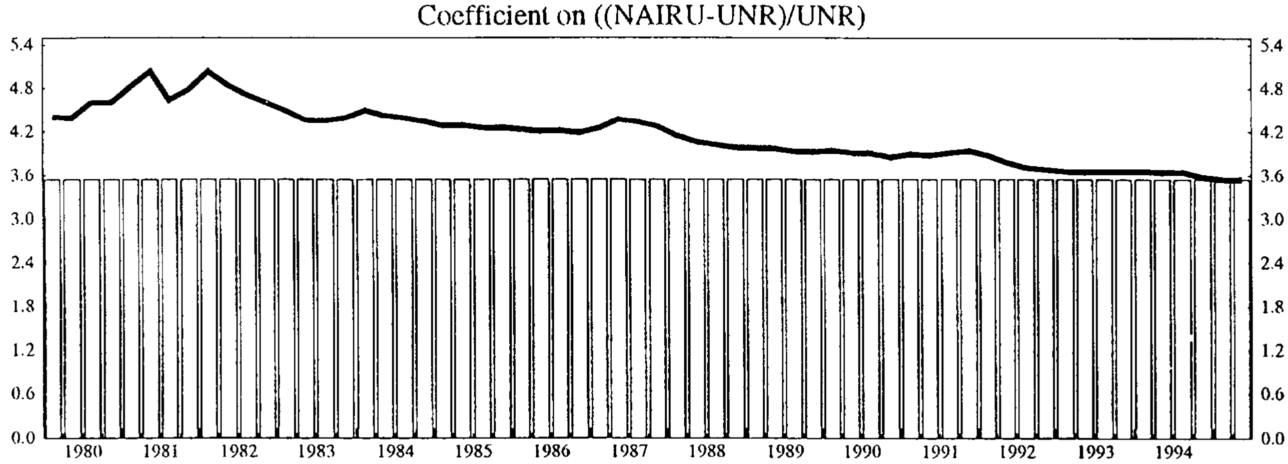


Chart 7a: Recursive Estimates of NAIRU and Parameters for United States

Nonlinear Phillips Curve Model

Recursive Estimates (solid thick line), Full Sample Estimates (bar), Standard Error Bands (dotted line)





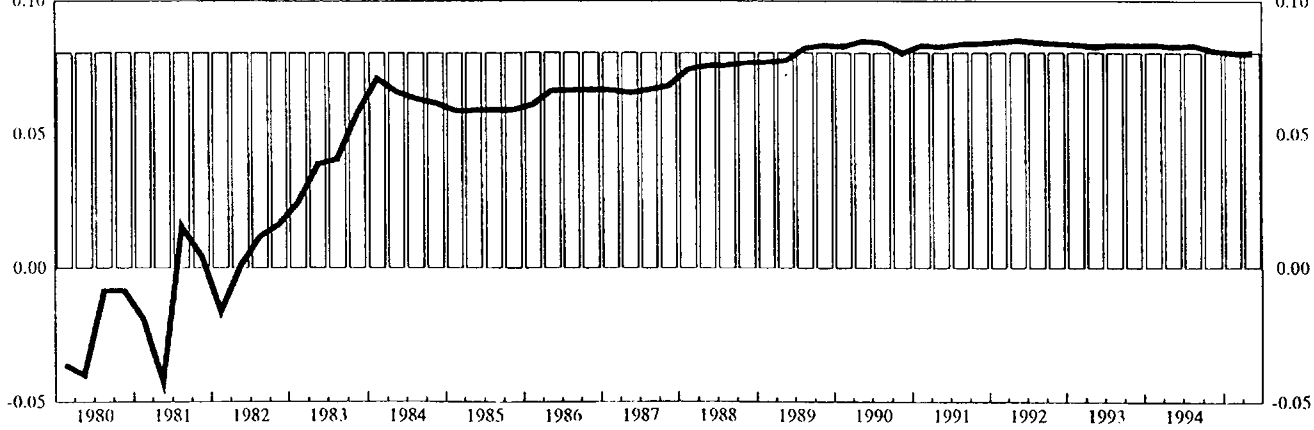
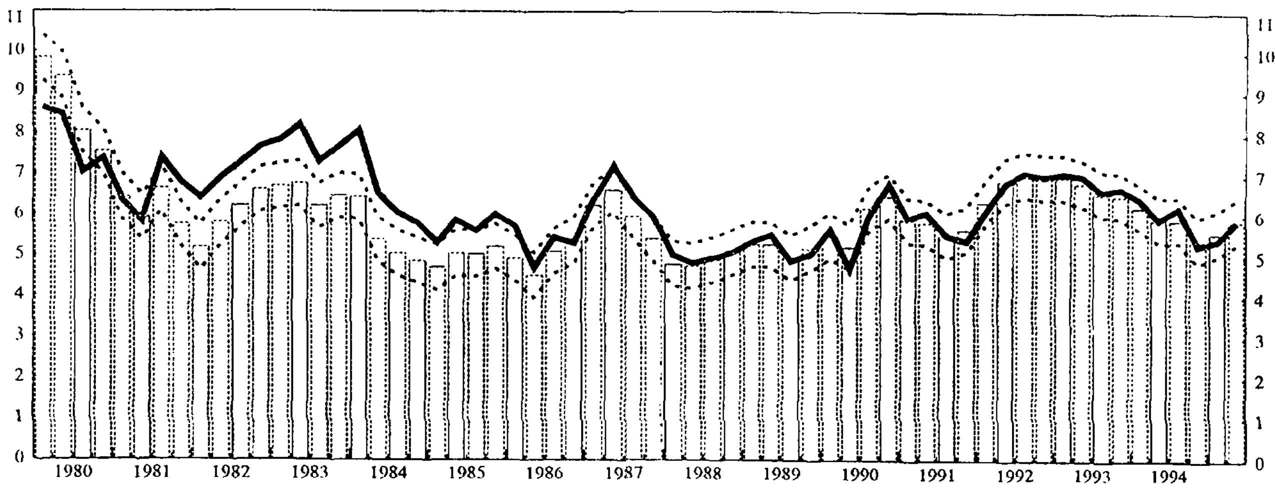


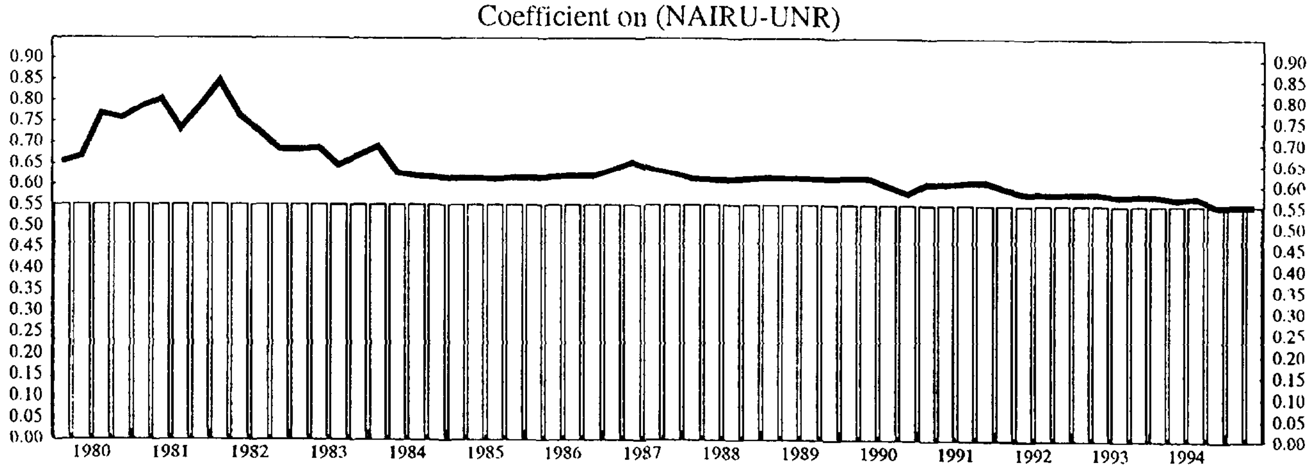
Chart 7b: Recursive Estimates of NAIRU and Parameters for United States

Linear Phillips Line Model

Recursive Estimates (solid thick line), Full Sample Estimates (bar), Standard Error Bands (dotted line)



Coefficient on (NAIRU-UNR)



Coefficient on expectation proxy from bond market

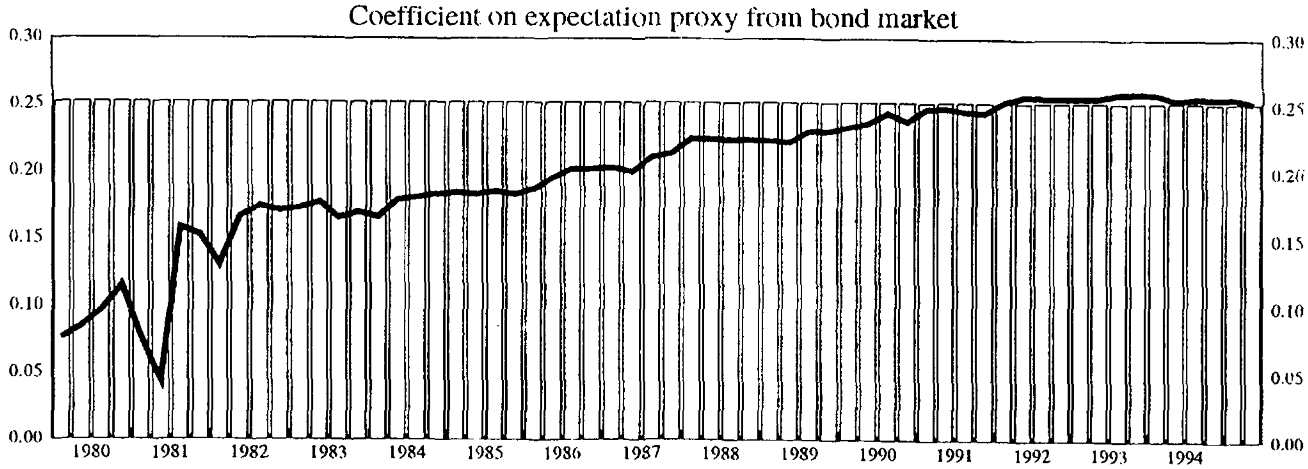


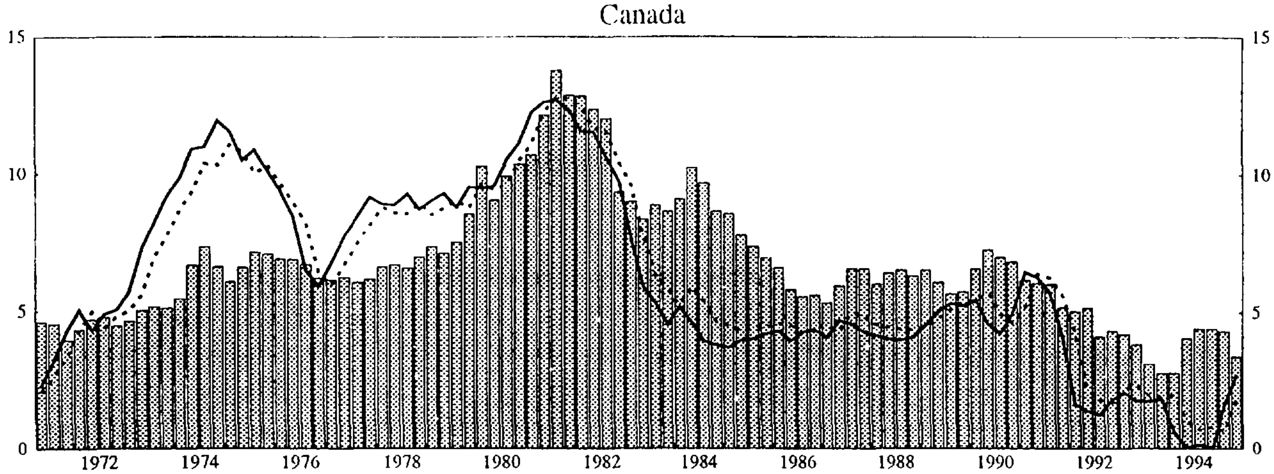
Chart 8: CPI Inflation and Inflation Expectations (1971Q2 to 1995Q2)

CPI Inflation (SOLID LINE: π)

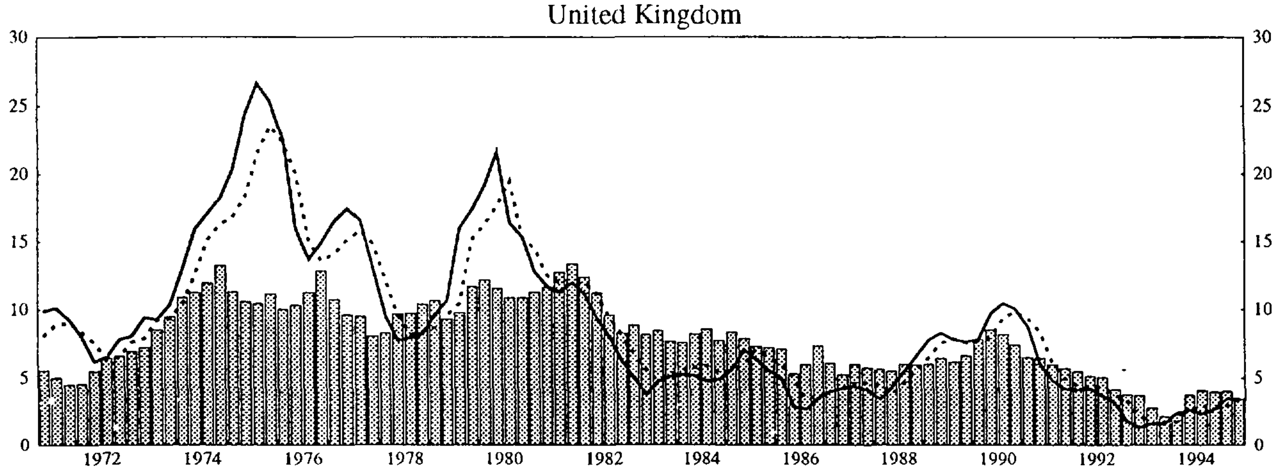
Short-Term Inflation Expectations (DASHED LINE: $\pi^e = \delta\pi^{LTE} + (1-\delta)\,\pi_{t-1}$)

Long-Term Inflation Expectations (BAR: π^{LTE})

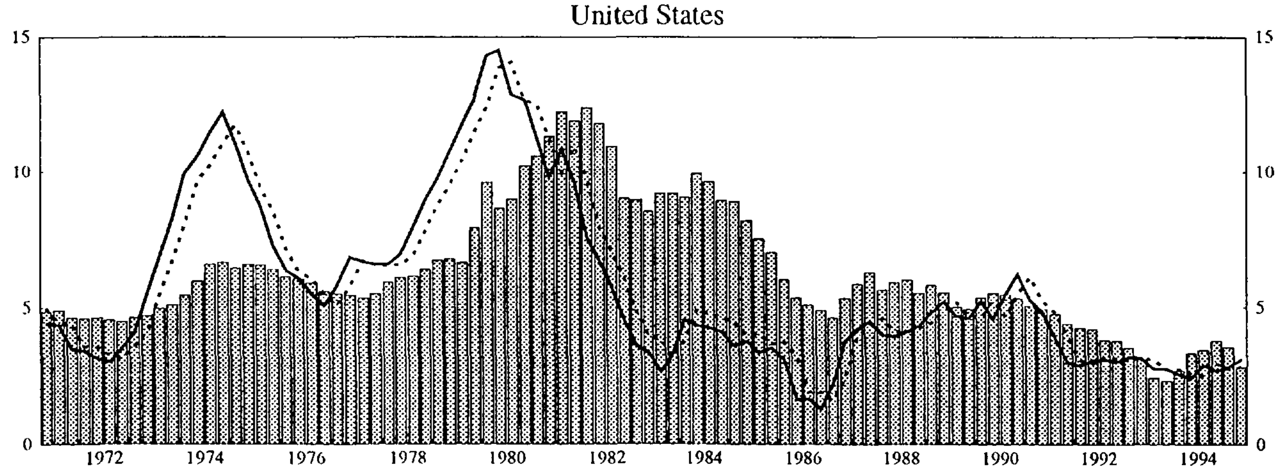
Canada



United Kingdom



United States



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