Methods of Determining the Amounts of Gross Salary Payments Required to Yield the Desired Net Salary Payments after Federal Income Tax

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I. Introduction

According to a notice circulated to its employees, the United Nations Organization has arranged to refund to them "any taxation levied on ... [their] salary by ... [their] ... national government ..."1/ This provision may or may not grant relief from the burden of Federal income taxes to UNO employees who are United States citizens, depending on whether the net wage after tax is the same as the prevailing gross wage or whether it is lower than the prevailing gross wage by approximately the amount of the tax.

The purpose of this memorandum is to consider the tax determination problems in two different types of tax situations which can arise: (a) The employer contracts to pay an amount of gross salary as will be sufficient to include the current tax liability for such amount and leave the stipulated amount of net salary after the tax is paid; (b) the employer pays to the employee in the first instance only the stipulated amount of net salary, not including the tax thereon, and subsequently reimburses the employee for all tax liability arising out of the net salary payment.

Under either of the above two situations there are four general methods of readily determining the amount to be paid in addition to net salary after tax: (a) Directly from Supplement T tax table where gross income before tax does not exceed $5,000; (b) from special table constructed in part from the Supplement T table; (c) by means of a precise algebraic formula applicable to any income levels and adaptable to take account of any factors; and (d) where deductions are a constant amount, by means of a simple arithmetic rule in conjunction with a specially adapted tax schedule.

The formula developed will hold for all employers who undertake to pay their employees' tax, as well as for the international organizations.

II. Method whereby tax is included in gross wage payment

The employer may decide to pay the amount of tax to the employee currently with the wage payment. In that event the employer must first compute that amount which when added to salary is equal to the tax on

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1/ Quoted in the New York Post, March 4, 1946.

the resulting gross income. Although the four methods described below assume that the determination of tax is made on the basis of a yearly period, they can be readily modified to accommodate quarterly, weekly, or any other periods. It is also assumed in this discussion that the employer and employee are agreed as to the exemption status and amount of deductions which shall be used in determining gross salary. If this is not the arrangement then the gross salary determination will depend on the final liability of the taxpayer, which in turn will depend upon intra-annual changes in the employee's exemption status and whether he uses the standard or itemized deductions.

Other changes which will affect the determination and the nature of the arrangement by the employer and employee involve changes in the income tax law, in the rates of remuneration, and in the various allowances. 1/ Finally, the exact wage payment by the employer for a regular pay-roll period would also depend upon whether the employee finds his tax by using (a) the Supplement T tax table, or (b) the exact method for determining his annual liability, and on (c) whether the wages are withheld on or not, and (d) if withheld on, whether by the wage-bracket tables or by the percentage method.

The formulas which are developed for determination of the required gross salary payments and tax thereon are based on two assumptions: (a) that the gross salary payment must be the precise amount which after payment of tax thereon will yield the exact amount of net salary contracted for; (b) that if the employee has other income, the gross salary determination disregards such other income so that in effect the amount of tax contracted for by the employer is that amount found by putting the wage or salary payment in the lowest possible tax brackets. The latter assumption is made in order to simplify the computations since each different amount of income received by the taxpayer other than salary will require separate computation.

Since the immediate purpose of this memorandum is to deal with some of the considerations pertaining to international organizations, it is further assumed that the employer referred to herein is exempt from the requirement of withholding on his wage payments to employees, as are international organizations under the International Organizations Immunities Act, approved December 29, 1945. 2/

1/ For an outline of some of these problems, see Part IV below.

2/ This act, in Section 4(c), amends Section 1621(a)(5) to the effect that wages subject to withholding shall not include remuneration paid for services by a United States citizen for a foreign government or international organization. Section 1 of the act defines an international organization as one so designated by Executive order.

A. Use of Supplement T tax table

A very simple way of finding the amount to be paid in addition to net salary, for all taxpayers where the gross salary before tax does not exceed $5,000, is to use the Supplement T table. To determine the amount of gross income required to yield a given amount of net income after tax, for a given exemption status, it is necessary merely to compare the amounts of adjusted gross income in the stub of the table with the tax amounts in the appropriate exemption column, in order to find that tax for which the difference between income and tax first 1/ reaches or just exceeds the given net income after tax. The gross salary desired is then obtained as the sum of the tax so found and the given net salary after tax.

Thus, suppose it is desired to find by means of the Supplement T table the gross salary before tax equivalent to a tax-free salary of $2,000, assuming 2 exemptions. Obviously the required amount cannot be less than $2,000. For the bracket $2,000-$2,025, the tax is $154, and the amount after tax for the lower limit is only $1,846. The amount of income after tax increases by about $20 with each succeeding bracket, since the brackets progress in steps of $25 and the tax amounts increase by roughly $5, in this range of income. By simple trial it is found that $2,000 of income after tax corresponds to the gross income bracket $2,175-$2,200, for which the tax is $184 and the income after tax $1,991 to $2,016. Hence, the desired gross income is contained in this wage bracket, and is obtained by adding $2,000 to $184, giving $2,184. As a check, the Supplement T table gives the tax as $184 for this amount, leaving exactly $2,000 after tax as desired.

1/ There are a number of instances where two Supplement T brackets, and hence two amounts of tax will be found to satisfy the requirments. In such cases it will be advisable to adopt an arbitrary rule in order to obtain a single determination of tax. The rule will be to take the smaller tax and gross income in each such case. Thus, if it is desired to find what gross income before tax leaves just $1,464 after tax, assuming 2 exemptions, we have the following information from the Supplement T table:

Supplement T bracket (adjusted gross income)	Supplement T tax from table	Difference, net income after tax
$1,625-$1,650	$185	$1,440 to $1,465
1,650- 1,675	189	1,461 to 1,486

The given amount of $1,464 after tax lies within each of the two overlapping ranges in the third column and therefore corresponds to the two gross income brackets shown, with the two corresponding taxes. Thus, both $1,649 ($1,464+$185) and $1,653 ($1,464+$189) yield the desired net income after tax of $1,464, and by the rule we take the smaller amount, $1,649.

Since the table ends at $5,000 gross income, it cannot be used in the above manner for net amounts after tax above levels of about $4,200 to $4,900, depending on the number of exemptions. The maximum tax-free salaries for which the Supplement T tax table can be used to obtain the corresponding gross salaries before tax are readily obtainable by subtraction of tax from $5,000, and are as follows:

Number of exemptions	Gross salary before tax	Net salary after tax
1	$5,000	$4,207
2	5,000	4,311
3	5,000	4,416
4	5,000	4,520
5	5,000	4,624
6	5,000	4,719
7	5,000	4,814
8	5,000	4,909
9 or more	5,000	5,000

B. Use of a table derived from the Supplement T table

To avoid separate determination of gross salary for each individual employee, a special table could be constructed to be used in a manner similar to the present tax table in determining ordinary liability. This table would show in the income columns brackets of net salary after tax. In the body of the table would appear, for the various numbers of exemptions, the amounts of income to be added to net salary in order to obtain gross salary. These entries would be the taxes to be paid on gross salary.

An excerpt from such a special table derived from the 1946 Supplement T tax table is given below. For example, if it is desired to pay $2,000 of net income after tax, the table shows that for a person with one exemption, $2,300 gross salary would have to be paid; with 2 exemptions the amount would be $2,184; and with 3 exemptions, $2,072. For all higher statuses the gross amount would be $2,000, since no tax liability would be incurred. As indicated in the table, the amounts of net salary after tax could be extended to any desired level and any width of bracket. 1/ The narrower the bracket the less would be the discrepancy between the amount given by the table and by the precise formula computation described in the following section.

1/ For gross incomes outside the limits of the Supplement T table the entries would be computed by applying the formula for T given on p. 8 below to the bracket midpoint, assuming deductions equal to 10 percent of adjusted gross income.

Amounts to be added to net salary after tax in order to pay the tax on the resulting gross salary

If net salary after tax is to be		And the number of 1946 exemptions claimed is				
At least	But less than	1	2	3	4	5
		The amount to be added to net salary to compensate for tax is a/				
$ 550	$ 575	$ 1	$ 0	$ 0	$ 0	$ 0
1,000	1,025	95	0	0	0	0
1,050	1,075	104	0	0	0	0
2,000	2,025	300	184	72	0	0
2,500	2,525	405	291	175	58	0
2,975	3,000	511	388	275	154	42
3,700	3,750	690	557	434	308	196
4,450	4,500	876 b/	733 b/	604 b/	470	350
4,950	5,000 b/	1,019	860	720	591	462
8,000	8,100 b/	2,054	1,826	1,640	1,448	1,257
16,000	16,500 b/	7,158	6,726	6,215	5,822	5,362

a/ Amounts are based on use of midpoint of bracket in conjunction with the Supplement T table, except as noted in footnote b.

b/ Amounts are computed by using bracket midpoint for P in the formula for T, p. 8, assuming total deductions equal 10 percent of gross salary before tax.

C. Substitution in algebraic formula

The preceding method involves some discrepancy on account of the use of brackets. By using narrower widths, the method may be made more precise, but if an exact computation of gross salary is necessary, it must be made either by means of a formula or by means of a rule equivalent to it. The present section explains the use of a formula. The following section gives an arithmetical rule which is simpler, but can be used only when total deductions are fixed in amount.

Let

P = net salary payment after tax;
P' = P expressed on a surtax net income basis, by subtracting deductions and exemptions;
G = desired gross salary before tax;
G' = G expressed on a surtax net income basis; and
T = present law liability on G.

We then have the fundamental relation

$$G - T = P,$$

by means of which, as shown in Appendix A, both G and T can be derived. The expression for T, the amount to be added to net salary after tax, is simpler and will be used here. The desired gross salary G is then obtained from the fundamental relation, since P is given.

There are two different cases, according as total deductions are constant or depend on P, the net salary after tax.

1. Total deductions are constant

This is the situation when, for example, the $500 standard deduction for gross salary (before tax) above $5,000 is used. The formula for the tax is then

$$T = \frac{rP' + (T_1 - rS_1)}{1 - r},$$

Where S_1 is the lower boundary of the surtax net income bracket S_1 to S_2 in which will fall a gross amount, G', obtained from G by subtracting deductions and exemptions; T_1 is the combined normal and surtax corresponding to S_1, and r is the combined normal and surtax

marginal rate, after the flat 5 percent cut of present law, for the bracket S_1 to S_2. This bracket has to be determined by trial, as is illustrated in the example below.

After T is found the gross income G is readily obtained by adding the net income after tax, P_1 to the tax, T.

Example: Let the net salary after tax be $8,000, with 4 exemptions, and suppose the amount of tax to be added is desired. With the $500 deduction and $2,000 exemption, we have P' = $5,500 [$8,000 - $2,500]. The bracket S_1 to S_2 is guessed to be $6,000 - $8,000. Substituting in the formula for T then gives

$$T = \frac{.285 \times 5,500 + (1,292 - .285 \times 6,000)}{1 - .285}$$

$$= \$1,607.69 \text{ 1/}$$

The gross salary before tax is found by adding this to $8,000, giving $9,607.69. Since the corresponding surtax net income is $7,107.69, the guessed bracket of $6,000 - $8,000 was the correct one. 2/ As a check on the computation, the tax on the gross salary of $9,607.69 computed in the usual way is $1,607.69, agreeing with T.

1/ This is, of course, appreciably greater than the value $1,448 given by the table, p. 5, since the table value is based on a deduction equal to 10 percent of gross income, or $961, as compared with only $500 in the above example.

2/ If the computed surtax net income fell outside the guessed bracket, it would be necessary to guess another bracket and try the formula again.

2. Total deductions are a constant percentage of income

If total deductions are not constant, the preceding formula requires modification. For example, if a ten-percent deduction is used, the formula becomes, as is shown in Appendix A,

$$T = \frac{{}_rP' + (T_1 - {}_rS_1)}{1 - .9r}$$

The symbols retain their meaning previously given, except that P' and G' are now obtained by deducting 10 percent from P and G, respectively, before subtracting the exemption. This formula differs from the preceding only in having in place of r in the denominator, the quantity .9r resulting from the 10-percent deduction. The decimal .9 is the complement of the 10-percent deduction ratio and, in general, for a deduction ratio of p, it would only be necessary to replace .9 in the denominator by the quantity (1-p). As before, gross income before tax, G, is obtained by adding the computed tax T to the given net income after tax, P.

Example: Using the same example as before, $8,000 net salary after tax and 4 exemptions, but this time with a deduction of 10 percent of the unknown gross income before tax instead of the known amount of $500, P' is found as .9 (8,000) - 2,000, or 5,200. Guessing the bracket S_1 to S_2 to be the same as before, $6,000 - $8,000, gives

$$T = \frac{.285 \times 5{,}200 + (1{,}292 - .285 \times 6{,}000)}{1 - .9\,(.285)}$$

$$= \$1{,}431.07.$$

This is less than the preceding amount ($1,607.69(since deductions are now greater. Gross salary before tax is now $9,431.07. The corresponding surtax net income is $6,487.96, which shows that the bracket $6,000 - $8,000 was a correct guess. The corresponding liability under present law is $1,431.07, which checks with T.

D. Arithmetic rule

For those who think the formula too complicated and the wage-bracket table method too imprecise, there is available a method which combines the benefits of the formula with the simplicity of the table method. By using the table given below, page 10, the process can be reduced to a few simple steps which can be performed mechanically by any clerk. This simple rule, however, is available only for the case where total deductions are constant. Where deductions are a certain percentage of income either the wage-bracket table (method C above) or the precise formula, method D, must be used.

Unlike the formula method, the use of the rule makes it simpler to find the desired gross salary without first finding the tax. If

the tax is desired, it can then be obtained as the difference between gross and net salaries. For derivation of the rule, see Appendix B.

The steps of the rule for finding the gross salary before tax are as follows:

(1) Subtract from the given net salary after tax the sum of the given total deductions and exemptions.
(2) Find in Column 6 of the table below the amount of net income after tax next below the amount determined in (1). Obtain the difference between these two quantities.
(3) Divide this difference by the corresponding complement of the marginal rate (Column 5).
(4) Add the result of step (3) to the bottom of the corresponding surtax bracket (column 1) and add back the deductions and exemptions. The result is the desired gross salary before tax.

Example: The results of applying the above rule to the previous example of $8,000 net salary after tax, 4 exemptions, and $500 de-deduction are as follows, where the first column indicates the steps of the rule, and the second gives the results of applying them to the columns of the tax schedule below:

Step	Result
(1)	8,000 - 2,500 = 5,500
(2)	4,708; 5,500 - 4,708 = 792
(3)	792 ÷ .715 = 1,107.69
(4)	(6,000 + 1,107.69) + 2,500 = 9,607.69 desired gross salary before taxes
Tax	9,607.69 - 8,000 = 1,607.69
Check: computed tax on gross salary	1,607.69

Table for use in connection with arithmetical rule for determining gross salary before tax 1/

Surtax net income bracket		Total normal and surtax, after 5-percent reduction			
Over	But not over	Cumulative tax to bottom of bracket 2/	Marginal rate within bracket	Complement of marginal rate	Income after tax, col.(1) less col.(3)
(1)	(2)	(3)	(4)	(5)	(6)
$0	$2,000	$0	19.00%	81.00%	$0
2,000	4,000	380.00	20.90	79.10	1,620.00
4,000	6,000	798.00	24.70	75.30	3,202.00
6,000	8,000	1,292.00	28.50	71.50	4,708.00
8,000	10,000	1,862.00	32.30	67.70	6,138.00
10,000	12,000	2,508.00	36.10	63.90	7,492.00
12,000	14,000	3,230.00	40.85	59.15	8,770.00
14,000	16,000	4,047.00	44.65	55.35	9,953.00
16,000	18,000	4,940.00	47.50	52.50	11,060.00
18,000	20,000	5,890.00	50.35	49.65	12,110.00
20,000	22,000	6,897.00	53.20	46.80	13,103.00
22,000	26,000	7,961.00	56.05	43.95	14,039.00
26,000	32,000	10,203.00	58.90	41.10	15,797.00
32,000	38,000	13,737.00	61.75	38.25	18,263.00
38,000	44,000	17,442.00	65.55	34.45	20,558.00
44,000	50,000	21,375.00	68.40	31.60	22,625.00
50,000	60,000	25,479.00	71.25	28.75	24,521.00
60,000	70,000	32,604.00	74.10	25.90	27,396.00
70,000	80,000	40,014.00	76.95	23.05	29,986.00
80,000	90,000	47,709.00	79.80	20.20	32,291.00
90,000	100,000	55,689.00	82.65	17.35	34,311.00
100,000	150,000	63,954.00	84.55	15.45	36,046.00
150,000	200,000	106,229.00	85.50	14.50	43,771.00
200,000	.....	148,979.00	86.45	13.55	51,021.00

1/ See text pp. 8 and 9, for explanation of rule.
2/ Total tax may not exceed 85.5 percent of net income.

III. Method whereby tax is paid separately from salary

If the tax is not included as part of the current wage payment, but is paid separately, it is assumed that the payment is made in the following year and only on the basis of reimbursing the employee for amounts of tax actually paid by him attributable to his "tax-free" income. 1/ If this payment is in turn treated as income by the Bureau, it is added to the income in the following year and the tax for that year is computed on the resulting total. Even if the salary remains unchanged, the tax for the second year will thus be greater than for the first. If this process is continued, assuming that each year's tax payment is taxable as income in the following year, it is apparent that each year's payment of salary plus tax is greater than that in the preceding year. The total payment for the year, however, does not, in practice increase indefinitely, but soon levels off to a constant amount, 2/ if salary and all the other factors remain constant.

The constant yearly amount which total payments reach is precisely equal to the gross income before tax determined by one of the four methods of Part II above. Unlike the basis whereby tax is currently included in gross wage payments, however, repayment of tax to the employee would be postponed by the employer to the following year under the separate-payment method. The latter would thus be equivalent to a different contract from one specifying current payment.

If any of the factors such as salary, tax provisions, personal status, etc. change from one year to another, the employee's liability will continue to be covered, although the expense to the employer of meeting the employee's income tax liability will, in general, be different than if he were paying the tax currently according to one of the appropriate methods of Part II. Thus, if taxes should be lower in 1947 than in 1946, then the series of tax payments generated by 1946 (net) salary would evidently be at lower levels if paid in 1947 and later years than if a single payment were computed and paid on the basis of 1946 salary and tax levels. Similarly, in years when salary or tax levels are increasing, it will cost more for the employer to pay the liability on a "deferred" basis.

1/ Where the employer wishes to pay the tax separately but pays it all in one year, the problem of tax determination is the same as that covered in Part II above.

2/ Cf. the columns (5) and (6) in table on page 12.

If the employee leaves the employer at the end of a year, 1/ or the tax-free arrangement ceases then, there will always be a lump-sum adjustment required which will be greater than the employer's previous annual payments, assuming no change in salary or tax levels. This is most simple shown in the following table giving the series of payments in the hypothetical case of an employee with a net salary of $2,000 subject to a flat rate of 20 percent, assuming no rate progression:

Comparison of two methods of tax reimbursement by employer

	I. Tax payment included in current year wage			II. Tax payment reimbursed by employer in following year		
	Current year gross salary including tax (1)	Liability on preceding year salary at 20 percent (2)	Employee's current year gross income col.(1) (3)	Current year tax-free salary (4)	Reimbursible tax on preceding year salary at 20 percent (5)	Employee's current year gross income col.(4) + col.(5) (6)
1	$2,500	-	$2,500	$2,000	-	$2,000
2	2,500	$500	2,500	2,000	$400	2,400
3	2,500	500	2,500	2,000	480	2,480
4	2,500	500	2,500	2,000	496	2,496
5	2,500	500	2,500	2,000	499	2,499
6	2,500	500	2,500	2,000	500	2,500
7	2,500	500	2,500	2,000	500	2,500
	(Employee leaves at end of 7th year)					
8	-	$500	-	-	$500	$500
9	-	-	-	-	100	100
10	-	-	-	-	20	20
11	-	-	-	-	4	4
12	-	-	-	-	1	1
13	-	-	-	-	-	-
Total, years 8-13	-	$500	-	-	$625	$625

It is interesting to note how the tax payments (column (5)), though increasing, finally level off under the second method. From the above tabulation it is clear that under the first method of payment the employee will have upon

1/ The case where the employee leaves during the year is briefly considered below, Part IV.

leaving the employment overhanging liability of $500 on his 7th year salary of $2,500 but since the employer pays tax in advance, this amount will have been included in his 7th year salary, leaving him actually $2,000 clear of tax with no further liability. Under method II, however, the total overhanging liability will be $625, spread over 5 years as shown (assuming the tax series is not terminated before it dies out 1/), and not the level yearly amount of $500 previously paid by the employer. An examination of the payments in the first 5 years shows that the additional $125 is exactly the amount that was, in effect, deferred in the early years as compared with the full level payment under method I. The amount of $625 can also be obtained from the formula of Part II, page 6, by substituting for P' therein the tax-free payment of $500. 2/

It is apparent that even if the employee leaves before the tax payment to him has reached the level amount of $500 (in this case, prior to the 6th year) the lump-sum adjustment can likewise be computed by substituting for P' in the formula the amount of liability incurred by him during his last year of employment. Thus, if he leaves at the end of the third year, his liability is 20 percent of $2,480, or $496. For this P', G is $620, of which $120 represents the amount of tax deferred in the 2nd and 3rd years' tax reimbursement by the employer.

In an actual situation, the calculations in the above cases would tend to be somewhat more complicated, but the principles would be identical.

IV. Other considerations

In this part of the memorandum there will be outlined, without any attempt at discussion, some of the miscellaneous types of problems bearing on tax determination.

A. Changes in employee's circumstances

The changes in the factors affecting liability considered under this heading are as follows:

1. Change of exemption status.
2. Change in claimed deductions.
3. Change in salary rates.

1/ This argument would not be altered even if the taxing stopped with tax on tax, leaving $500 for the employer to pay in the 8th year and $100 in the 9th. The point is that the level amount of $500, which was adequate so long as the employee was working, would be insufficient to discharge the contract obligation when the employee leaves.

2/ Under the assumption of no rate progression, there is, in effect, just one surtax net income bracket, with S_1 and T_1 both zero, and $r = 20$. The formula then reduces to $G = P'/(1-4) = 1.25P'$. For $P' = 500$, $G = 625$.

These changes might cause discrepancies, analogous to overwithholding and underwithholding, between the employer's tax payment to the employee and the latter's tax payment to the Government. In general, if tax is paid by the employer currently with the wage payment, the issue arises as to whether such payment shall be changed with each change in any of the above factors affecting liability or whether the current payment shall be changed only a limited number of times during the year, at certain "status determination" dates.

If adjustments are made by the employee to the employer or vice versa to correct discrepancies between the employee's final liability for the year and the amount of tax received from the employer, the question arises as to how such reconciling payments should be treated. Should a repayment by the employee to the employer on account of overpayment to the employee (a) be deducted from the preceding year's income and the liability for that year recomputed, entailing a refund on the part of the Bureau? or (b) should such repayment be deducted from income in the year in which it is made, reducing the liability for that year instead? Similar questions arise in connection with an adjustment in the opposite direction.

B. Employee leaves the employment during the year

Part III above described the adjustment the employer would have to make if he regularly pays the employee's tax in the year after it is incurred and the employee leaves at the end of a year. It was shown that such adjustment would be unnecessary if the employer used the basis discussed in Part II, of including the tax in the current wage payment. Under this method, there would likewise be no problem if the employee leaves during, instead of at the end of, the year.

Under the separate tax payment procedure (method II), however, there would be further problems if the employee left in midyear, in addition to those of the type indicated in Part III. If the employer wishes to settle with the employee without waiting until the end of the year in which he leaves, he will not yet have available all the necessary facts. In most cases the liability may be accurately determined on the basis of salary paid up to the time of leaving and the facts prevalent then. Situations may arise, however, where due to a change of conditions during the remainder of the year the employee will have/under - or overpaid on the basis of his circumstances at the time of leaving. The problem then arises as to whether or how the employee should make an adjustment subsequent to leaving.

C. Employee's declaration of estimated tax

Since wages received from employers referred to in this memorandum are not subject to withholding, all employees involved must, under the law, file declarations of estimated tax unless total income is less than $500. Two problems then arise:

1. What shall the employee report as income on Form 1040-ES? If receiving tax currently (or separately, but during the current year), he should presumably report as income the total of expected gross wage payments, including tax. If receiving tax on a deferred basis, however, can such tax be considered to be constructively received prior to the year in which the employer actually pays it out to the employee? What about amounts received from the employer in the nature of post and other allowances? Which of these are income and which, if any, are exempt from tax?

2. If the employer pays tax separately to the employee in the year following that in which the tax is incurred, might not some employees experience hardship in meeting their quarterly payments on declaration in advance of reimbursement by the employer?

D. Direct payment of tax by the employer to the Government

Another plan for meeting tax liability is for the employer to turn the tax directly over to the Government, instead of indirectly via the employee. This plan might be effected by having the employee fill out his tax return in the usual way and give it to the employer who then pays the tax directly to the Government. Any such novel method of paying taxes would probably raise a whole group of problems including some of the type already mentioned. A thorough treatment cannot, however, be undertaken within the limits of the present memorandum.

Treasury Department, Division of Tax Research April 9, 1946

APPENDIX A

Derivation of Formulas for Determining Gross Salary Payments and Tax

Below is given the derivation of the algebraic formula discussed in Part II, section C, in the two cases (a) when total deductions are constant and (b) when total deductions are a fixed proportion of income.

The fundamental quantities are as defined in Part II, section C, namely,

P = net salary payment after tax;
P' = P expressed on a surtax net income basis, by subtracting deductions and exemptions;
G = desired gross salary before tax;
G' = G expressed on a surtax net income basis;
T = present law liability on G

We are given P, net salary after tax, and desire to find G, the gross salary before tax, and T, the tax on this gross salary, which is to be added to P. The definition of net salary yields the fundamental relation

$$G - T = P \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Further development depends upon whether there is a relation between deductions and income.

A. Total deductions are a constant amount

Let the sum of deductions and exemptions be denoted by K. Expressing P and G in terms of their corresponding surtax net incomes P' and G', we have

$$P = P' \neq K$$
$$G = G' \neq K$$

so that equation (1) becomes

$$G' - T = P' \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where P' is known, G' is sought, and T depends on G'. Equation (2) is solved for G' by first finding the expression for the tax T.

Let the surtax net income G' fall into the bracket S_1 to S_2, in which the total marginal rate is r and the cumulative tax to the bottom

of the bracket S_1 is T_1. Then the total tax T can be written

$$T = T_1 + r\,(G' - S_1).$$

Substitution in (2) gives

$$G' - [T_1 + r\,(G' - S_1)] = P',$$

which, solved for G', gives

$$G' = \frac{P' + (T_1 - rS_1)}{1 - r}.$$

The expression for the tax T is obtained by means of (2), giving

$$T = G' - P' = \frac{P' + (T_1 - S_1)}{1 - r} - P' = \frac{rP' + (T_1 - rS_1)}{1 - r} \quad . \quad . \quad (3)$$

This is the formula used in the memorandum.

The gross salary is obtained either by adding P to the tax T just found, or by adding back K to the expression for G' above (equation (2)), obtaining

$$G = \frac{P' + (T_1 - rS_1)}{1 - r} + K.$$

The difficulty in using the formulas for G and T lies in determining the surtax bracket S_1 to S_2 in which G' falls. This can be guessed by trial and error, but two or more trials may be necessary. A table can, however, be constructed once and for all which will eliminate guessing. But if such a table is used it is simpler to interpolate arithmetically in the manner described in the text, Part II, Method D, page 8 than to use the above formulas.

B. Total deductions are a constant percentage of income

Let p be this percentage. Then, if E is the exemption, we have

$$G = G' + (pG + E)$$

and

$$P = P' + (pP + E).$$

From these relations G and P must be expressed solely in terms of G' and P', respectively, giving

$$G = \frac{G' + E}{1 - p'} \qquad P = \frac{P' + E}{1 - p}.$$

Substituting in the fundamental relation (1) we have, using expression (3) for the tax T on G',

$$\text{or} \qquad \frac{G' \neq E}{1 - p} - [\, T_1 \neq r\,(G' - S_1)] \;=\; P,$$

$$G' \neq E - (1 - p)T_1 - (1 - p)r(G' - S1) = (1 - p)\, P.$$

From this G' is found as

$$G' = \frac{(\,(1 - p)\, P - E\,) \neq (1 - p)\,(T_1 - rS_1)}{1 - (1 - p)r}$$

where $P' = (1 - p)\, P - E$ is the known amount P converted to a surtax net income basis, but with a deduction, $(1 - p)\, P$, that now depends upon P, instead of being constant.

We have, finally, using the expression just found,

$$G = \frac{G' \neq E}{1 - p} = \frac{P' \neq (1 - p)\,(T_1 - rS_1)}{(1 - p)\,(1 - (1 - p)r)} \neq \frac{1}{1 - p} E, \quad \ldots\ldots (4)$$

and

$$T = G - P = \frac{G' \neq E}{1 - p} - \frac{P' \neq E}{1 - p} = \frac{G' - P'}{1 - p}$$

$$= \left[\frac{P' \neq (1 - p)\,(T_1 - rS_1)}{1 - (1 - p)r}\right] \frac{1}{1 - p} - \frac{P'}{1 - p}$$

$$= \frac{rP' \neq (T_1 - rS_1)}{1 - (1 - p)r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

In the usual situation of the 10-percent deduction ratio, $p = .10$, $1 - p = .90$, and formulas (4) and (5) become

$$G = \frac{P' \neq .9(T_1 - rS_1)}{.9\,(1 - .9r)} \neq \frac{1}{.9} E$$

and

$$T = \frac{rP' \neq (T_1 - rS_1)}{1 - .9r},$$

respectively.

APPENDIX B

Derivation of Arithmetic Rule for Determining Gross Salary

The rule given in the text, Part II, section D, for finding the gross salary required to yield a given net salary after tax, assuming constant total deductions, is as follows:

(a) Subtract the sum of the given total deductions and exemptions from the given net salary after tax.

(b) Find in column (6) of the table on page 10, the amount of net income after tax next below the amount determined in (a). Obtain the difference between these two quantities.

(c) Divide this difference by the corresponding complement of the marginal rate (column (5)).

(d) Add the result of step (c) to the bottom of the corresponding surtax bracket (column (1)) and add back the deductions and exemptions. The result is the desired gross salary before tax.

The tax on the gross amount so found is then the difference between this amount and the given net salary after tax.

The above rule is essentially a matter of interpolation in the tax schedule to find what income before tax corresponds to a given income after tax. The schedule, however, is in terms of income after exemptions and deductions. Before using the schedule, therefore, the given income must be converted to surtax net income by subtracting exemptions and deductions. This is the first step of the rule.

If the given net salary after tax, after subtracting exemptions and deductions, happened to be the same as one of the amounts listed in column (6), the corresponding surtax net income before tax could be read immediately from column (1). If this is not the case, then obtain the difference between the desired income after tax (step (a)) and the nearest amount below it in column (6). This is step (b).

Step (c) is explained as follows: Each dollar of income into the surtax bracket beyond the lower limit in column (1) pays r dollars in tax, leaving (1 - r) dollars after tax, where r is the marginal rate. Therefore, $\frac{1}{1 - r}$ dollars of surtax net income in the bracket

will leave exactly 1 dollar after tax, but instead of 1 dollar we have the number of dollars found in step (b). The number of dollars into the bracket is therefore $\frac{1}{1 - r}$ times this amount. Column (5) giving (1 - r) for each surtax bracket is provided in the table on page 10 to facilitate this step.

Adding the number of dollars into the bracket to the lower boundary gives the surtax net income, and adding back deductions and exemptions gives the desired gross salary before tax. This is step (d) which completes the rule.

The tax on gross salary is found by subtracting net from gross as already mentioned. If desired, this may be checked by actual computation.

JL:lcf - 4/8/46